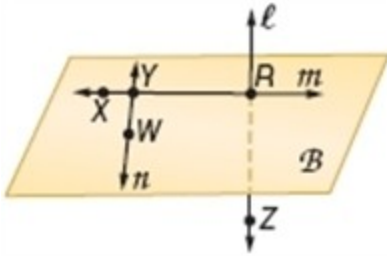


## 1-1 Points, Lines, and Planes

Use the figure to name each of the following.



1. a line containing point  $X$

**SOLUTION:**

The point  $X$  lies on the line  $m$ ,  $\overleftrightarrow{RX}$ , or  $\overleftrightarrow{RY}$ .

**ANSWER:**

Sample answer:  $m$

2. a line containing point  $Z$

**SOLUTION:**

The point  $Z$  lies on the line  $l$  or  $\overleftrightarrow{RZ}$ .

**ANSWER:**

Sample answer:  $l$

3. a plane containing points  $W$  and  $R$

**SOLUTION:**

A plane is a flat surface made up of points that extends infinitely in all directions. Here, the plane containing the points  $W$  and  $R$  is  $B$ .

**ANSWER:**

$B$

**Name the geometric term modeled by each object.**

4. a beam from a laser

**SOLUTION:**

A beam from a laser models a line.

**ANSWER:**

line

5. a floor

**SOLUTION:**

A floor is a flat surface that extends in all directions. So, it models a plane.

**ANSWER:**

plane

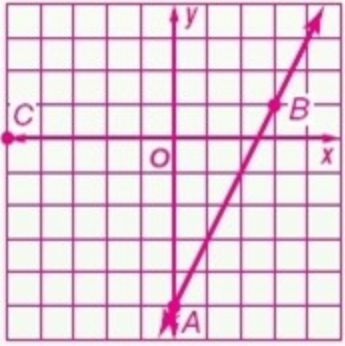
## 1-1 Points, Lines, and Planes

**Draw and label a figure for each relationship.**

6. A line in a coordinate plane contains  $A(0, -5)$  and  $B(3, 1)$  and a point  $C$  that is not collinear with  $\overline{AB}$ .

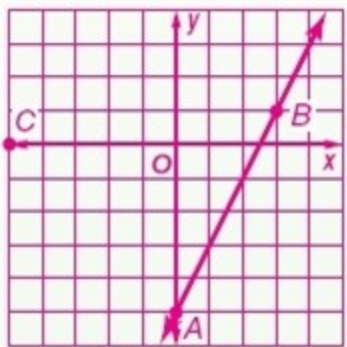
**SOLUTION:**

Plot the points  $A(0, -5)$  and  $B(3, 1)$  on a coordinate plane and draw a line through them. Mark the point  $C$  anywhere on the coordinate plane except on the line  $\overline{AB}$ .



**ANSWER:**

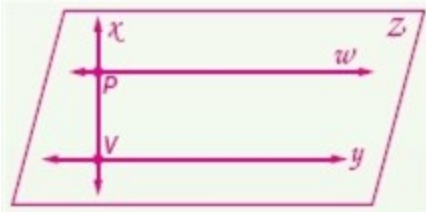
Sample answer:



7. Plane  $Z$  contains lines  $x$ ,  $y$ , and  $w$ . Lines  $x$  and  $y$  intersect at point  $V$  and lines  $x$  and  $w$  intersect at point  $P$ .

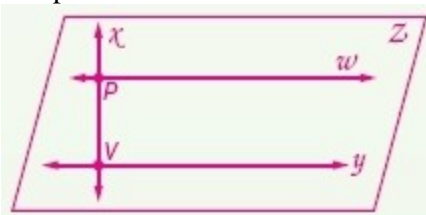
**SOLUTION:**

First draw two lines  $w$  and  $y$  on a plane  $Z$ . Draw another line  $x$  intersecting the line  $w$  at  $P$  and the line  $y$  at  $V$ .



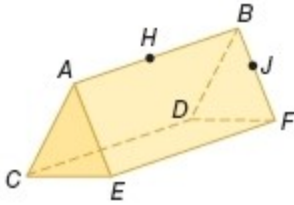
**ANSWER:**

Sample answer:



## 1-1 Points, Lines, and Planes

Refer to the figure.



8. How many planes are shown in the figure?

**SOLUTION:**

A plane is a flat surface made up of points that extends infinitely in all directions. Here, each side of the solid is a flat surface that extends to all directions. So, there are 5 planes (plane  $ACE$ , plane  $BDF$ , plane  $ABFE$ , plane  $CABD$ , plane  $CDEF$ ); 2 of which form triangular faces (plane  $ACE$ , plane  $BDF$ ) and 3 which form rectangles (plane  $ABFE$ , plane  $CABD$ , plane  $CDEF$ ).

**ANSWER:**

5

9. Name three points that are collinear.

**SOLUTION:**

Collinear points are points that lie on the same line. Here, the point  $H$  lies on the line  $\overline{AB}$ . So, the points  $A$ ,  $H$ , and  $B$  are collinear points. Also, the points  $B$ ,  $J$ , and  $F$  are collinear points.

**ANSWER:**

Sample answer:  $A$ ,  $H$ , and  $B$

10. Are points  $A$ ,  $H$ ,  $J$ , and  $D$  coplanar? Explain.

**SOLUTION:**

Coplanar points are points that lie in the same plane. Here, the points  $A$ ,  $H$ , and  $J$  lie in plane  $ABF$ , but point  $D$  does not lie in plane  $ABF$ . So the answer is no, the points are not coplanar.

**ANSWER:**

No; points  $A$ ,  $H$ , and  $J$  lie in plane  $ABF$ , but point  $D$  does not lie in plane  $ABF$ .

11. Are points  $B$ ,  $D$ , and  $F$  coplanar? Explain.

**SOLUTION:**

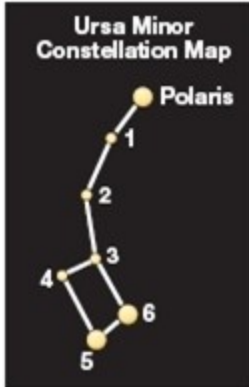
Coplanar points are points that lie in the same plane. Here, the points  $B$ ,  $D$ , and  $F$  all lie in plane  $BDF$ . So the answer is yes, the points are coplanar.

**ANSWER:**

Yes; points  $B$ ,  $D$ , and  $F$  lie in plane  $BDF$ .

## 1-1 Points, Lines, and Planes

12. **ASTRONOMY** Ursa Minor, or the Little Dipper, is a constellation made up of seven stars in the northern sky including the star Polaris.
- What geometric figures are modeled by the stars?
  - Are Star 1, Star 2, and Star 3 collinear on the constellation map? Explain.
  - Are Polaris, Star 2, and Star 6 coplanar on the map?



### **SOLUTION:**

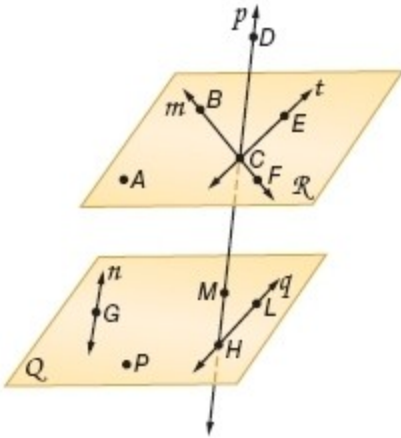
- The stars in the diagram represent specific coordinate locations. So, they model points with no shape or size.
- Collinear points are points that lie on the same line. There are lines drawn *from* each of the points. However, there is no single line drawn *through* each of the points that represent Star 1, Star 2, and Star 3. Therefore, they are not collinear.
- There is exactly one plane through any three points not on the same line. Since Polaris, Star 2, and Star 6 are not collinear, they are coplanar.

### **ANSWER:**

- points
- No; there is not a straight line through all three points. So the points are not collinear.
- Yes; there is exactly one plane that passes through any three noncollinear points. Since Polaris, Star 2, and Star 6 are noncollinear, they are coplanar.

## 1-1 Points, Lines, and Planes

Refer to the figure.



13. Name the lines that are only in plane  $Q$ .

**SOLUTION:**

There are only two lines that lie in plane  $Q$ :  $n$  and  $q$ .

**ANSWER:**

Sample answer:  $n$  and  $q$

14. How many planes are labeled in the figure?

**SOLUTION:**

There are two planes labeled in the figure, namely  $Q$  and  $R$ .

**ANSWER:**

2

15. Name the plane containing the lines  $m$  and  $t$ .

**SOLUTION:**

A plane is a flat surface made up of points that extends infinitely in all directions. Here, the plane containing the lines  $m$  and  $t$  is  $R$ .

**ANSWER:**

$R$

16. Name the intersection of lines  $m$  and  $t$ .

**SOLUTION:**

The two lines  $m$  and  $t$  intersect at the point  $C$  on the plane  $R$ .

**ANSWER:**

$C$

## 1-1 Points, Lines, and Planes

17. Name a point that is not coplanar with points  $A$ ,  $B$ , and  $C$ .

**SOLUTION:**

Coplanar points are points that lie in the same plane. Here, the points  $A$ ,  $B$ , and  $C$  lie on the plane  $R$ . So, to find a point which is NOT coplanar with  $A$ ,  $B$ , and  $C$ , consider any point on the plane  $Q$ . The point  $P$ , which is on the plane  $Q$ , is not coplanar with the points  $A$ ,  $B$ , and  $C$ .

**ANSWER:**

Sample answer: point  $P$

18. Are points  $F$ ,  $M$ ,  $G$ , and  $P$  coplanar? Explain.

**SOLUTION:**

Coplanar points are points that lie in the same plane. Here, the points  $G$  and  $P$  lie on the plane  $Q$ . But the point  $M$  lies between the planes  $Q$  and  $R$  and the point  $F$  lies on the plane  $R$ .

**ANSWER:**

No; point  $F$  lies in plane  $R$ , while points  $G$  and  $P$  lie in plane  $Q$  and  $M$  lies between planes  $R$  and  $Q$ .

19. Name the points not contained in a line shown.

**SOLUTION:**

The points  $A$  and  $P$  do not lie in any of the lines shown on the planes  $Q$  and  $R$ .

**ANSWER:**

points  $A$  and  $P$

20. What is another name for line  $t$ ?

**SOLUTION:**

There are two points  $C$  and  $E$  marked on the line  $t$ . So, the line  $t$  can also be named as  $\overline{CE}$ .

**ANSWER:**

$\overline{CE}$

21. Does line  $n$  intersect line  $q$ ? Explain.

**SOLUTION:**

A line and a plane can be extended infinitely. Lines  $n$  and  $q$  are coplanar but not parallel. So, when the lines  $n$  and  $q$  are extended on the plane  $Q$ , they will intersect.

**ANSWER:**

Yes; line  $n$  intersects line  $q$  when the lines are extended.

## 1-1 Points, Lines, and Planes

Name the geometric term(s) modeled by each object.



**SOLUTION:**

The tip of a pen denotes a location. So, it models a point.

**ANSWER:**

point

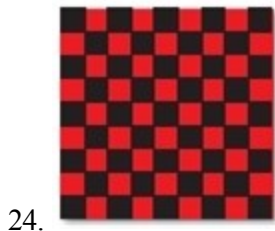


**SOLUTION:**

The roof segment is a flat surface that extends in all directions, so it is a plane. The edges of a roof model lines, which intersect at the corners. So, the highlighted parts model intersecting lines.

**ANSWER:**

plane; intersecting lines



**SOLUTION:**

The chessboard is a flat surface that extends in all directions. So, it is a plane. Also it has lines that intersect on the plane. So, it also models intersecting lines.

**ANSWER:**

plane; intersecting lines

## 1-1 Points, Lines, and Planes



25.

**SOLUTION:**

In the figure two flat surfaces intersect each other. The two flat surfaces model two planes, so the whole figure models two planes intersecting to form a line.

**ANSWER:**

two planes intersecting in a line

26. a blanket

**SOLUTION:**

A blanket is a flat surface that extends in all directions. So, it models a plane.

**ANSWER:**

plane

27. a knot in a rope

**SOLUTION:**

A knot in a rope denotes a location. So, it models a point.

**ANSWER:**

point

28. a telephone pole

**SOLUTION:**

A telephone pole models a line.

**ANSWER:**

line

29. the edge of a desk

**SOLUTION:**

The edge of a desk models a line.

**ANSWER:**

line



## 1-1 Points, Lines, and Planes

30. two connected walls

**SOLUTION:**

Each wall models a plane. Two connected walls model intersecting planes.

**ANSWER:**

intersecting planes

31. a partially opened folder

**SOLUTION:**

Each side of a folder is a flat surface that extends in all directions. So, a partially opened folder models two intersecting planes.

**ANSWER:**

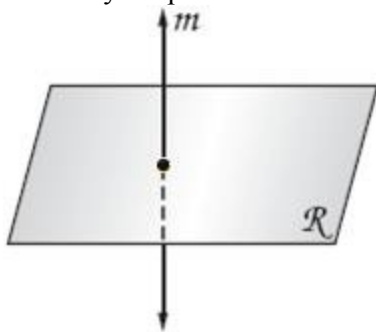
intersecting planes

**Draw and label a figure for each relationship.**

32. Line  $m$  intersects plane  $R$  at a single point.

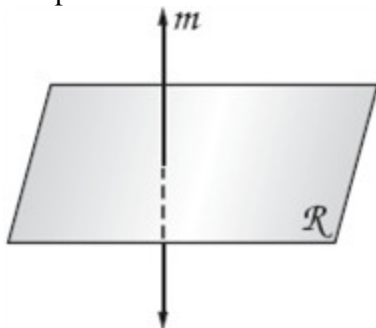
**SOLUTION:**

Draw a plane  $R$  and add only one point. Draw line  $m$  vertically through the point. Dash the line to indicate the portion hidden by the plane.



**ANSWER:**

Sample answer:



## 1-1 Points, Lines, and Planes

33. Two planes do not intersect.

**SOLUTION:**

Draw two planes that do not intersect, that is, two planes that are parallel.



**ANSWER:**

Sample answer:



34. Points  $X$  and  $Y$  lie on  $\overline{CD}$ .

**SOLUTION:**

Draw a line  $\overline{CD}$  and plot two points  $X$  and  $Y$  on the line.



**ANSWER:**

Sample answer:

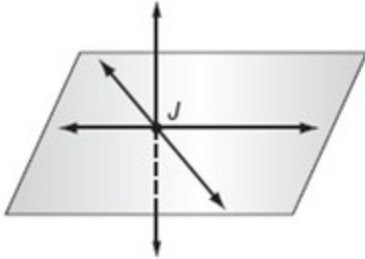


## 1-1 Points, Lines, and Planes

35. Three lines intersect at point J but do not all lie in the same plane.

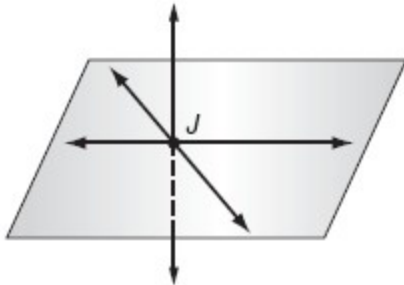
*SOLUTION:*

Draw two lines that intersect at a point on a plane. Then draw a line perpendicular to the plane so the line does not lie in the same plane. Dash the portion of the vertical line to indicate that it is hidden by the plane.



*ANSWER:*

Sample answer:

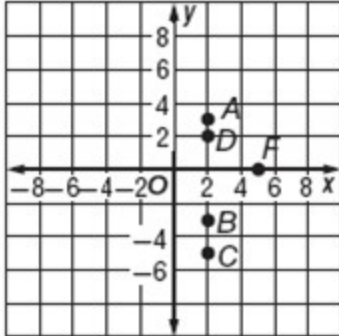


## 1-1 Points, Lines, and Planes

36. Points  $A(2, 3)$ ,  $B(2, -3)$ ,  $C$  and  $D$  are collinear, but  $A, B, C, D,$  and  $F$  are not.

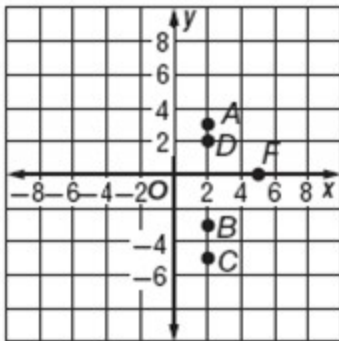
**SOLUTION:**

First, plot the point  $A(2, 3)$ , and  $B(2, -3)$  on a coordinate plane. Now plot the points  $C$  and  $D$  on the same line as  $A$  and  $B$ . Plot another point  $F$  which is not in the same line as  $A$  and  $B$ .



**ANSWER:**

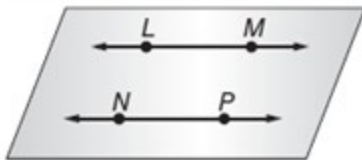
Sample answer:



37. Lines  $\overline{LM}$  and  $\overline{NP}$  are coplanar but do not intersect.

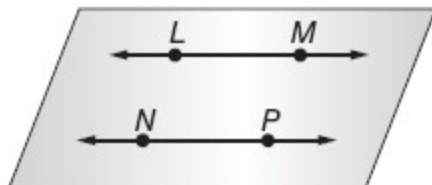
**SOLUTION:**

Draw two parallel lines  $\overline{LM}$  and  $\overline{NP}$  on a plane.



**ANSWER:**

Sample answer:

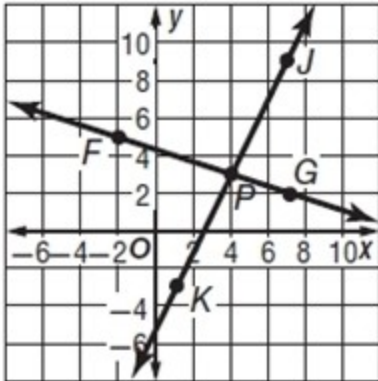


## 1-1 Points, Lines, and Planes

38.  $\overline{FG}$  and  $\overline{JK}$  intersect at  $P(4, 3)$ , where point  $F$  is at  $(-2, 5)$  and point  $J$  is at  $(7, 9)$ .

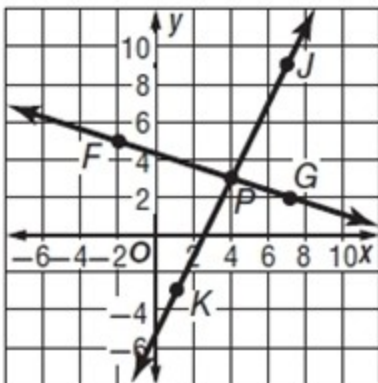
**SOLUTION:**

Plot and label the points  $P(4, 3)$ ,  $F(-2, 5)$ , and  $J(7, 9)$  on a coordinate plane. Draw a straight line through points  $P$  and  $J$ . Similarly, draw a straight line through points  $P$  and  $F$ .



**ANSWER:**

Sample answer:

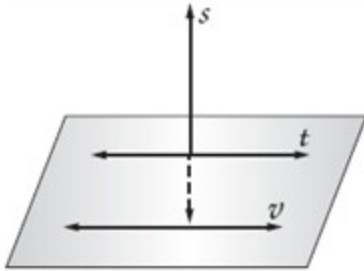


## 1-1 Points, Lines, and Planes

39. Lines  $s$  and  $t$  intersect, and line  $v$  does not intersect either one.

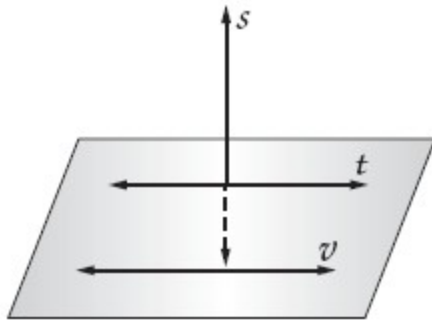
**SOLUTION:**

Draw two parallel lines  $t$  and  $v$  on a coordinate plane. Then draw a line  $s$  perpendicular to both the plane and the line  $t$ , but does not intersect the line  $v$ . Dash the portion of line  $s$  to indicate it is hidden by the plane.



**ANSWER:**

Sample answer:



**PACKING** When packing breakable objects such as glasses, movers frequently use boxes with inserted dividers like the one shown.



40. How many planes are modeled in the picture?

**SOLUTION:**

There are 5 planes that give the box the shape of a rectangular prism: the bottom and four sides. Then there are 4 more planes modeled by the partially opened top flaps. Inside the box, the space is divided by 6 planes which intersect each other. So, there are a total of 15 planes in the figure.

**ANSWER:**

15

## 1-1 Points, Lines, and Planes

41. What parts of the box model lines?

**SOLUTION:**

The edges of the sides of the box model lines. The dividers represent planes and the intersection of planes are lines. So, the edges of the dividers also model lines.

**ANSWER:**

edges

42. What parts of the box model points?

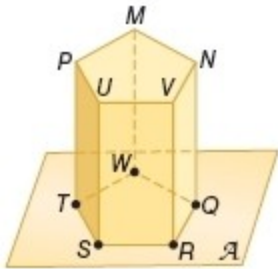
**SOLUTION:**

Each vertex of the box is a location which models a point.

**ANSWER:**

vertices

**Refer to the figure below.**



43. Name two collinear points.

**SOLUTION:**

Collinear points are points that lie on the same line. Here, points  $M$  and  $N$  lie on the same line, so they are collinear points. (Note that there are many other pairs of collinear points.)

**ANSWER:**

Sample answer:  $M$  and  $N$

44. How many planes appear in the figure?

**SOLUTION:**

A plane is a flat surface that extends infinitely in all directions. Here, the 5 rectangular sides of the figure represent 5 planes, the top pentagonal face represents another plane. Since the base of the prism lies on plane  $A$ , it only represents one additional plane, the seventh plane. So, 7 planes appear in the figure.

**ANSWER:**

7

45. Do plane  $A$  and plane  $MNP$  intersect? Explain.

**SOLUTION:**

$MNP$  is the top face of the solid, and does not have any common lines with the plane  $A$ . So, they do not intersect.

**ANSWER:**

No; they do not have any lines in common.

## 1-1 Points, Lines, and Planes

46. In what line do planes  $A$  and  $QRV$  intersect?

**SOLUTION:**

The plane  $QRV$  contains the rectangle  $QRVN$ . This rectangle intersects the plane  $A$  in the line  $\overline{QR}$ .

**ANSWER:**

$\overline{QR}$

47. Are points  $T, S, R, Q,$  and  $V$  coplanar? Explain.

**SOLUTION:**

Coplanar points are points that lie in the same plane. Here, the points  $T, S, R,$  and  $Q$  all lie on the plane  $A$ ; there is no other plane which contains all four of them. But the point  $V$  does not lie on plane  $A$ . Therefore, they are not coplanar.

**ANSWER:**

No;  $V$  does not lie in the same plane.

48. Are points  $T, S, R, Q,$  and  $W$  coplanar? Explain.

**SOLUTION:**

Coplanar points are points that lie in the same plane. Here, the points  $T, S, R, Q,$  and  $W$  all lie on the plane  $A$ . Therefore, they are coplanar.

**ANSWER:**

Yes; all of the points lie in plane  $TSR$ .

49. **FINITE PLANES** A *finite plane* is a plane that has boundaries, or does not extend indefinitely. The street signs shown are finite planes.

Refer to Page 10.

**a.** If the pole models a line, name the geometric term that describes the intersection between the signs and the pole.

**b.** What geometric term(s) describes the intersection between the two finite planes? Explain your answer with a diagram if necessary.

**SOLUTION:**

**a.** The pole models a line that passes through the point of intersection of the two planes. The intersection of the pole and signs is a line.

**b.** If the two finite planes intersect each other, their intersection will be modeled by a line. However, in this case, they are finite planes, and do not intersect each other so they only meet at a point.



**ANSWER:**

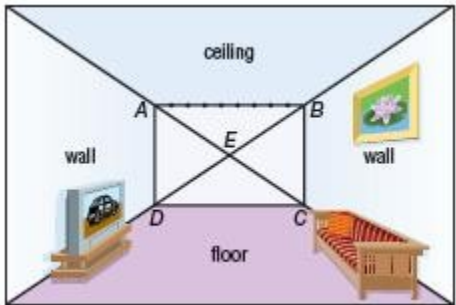
**a.** line

**b.** point



## 1-1 Points, Lines, and Planes

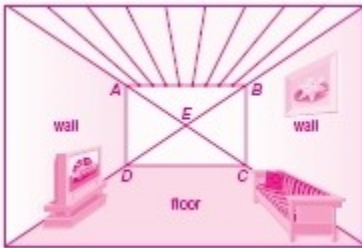
50. **ONE-POINT PERSPECTIVE** One-point perspective drawings use lines to convey depth. Lines representing horizontal lines in the real object can be extended to meet at a single point called the *vanishing point*. Suppose you want to draw a tiled ceiling in the room below with eight tiles across.



- What point represents the vanishing point in the drawing?
- Trace the figure. Then draw lines from the vanishing point through each of the eight points between  $A$  and  $B$ . Extend these lines to the top edge of the drawing.
- How could you change the drawing to make the back wall of the room appear farther away?

### SOLUTION:

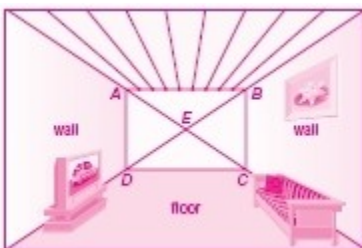
- In the one-point perspective drawn, the horizontal lines representing the top and bottom edges of the room seem to meet at a single point called the *vanishing point*. Here, the vanishing point will be the center of the rectangle, the point  $E$ .
- The line segment  $\overline{AB}$  is divided into 9 equal segments using 8 points. Draw 8 horizontal lines through each of these points that meet at the vanishing point  $E$ .



- Moving the points  $A$ ,  $B$ ,  $C$ , and  $D$  closer to the point  $E$  (but still as the vertices of a rectangle with  $E$  as its center) would make the back of the room appear further away.

### ANSWER:

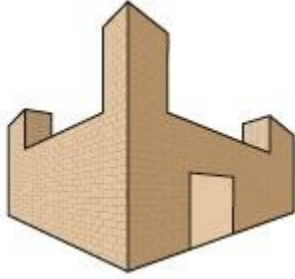
- Point  $E$
- 



- Sample answer: Draw points  $A$ ,  $B$ ,  $C$ , and  $D$  closer to the vanishing point.

51. **TWO-POINT PERSPECTIVE** Two-point perspective drawings use two vanishing points to convey depth.

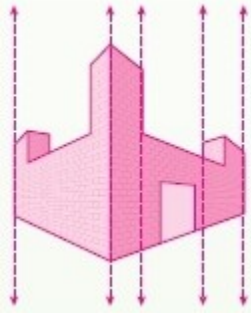
## 1-1 Points, Lines, and Planes



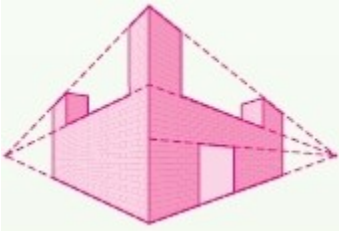
- Trace the drawing of the castle shown. Draw five of the vertical lines used to create the drawing.
- Draw and extend the horizontal lines to locate the vanishing points and label them.
- What do you notice about the vertical lines as they get closer to the vanishing point?
- Draw a two-point perspective of a home or a room in a home.

**SOLUTION:**

- Draw any five of the vertical lines.



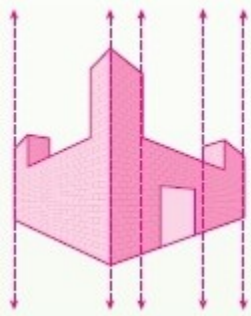
- Draw the horizontal lines.



- As we near the vanishing point, the vertical lines get closer to each other.
- See students' work.

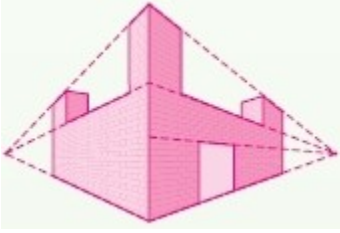
**ANSWER:**

- Sample answer:



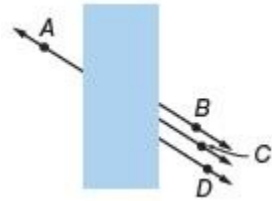
-

## 1-1 Points, Lines, and Planes



- c. Sample answer: They get closer together.
- d. See students' work.

52. **OPTICAL ILLUSION** Name two points on the same line in the figure. How can you support your assertion?



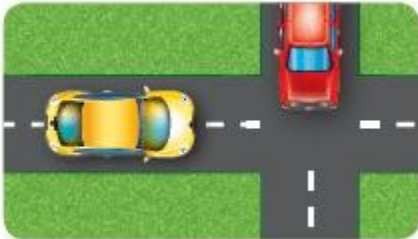
**SOLUTION:**

Using a ruler we can figure out that the line containing the point  $C$  is an extension of the line containing the point  $A$ . Therefore, the points  $A$  and  $C$  are collinear.

**ANSWER:**

Sample answer:  $A$  and  $C$ ; use a ruler to figure out which point would be the end of the line if it continued.

53. **TRANSPORTATION** When two cars enter an intersection at the same time on opposing paths, one of the cars must adjust its speed or direction to avoid a collision. Two airplanes, however, can cross paths while traveling in different directions without colliding. Explain how this is possible.



**SOLUTION:**

The two cars are in the same horizontal plane. But the airplanes are in different horizontal planes, so they can cross without colliding.

**ANSWER:**

Sample answer: The airplanes are in different horizontal planes.

54. **MULTIPLE REPRESENTATIONS** Another way to describe a group of points is called a locus. A locus is a set of points that satisfy a particular condition. In this problem, you will explore the locus of points that satisfy an equation.

- a. **TABULAR** Represent the locus of points satisfying the equation  $2 + x = y$  using a table of at least five values.
- b. **GRAPHICAL** Represent this same locus of points using a graph.
- c. **VERBAL** Describe the geometric figure that the points suggest.

**SOLUTION:**

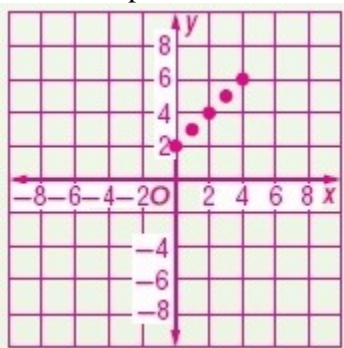
- a. Choose 5 random values for  $x$  and substitute the values in the expression  $2 + x = y$  to find the corresponding  $y$ -

## 1-1 Points, Lines, and Planes

value. Use a table with first column for the  $x$ -values and the second column for the  $y$ -values.

$x$	$y$
0	2
1	3
2	4
3	5
4	6

b. Plot the points on a coordinate plane.



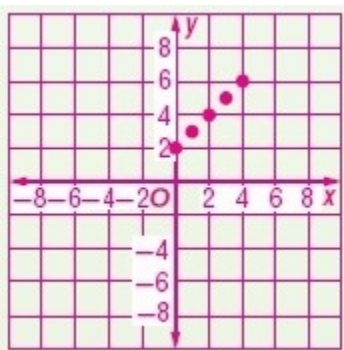
c. The plotted points appear collinear. So, the points suggest a line.

ANSWER:

a. Sample answer:

$x$	$y$
0	2
1	3
2	4
3	5
4	6

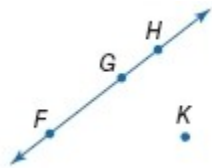
b.



c. a line

## 1-1 Points, Lines, and Planes

55. **PROBABILITY** Three of the labeled points are chosen at random.



- What is the probability that the points chosen are collinear?
- What is the probability that the points chosen are coplanar?

**SOLUTION:**

- We can choose three points in 4 different ways. ( $FGH$ ,  $FGK$ ,  $GHK$ ,  $FHK$ ) Among these 4 ways, in only one choice of points,  $F$ ,  $G$ ,  $H$  the points are collinear. Therefore, the probability is  $\frac{1}{4}$ .
- There is exactly one plane through any three noncollinear points and infinitely many planes through three collinear points. Therefore, the probability that the three points chosen are coplanar is 1.

**ANSWER:**

- $\frac{1}{4}$
- 1

56. **MULTIPLE REPRESENTATIONS** In this problem, you will explore the locus of points that satisfy an inequality.
- TABULAR** Represent the locus of points satisfying the inequality  $y < -3x - 1$  using a table of at least ten values.
  - GRAPHICAL** Represent this same locus of points using a graph.
  - VERBAL** Describe the geometric figure that the points suggest.

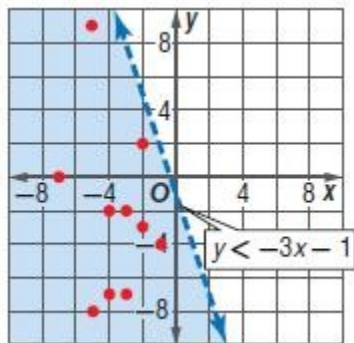
**SOLUTION:**

- Choose 10 random values for  $x$  and substitute the values in the expression  $y < -3x - 1$  to find the corresponding  $y$ -value. Use a table with first column for the  $x$ -values and the second column for the  $y$ -values.

$x$	$y$
-1	-4
-2	-3
-2	2
-3	-2
-3	-7
-4	-2
-4	-7
-5	-8
-5	9
-7	0

- Plot the points on a coordinate plane and identify the solution region.

## 1-1 Points, Lines, and Planes



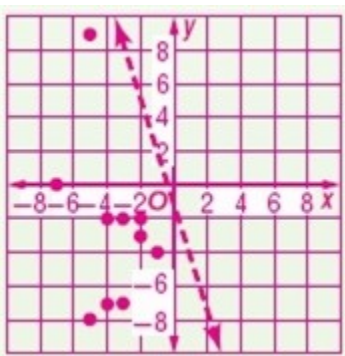
c. The solution includes the part of the coordinate plane below the line  $y = -3x - 1$ .

ANSWER:

a. Sample answer:

$x$	$y$
-1	-4
-2	-3
-2	2
-3	-2
-3	-7
-4	-2
-4	-7
-5	-8
-5	9
-7	0

b.



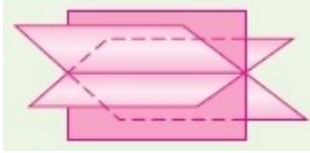
c. the part of the coordinate plane below the line  $y = -3x - 1$

## 1-1 Points, Lines, and Planes

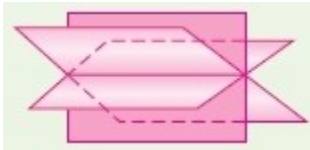
57. **OPEN ENDED** Sketch three planes that intersect in a line.

**SOLUTION:**

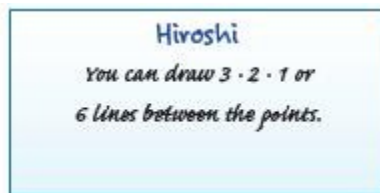
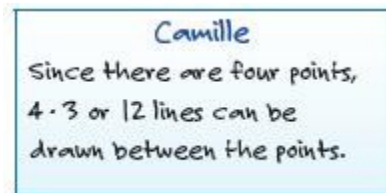
Sketch the three planes so that they intersect through the same line.



**ANSWER:**

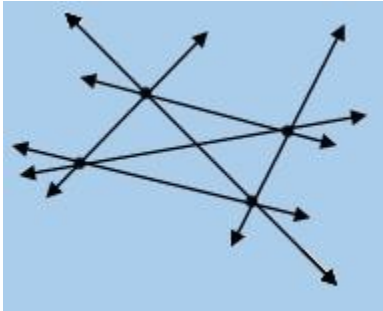


58. **ERROR ANALYSIS** Camille and Hiroshi are trying to determine the most number of lines that can be drawn using any two of four random points. Is either correct? Explain.



**SOLUTION:**

After you draw the line from the first point to the other three, one of the lines from the second point is already drawn. So, the total number of lines is  $3 \cdot 2 \cdot 1 = 6$ . Therefore, Hiroshi is correct.



**ANSWER:**

Sample answer: Hiroshi is correct; after you draw the line from the first point to the other three, one of the lines from the second point is already drawn.

59. **CCSS ARGUMENTS** What is the greatest number of planes that can be determined using any three of the points  $A$ ,  $B$ ,  $C$ , and  $D$  if no three points are collinear?

**SOLUTION:**

Any three noncollinear points will determine a plane. There are 4 planes that can be determined by using only 3 of the points. Planes  $ABC$ ,  $ACD$ ,  $ABD$ , and  $BCD$ .

**ANSWER:**

4

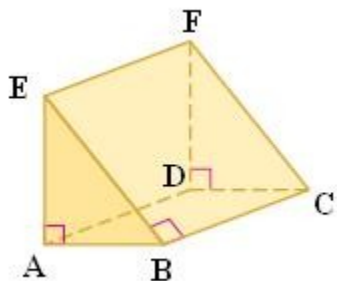
## 1-1 Points, Lines, and Planes

60. **REASONING** Is it possible for two points on the surface of a prism to be neither collinear nor coplanar? Justify your answer.

**SOLUTION:**

No. Sample answer: There is exactly one line through any two points and exactly one plane through any three points not on the same line. Therefore, any two points on the prism must be collinear and coplanar.

For example, in triangular prism  $ABCDEF$ , two points can be chosen that are on the same face or on any two of the five different faces. If the two points are both on the triangular face  $ABE$ , then a line could be drawn through those two points. If one point is on the rectangular face  $BCFE$  and the other point is on rectangular face  $ADFE$ , then the line connecting the two points might be in plane  $BCFE$ , plane  $ADFE$ , or it might pass through the interior points of the prism. This line would not be in the plane determined by one of the five faces of the prism, but there are an infinite number of planes that could contain the line. No matter where the points are located on the prism, the points will be collinear and coplanar.



**ANSWER:**

No. Sample answer: There is exactly one line through any two points and exactly one plane through any three points not on the same line. Therefore, any two points on the prism must be collinear and coplanar.

61. **WRITING IN MATH** Refer to Exercise 49. Give a real-life example of a finite plane. Is it possible to have a real-life object that is an infinite plane? Explain your reasoning.

**SOLUTION:**

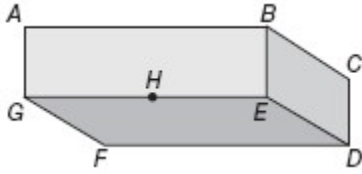
A table is a finite plane. It is not possible to have a real-life object that is an infinite plane because all real-life objects have boundaries.

**ANSWER:**

Sample answer: A table is a finite plane. It is not possible to have a real-life object that is an infinite plane because all real-life objects have boundaries.



## 1-1 Points, Lines, and Planes



62. Which statement about the figure below is *not* true?
- A Point  $H$  lies in planes  $AGE$  and  $GED$ .
  - B Planes  $GAB$ ,  $GFD$  and  $BED$  intersect at point  $E$ .
  - C Points  $F$ ,  $E$ , and  $B$  are coplanar.
  - D Points  $A$ ,  $H$ , and  $D$  are collinear.

**SOLUTION:**

The point  $H$  lies on the edge  $\overline{GE}$  which is common to the planes  $AGE$  and  $GED$ . So, the statement in option A is true.

The common vertex to the planes  $GAB$ ,  $GFD$  and  $BED$  is  $E$ . So, the statement in option B is also correct.

Any three points are coplanar. Since the points  $B$ ,  $E$  and  $F$  are noncollinear, they are contained in exactly one plane. Option C is also correct.

The points  $A$ ,  $H$ , and  $D$  do not lie on the same line. So, they are not collinear. Therefore, the statement in option D is not true. So, the correct choice is D.

**ANSWER:**

D

63. **ALGEBRA** What is the value of  $x$  if  $3x + 2 = 8$ ?

- F -2
- G 0
- H 2
- J 6

**SOLUTION:**

$$3x + 2 = 8 \quad \text{Original equation}$$

$$3x + 2 - 2 = 8 - 2 \quad \text{Subtract 2 from each side.}$$

$$3x = 6 \quad \text{Simplify}$$

$$\frac{3x}{3} = \frac{6}{3} \quad \text{Divide each side by 3.}$$

$$x = 2 \quad \text{Simplify}$$

Therefore, the correct choice is H.

**ANSWER:**

H

## 1-1 Points, Lines, and Planes

64. **GRIDDED RESPONSE** An ice chest contains 3 types of drinks: 10 apple juices, 15 grape juices, and 15 bottles of water. What is the probability that a drink selected randomly from the ice chest does not contain fruit juice?

**SOLUTION:**

There are 40 bottles in the ice chest and 15 of them are not fruit juice. So, the probability that a drink selected randomly from the ice chest contains no fruit juice is  $\frac{15}{40}$  or  $\frac{3}{8}$ .

**ANSWER:**

$\frac{3}{8}$  or 0.625

65. **SAT/ACT** A certain school's enrollment increased 6% this year over last year's enrollment. If the school now has 1378 students enrolled, how many students were enrolled last year?

- A 1295
- B 1300
- C 1350
- D 1460
- E 1500

**SOLUTION:**

The enrollment has increased by 6% this year. Let  $x$  be last year's enrollment, then, solve for  $x$ .

$$\frac{\text{with increase}}{\text{Original}} \frac{106}{100} = \frac{1378}{x} \frac{\text{this year}}{\text{last year}}$$

$$106x = 1378(100)$$

Find the cross products.

$$106x = 137800$$

Simplify

$$\frac{106x}{106} = \frac{137800}{106}$$

Divide each side by 106.

$$x = 1300$$

Simplify

Therefore, the correct choice is B.

**ANSWER:**

B

## 1-1 Points, Lines, and Planes

**Simplify.**

66.  $\sqrt{72}$

**SOLUTION:**

$$\sqrt{72} = \sqrt{2 \cdot 2 \cdot 2 \cdot 3 \cdot 3} \quad \text{Prime factorization}$$

$$= \sqrt{2^2} \sqrt{2} \sqrt{3^2} \quad \text{Product Property}$$

$$= 2 \cdot 3 \sqrt{2} \quad \text{Simplify}$$

$$= 6\sqrt{2} \quad \text{Simplify.}$$

**ANSWER:**

$$6\sqrt{2}$$

67.  $\sqrt{18} \cdot \sqrt{14}$

**SOLUTION:**

Write the prime factorizations of 18 and 14.

$$\sqrt{18} = \sqrt{2 \cdot 3 \cdot 3}$$

$$\sqrt{14} = \sqrt{2 \cdot 7}$$

Use the Product Property of Square Roots.

$$\sqrt{18} \cdot \sqrt{14}$$

$$= \sqrt{18 \cdot 14} \quad \text{Product Property}$$

$$= \sqrt{2 \cdot 3 \cdot 3 \cdot 2 \cdot 7} \quad \text{Prime Factorization}$$

$$= \sqrt{2^2 \cdot 3^2 \cdot 7} \quad \text{Group Factors.}$$

$$= \sqrt{2^2} \cdot \sqrt{3^2} \cdot \sqrt{7} \quad \text{Product Property}$$

$$= 2 \cdot 3 \sqrt{7} \quad \text{Take Square roots.}$$

$$= 6\sqrt{7} \quad \text{Multiply.}$$

**ANSWER:**

$$6\sqrt{7}$$

## 1-1 Points, Lines, and Planes

68.  $\sqrt{44x^4y^3}$

SOLUTION:

$$\begin{aligned}\sqrt{44x^4y^3} &= \sqrt{2 \cdot 2 \cdot 11 \cdot x \cdot x \cdot x \cdot x \cdot y \cdot y \cdot y} && \text{Prime factorization} \\ &= \sqrt{2^2 \cdot 11 \cdot x^2 \cdot x^2 \cdot y^2 \cdot y} && \text{Group Factors} \\ &= \sqrt{2^2} \sqrt{11} \sqrt{x^2} \sqrt{x^2} \sqrt{y^2} \sqrt{y} && \text{Product Property} \\ &= 2 \cdot \sqrt{11} \cdot x \cdot x \cdot y \sqrt{y} && \text{Take square roots.} \\ &= 2x^2y\sqrt{11y} && \text{Multiply.}\end{aligned}$$

ANSWER:

$$2x^2y\sqrt{11y}$$

69.  $\frac{3}{\sqrt{18}}$

SOLUTION:

$$\begin{aligned}\frac{3}{\sqrt{18}} &= \frac{3}{\sqrt{2 \cdot 3 \cdot 3}} && \text{Prime factorization} \\ &= \frac{3}{\sqrt{2 \cdot 3^2}} && \text{Product Property} \\ &= \frac{3}{\sqrt{2} \sqrt{3^2}} && \text{Product Property} \\ &= \frac{3}{3\sqrt{2}} && \text{Simplify.} \\ &= \frac{1}{\sqrt{2}} && \text{Simplify.} \\ &= \frac{1}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} && \text{Multiply by } \frac{\sqrt{2}}{\sqrt{2}} \\ &= \frac{\sqrt{2}}{2} && \text{Simplify.}\end{aligned}$$

ANSWER:

$$\frac{\sqrt{2}}{2}$$

## 1-1 Points, Lines, and Planes

70.  $\sqrt{\frac{28}{75}}$

**SOLUTION:**

$$\begin{aligned}\sqrt{\frac{28}{75}} &= \frac{\sqrt{28}}{\sqrt{75}} && \text{Quotient Property} \\ &= \frac{\sqrt{2 \cdot 2 \cdot 7}}{\sqrt{3 \cdot 5 \cdot 5}} && \text{Prime factorization} \\ &= \frac{\sqrt{2^2} \sqrt{7}}{\sqrt{3} \sqrt{5^2}} && \text{Product Property} \\ &= \frac{2\sqrt{7}}{5\sqrt{3}} && \text{Simplify.} \\ &= \frac{2\sqrt{7}}{5\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} && \text{Simplify.} \\ &= \frac{2\sqrt{21}}{5 \cdot 3} && \text{Multiply by } \frac{\sqrt{3}}{\sqrt{3}}. \\ &= \frac{2\sqrt{21}}{15} && \text{Simplify.}\end{aligned}$$

**ANSWER:**

$$\frac{2\sqrt{21}}{15}$$

## 1-1 Points, Lines, and Planes

71.  $\frac{\sqrt{8a^6}}{\sqrt{108}}$

SOLUTION:

$$\frac{\sqrt{8a^6}}{\sqrt{108}} = \frac{\sqrt{2 \cdot 2 \cdot 2 \cdot a \cdot a \cdot a \cdot a \cdot a \cdot a}}{\sqrt{2 \cdot 2 \cdot 3 \cdot 3 \cdot 3}} \quad \text{Prime factorization}$$

$$= \frac{\sqrt{2^2 \cdot 2 \cdot a^6}}{\sqrt{2^2 \cdot 3^2 \cdot 3}} \quad \text{Group factors.}$$

$$= \frac{\sqrt{2^2} \sqrt{2} \sqrt{a^6}}{\sqrt{2^2} \sqrt{3^2} \sqrt{3}} \quad \text{Product Property}$$

$$= \frac{2a^3 \sqrt{2}}{2 \cdot 3 \sqrt{3}} \quad \text{Take square roots.}$$

$$= \frac{a^3 \sqrt{2}}{3 \sqrt{3}} \quad \text{Division.}$$

$$= \frac{a^3 \sqrt{2}}{3 \sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} \quad \text{Multiply by } \frac{\sqrt{3}}{\sqrt{3}}.$$

$$= \frac{|a^3| \sqrt{6}}{3 \cdot 3} \quad \text{Simplify.}$$

$$= \frac{|a^3| \sqrt{6}}{9} \quad \text{Multiply.}$$

ANSWER:

$$\frac{|a^3| \sqrt{6}}{9}$$

## 1-1 Points, Lines, and Planes

$$72. \frac{5}{4-\sqrt{2}}$$

**SOLUTION:**

Multiply both numerator and the denominator by the conjugate of the denominator  $4+\sqrt{2}$ .

$$\begin{aligned}\frac{5}{4-\sqrt{2}} &= \frac{5}{4-\sqrt{2}} \left( \frac{4+\sqrt{2}}{4+\sqrt{2}} \right) && \text{Multiply by } \frac{4+\sqrt{2}}{4+\sqrt{2}}. \\ &= \frac{5(4+\sqrt{2})}{4^2-(\sqrt{2})^2} && \text{Simplify.} \\ &= \frac{20+5\sqrt{2}}{14} && \text{Simplify.}\end{aligned}$$

**ANSWER:**

$$\frac{20+5\sqrt{2}}{14}$$

$$73. \frac{4\sqrt{3}}{2+\sqrt{5}}$$

**SOLUTION:**

Multiply both numerator and the denominator by the conjugate of the denominator  $2-\sqrt{5}$ .

$$\begin{aligned}\frac{4\sqrt{3}}{2+\sqrt{5}} &= \frac{4\sqrt{3}}{2+\sqrt{5}} \left( \frac{2-\sqrt{5}}{2-\sqrt{5}} \right) && \text{Multiply by } \frac{2-\sqrt{5}}{2-\sqrt{5}}. \\ &= \frac{4\sqrt{3}(2-\sqrt{5})}{2^2-(\sqrt{5})^2} && \text{Simplify.} \\ &= \frac{8\sqrt{3}-4\sqrt{15}}{4-5} && \text{Simplify.} \\ &= 4\sqrt{15}-8\sqrt{3} && \text{Simplify.}\end{aligned}$$

**ANSWER:**

$$4\sqrt{15}-8\sqrt{3}$$

## 1-1 Points, Lines, and Planes

74. **SHOPPING** Suppose you buy 3 shirts and 2 pairs of slacks on sale at a clothing store for \$72. The next day, a friend buys 2 shirts and 4 pairs of slacks for \$96. If the dress shirts you each bought were all the same price and the slacks were also all the same price, then what was the cost of each shirt and each pair of slacks?

### **SOLUTION:**

The scenario can be represented by a system of linear equations. Each equation represents the purchases for the day. Let  $x$  be the cost of a shirt and  $y$  the cost of a pair of slacks. Represent the first day as  $3x + 2y = 72$  and the second as  $2x + 4y = 96$ .

Solve the system using elimination.

Multiply both sides of the first equation by 2 to get the same  $y$ -coefficient.

$$6x + 4y = 144$$

Subtract the second equation from the new equation.

$$6x + 4y = 144 \quad \text{Multiply equation 1 by 2}$$

$$\underline{2x + 4y = 96} \quad (-) \quad \text{Subtract equations.}$$

$$4x = 48 \quad \text{Simplify}$$

$$\frac{4x}{4} = \frac{48}{4} \quad \text{Divide each side by 4.}$$

$$x = 12 \quad \text{Simplify.}$$

Use the value of  $x$  to find the value of  $y$ .

$$3x + 2y = 72 \quad \text{Equation 1}$$

$$3(12) + 2y = 72 \quad \text{Replace } x \text{ with 12.}$$

$$36 + 2y = 72 \quad \text{Simplify}$$

$$36 - 36 + 2y = 72 - 36 \quad \text{Subtract 36 from each side.}$$

$$2y = 36 \quad \text{Simplify.}$$

$$\frac{2y}{2} = \frac{36}{2} \quad \text{Divide each side by 2.}$$

$$y = 18 \quad \text{Simplify.}$$

Therefore, each shirt costs \$12 and each pair of slacks costs \$18.

### **ANSWER:**

shirt: \$12; slacks: \$18



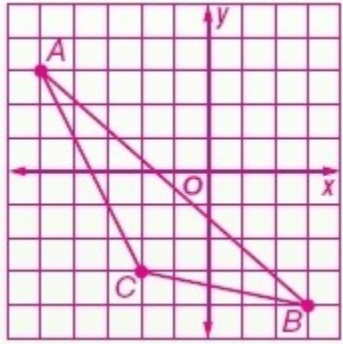
## 1-1 Points, Lines, and Planes

**Graph the following points and connect them in order to form a figure.**

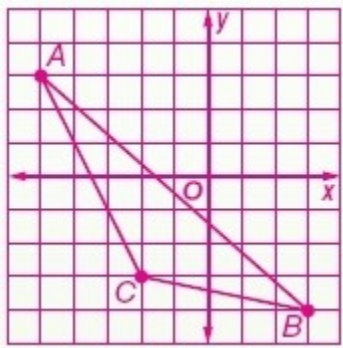
75.  $A(-5, 3)$ ,  $B(3, -4)$ ,  $C(-2, -3)$

**SOLUTION:**

Plot the points  $A(-5, 3)$ ,  $B(3, -4)$ , and  $C(-2, -3)$  on a coordinate plane and join them with straight lines.



**ANSWER:**

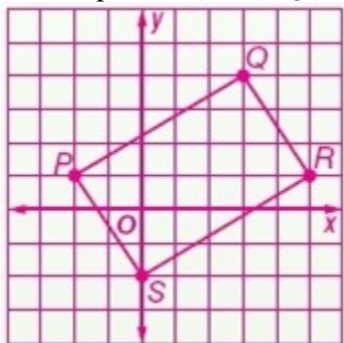


## 1-1 Points, Lines, and Planes

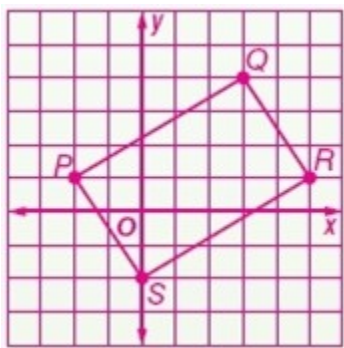
76.  $P(-2, 1)$ ,  $Q(3, 4)$ ,  $R(5, 1)$ ,  $S(0, -2)$

**SOLUTION:**

Plot the points  $P(-2, 1)$ ,  $Q(3, 4)$ ,  $R(5, 1)$ , and  $S(0, -2)$  on a coordinate plane and join them with straight lines.



**ANSWER:**



**GROCERIES** Find an approximate metric weight for each item.



77. Net Wt: 15 oz

**SOLUTION:**

Sample answer: Use dimensional analysis and one ounce is equivalent to about 28.3 grams.

$$15 \cancel{\text{oz}} \times \frac{28.3 \text{ g}}{1 \cancel{\text{oz}}} = 424.5 \text{ g}$$

So, 15 oz is approximately 424.5 g.

**ANSWER:**

Sample answer: 424.5 g

## 1-1 Points, Lines, and Planes



Net Wt: 8.2 oz

78.

**SOLUTION:**

Sample answer: Use dimensional analysis and one ounce is equivalent to about 28.3 grams.

$$8.2 \cancel{\text{ oz}} \times \frac{28.3 \text{ g}}{1 \cancel{\text{ oz}}} = 232.06 \text{ g}$$

So, 8.2 oz is approximately 232.1 g.

**ANSWER:**

Sample answer: 232.1 g



Net Wt: 2.5 lb

79.

**SOLUTION:**

Sample answer: Use dimensional analysis and one kilogram is equivalent to about 2.2 pounds.

$$2.5 \cancel{\text{ lb}} \times \frac{1 \text{ kg}}{2.2 \cancel{\text{ lb}}} \approx 1.136 \text{ kg}$$

So, 2.5 lb is approximately 1.1 kg.

**ANSWER:**

Sample answer: 1.1 kg

**Replace each  $\circ$  with  $>$ ,  $<$ , or  $=$  to make a true statement.**

80.  $\frac{1}{4}$  in.  $\circ$   $\frac{1}{2}$  in.

**SOLUTION:**

To compare these fractions, write the fractions with a common denominator.

$$\frac{1}{2} \cdot \frac{2}{2} = \frac{2}{4}$$

$$\frac{1}{4} < \frac{2}{4}$$

$$\frac{1}{4} < \frac{1}{2}$$

So,  $\frac{1}{4}$  in.  $<$   $\frac{1}{2}$  in.

**ANSWER:**

$<$

## 1-1 Points, Lines, and Planes

81.  $\frac{3}{4}$  in.  $\circ$   $\frac{5}{8}$  in.

**SOLUTION:**

To compare these fractions, write the fractions with a common denominator.

$$\frac{3}{4} \cdot \frac{2}{2} = \frac{6}{8}$$

$$\frac{6}{8} > \frac{5}{8}$$

$$\frac{3}{4} > \frac{5}{8}$$

So,  $\frac{3}{4}$  in.  $>$   $\frac{5}{8}$  in.

**ANSWER:**

$>$

82.  $\frac{3}{8}$  in.  $\circ$   $\frac{6}{16}$  in.

**SOLUTION:**

To compare these fractions, write the fractions with a common denominator.

$$\frac{3}{8} \cdot \frac{2}{2} = \frac{6}{16}$$

$$\frac{6}{16} = \frac{6}{16}$$

$$\frac{3}{8} = \frac{6}{16}$$

So,  $\frac{3}{8}$  in.  $=$   $\frac{6}{16}$  in.

**ANSWER:**

$=$

83. 18 mm  $\circ$  2 cm

**SOLUTION:**

$$\begin{aligned} 2 \text{ cm} &= \underline{2} \text{ mm} && \text{larger unit} \rightarrow \text{smaller unit} \\ &= 2 \times 10 && 1 \text{ cm} = 10 \text{ mm, } \times \text{ by } 10 \\ &= 20 \text{ mm} && \text{Multiply.} \end{aligned}$$

So, 2 cm is equivalent to 20 mm.

$$18 < 20$$

Thus, 18 mm  $<$  2 cm

**ANSWER:**

$<$

## 1-1 Points, Lines, and Planes

84.  $32\text{ mm} \bigcirc 3.2\text{ cm}$

**SOLUTION:**

$$\begin{aligned} 3.2\text{ cm} &= \underline{\quad?} \text{ mm} && \text{larger unit} \rightarrow \text{smaller unit} \\ &= 3.2 \times 10 && 1\text{ cm} = 10\text{ mm, } \times \text{ by } 10 \\ &= 32\text{ mm} && \text{Simplify} \end{aligned}$$

So, 3.2 cm is equivalent to 32 mm.

Thus,  $32\text{ mm} = 3.2\text{ cm}$

**ANSWER:**

=

85.  $0.8\text{ m} \bigcirc 8\text{ cm}$

**SOLUTION:**

$$\begin{aligned} 0.8\text{ m} &= \underline{\quad?} \text{ cm} && \text{larger unit} \rightarrow \text{smaller unit} \\ &= 0.8 \times 100 && 1\text{ m} = 100\text{ cm, } \times \text{ by } 100 \\ &= 80\text{ cm} && \text{Simplify} \end{aligned}$$

So, 0.8 m is equivalent to 80 cm.

$80 > 8$

Thus,  $0.8\text{ m} > 8\text{ cm}$ .

**ANSWER:**

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