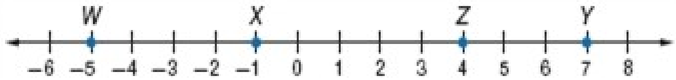
1-3 Distance and Midpoints

Use the number line to find each measure.



1. *XY*

SOLUTION:

$$\begin{aligned} XY &= |x_2 - x_1| && \text{Distance Formula} \\ &= |7 - (-1)| && \text{Replace } x_2 \text{ with } 7 \text{ and } x_1 \text{ with } -1. \\ &= |7 + 1| && \text{Simplify.} \\ &= |8| && \text{Addition.} \\ &= 8 && |8| \text{ is } 8. \end{aligned}$$

The distance between *X* and *Y* is 8 units. So, *XY* = 8.

ANSWER:

8

2. *WZ*

SOLUTION:

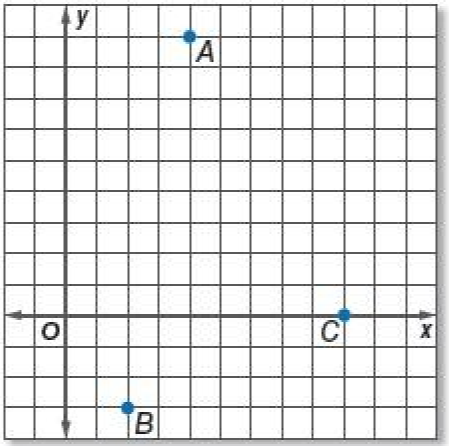
$$\begin{aligned} WZ &= |x_2 - x_1| && \text{Distance Formula} \\ &= |4 - (-5)| && \text{Replace } x_2 \text{ with } 4 \text{ and } x_1 \text{ with } -5. \\ &= |4 + 5| && \text{Simplify.} \\ &= |9| && \text{Addition.} \\ &= 9 && |9| \text{ is } 9. \end{aligned}$$

The distance between *W* and *Z* is 9 units. So, *WZ* = 9.

ANSWER:

9

TIME CAPSULE Graduating classes have buried time capsules on the campus of East Side High School for over twenty years. The points on the diagram show the position of three time capsules. Find the distance between each pair of time capsules.



3. $A(4, 9)$, $B(2, –3)$

SOLUTION:

Use the Distance Formula.

$$\begin{aligned} AB &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} && \text{Distance formula.} \\ &= \sqrt{(2-4)^2 + (-3-9)^2} && \text{Substitution.} \\ &= \sqrt{(-2)^2 + (-12)^2} && \text{Subtraction.} \\ &= \sqrt{148} && \text{Square terms.} \\ &\approx 12.2 && \text{Simplify.} \end{aligned}$$

The distance between A and B is $\sqrt{148}$ or about 12.2 units.

ANSWER:

$\sqrt{148}$ or about 12.2 units

4. $A(4, 9)$, $C(9, 0)$

SOLUTION:

Use the Distance Formula.

$$\begin{aligned}
AC &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} && \text{Distance Formula} \\
&= \sqrt{(9-4)^2 + (0-9)^2} && \text{Substitution.} \\
&= \sqrt{(5)^2 + (-9)^2} && \text{Subtraction.} \\
&= \sqrt{25 + 81} && \text{Square terms.} \\
&= \sqrt{106} && \text{Simplify.} \\
&\approx 10.3
\end{aligned}$$

The distance between A and C is $\sqrt{106}$ or about 10.3 units.

ANSWER:

$\sqrt{106}$ or about 10.3 units

5. $B(2, –3)$, $C(9, 0)$

SOLUTION:

Use the Distance Formula.

$BC = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

Substitute.

$$\begin{aligned}
BC &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} && \text{Distance Formula} \\
&= \sqrt{(9-2)^2 + (0-(-3))^2} && \text{Substitution.} \\
&= \sqrt{(7)^2 + 3^2} && \text{Subtraction.} \\
&= \sqrt{49 + 9} && \text{Square terms.} \\
&= \sqrt{58} && \text{Simplify.} \\
&\approx 7.6
\end{aligned}$$

The distance between B and C is $\sqrt{58}$ or about 7.6 units.

ANSWER:

$\sqrt{58}$ or about 7.6 units

6. **CCSS REASONING** Which two time capsules are the closest to each other? Which are farthest apart?

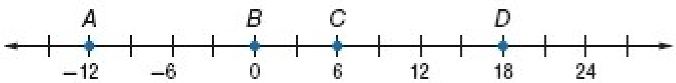
SOLUTION:

closest: *B* and *C*; farthest: *A* and *C*

ANSWER:

closest: *B* and *C*; farthest: *A* and *C*

Use the number line to find the coordinate of the midpoint of each segment.



7. $\overline{AC}$

SOLUTION:

Here, the number line is marked with an interval of 3. The point *A* is at –12 and *C* is at 6.

$$\begin{aligned} M &= \frac{x_1+x_2}{2} && \text{Midpoint Formula} \\ &= \frac{-12+6}{2} && \text{Substitution.} \\ &= \frac{-6}{2} && \text{Simplify} \\ &= -3 && \text{Simplify.} \end{aligned}$$

So, the midpoint of $\overline{AC}$ is -3 which is the point between –6 and 0.

ANSWER:

–3

8. $\overline{BD}$

SOLUTION:

Here, the number line is marked with an interval of 3. The point *B* is at 0 and *D* is at 18.

$$\begin{aligned} M &= \frac{x_1+x_2}{2} && \text{Midpoint Formula} \\ &= \frac{0+18}{2} && \text{Substitution.} \\ &= \frac{18}{2} && \text{Simplify} \\ &= 9 && \text{Simplify.} \end{aligned}$$

So, the midpoint of $\overline{BD}$ is 9 which is the point between 6 and 12.

ANSWER:

9

Find the coordinates of the midpoint of a segment with the given endpoints.

9. $J(5, -3)$, $K(3, -8)$

SOLUTION:

Use the Midpoint Formula

$$\begin{aligned}&\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)\\&=\left(\frac{5+3}{2}, \frac{-3-8}{2}\right) && \text{Substitution.}\\&=\left(\frac{8}{2}, \frac{-11}{2}\right) && \text{Simplify.}\\&=(4, -5.5) && \text{Simplify.}\end{aligned}$$

The midpoint of $\overline{JK}$ is (4, –5.5).

ANSWER:

(4, –5.5)

10. $M(7, 1)$, $N(4, -1)$

SOLUTION:

Use the Midpoint Formula.

$$\begin{aligned}&\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)\\&=\left(\frac{7+4}{2}, \frac{1-1}{2}\right) && \text{Substitution.}\\&=\left(\frac{13}{2}, \frac{0}{2}\right) && \text{Simplify.}\\&=(5.5, 0) && \text{Division.}\end{aligned}$$

The midpoint of $\overline{MN}$ is (5.5, 0).

ANSWER:

(5.5, 0)

11. Find the coordinates of G if $F(1, 3.5)$ is the midpoint of $\overline{GJ}$ and J has coordinates $(6, -2)$.

SOLUTION:

Let the coordinates of G be (x, y).
Then by the Midpoint Formula,
$\left(\frac{x+6}{2}, \frac{y-2}{2}\right) = (1, 3.5)$.

Write two equations to find the coordinates of G.

$$\frac{x+6}{2} = 1 \quad \text{Midpoint formula for } x.$$
$$2\left(\frac{x+6}{2}\right) = 2 \cdot 1 \quad \text{Multiply each side by 2.}$$
$$x + 6 = 2 \quad \text{Simplify.}$$
$$x + 6 - 6 = 2 - 6 \quad \text{Subtract 6 from each side.}$$
$$x = -4 \quad \text{Simplify.}$$

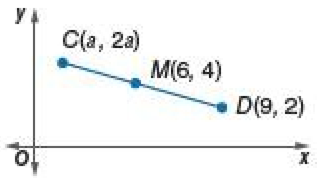
$$\frac{y-2}{2} = 3.5 \quad \text{Midpoint formula for } y.$$
$$2 \cdot \left(\frac{y-2}{2}\right) = 2 \cdot 3.5 \quad \text{Simplify.}$$
$$y - 2 = 7 \quad \text{Multiply each side by 2.}$$
$$y - 2 + 2 = 7 + 2 \quad \text{Subtract 2 from each side.}$$
$$y = 9 \quad \text{Simplify.}$$

The coordinates of G are $(-4, 9)$.

ANSWER:

$(-4, 9)$

12. **ALGEBRA** Point M is the midpoint of $\overline{CD}$. What is the value of a in the figure?



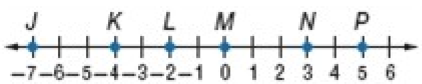
SOLUTION:
Use the Midpoint Formula to determine the value of a.

$$\begin{aligned} M &= \left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right) && \text{Midpoint Formula} \\ (6,4) &= \left(\frac{a+9}{2}, \frac{2a+2}{2}\right) && (x_1, y_1) = (a, 2a), (x_2, y_2) = (9, 2) \\ 6 &= \frac{a+9}{2} && \text{Set the } x-\text{coordinates equal.} \\ 12 &= a+9 && \text{Multiply each side by 2.} \\ 3 &= a && \text{Subtract 9 from each side.} \end{aligned}$$

The value of a is 3.

ANSWER:
3

Use the number line to find each measure.



13. JL

SOLUTION:

$$\begin{aligned} JL &= |x_2 - x_1| && \text{Distance Formula} \\ &= |-2-(-7)| && \text{Replace } x_2 \text{ with } -2 \text{ and } x_1 \text{ with } -7. \\ &= |-2+7| && \text{Simplify.} \\ &= |5| && \text{Addition.} \\ &= 5 && |5| \text{ is } 5. \end{aligned}$$

$JL = 5$

ANSWER:
5

1-3 Distance and Midpoints

14. *JK*

SOLUTION:

$$\begin{aligned} JK &= |x_2 - x_1| && \text{Distance Formula} \\ &= |-4-(-7)| && \text{Replace } x_2 \text{ with } -4 \text{ and } x_1 \text{ with } -7. \\ &= |-4+7| && \text{Simplify.} \\ &= |3| && \text{Simplify.} \\ &= 3 && |3| \text{ is } 3. \end{aligned}$$

JK = 3

ANSWER:

3

15. *KP*

SOLUTION:

$$\begin{aligned} KP &= |x_2 - x_1| && \text{Distance Formula} \\ &= |5-(-4)| && \text{Replace } x_2 \text{ with } 5 \text{ and } x_1 \text{ with } -4. \\ &= |5+4| && \text{Simplify.} \\ &= |9| && \text{Simplify.} \\ &= 9 && |9| \text{ is } 9. \end{aligned}$$

KP = 9

ANSWER:

9

16. *NP*

SOLUTION:

$$\begin{aligned} NP &= |x_2 - x_1| && \text{Distance Formula} \\ &= |5-3| && \text{Replace } x_2 \text{ with } 5 \text{ and } x_1 \text{ with } 3. \\ &= |2| && \text{Simplify.} \\ &= 2 && |2| \text{ is } 2. \end{aligned}$$

NP = 2

ANSWER:

2

17. *JP*

SOLUTION:

$$\begin{aligned} JP &= |x_2 - x_1| && \text{Distance Formula} \\ &= |5 - (-7)| && \text{Replace } x_2 \text{ with } 5 \text{ and } x_1 \text{ with } -7. \\ &= |5 + 7| && \text{Simplify.} \\ &= |12| && \text{Simplify.} \\ &= 12 && |12| \text{ is } 12. \end{aligned}$$

JP = 12

ANSWER:

12

18. *LN*

SOLUTION:

$$\begin{aligned} LN &= |x_2 - x_1| && \text{Distance Formula} \\ &= |3 - (-2)| && \text{Replace } x_2 \text{ with } 3 \text{ and } x_1 \text{ with } -2. \\ &= |3 + 2| && \text{Simplify.} \\ &= |5| && \text{Simplify.} \\ &= 5 && |5| \text{ is } 5. \end{aligned}$$

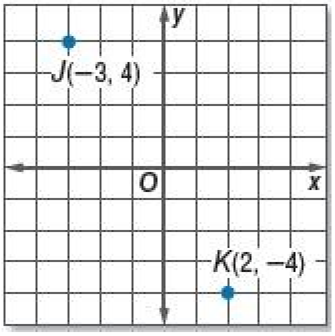
LN = 5

ANSWER:

5

1-3 Distance and Midpoints

Find the distance between each pair of points.

19. 

SOLUTION:

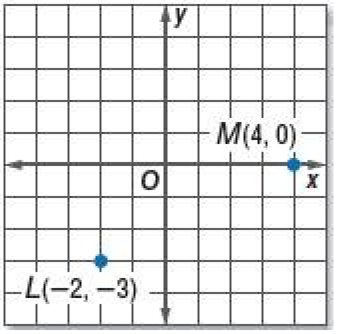
Use the Distance Formula.

$$\begin{aligned} JK &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} && \text{Distance Formula} \\ &= \sqrt{(2 - (-3))^2 + (-4 - 4)^2} && \text{Substitution.} \\ &= \sqrt{5^2 + (-8)^2} && \text{Subtraction.} \\ &= \sqrt{25 + 64} && \text{Square terms.} \\ &= \sqrt{89} && \text{Addition.} \\ &\approx 9.4 \end{aligned}$$

The distance between *J* and *K* is $\sqrt{89}$ or about 9.4 units.

ANSWER:

$\sqrt{89}$ or about 9.4 units

20. 

SOLUTION:

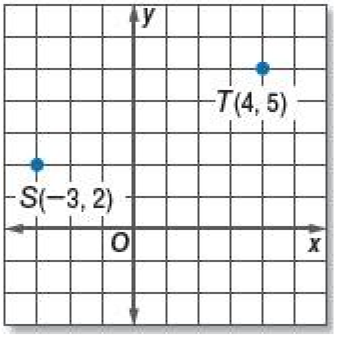
Use the Distance Formula.

$$\begin{aligned} ML &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} && \text{Distance Formula} \\ &= \sqrt{(-2-4)^2 + (-3-0)^2} && \text{Substitution.} \\ &= \sqrt{(-6)^2 + (-3)^2} && \text{Subtraction.} \\ &= \sqrt{36+9} && \text{Square terms.} \\ &= \sqrt{45} && \text{Addition.} \\ &\approx 6.7 \end{aligned}$$

The distance between M and L is $\sqrt{45}$ or about 6.7 units.

ANSWER:

$\sqrt{45}$ or about 6.7 units

21. 

SOLUTION:

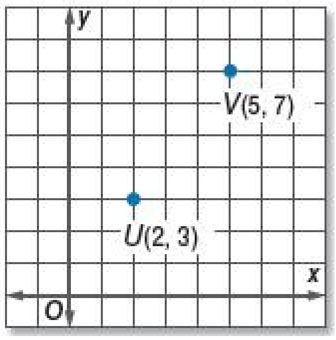
Use the Distance Formula.

$$\begin{aligned} ST &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} && \text{Distance Formula} \\ &= \sqrt{(4 - (-3))^2 + (5 - 2)^2} && \text{Substitution.} \\ &= \sqrt{7^2 + 3^2} && \text{Subtraction.} \\ &= \sqrt{49 + 9} && \text{Square terms} \\ &= \sqrt{58} && \text{Addition.} \\ &\approx 7.6 \end{aligned}$$

The distance between *S* and *T* is $\sqrt{58}$ or about 7.6 units.

ANSWER:

$\sqrt{58}$ or about 7.6 units

22. 

SOLUTION:

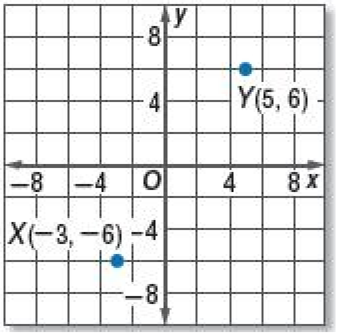
Use the Distance Formula.

$$\begin{aligned}
UV &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} && \text{Distance Formula} \\
&= \sqrt{(2-5)^2 + (3-7)^2} && \text{Substitution.} \\
&= \sqrt{(-3)^2 + (-4)^2} && \text{Subtraction.} \\
&= \sqrt{9+16} && \text{Square terms.} \\
&= \sqrt{25} && \text{Addition.} \\
&= 5
\end{aligned}$$

The distance between *U* and *V* is 5 units.

ANSWER:

5 units

23. 

SOLUTION:

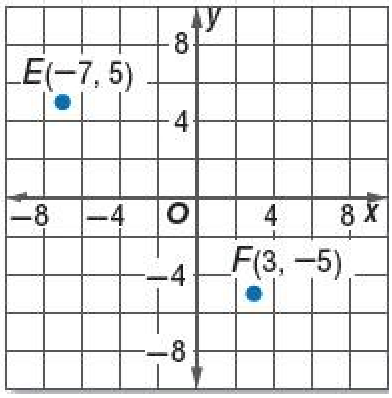
Use the Distance Formula.

$$\begin{aligned} XY &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} && \text{Distance Formula} \\ &= \sqrt{(5-(-3))^2 + (6-(-6))^2} && \text{Substitution.} \\ &= \sqrt{8^2 + 12^2} && \text{Simplify inside parenthesis.} \\ &= \sqrt{64 + 144} && \text{Square terms.} \\ &= \sqrt{208} && \text{Addition.} \\ &\approx 14.4 \end{aligned}$$

The distance between X and Y is $\sqrt{208}$ or about 14.4 units.

ANSWER:

$\sqrt{208}$ or about 14.4 units

24. 

SOLUTION:

Use the Distance Formula.

$$
\begin{aligned}
EF &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} && \text{Distance Formula} \\
&= \sqrt{(3 - (-7))^2 + (-5 - 5)^2} && \text{Substitution} \\
&= \sqrt{10^2 + (-10)^2} && \text{Subtraction.} \\
&= \sqrt{100 + 100} && \text{Square terms} \\
&= \sqrt{200} && \text{Addition.} \\
&\approx 14.1
\end{aligned}
$$

The distance between *E* and *F* is $\sqrt{200}$ or about 14.1 units.

ANSWER:

$\sqrt{200}$ or about 14.1 units

1-3 Distance and Midpoints

25. $X(1, 2)$, $Y(5, 9)$

SOLUTION:

Use the Distance Formula.

$$\begin{aligned}
XY &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} && \text{Distance Formula} \\
&= \sqrt{(5-1)^2 + (9-2)^2} && \text{Substitution.} \\
&= \sqrt{4^2 + 7^2} && \text{Subtraction.} \\
&= \sqrt{16 + 49} && \text{Square terms.} \\
&= \sqrt{65} && \text{Simplify.} \\
&\approx 8.1
\end{aligned}$$

The distance between X and Y is $\sqrt{65}$ or about 8.1 units.

ANSWER:

$\sqrt{65}$ or about 8.1 units

26. $P(3, 4)$, $Q(7, 2)$

SOLUTION:

Use the Distance Formula.

$$\begin{aligned}
PQ &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} && \text{Distance Formula} \\
&= \sqrt{(7-3)^2 + (2-4)^2} && \text{Substitution.} \\
&= \sqrt{4^2 + (-2)^2} && \text{Subtraction.} \\
&= \sqrt{16 + 4} && \text{Square terms.} \\
&= \sqrt{20} && \text{Addition.} \\
&\approx 4.5
\end{aligned}$$

The distance between P and Q is $\sqrt{20}$ or about 4.5 units.

ANSWER:

$\sqrt{20}$ or about 4.5 units

1-3 Distance and Midpoints

27. $M(-3, 8)$, $N(-5, 1)$

SOLUTION:

Use the Distance Formula.

$$\begin{aligned} MN &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} && \text{Distance Formula} \\ &= \sqrt{(-5-(-3))^2 + (1-8)^2} && \text{Subsiitution.} \\ &= \sqrt{(-2)^2 + (-7)^2} && \text{Subtraction.} \\ &= \sqrt{4+49} && \text{Square terms.} \\ &= \sqrt{53} && \text{Addition.} \\ &\approx 7.3 \end{aligned}$$

The distance between M and N is $\sqrt{53}$ or about 7.3 units.

ANSWER:

$\sqrt{53}$ or about 7.3 units

28. $Y(-4, 9)$, $Z(-5, 3)$

SOLUTION:

Use the Distance Formula.

$$\begin{aligned} YZ &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} && \text{Distance Formula} \\ &= \sqrt{(-5-(-4))^2 + (3-9)^2} && \text{Substitution.} \\ &= \sqrt{(-1)^2 + (-6)^2} && \text{Subtraction.} \\ &= \sqrt{1+36} && \text{Square terms.} \\ &= \sqrt{37} && \text{Addition.} \\ &\approx 6.1 \end{aligned}$$

The distance between X and Y is $\sqrt{37}$ or about 6.1 units.

ANSWER:

$\sqrt{37}$ or about 6.1 units

1-3 Distance and Midpoints

29. $A(2, 4)$, $B(5, 7)$

SOLUTION:
Use the Distance Formula.

$$
\begin{aligned}
AB &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} && \text{Distance Formula} \\
&= \sqrt{(5-2)^2 + (7-4)^2} && \text{Substitution.} \\
&= \sqrt{3^2 + 3^2} && \text{Subtraction.} \\
&= \sqrt{9+9} && \text{Square terms.} \\
&= \sqrt{18} && \text{Addition.} \\
&\approx 4.2
\end{aligned}
$$

The distance between A and B is $\sqrt{18}$ or about 4.2 units.

ANSWER:
$\sqrt{18}$ or about 4.2 units

30. $C(5, 1)$, $D(3, 6)$

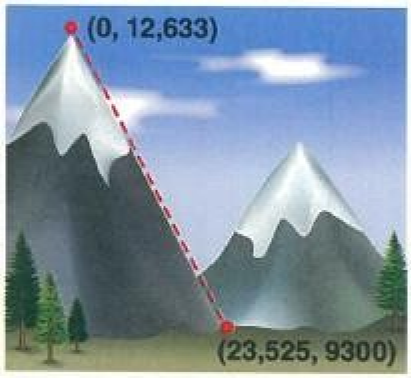
SOLUTION:
Use the Distance Formula.

$$
\begin{aligned}
CD &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} && \text{Distance Formula} \\
&= \sqrt{(3-5)^2 + (6-1)^2} && \text{Substitution.} \\
&= \sqrt{(-2)^2 + 5^2} && \text{Subtraction.} \\
&= \sqrt{4+25} && \text{Square terms.} \\
&= \sqrt{29} && \text{Addition.} \\
&\approx 5.4
\end{aligned}
$$

The distance between C and D is $\sqrt{29}$ or about 5.4 units.

ANSWER:
$\sqrt{29}$ or about 5.4 units

31. **CCSS REASONING** Vivian is planning to hike to the top of Humphreys Peak on her family vacation. The coordinates of the peak of the mountain and of the base of the trail are shown. If the trail can be approximated by a straight line, estimate the length of the trail. (*Hint*: 1 mi = 5280 ft)



SOLUTION:

Use the Distance Formula.

$$\begin{aligned} d &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} && \text{Distance Formula} \\ &= \sqrt{(23,525 - 0)^2 + (9300 - 12,633)^2} && \text{Substitution.} \\ &= \sqrt{(23,525)^2 + (-3333)^2} && \text{Subtraction.} \\ &= \sqrt{553,425,625 + 1,108,89} && \text{Square terms.} \\ &= \sqrt{564,534,514} && \text{Addition.} \\ &\approx 23,760 \end{aligned}$$

Divide by 5280 to convert to miles.

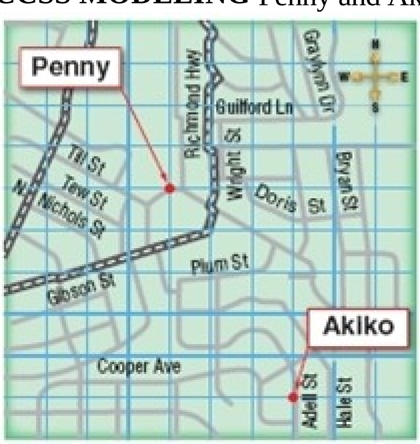
$23760 \div 5280 = 4.5$

The length of the trail is about 4.5 mi.

ANSWER:

4.5 mi

32. **CCSS MODELING** Penny and Akiko live in the locations shown on the map below.



a. If each square on the grid represents one block and the bottom left corner of the grid is the location of the origin,

what is the straight-line distance from Penny's house to Akiko's?
b. If Penny moves three blocks to the north and Akiko moves 5 blocks to the west, how far apart will they be?

SOLUTION:

a. The coordinates of the houses are:
Penny (4, 6) and Akiko (7, 1)

Use the Distance Formula to find the distance between their houses.

$$\begin{aligned}
d &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} && \text{Distance Formula} \\
&= \sqrt{(7-4)^2 + (1-6)^2} && \text{Substitution.} \\
&= \sqrt{3^2 + (-5)^2} && \text{Subtraction.} \\
&= \sqrt{9 + 25} && \text{Square terms.} \\
&= \sqrt{34} && \text{Addition.} \\
&\approx 5.8
\end{aligned}$$

The distance between their houses is about 5.8 blocks.

b. If Penny moves 3 blocks to the north, then the coordinates of her house become (4, 9). If Akiko moves 5 blocks to the west, then the coordinates of her house become (2, 1).

Use the Distance Formula to find the new distance.

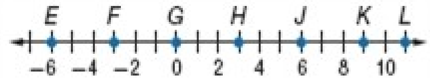
$$\begin{aligned}
d &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} && \text{Distance Formula} \\
&= \sqrt{(2-4)^2 + (1-9)^2} && \text{Substitution.} \\
&= \sqrt{(-2)^2 + (-8)^2} && \text{Subtraction.} \\
&= \sqrt{4 + 64} && \text{Square terms.} \\
&= \sqrt{68} && \text{Addition.} \\
&\approx 8.2
\end{aligned}$$

The distance between their houses after their move is about 8.2 blocks.

ANSWER:

a. 5.8 blocks
b. 8.2 blocks

Use the number line to find the coordinate of the midpoint of each segment.



33. $\overline{HK}$

SOLUTION:

H is at 3, and *K* is at 9.

$$\begin{aligned} M &= \frac{x_1+x_2}{2} && \text{Midpoint Formula} \\ &= \frac{3+9}{2} && \text{Substitution.} \\ &= \frac{12}{2} && \text{Addition} \\ &= 6 && \text{Division.} \end{aligned}$$

The midpoint is 6.

ANSWER:

6

34. $\overline{JL}$

SOLUTION:

J is at 6 and *L* is at 11.

$$\begin{aligned} M &= \frac{x_1+x_2}{2} && \text{Midpoint Formula} \\ &= \frac{6+11}{2} && \text{Substitution.} \\ &= \frac{17}{2} && \text{Addition} \\ &= 8.5 && \text{Division.} \end{aligned}$$

The midpoint is 8.5.

ANSWER:

8.5

35. $\overline{EF}$

SOLUTION:

E is at –6 and *F* is at –3.

$$\begin{aligned} M &= \frac{x_1 + x_2}{2} && \text{Midpoint Formula} \\ &= \frac{-6 + (-3)}{2} && \text{Substitution.} \\ &= \frac{-9}{2} && \text{Addition} \\ &= -4.5 && \text{Division.} \end{aligned}$$

The midpoint is –4.5.

ANSWER:

–4.5

36. $\overline{FG}$

SOLUTION:

F is at –3 and *G* is at 0.

$$\begin{aligned} M &= \frac{x_1 + x_2}{2} && \text{Midpoint Formula} \\ &= \frac{-3 + 0}{2} && \text{Substitution.} \\ &= \frac{-3}{2} && \text{Addition} \\ &= -1.5 && \text{Division.} \end{aligned}$$

The midpoint is –1.5.

ANSWER:

–1.5

37. $\overline{FK}$

SOLUTION:

F is at –3 and *K* is at 9.

$$\begin{aligned} M &= \frac{x_1 + x_2}{2} && \text{Midpoint Formula} \\ &= \frac{-3 + 9}{2} && \text{Substitution.} \\ &= \frac{6}{2} && \text{Addition} \\ &= 3 && \text{Division.} \end{aligned}$$

The midpoint is 3.

ANSWER:

3

38. $\overline{EL}$

SOLUTION:

E is at –6 and *L* is at 11.

$$\begin{aligned} M &= \frac{x_1 + x_2}{2} && \text{Midpoint Formula} \\ &= \frac{-6 + 11}{2} && \text{Subsitution.} \\ &= \frac{5}{2} && \text{Simplify} \\ &= 2.5 && \text{Simplify.} \end{aligned}$$

The midpoint is 2.5.

ANSWER:

2.5

Find the coordinates of the midpoint of a segment with the given endpoints.

39. *C*(22, 4), *B*(15, 7)

SOLUTION:

Use the Midpoint Formula

$$\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right) \quad \text{Midpoint Formula}$$
$$= \left(\frac{22+15}{2}, \frac{4+7}{2}\right) \quad \text{Substitution.}$$
$$= \left(\frac{37}{2}, \frac{11}{2}\right) \quad \text{Addition.}$$
$$= (18.5, 5.5) \quad \text{Division}$$

The midpoint of $\overline{BC}$ is (18.5, 5.5).

ANSWER:

(18.5, 5.5)

40. *W*(12, 2), *X*(7, 9)

SOLUTION:

Use the Midpoint Formula

$$\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right) \quad \text{Midpoint Formula}$$
$$= \left(\frac{12+7}{2}, \frac{2+9}{2}\right) \quad \text{Substitution.}$$
$$= \left(\frac{19}{2}, \frac{11}{2}\right) \quad \text{Addition.}$$
$$= (9.5, 5.5) \quad \text{Division.}$$

The midpoint of $\overline{WX}$ is (9.5, 5.5).

ANSWER:

(9.5, 5.5)

41. $D(-15, 4)$, $E(2, -10)$

SOLUTION:

Use the Midpoint Formula.

$$\begin{aligned}
&\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right) && \text{Midpoint Formula}\\
&=\left(\frac{-15+2}{2}, \frac{4-10}{2}\right) && \text{Substitution.}\\
&=\left(\frac{-13}{2}, \frac{-6}{2}\right) && \text{Addition.}\\
&=(-6.5,\ -3) && \text{Division.}
\end{aligned}$$

The midpoint of $\overline{DE}$ is (–6.5, –3).

ANSWER:

(–6.5, –3)

42. $V(-2, 5)$, $Z(3, -17)$

SOLUTION:

Use the Midpoint Formula.

$$\begin{aligned}
&\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right) && \text{Midpoint Formula}\\
&=\left(\frac{-2+3}{2}, \frac{5+(-17)}{2}\right) && \text{Substitution.}\\
&=\left(\frac{1}{2}, \frac{-12}{2}\right) && \text{Addition.}\\
&=(0.5,\ -6) && \text{Division.}
\end{aligned}$$

The midpoint of $\overline{VZ}$ is (0.5, –6).

ANSWER:

(0.5, –6)

43. $X(-2.4, -14)$, $Y(-6, -6.8)$

SOLUTION:
Use the Midpoint Formula.

$$\begin{aligned}
&\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right) && \text{Midpoint Formula} \\
&= \left(\frac{-2.4+(-6)}{2}, \frac{-14+(-6.8)}{2}\right) && \text{Substitution.} \\
&= \left(\frac{-8.4}{2}, \frac{-20.8}{2}\right) && \text{Addition.} \\
&= (-4.2, -10.4) && \text{Division.}
\end{aligned}$$

The midpoint of $\overline{XY}$ is (–4.2, –10.4).

ANSWER:
(–4.2, –10.4)

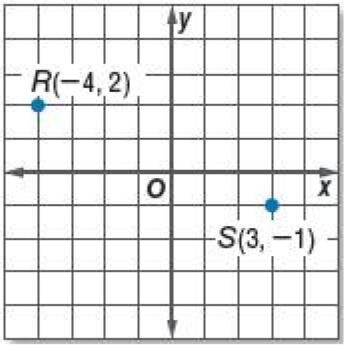
44. $J(-11.2, -3.4)$, $K(-5.6, -7.8)$

SOLUTION:
Use the Midpoint Formula.

$$\begin{aligned}
&\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right) && \text{Midpoint Formula} \\
&= \left(\frac{-11.2+(-5.6)}{2}, \frac{-3.4+(-7.8)}{2}\right) && \text{Substitution.} \\
&= \left(\frac{-16.8}{2}, \frac{-11.2}{2}\right) && \text{Addition.} \\
&= (-8.4, -5.6) && \text{Division.}
\end{aligned}$$

The midpoint of $\overline{JK}$ is (–8.4, –5.6).

ANSWER:
(–8.4, –5.6)

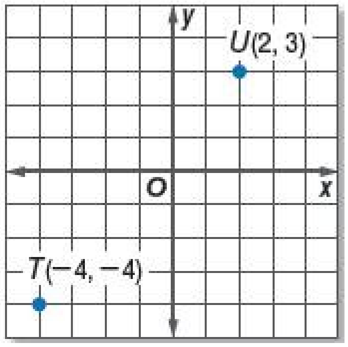
45. 

SOLUTION:
Use the Midpoint Formula.

$$\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right) \quad \text{Midpoint Formula}$$
$$=\left(\frac{-4+3}{2}, \frac{2+(-1)}{2}\right) \quad \text{Substitution.}$$
$$=\left(-\frac{1}{2}, \frac{1}{2}\right) \quad \text{Addition.}$$

The midpoint of $\overline{RS}$ is $\left(-\frac{1}{2}, \frac{1}{2}\right)$.

ANSWER:
$\left(-\frac{1}{2}, \frac{1}{2}\right)$

46. 

SOLUTION:

Use the Midpoint Formula.

$$\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right) \quad \text{Midpoint Formula}$$
$$= \left(\frac{-4+2}{2}, \frac{-4+3}{2}\right) \quad \text{Substitution.}$$
$$= \left(\frac{-2}{2}, \frac{-1}{2}\right) \quad \text{Addition.}$$
$$= \left(-1, -\frac{1}{2}\right) \quad \text{Simplify.}$$

The midpoint of $\overline{TU}$ is $\left(-1, -\frac{1}{2}\right)$.

ANSWER:

$\left(-1, -\frac{1}{2}\right)$

Find the coordinates of the missing endpoint if *B* is the midpoint of $\overline{AC}$.

47. $C(–5, 4)$, $B(–2, 5)$

SOLUTION:

Let A be (x, y).
Then by the Midpoint Formula,
$\left(\frac{x-5}{2}, \frac{y+4}{2}\right) = (-2, 5)$.

Write two equations to find the coordinates of A.

$$\begin{aligned} \frac{x-5}{2} &= -2 && \text{Midpoint Formula} \\ 2\left(\frac{x-5}{2}\right) &= 2(-2) && \times \text{each side by 2.} \\ x-5 &= -4 && \text{Multiply.} \\ x-5+5 &= -4+5 && +5 \text{ to each side.} \\ x &= 1 && \text{Simplify.} \end{aligned}$$

$$\begin{aligned} \frac{y+4}{2} &= 5 && \text{Midpoint Formula} \\ 2\left(\frac{y+4}{2}\right) &= 2(5) && \times \text{each side by 2.} \\ y+4 &= 10 && \text{Simplify.} \\ y+4-4 &= 10-4 && -4 \text{ from each side.} \\ y &= 6 && \text{Simplify.} \end{aligned}$$

The coordinates of A are (1, 6).

ANSWER:

$A(1, 6)$

48. $A(1, 7)$, $B(–3, 1)$

SOLUTION:

Let the coordinates of C be (x, y).
Then by the Midpoint Formula,
$\left(\frac{1+x}{2}, \frac{7+y}{2}\right) = (-3, 1)$.

Write two equations to find the coordinates of C.

$$\begin{aligned} \frac{1+x}{2} &= -3 && \text{Midpoint Formula} \\ 2\left(\frac{1+x}{2}\right) &= 2(-3) && \text{Multiply each side by 2.} \\ 1+x &= -6 && \text{Simplify.} \\ 1-1+x &= -6-1 && \text{Subtract 1 to each side.} \\ x &= -7 && \text{Simplify.} \end{aligned}$$

$$\begin{aligned} \frac{7+y}{2} &= 1 && \text{Midpoint formula} \\ 2\left(\frac{7+y}{2}\right) &= 2(1) && \text{Multiply each side by 2.} \\ 7+y &= 2 && \text{Simplify.} \\ 7-7+y &= 2-7 && \text{Subtract 7 from each side.} \\ y &= -5 && \text{Simplify.} \end{aligned}$$

The coordinates of C are $(–7, –5)$.

ANSWER:

$C(–7, –5)$

49. $A(-4, 2)$, $B(6, -1)$

SOLUTION:

Let the coordinates of C be (x, y).

Then by the Midpoint Formula,
$\left(\frac{-4+x}{2}, \frac{2+y}{2}\right) = (6, -1)$.

Write two equations to find the coordinates of C.

$$\begin{aligned} \frac{-4+x}{2} &= 6 && \text{Midpoint Formula} \\ 2\left(\frac{-4+x}{2}\right) &= 2(6) && \text{Multiply each side by 2.} \\ -4+x &= 12 && \text{Simplify.} \\ -4+4+x &= 12+4 && \text{Add 4 to each side.} \\ x &= 16 && \text{Simplify.} \end{aligned}$$

$$\begin{aligned} \frac{2+y}{2} &= -1 && \text{Midpoint Formula} \\ 2\left(\frac{2+y}{2}\right) &= 2(-1) && \text{Multiply each side by 2.} \\ 2+y &= -2 && \text{Simplify.} \\ 2-2+y &= -2-2 && \text{Subtract 2 from each side.} \\ y &= -4 && \text{Simplify.} \end{aligned}$$

The coordinates of C are $(16, -4)$.

ANSWER:

$C(16, -4)$

50. $C(-6, -2)$, $B(-3, -5)$

SOLUTION:

Let the coordinates of A be (x, y).

Then by the Midpoint Formula,
$\left(\frac{x-6}{2}, \frac{y-2}{2}\right) = (-3, -5)$.

Write two equations to find the coordinates of A.

$$\begin{aligned} \frac{x-6}{2} &= -3 && \text{Midpoint Formula} \\ 2\left(\frac{x-6}{2}\right) &= 2(-3) && \times \text{ each side by 2.} \\ x-6 &= -6 && \text{Simplify.} \\ x-6+6 &= -6+6 && +6 \text{ to each side.} \\ x &= 0 && \text{Simplify.} \end{aligned}$$

$$\begin{aligned} \frac{y-2}{2} &= -5 && \text{Midpoint Formula} \\ 2\left(\frac{y-2}{2}\right) &= 2(-5) && \times \text{ each side by 2.} \\ y-2 &= -10 && \text{Simplify.} \\ y-2+2 &= -10+2 && +2 \text{ to each side.} \\ y &= -8 && \text{Simplify.} \end{aligned}$$

The coordinates of A are $(0, -8)$.

ANSWER:

$A(0, -8)$

51. $A(4, –0.25)$, $B(–4, 6.5)$

SOLUTION:

Let the coordinates of C be (x, y).

Then by the Midpoint Formula,
$\left(\frac{4+x}{2}, \frac{-0.25+y}{2}\right) = (-4, 6.5)$.

Write two equations to find the coordinates of C.

$$\begin{aligned}
\frac{4+x}{2} &= -4 && \text{Midpoint Formula} \\
2\left(\frac{4+x}{2}\right) &= 2(-4) && \times \text{each side by 2.} \\
4+x &= -8 && \text{Simplify.} \\
4-4+x &= -8-4 && -4 \text{ to each side.} \\
x &= -12 && \text{Simplify.}
\end{aligned}$$

$$\begin{aligned}
\frac{-0.25+y}{2} &= 6.5 && \text{Midpoint Formula} \\
2\left(\frac{-0.25+y}{2}\right) &= 2(6.5) && \times \text{each side by 2} \\
-0.25+y &= 13 && \text{Simplify.} \\
-0.25+0.25+y &= 13+0.25 && +0.25 \text{ to each side.} \\
y &= 13.25 && \text{Simplify.}
\end{aligned}$$

The coordinates of C are (–12, 13.25).

ANSWER:

$C(–12, 13.25)$

52. $C\left(\frac{5}{3},-6\right), B\left(\frac{8}{3},4\right)$

SOLUTION:

Let the coordinates of A be (x, y).

Then by the Midpoint Formula,
$\left(\frac{x+\frac{5}{3}}{2}, \frac{y-6}{2}\right) = \left(\frac{8}{3}, 4\right)$.

Write two equations to find the coordinates of A.

$$\begin{aligned}
\frac{x+\frac{5}{3}}{2} &= \frac{8}{3} && \text{Midpoint Formula} \\
2\left(\frac{x+\frac{5}{3}}{2}\right) &= 2\left(\frac{8}{3}\right) && \times \text{ each side by 2.} \\
x+\frac{5}{3} &= \frac{16}{3} && \text{Simplify.} \\
x+\frac{5}{3}-\frac{5}{3} &= \frac{16}{3}-\frac{5}{3} && -\frac{5}{3} \text{ to each side.} \\
x &= \frac{11}{3} && \text{Simplify.}
\end{aligned}$$

$$\begin{aligned}
\frac{y-6}{2} &= 4 && \text{Midpoint Formula} \\
2\left(\frac{y-6}{2}\right) &= 2(4) && \times \text{ each side by 2.} \\
y-6 &= 8 && \text{Simplify.} \\
y-6+6 &= 8+6 && +6 \text{ to each side.} \\
y &= 14 && \text{Simplify.}
\end{aligned}$$

The coordinates of A are $\left(\frac{11}{3}, 14\right)$.

ANSWER:

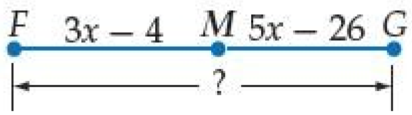
$A\left(\frac{11}{3}, 14\right)$

ALGEBRA Suppose M is the midpoint of $\overline{FG}$. Use the given information to find the missing measure or value.

53. $FM = 3x - 4$, $MG = 5x - 26$, $FG = ?$

SOLUTION:

If M is the midpoint, then $FM = MG$.



$$
\begin{aligned}
FM &= MG && \text{Given.}\\
3x - 4 &= 5x - 26 && \text{Subsitution.}\\
3x - 4 + 4 &= 5x - 26 + 4 && \text{Add 4 to each side.}\\
3x &= 5x - 22 && \text{Simplify.}\\
3x - 5x &= 5x - 5x - 22 && -5x \text{ from each side.}\\
-2x &= -22 && \text{Simplify.}\\
\frac{-2x}{-2} &= \frac{-22}{-2} && \text{Divide each side by } -2.\\
x &= 11 && \text{Simplify.}
\end{aligned}
$$

Then, $x = 11$.

$$
\begin{aligned}
FM &= 3x - 4\\
&= 3(11) - 4\\
&= 29\\
MG &= 5x - 26\\
&= 5(11) - 26\\
&= 29\\
FG &= FM + MG\\
&= 29 + 29\\
&= 58
\end{aligned}
$$

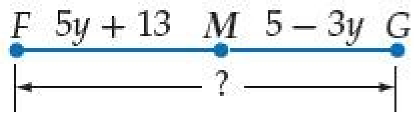
ANSWER:

58

54. $FM = 5y + 13$, $MG = 5 - 3y$, $FG = ?$

SOLUTION:

If M is the midpoint, then $FM = MG$.



$FM = MG$	Given.
$5y + 13 = 5 - 3y$	Substitution.
$5y + 3y + 13 = 5 - 3y + 3y$	Add $3y$ to each side.
$8y + 13 = 5$	Simplify.
$8y + 13 - 13 = 5 - 13$	Subtract 13 from each side.
$8y = -8$	Simplify.
$\frac{8y}{8} = \frac{-8}{8}$	Divide each side by 8.
$y = -1$	Simplify.

Then $y = -1$.

$FM = 5y + 13$
$= 5(-1) + 13$
$= 8$
$MG = 5 - 3y$
$= 5 - 3(-1)$
$= 8$
$FG = FM + MG$
$= 8 + 8$
$= 16$

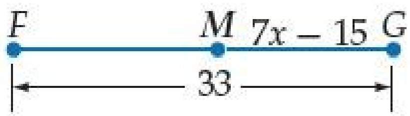
ANSWER:

16

55. $MG = 7x - 15$, $FG = 33$, $x = ?$

SOLUTION:

If M is the midpoint, then $MG = \frac{FG}{2}$.



Substitute.

$$MG = \frac{33}{2}$$
$$= 16.5$$

Thus $MG = 16.5$.

Find x,

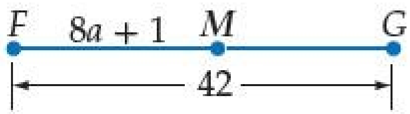
$$\begin{aligned} MG &= MG && \text{Given.} \\ 7x - 15 &= 16.5 && \text{Substitution.} \\ 7x - 15 + 15 &= 16.5 + 15 && +15 \text{ to each side.} \\ 7x &= 31.5 && \text{Simplify.} \\ \frac{7x}{7} &= \frac{31.5}{7} && \div \text{ each side by } 7. \\ x &= 4.5 && \text{Simplify.} \end{aligned}$$

ANSWER:

4.5

56. $FM = 8a + 1$, $FG = 42$, $a = ?$

SOLUTION:



If M is the midpoint, then $FM = \frac{FG}{2}$

Substitute.

$$\begin{aligned} FM &= \frac{42}{2} \\ &= 21 \end{aligned}$$

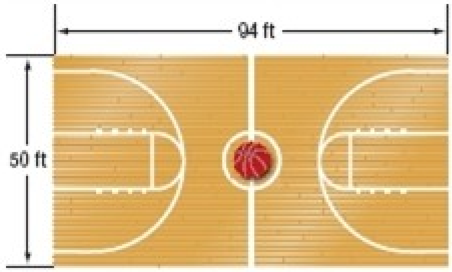
So, $FM = 21$.

$$\begin{aligned} FM &= FM && \text{Given.} \\ 8a + 1 &= 21 && \text{Substitution.} \\ 8a + 1 - 1 &= 21 - 1 && -1 \text{ from each side.} \\ 8a &= 20 && \text{Simplify.} \\ \frac{8a}{8} &= \frac{20}{8} && \div \text{ each side by 8.} \\ a &= 2.5 && \text{Simplify.} \end{aligned}$$

ANSWER:

2.5

57. **BASKETBALL** The dimensions of a basketball court are shown below. Suppose a player throws the ball from a corner to a teammate standing at the center of the court.



a. If center court is located at the origin, find the ordered pair that represents the location of the player in the bottom right corner.
b. Find the distance that the ball travels.

SOLUTION:

a. The center court is located at the origin. Since the court is 94 feet long, each end line is $\frac{1}{2}(94)$ or 47 feet from center court. So, the right corners will have *x*-coordinate values of 47.
Since the court is 50 feet wide, each side line be $\frac{1}{2}(50)$ or 25 feet from center court.
So, the bottom corners will have *y*-coordinate values of –25. Therefore, the coordinates of the bottom right corner are (47, –25).

b. Use the Distance Formula to find the distance between (0, 0) and (47, –25).

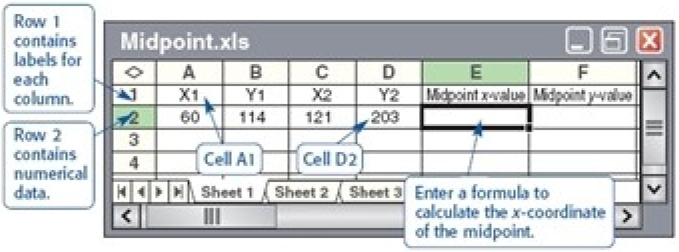
$$
\begin{aligned}
d &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\
d &= \sqrt{(47 - 0)^2 + (-25 - 0)^2} && \text{Substitution.} \\
&= \sqrt{47^2 + (-25)^2} && \text{Subtraction.} \\
&= \sqrt{2209 + 625} && \text{Square terms.} \\
&= \sqrt{2834} && \text{Addition.} \\
&\approx 53.2
\end{aligned}
$$

The distance between the center of the court and the bottom right corner is about 53.2 ft. The ball will travel about 53.2 ft.

ANSWER:

a. (47, –25)
b. ≈53.2 ft

CCSS TOOLS Spreadsheets can be used to perform calculations quickly. The spreadsheet below can be used to calculate the distance between two points. Values are used in formulas by using a specific cell name. The value of x_1 is used in a formula using its cell name, A2.



Write a formula for the indicated cell that could be used to calculate the indicated value using the coordinates (x_1 , y_1) and (x_2 , y_2) as the endpoint of a segment.

58. E2; the *x*-value of the midpoint of the segment

SOLUTION:

To find the midpoint of the segment, use the **AVERAGE** function. The **AVERAGE** function sums the specified cells and divides by the number of cells. We want to sum A2 and C2 and divide by two.

=AVERAGE(A2, C2)

ANSWER:

=AVERAGE(A2, C2)

59. F2; the *y*-value of the midpoint of the segment

SOLUTION:

To find the midpoint of the segment, use the **AVERAGE** function. The **AVERAGE** function sums the specified cells and divides by the number of cells. We want to sum B2 and D2 and divide by two.

=AVERAGE(B2, D2)

ANSWER:

=AVERAGE(B2, D2)

60. G2; the length of the segment

SOLUTION:

To find the distance of the segment, you need to use the distance formula. The distance formula is not a built in function on the spreadsheet. Remember that (x_2, y_2) are stored in (C2, D2) and (x_1, y_1) are stored in (A2, B2). Use the **SQRT** function for the square root. Use the ^ key to raise to a power of 2. You will need to have several sets parenthesis.

=SQRT((C2 – A2)^2 + (D2 – B2)^2)

ANSWER:

=SQRT((C2 – A2)^2 + (D2 – B2)^2)

Name the point(s) that satisfy the given condition.

61. two points on the *x*-axis that are 10 units from (1, 8)

SOLUTION:

The *y*-coordinate of the point on the *x*-axis is 0.
So, the point would be of the form (*x*, 0).

Use the Distance Formula to find an expression for the distance between the points (*x*, 0) and (1, 8) and equate it to 10.

$$\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}=d \quad \text{Distance Formula}$$
$$\sqrt{(1-x)^2+(8-0)^2}=10 \quad \text{Substitution.}$$
$$\left(\sqrt{(1-x)^2+(8-0)^2}\right)^2=(10)^2 \quad \text{Square each side}$$
$$(1-x)^2+(8-0)^2=100 \quad \text{Simplify.}$$
$$1-2x+x^2+64=100 \quad \text{Square terms.}$$
$$x^2-2x+65=100 \quad \text{Simplify.}$$
$$x^2-2x+65-100=100-100 \quad \text{Subtract 100 from each side.}$$
$$x^2-2x-35=0 \quad \text{Simplify.}$$
$$(x-7)(x+5)=0 \quad \text{Factor.}$$
$$x=7 \text{ or } x=-5 \quad \text{Solve for } x.$$

There are two possible values for *x*, –5 and 7.
So, the two points are (–5, 0) and (7, 0).

ANSWER:

(–5, 0), (7, 0)

62. two points on the y-axis that are 25 units from (–24, 3)

SOLUTION:

The x-coordinate of the point on the y-axis is 0.
So, the point would be of the form $(0, y)$.

Use the Distance Formula to find an expression for the distance between the points $(0, y)$ and (–24, 3) and equate it to 25.

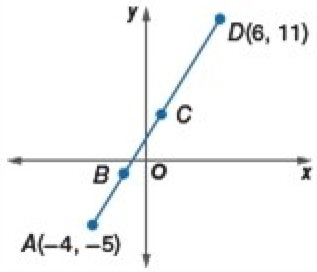
$$
\begin{aligned}
\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} &= d && \text{Distance Formula} \\
\sqrt{(-24 - 0)^2 + (3 - y)^2} &= 25 && \text{Substitution.} \\
\left(\sqrt{(-24 - 0)^2 + (3 - y)^2}\right)^2 &= (25)^2 && \text{Square each side} \\
(-24 - 0)^2 + (3 - y)^2 &= 625 && \text{Simplify.} \\
576 + 9 + y^2 - 6y &= 625 && \text{Square terms.} \\
y^2 - 6y + 585 &= 625 && \text{Simplify.} \\
y^2 - 6y + 585 - 625 &= 625 - 625 && -625 \text{ from each side.} \\
y^2 - 6y - 40 &= 0 && \text{Simplify.} \\
(y - 10)(y + 4) &= 0 && \text{Factor.} \\
y = 10 \text{ or } y &= -4 && \text{Zero Product Property}
\end{aligned}
$$

There are two possible values for y, –4 and 10.
So, the two points are (0, –4) and (0, 10).

ANSWER:

(0, –4), (0, 10)

63. **COORDINATE GEOMETRY** Find the coordinates of *B* if *B* is the midpoint of $\overline{AC}$ and *C* is the midpoint of $\overline{AD}$.



SOLUTION:

Use the Midpoint Formula to find the coordinates of *C*.

$$\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right) \quad \text{Midpoint Formula}$$
$$= \left(\frac{-4+6}{2}, \frac{-5+11}{2}\right) \quad \text{Substitution.}$$
$$= \left(\frac{2}{2}, \frac{6}{2}\right) \quad \text{Addition.}$$
$$= (1, 3) \quad \text{Division.}$$

Use the Midpoint Formula to find the coordinates of *B*.

$$\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right) \quad \text{Midpoint Formula/}$$
$$= \left(\frac{-4+1}{2}, \frac{-5+3}{2}\right) \quad \text{Substitution.}$$
$$= \left(-\frac{3}{2}, \frac{-2}{2}\right) \quad \text{Addition.}$$
$$= (-1.5, -1) \quad \text{Division.}$$

ANSWER:

$$\left(-1\frac{1}{2}, -1\right)$$

ALGEBRA Determine the value(s) of *n*.

64. $J(n, n+2)$, $K(3n, n-1)$, $JK = 5$

SOLUTION:

Use the Distance Formula to find an expression for *JK* and equate it to 5.

$$
\begin{aligned}
\sqrt{(x_2-x_1)^2+(y_2-y_1)^2} &= d && \text{Distance Formula}\\
\sqrt{(3n-n)^2+((n-1)-(n+2))^2} &= 5 && \text{Substitution.}\\
\sqrt{(2n)^2+(-3)^2} &= 5 && \text{Simplify.}\\
\left(\sqrt{(2n)^2+(-3)^2}\right)^2 &= (5)^2 && \text{Square each side}\\
(2n)^2+(-3)^2 &= 25 && \text{Simplify.}\\
4n^2+9 &= 25 && \text{Square each term.}\\
4n^2+9-9 &= 25-9 && -9 \text{ from each side.}\\
4n^2 &= 16 && \text{Simplify.}\\
\frac{4n^2}{4} &= \frac{16}{4} && \text{Divide each side by 4.}\\
n^2 &= 4 && \text{Simplify.}\\
\sqrt{n^2} &= \sqrt{4} && \text{Take square root of each side}\\
n &= \pm 2 && \text{Simplify.}
\end{aligned}
$$

ANSWER:

± 2

65. $P(3n, n - 7)$, $Q(4n, n + 5)$, $PQ = 13$

SOLUTION:

Use the Distance Formula to find an expression for *PQ* and equate it to 13.

$\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} = d$	Distance Formula
$\sqrt{(4n - 3n)^2 + ((n + 5) - (n - 7))^2} = 13$	Substitution.
$\sqrt{n^2 + 12^2} = 13$	Simplify.
$\left(\sqrt{n^2 + 12^2}\right)^2 = (13)^2$	Square each side
$n^2 + 12^2 = 169$	Simplify.
$n^2 + 144 = 169$	Square each term.
$n^2 + 144 - 144 = 169 - 144$	-144 from each side.
$n^2 = 25$	Simplify.
$\sqrt{n^2} = \sqrt{25}$	Take square root of each side
$n = \pm 5$	Simplify.

ANSWER:

± 5

66. **GEOGRAPHY** Wilmington, North Carolina, is located at (34.3°, 77.9°), which represents north latitude and west longitude. Winston-Salem is in the northern part of the state at (36.1°, 80.2°).

a. Find the latitude and longitude of the midpoint of the segment between Wilmington and Winston-Salem.

b. Use an atlas or the Internet to find a city near the location of the midpoint.

c. If Winston-Salem is the midpoint of the segment with one endpoint at Wilmington, find the latitude and longitude of the other endpoint.

d. Use an atlas or the Internet to find a city near the location of the other endpoint.

SOLUTION:

a. Use the Midpoint Formula.

$$\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$$
$$=\left(\frac{34.3+36.1}{2}, \frac{77.9+80.2}{2}\right)$$
$$=\left(\frac{70.4}{2}, \frac{158.1}{2}\right)$$
$$=(35.2, 79.05)$$

Round to the nearest tenth.
= (35.2, 70.1)
b. Sample answer: Fayetteville

c. Let (x, y) be the location of the other end point. Then by the Midpoint Formula,
$$\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$$
$$=\left(\frac{34.3+x}{2}, \frac{77.9+y}{2}\right)$$
$$=(36.1, 80.2)$$

Write two equations to find the values of x and y.
$$\frac{34.3+x}{2}=36.1$$
$$34.3+x=72.2$$
$$x=37.9$$

$$\frac{77.9+y}{2}=80.2$$
$$77.9+y=160.4$$
$$y=82.5$$
The latitude and longitude of the other end point are (37.9, 82.5).

d. Sample answer: Prestonburg, Kentucky

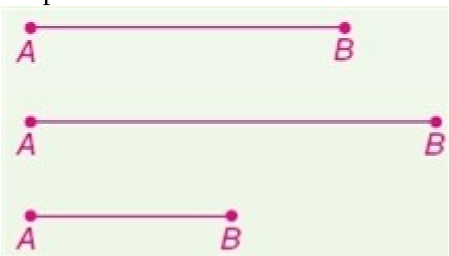
ANSWER:
a. (35.2°, 79.1°)
b. Sample answer: Fayetteville
c. (37.9°, 82.5°)
d. Sample answer: Prestonburg, Kentucky

67. **MULTIPLE REPRESENTATIONS** In this problem, you will explore the relationship between a midpoint of a segment and the midpoint between the endpoint and the midpoint.
a. GEOMETRIC Use a straightedge to draw three different line segments. Label the endpoints *A* and *B*.
b. GEOMETRIC On each line segment, find the midpoint of $\overline{AB}$ and label it *C*. Then find the midpoint of $\overline{AC}$ and label it *D*.
c. TABULAR Measure and record *AB*, *AC*, and *AD* for each line segment. Organize your results into a table.
d. ALGEBRAIC If *AB* = *x*, write an expression for the measures *AC* and *AD*.
e. VERBAL Make a conjecture about the relationship between *AB* and each segment if you were to continue to

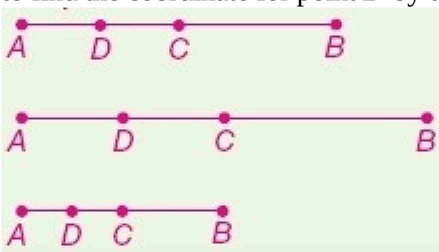
find the midpoints of a segment and a midpoint you previously found.

SOLUTION:

a. Sample answer: Use a ruler to draw 3 line segments between any two numbered marks, x_1 and x_2 . Label the endpoints A and B.



b. Sample answer: Find $C = \frac{x_1+x_2}{2}$ and place a point on each line segment at this coordinate. Repeat the process to find the coordinate for point D by using the coordinates for points A and C.



c. Sample answer: For each line segment, use the ruler to measure the length of AB, AC, and AD. Record your results in a table.

line	AB	AC	AD
1	4	2	1
2	6	3	1.5
3	3	1.5	0.75

d.

$AC = \frac{1}{2}AB$ Definition of midpoint

$AC = \frac{1}{2}x$ $AB = x$

$AD = \frac{1}{2}AC$ Definition of midpoint

$AD = \frac{1}{2}\left(\frac{1}{2}x\right)$ Substitution

$AD = \frac{1}{4}x$ Simplify.

e. Let $AB = x$ and look for a pattern.

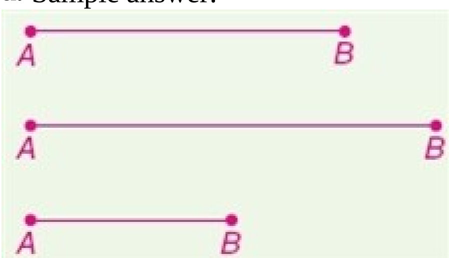
Number of Midpoints	Length of Smallest Segment

1	$\frac{1}{2}x$
2	$\frac{1}{2} \cdot \frac{1}{2}x = \frac{1}{2(2)}x$
3	$\frac{1}{2} \cdot \frac{1}{2(2)}x = \frac{1}{2^3}x$
4	$\frac{1}{2} \cdot \frac{1}{2^3}x = \frac{1}{2^4}x$
n	$\frac{1}{2^n}x$

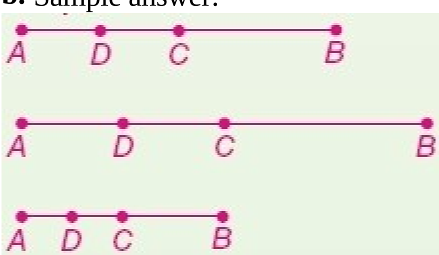
Sample answer: If $AB = x$ and n midpoints are found, then the smallest segment will have a measure of $\frac{1}{2^n}x$.

ANSWER:

a. Sample answer:



b. Sample answer:



c. Sample answer:

line	AB	AC	AD
1	4	2	1
2	6	3	1.5
3	3	1.5	0.75

d. $AC = \frac{1}{2}x, AD = \frac{1}{4}x$

e. Sample answer: If $AB = x$ and n midpoints are found, then the smallest segment will have a measure of $\frac{1}{2^n}x$.

1-3 Distance and Midpoints

68. **WRITING IN MATH** Explain how the Pythagorean Theorem and the Distance Formula are related.

SOLUTION:

Sample answer: The Pythagorean Theorem relates the lengths of the legs of a right triangle to the length of the hypotenuse using the formula $c^2 = a^2 + b^2$. If you take the square root of the formula, you get $c = \sqrt{a^2 + b^2}$. Think of the hypotenuse of the triangle as the distance between the two points, the a value as the horizontal distance $x_2 - x_1$, and the b value as the vertical distance $y_2 - y_1$. If you substitute, the Pythagorean Theorem becomes the Distance Formula, $c = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$.

ANSWER:

Sample answer: The Pythagorean Theorem relates the lengths of the legs of a right triangle to the length of the hypotenuse using the formula $c^2 = a^2 + b^2$. If you take the square root of the formula, you get $c = \sqrt{a^2 + b^2}$. Think of the hypotenuse of the triangle as the distance between the two points, the a value as the horizontal distance $x_2 - x_1$, and the b value as the vertical distance $y_2 - y_1$. If you substitute, the Pythagorean Theorem becomes the Distance Formula, $c = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$.

69. **REASONING** Is the point one third of the way from (x_1, y_1) to (x_2, y_2) *sometimes*, *always*, or *never* the point $\left(\frac{x_1 + x_2}{3}, \frac{y_1 + y_2}{3}\right)$? Explain.

SOLUTION:

Sample answer: Choose some points that lie on horizontal, vertical, and diagonal line segments. Use the distance for distance between the first pair of points and the first point and the new point.

(x_1, y_1)	(x_2, y_2)	$\left(\frac{x_1+x_2}{3}, \frac{y_1+y_2}{3}\right)$	Distance between first pair of points	Distance between first p
(–3, 0)	(6, 0)	$\left(\frac{-3+6}{3}, \frac{0+0}{3}\right) = (1, 0)$	$\sqrt{(-3-6)^2 + (0-0)^2} = 9$	$\sqrt{(-3-1)^2 + (0-0$
(0, 1)	(0, 13)	$\left(\frac{0+0}{3}, \frac{1+13}{3}\right) = \left(0, \frac{14}{3}\right)$	$\sqrt{(0-0)^2 + (1-13)^2} = 12$	$\sqrt{(0-0)^2 + \left(1-\frac{14}{3}\right)$
(0, 0)	(6, 0)	$\left(\frac{0+6}{3}, \frac{0+0}{3}\right) = (2, 0)$	$\sqrt{(0-6)^2 + (0-0)^2} = 6$	$\sqrt{(0-2)^2 + (0-0)^2$
(9,12)	(0, 0)	$\left(\frac{9+0}{3}, \frac{12+0}{3}\right) = (3, 4)$	$\sqrt{(9-0)^2 + (12-0)^2} = 15$	$\sqrt{(9-3)^2 + (12-4)$
(0, 0)	(12, 9)	$\left(\frac{0+12}{3}, \frac{0+9}{3}\right) = (4, 3)$	$\sqrt{(0-12)^2 + (0-9)^2} = 15$	$\sqrt{(0-4)^2 + (0-3)^2$
(–4, –5)	(5, 7)	$\left(\frac{-4+5}{3}, \frac{-5+7}{3}\right) = \left(\frac{1}{3}, \frac{2}{3}\right)$	$\sqrt{(-4-5)^2 + (-5-7)^2} = 15$	$\sqrt{\left(-4-\frac{1}{3}\right)^2 + \left(-5$
(3, –2)	(3, 4)	$\left(\frac{3+3}{3}, \frac{-2+4}{3}\right) = \left(2, \frac{2}{3}\right)$	$\sqrt{(3-3)^2 + (-2-4)^2} = 6$	$\sqrt{(3-2)^2 + \left(-2-\right.$

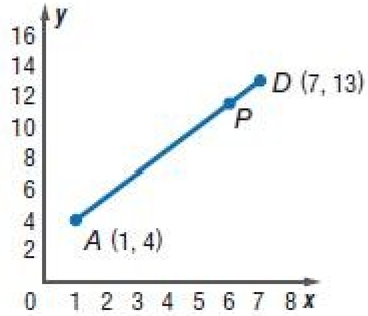
Test each pair of distances. Only $2 = \frac{1}{3}(6)$ and $5 = \frac{1}{3}(15)$. So when $(x_1, y_1) = (0, 0)$, the point $\left(\frac{x_1+x_2}{3}, \frac{y_1+y_2}{3}\right.$
way from (x_1, y_1) to (x_2, y_2). Therefore, the correct answer is sometimes.

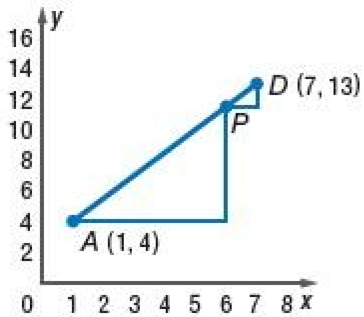
ANSWER:

Sample answer: Sometimes; when the point (x_1, y_1) has coordinates (0, 0).

70. **CHALLENGE** Point *P* is located on the segment between point *A*(1, 4) and point *D*(7, 13). The distance from *A* to *P* is twice the distance from *P* to *D*. What are the coordinates of point *P*?

SOLUTION:





The point *P* divides the line segment *AD* in the ratio 2:1.

Find the length of *AD*.

$$AD = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \quad \text{Distance Formula}$$
$$= \sqrt{(7-1)^2 + (13-4)^2} \quad \text{Substitution.}$$
$$= \sqrt{36 + 81} \quad \text{Subtraction.}$$
$$= \sqrt{117} \text{ or } 3\sqrt{13} \quad \text{Addition.}$$

Since the ratio of *AP* to *PD* is 2 to 1, the ratio of *AP* to *AD* is 2 to 3. Find the length of *AP*.

$$\frac{AP}{AB} = \frac{2}{3}$$
$$\frac{AP}{3\sqrt{13}} = \frac{2}{3} \quad \text{Substitution}$$
$$AP = 2\sqrt{13} \quad \text{Multiply each side by } 3\sqrt{13}.$$

If point *P* is on segment *AD*, then it must satisfy the equation of $\overleftrightarrow{AD}$. Find the slope and equation of $\overleftrightarrow{AD}$.

$$m = \frac{y_2 - y_1}{x_2 - x_1} \quad \text{Slope formula}$$
$$m = \frac{13-4}{7-1} \quad \text{Substitution.}$$
$$m = \frac{9}{6} \text{ or } \frac{3}{2} \quad \text{Subtraction / Simplify.}$$

Write the equation for the line.

$$\begin{aligned}(y-y_1) &= m(x-x_1) && \text{Point-slope form}\\ y-4 &= \tfrac{3}{2}(x-1) && m=\tfrac{3}{2},\ (x_1,y_1)=(1,4)\\ y-4 &= \tfrac{3}{2}x-\tfrac{3}{2} && \text{Mulitply.}\\ y &= \tfrac{3}{2}x+\tfrac{5}{2} && \text{Add 4 to each side.}\end{aligned}$$

Use the distance formula and substitution to solve for x if $P = (x, y)$.

$$\begin{aligned}\sqrt{(x-1)^2+(y-4)^2} &= 2\sqrt{13} && PD=2\sqrt{13}\\ (x-1)^2+(y-4)^2 &= \left(2\sqrt{13}\right)^2 && \text{Square each side.}\\ (x-1)^2+\left(\tfrac{3}{2}x+\tfrac{5}{2}-4\right)^2 &= \left(2\sqrt{13}\right)^2 && \text{Substitute}\tfrac{3}{2}x+\tfrac{5}{2}\text{for}y.\\ x^2-2x+1+\tfrac{9}{4}x^2-\tfrac{9}{2}y+\tfrac{9}{4} &= 52 && \text{Multiply.}\\ \tfrac{13}{4}x^2-\tfrac{13}{2}x+\tfrac{13}{4} &= 52 && \text{Simplify.}\\ 13x^2-26x+13 &= 208 && \times\text{ each side by 4.}\\ 13x^2-26x-195 &= 0 && -208\text{ from each side.}\\ x^2-2x-15 &= 0 && \div\text{ each side by 13.}\\ (x-5)(x+3) &= 0 && \text{Factor.}\\ x &= 5\text{ or }-3 && \text{Zero Product Property}\end{aligned}$$

Since P is on $\overline{AD}$, it must be in quadrant I. So, x cannot equal –3. Use the equation of the line and $x = 5$ to determine the value of the y-coordinate for the point P.

$$\begin{aligned}y &= \tfrac{3}{2}x+\tfrac{5}{2}\\ y &= \tfrac{3}{2}(5)+\tfrac{5}{2}\\ y &= \tfrac{20}{2}\text{ or }10\end{aligned}$$

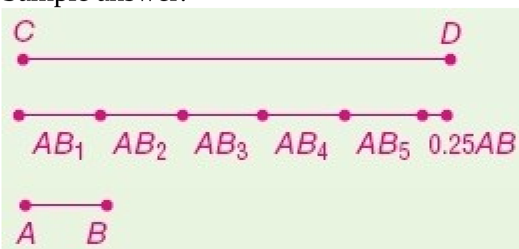
So, the coordinates of P are (5, 10).

ANSWER:
(5, 10)

71. **OPEN ENDED** Draw a segment and name it $\overline{AB}$. Using only a compass and a straightedge, construct a segment $\overline{CD}$ such that $CD = 5\frac{1}{4}AB.$ Explain and then justify your construction.

SOLUTION:

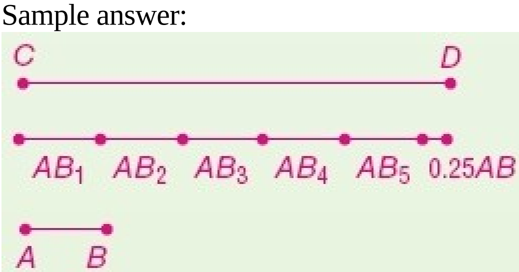
Sample answer:



Draw $\overline{AB}.$ Next, draw a construction line and place point *C* on it. From *C*, strike 6 arcs in succession of length *AB*. On the sixth $\overline{AB}$ length, perform a segment bisector two times to create a $\frac{1}{4}AB$ length. Label the endpoint *D*.

ANSWER:

Sample answer:



Draw $\overline{AB}.$ Next, draw a construction line and place point *C* on it. From *C*, strike 6 arcs in succession of length *AB*. On the sixth $\overline{AB}$ length, perform a segment bisector two times to create a $\frac{1}{4}AB$ length. Label the endpoint *D*.

1-3 Distance and Midpoints

72. **WRITING IN MATH** Describe a method of finding the midpoint of a segment that has one endpoint at (0, 0). Give an example using your method, and explain why your method works.

SOLUTION:

Sample answer: Divide each coordinate of the endpoint that is not located at the origin by 2.

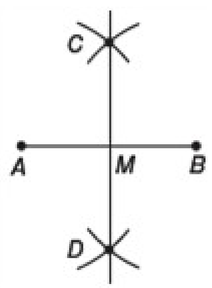
For example, if the segment has coordinates (0, 0) and (–10, 6), the midpoint is located at $\left(\frac{-10}{2}, \frac{6}{2}\right)$ or $(-5, 3)$.

Using the midpoint formula, if the endpoints of the segment are (0, 0) and (a, b), the midpoint is $\left(\frac{a-0}{2}, \frac{b-0}{2}\right)$ or $\left(\frac{a}{2}, \frac{b}{2}\right)$.

ANSWER:

Sample answer: Divide each coordinate of the endpoint that is not located at the origin by 2. For example, if the segment has coordinates (0, 0) and (–10, 6), the midpoint is located at $\left(\frac{-10}{2}, \frac{6}{2}\right)$ or $(-5, 3)$. Using the midpoint formula, if the endpoints of the segment are (0, 0) and (a, b), the midpoint is $\left(\frac{a-0}{2}, \frac{b-0}{2}\right)$ or $\left(\frac{a}{2}, \frac{b}{2}\right)$.

73. Which of the following best describes the first step in bisecting $\overline{AB}$?



A From point A, draw equal arcs on $\overline{CD}$ using the same compass width.

B From point A, draw equal arcs above and below $\overline{AB}$ using a compass width of $\frac{1}{3}\overline{AB}$.

C From point A, draw equal arcs above and below $\overline{AB}$ using a compass width greater than $\frac{1}{2}\overline{AB}$.

D From point A, draw equal arcs above and below $\overline{AB}$ using a compass width less than $\frac{1}{2}\overline{AB}$.

SOLUTION:

The first step in bisecting $\overline{AB}$ is from point A, draw equal arcs above and below $\overline{AB}$ using a compass width greater than $\frac{1}{2}\overline{AB}$. So, the correct choice is C.

ANSWER:

C

74. **ALGEBRA** Beth paid $74.88 for 3 pairs of jeans. All 3 pairs of jeans were the same price. How much did each pair of jeans cost?
F $24.96
G $37.44
H $74.88
J $224.64

SOLUTION:
Divide $74.88 by 3.
$74.88 \div 3 = 24.96$
The correct choice is F.

ANSWER:
F

75. **SAT/ACT** If $5^{2x-3} = 1,$ then x =
A 0.4
B 0.6
C 1.5
D 1.6
E 2

SOLUTION:
Write 1 as 5^0.
If the bases of an equation are equal, then the exponents are equal.

$$
\begin{aligned}
5^{2x-3} &= 1 && \text{Original equation} \\
5^{2x-3} &= 5^0 && \text{Replace 1 with } 5^0. \\
2x - 3 &= 0 && \text{Set exponents equal.} \\
2x - 3 + 3 &= 0 + 3 && \text{Add 3 to each side.} \\
2x &= 3 && \text{Simplify.} \\
\frac{2x}{2} &= \frac{3}{2} && \text{Divide each side by 2.} \\
x &= 1.5 && \text{Simplify.}
\end{aligned}
$$

The correct choice is C.

ANSWER:
C

76. **GRIDDED RESPONSE** One endpoint of $\overline{AB}$ has coordinates (–3, 5). If the coordinates of the midpoint of $\overline{AB}$ are (2, –6), what is the approximate length of $\overline{AB}$?

SOLUTION:

Let (x, y) be the location of the other end point. Then by the Midpoint Formula,

$$\left(\frac{-3+x}{2}, \frac{5+y}{2}\right) = (2, \ -6) \quad (x_2, y_2) = (-3, 5)$$

Write two equations to find the values of x and y.

$$\begin{aligned} \frac{-3+x}{2} &= 2 && \text{Midpoint Formula} \\ 2\left(\frac{-3+x}{2}\right) &= 2(2) && \text{Multiply each side by 2.} \\ -3+x &= 4 && \text{Simplify.} \\ -3+3+x &= 4+3 && \text{Add 3 to each side.} \\ x &= 7 && \text{Simplify.} \end{aligned}$$

$$\begin{aligned} \frac{5+y}{2} &= -6 && \text{Midpoint formula.} \\ 2\left(\frac{5+y}{2}\right) &= 2(-6) && \text{Mutliply each side by 2.} \\ 5+y &= -12 && \text{Simplify.} \\ 5-5+y &= -12-5 && \text{Subtract 5 from each side.} \\ y &= -17 && \text{Simplify.} \end{aligned}$$

The coordinates of the other end point are (7, –17)

Use the Distance Formula to find the distance between the endpoints.

$$\begin{aligned} d &= \sqrt{(x_2-x_1)^2+(y_2-y_1)^2} && \text{Distance Formula} \\ &= \sqrt{(7-(-3))^2+(-17-5)^2} && \text{Substitution.} \\ &= \sqrt{10^2+(-22)^2} && \text{Subtraction.} \\ &= \sqrt{100+484} && \text{Square each term.} \\ &= \sqrt{584} && \text{Addition.} \\ &\approx 24.2 && \text{Evaluate square root.} \end{aligned}$$

The length of $\overline{AB}$ is about 24.2.

ANSWER:

24.2

Find the length of each object.

77. Refer to Page 35.

SOLUTION:

$2\frac{1}{8}$ in .

ANSWER:

$2\frac{1}{8}$ in .

78. Refer to Page 35.

SOLUTION:

38 mm or 3.8 cm

ANSWER:

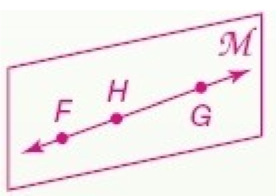
38 mm or 3.8 cm

Draw and label a figure for each relationship.

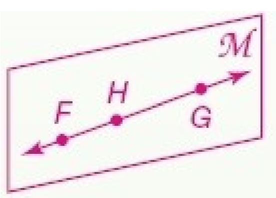
79. $\overleftrightarrow{FG}$ lies in plane *M* and contains point *H*.

SOLUTION:

Draw plane *M*. Place two points in the plane and label them *F* and *G*. Draw $\overleftrightarrow{FG}$. Place another point on $\overleftrightarrow{FG}$ and label it *H*.



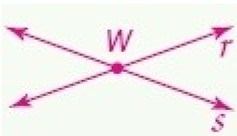
ANSWER:



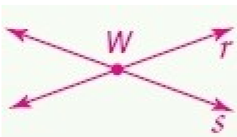
80. Lines r and s intersect at point W.

SOLUTION:

Draw a line and label it r. Draw a second line that crosses the first line and label it s. Label the point of intersection of the two lines W.



ANSWER:



81. **TRUCKS** A sport-utility vehicle has a maximum load limit of 75 pounds for its roof. You want to place a 38-pound cargo carrier and 4 pieces of luggage on top of the roof. Write and solve an inequality to find the average allowable weight for each piece of luggage.

SOLUTION:

Let x be the weight of each piece of luggage. Then the combined weight of the 4 pieces of luggage is $4x$.

The weight of the cargo carrier is 38 pounds.
The total weight is $4x + 38$.
The maximum weight is 75 pounds.
That is, $4x + 38 \le 75$.

Solve for x.

$$\begin{aligned} 4x + 38 &\le 75 && \text{Original inequality} \\ 4x + 38 - 38 &\le 75 - 38 && \text{Subtract 38 from each side.} \\ 4x &\le 37 && \text{Simplify.} \\ \frac{4x}{4} &\le \frac{37}{4} && \text{Divide each side by 4.} \\ x &\le 9.25 && \text{Simplify.} \end{aligned}$$

So, the allowable weight for each piece of luggage is 9.25 lb or less.

ANSWER:

$4x + 38 \le 75$; 9.25 lb or less

Solve each equation.

82. $8x - 15 = 5x$

SOLUTION:

Isolate *x*.

$$\begin{aligned} 8x - 15 &= 5x && \text{Original equation} \\ 8x - 15 + 15 &= 5x + 15 && \text{Add 15 to each side.} \\ 8x &= 5x + 15 && \text{Simplify.} \\ 8x - 5x &= 5x - 5x + 15 && \text{Subtract } 5x \text{ from each side.} \\ 3x &= 15 && \text{Simplify.} \\ \frac{3x}{3} &= \frac{15}{3} && \text{Divide each side by 3.} \\ x &= 5 && \text{Simplify.} \end{aligned}$$

ANSWER:

5

83. $5y - 3 + y = 90$

SOLUTION:

$$\begin{aligned} 5y - 3 + y &= 90 && \text{Original equation} \\ 6y - 3 &= 90 && \text{Simplify.} \\ 6y - 3 + 3 &= 90 + 3 && \text{Add 3 to each side.} \\ 6y &= 93 && \text{Simplify.} \\ \frac{6y}{6} &= \frac{93}{6} && \text{Divide each side by 6.} \\ y &= 15.5 && \text{Simplify.} \end{aligned}$$

ANSWER:

15.5

84. $16a + 21 = 20a - 9$

SOLUTION:

Isolate a.

$$\begin{aligned} 16a + 21 &= 20a - 9 && \text{Original equation} \\ 16a - 20a + 21 &= 20a - 20a - 9 && \text{Add } 20a \text{ to each side.} \\ -4a + 21 &= -9 && \text{Simplify.} \\ -4a + 21 - 21 &= -9 - 21 && \text{Subtract 21 from each side.} \\ -4a &= -30 && \text{Simplify.} \\ \frac{-4a}{-4} &= \frac{-30}{-4} && \text{Divide each side by } -4. \\ a &= 7.5 && \text{Simplify.} \end{aligned}$$

ANSWER:

7.5

85. $9k - 7 = 21 - 3k$

SOLUTION:

Isolate k.

$$\begin{aligned} 9k - 7 &= 21 - 3k && \text{Original equation} \\ 9k + 3k - 7 &= 21 - 3k + 3k && \text{Add} 3k \text{ to each side.} \\ 12k - 7 &= 21 && \text{Simplify.} \\ 12k - 7 + 7 &= 21 + 7 && \text{Add} 7 \text{ from each side.} \\ 12k &= 28 && \text{Simplify.} \\ \frac{12k}{12} &= \frac{28}{12} && \text{Divide each side by} 12. \\ k &= 2\frac{1}{3} && \text{Simplify.} \end{aligned}$$

ANSWER:

$2\frac{1}{3}$

86. $11z - 13 = 3z + 17$

SOLUTION:

Isolate z.

$$\begin{aligned} 11z - 13 &= 3z + 17 && \text{Original equation} \\ 11z - 3z - 13 &= 3z - 3z + 17 && \text{Add } 3z \text{ to each side.} \\ 8z - 13 &= 17 && \text{Simplify.} \\ 8z - 13 + 13 &= 17 + 13 && \text{Subtract } 13 \text{ from each side.} \\ 8z &= 30 && \text{Simplify.} \\ \frac{8z}{8} &= \frac{30}{8} && \text{Divide each side by } 8 \\ z &= 3\frac{3}{4} && \text{Simplify.} \end{aligned}$$

ANSWER:

$3\frac{3}{4}$

87. $15 + 6n = 4n + 23$

SOLUTION:

Isolate n.

$$\begin{aligned} 15 + 6n &= 4n + 23 && \text{Original equation} \\ 15 + 6n - 4n &= 4n - 4n + 23 && \text{Subtract } 4n \text{ from each side.} \\ 15 + 2n &= 23 && \text{Simplify.} \\ 15 - 15 + 2n &= 23 - 15 && \text{Subtract 15 from each side.} \\ 2n &= 8 && \text{Simplify.} \\ \frac{2n}{2} &= \frac{8}{2} && \text{Divide each side by 2.} \\ n &= 4 && \text{Simplify.} \end{aligned}$$

ANSWER:

4