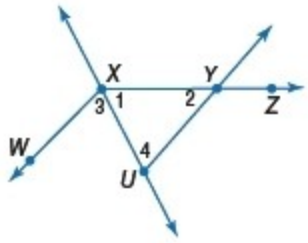
1-4 Angle Measure

Use the figure at the right.



1. Name the vertex of $\angle 4$.

   SOLUTION:
   U

   ANSWER:
   U

2. Name the sides of $\angle 3$.

   SOLUTION:
   $\overrightarrow{XW}$, $\overrightarrow{XU}$

   ANSWER:
   $\overrightarrow{XW}, \overrightarrow{XU}$

3. What is another name for $\angle 2$?

   SOLUTION:
   $\angle XYU$, $\angle UYX$

   ANSWER:
   $\angle XYU$, $\angle UYX$

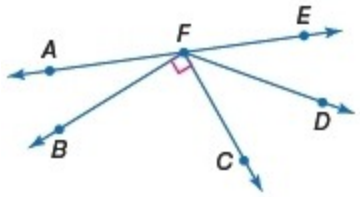
4. What is another name for $\angle UXY$?

   SOLUTION:
   $\angle 1$, $\angle YXU$

   ANSWER:
   $\angle 1$, $\angle YXU$

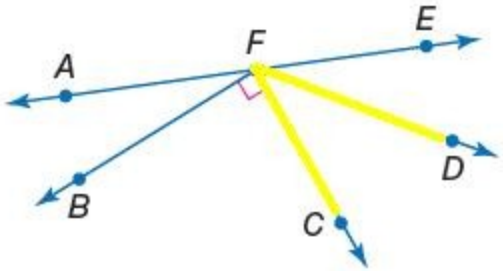
1-4 Angle Measure

Copy the diagram shown, and extend each ray. Classify each angle as *right, acute,* or *obtuse*. Then use a protractor to measure the angle to the nearest degree.



5. $\angle CFD$

SOLUTION:

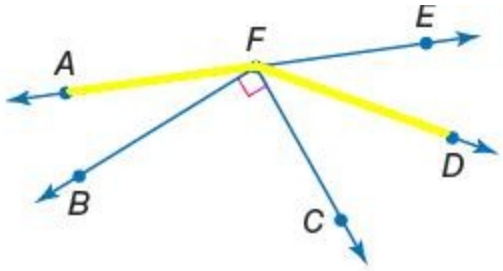


$\angle$CFD is an acute angle. The measure of $\angle$CFD is 40.

ANSWER:
acute; 40

6. $\angle AFD$

SOLUTION:

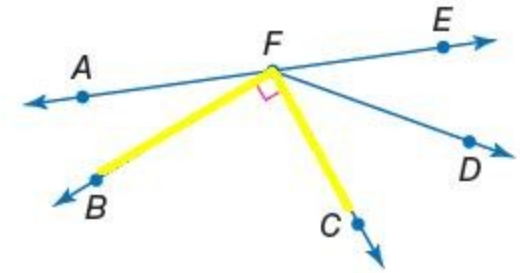


$\angle$ AFD is an obtuse angle. The measure of $\angle$AFD is 150.

ANSWER:
obtuse; 150

7. $\angle BFC$

SOLUTION:

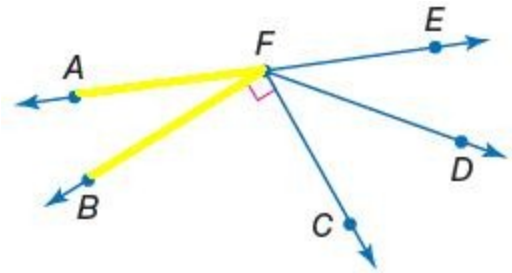


$\angle$BFC is a right angle. The measure of $\angle$BFC is 90.

ANSWER:
right; 90

8. $\angle AFB$

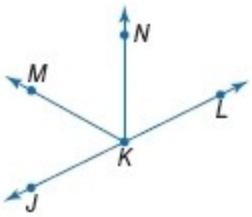
SOLUTION:



$\angle$ AFB is an acute angle. The measure of $\angle$AFB is 25

ANSWER:
acute; 25

ALGEBRA In the figure, $\overrightarrow{KJ}$ and $\overrightarrow{KL}$ are opposite rays, $\overrightarrow{KN}$ bisects $\angle LKM$.



9. If $m\angle LKM = 7x - 5$ and $m\angle NKM = 3x + 9$, find $m\angle LKM$.

SOLUTION:

$m\angle LKM = m\angle LKN + m\angle NKM$. Because of the definition of bisecting rays, $m\angle LKN = m\angle NKM$.

$m\angle LKM = m\angle LKN + m\angle NKM$	Definition of bisecting ray
$m\angle LKM = m\angle NKM + m\angle NKM$	Substitution
$m\angle LKM = 2(m\angle NKM)$	Addition.
$7x - 5 = 2(3x + 9)$	Substitution.
$7x - 5 = 6x + 18$	Distributive Property
$7x - 6x - 5 = 6x - 6x + 18$	Subtract $6x$ from each side.
$x - 5 = 18$	Simplify.
$x - 5 + 5 = 18 + 5$	Add 5 to each side.
$x = 23$	Simplify.

To find , $m\angle LKM$ substitute $x = 23$ into the equation $m\angle LKM = 7x - 5$,.

$$\begin{aligned} 7x - 5 &= 7(23) - 5 \\ &= 161 - 5 \\ &= 156 \end{aligned}$$

ANSWER:

156

10. If $m\angle NKL = 7x - 9$ and $m\angle JKM = x + 3$, find $m\angle JKN$.

SOLUTION:

By the Angle Addition Postulate, $m\angle MKJ + m\angle NKM + m\angle LKN = 180$.

Since $\overrightarrow{KN}$ bisects $\angle LKM$, $m\angle NKM = m\angle NKL$.
Thus, $m\angle MKJ + m\angle LKN + m\angle LKN = 180$.

$$
\begin{aligned}
m\angle MKJ + m\angle LKN + m\angle LKN &= 180 \\
x + 3 + 7x - 9 + 7x - 9 &= 180 && \text{Substitution.} \\
15x - 15 &= 180 && \text{Simplify.} \\
15x - 15 + 15 &= 180 + 15 && \text{Add 15 to each side.} \\
15x &= 195 && \text{Simplify.} \\
\frac{15x}{15} &= \frac{195}{15} && \text{Divide each side by 15.} \\
x &= 13 && \text{Simplify.}
\end{aligned}
$$

Find an expression for $m\angle JKN$.

$$
\begin{aligned}
m\angle JKN &= m\angle JKM + m\angle NKM \\
&= x + 3 + 7x - 9 \\
&= 8x - 6
\end{aligned}
$$

To find $8x - 6$, substitute the value 13 for x.

$$
\begin{aligned}
m\angle JKN &= 8(13) - 6 \\
&= 104 - 6 \\
&= 98
\end{aligned}
$$

ANSWER:

98

11. **CCSS PRECISION** A miter cut is used to build picture frames with corners that meet at right angles.
a. José miters the ends of some wood for a picture frame at congruent angles. What is the degree measure of his cut? Explain and classify the angle.
b. What does the joint represent in relation to the angle formed by the two pieces?

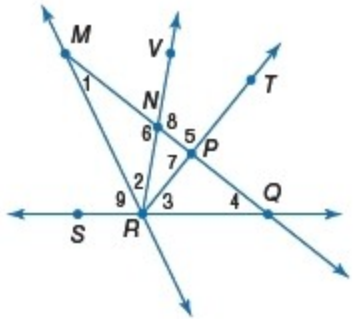
SOLUTION:

a. 45; When joined together, the angles form a right angle, which measures 90. If the two angles that form this right angle are congruent, then the measure of each angle is 90 ÷ 2 or 45. The angle of the cut is an acute angle.
b. The joint is the angle bisector of the frame angle.

ANSWER:

a. 45; When joined together, the angles form a right angle, which measures 90. If the two angles that form this right angle are congruent, then the measure of each angle is 90 ÷ 2 or 45. The angle of the cut is an acute angle.
b. The joint is the angle bisector of the frame angle.

For Exercises 12–29, use the figure below.



Name the vertex of each angle.

12. $\angle 4$

SOLUTION:
Q

ANSWER:
Q

13. $\angle 7$

SOLUTION:
P

ANSWER:
P

14. $\angle 2$

SOLUTION:
R

ANSWER:
R

15. $\angle 1$

SOLUTION:
M

ANSWER:
M

Name the sides of each angle.

16. $\angle TPQ$

SOLUTION:
$\overrightarrow{PT}, \overrightarrow{PQ}$

ANSWER:
$\overrightarrow{PT}, \overrightarrow{PQ}$

17. $\angle VNM$

SOLUTION:
$\overrightarrow{NV}, \overline{NM}$

ANSWER:
$\overrightarrow{NV}, \overline{NM}$

18. $\angle 6$

SOLUTION:
$\overline{NM}, \overline{NR}$

ANSWER:
$\overline{NM}, \overline{NR}$

19. $\angle 3$

SOLUTION:
$\overrightarrow{RP}, \overrightarrow{RQ}$

ANSWER:
$\overrightarrow{RP}, \overrightarrow{RQ}$

Write another name for each angle.

20. $\angle 9$

SOLUTION:

$\angle MRS$, $\angle SRM$

ANSWER:

$\angle MRS$, $\angle SRM$

21. $\angle QPT$

SOLUTION:

$\angle TPQ$

ANSWER:

$\angle TPQ$

22. $\angle MQS$

SOLUTION:

$\angle 4$, $\angle SQM$, $\angle MQR$, $\angle RQM$, $\angle NQS$, $\angle SQN$, $\angle NQR$, $\angle RQN$, $\angle PQR$, $\angle RQP$, $\angle PQS$, $\angle SQP$

ANSWER:

$\angle 4$, $\angle SQM$, $\angle MQR$, $\angle RQM$, $\angle NQS$, $\angle SQN$, $\angle NQR$, $\angle RQN$, $\angle PQR$, $\angle RQP$, $\angle PQS$, $\angle SQP$

23. $\angle 5$

SOLUTION:

$\angle TPN$, $\angle NPT$, $\angle TPM$, $\angle MPT$

ANSWER:

$\angle TPN$, $\angle NPT$, $\angle TPM$, $\angle MPT$

24. Name an angle with vertex N that appears obtuse.

SOLUTION:

Sample answer: $\angle VNQ$

ANSWER:

Sample answer: $\angle VNQ$

25. Name an angle with vertex Q that appears acute.

SOLUTION:

$\angle 4$

ANSWER:

$\angle 4$

26. Name a point in the interior of $\angle VRQ$.

SOLUTION:
P, *T*

ANSWER:
P, *T*

27. Name a point in the exterior of $\angle MRT$.

SOLUTION:
S, *Q*

ANSWER:
S, *Q*

28. Name a pair of angles that share exactly one point.

SOLUTION:
Sample answer: $\angle 6$ and $\angle 8$ share the vertex point *N*.

ANSWER:
Sample answer: $\angle 6$, $\angle 8$

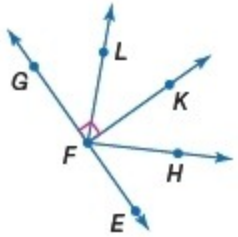
29. Name a pair of angles that share more than one point.

SOLUTION:
Sample answer: $\angle MPR$ and $\angle PRQ$ share points *P* and *R*.

ANSWER:
Sample answer: $\angle MPR$, $\angle PRQ$

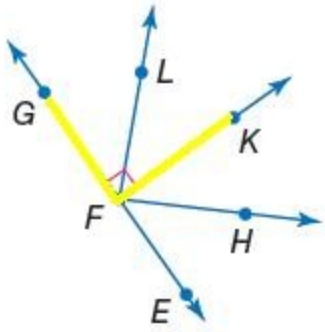
1-4 Angle Measure

Copy the diagram shown, and extend each ray. Classify each angle as *right, acute,* or *obtuse*. Then use a protractor to measure the angle to the nearest degree.



30. $\angle GFK$

SOLUTION:

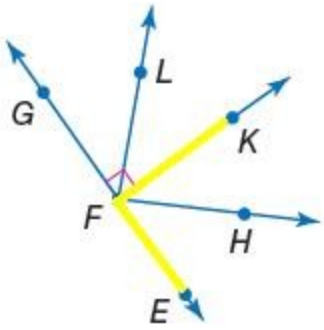


$\angle GFK$ is a right angle. The measure of $\angle GFK$ is 90.

ANSWER:
90, right

31. $\angle EFK$

SOLUTION:

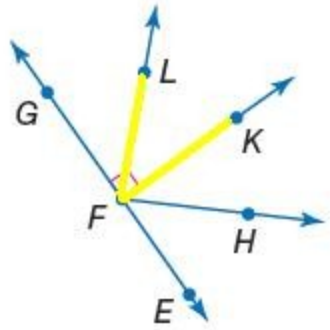


$\angle EFK$ is a right angle. The measure of $\angle EFK$ is 90.

ANSWER:
90, right

32. $\angle LFK$

SOLUTION:

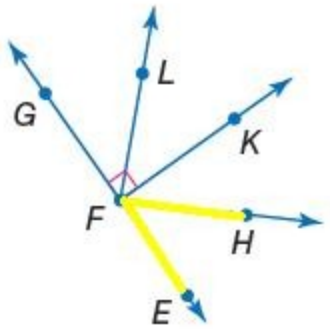


$\angle LFK$ is an acute angle. The measure of $\angle LFK$ is 45.

ANSWER:
45, acute

33. $\angle EFH$

SOLUTION:

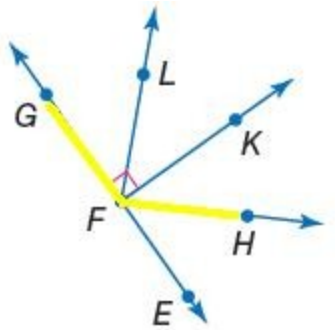


$\angle EFH$ is an acute angle. The measure of $\angle EFH$ is 45.

ANSWER:
45, acute

34. $\angle GFH$

SOLUTION:



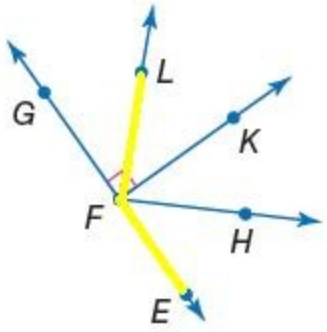
$\angle GFH$ is an obtuse angle. The measure of $\angle GFH$ is 135.

ANSWER:

135, obtuse

35. $\angle EFL$

SOLUTION:



$\angle EFL$ is an obtuse. The measure of $\angle EFL$ is 135.

ANSWER:

135, obtuse

36. **CLOCKS** Determine at least three different times during the day when the hands on a clock form each of the following angles. Explain.
    a. right angle
    b. obtuse angle
    c. congruent acute angles
    Refer to Page 42.

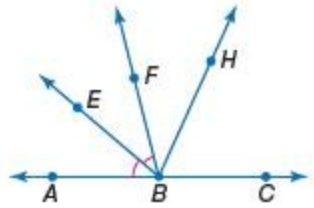
    SOLUTION:

    a. Sample answer: 3:00, 6:16, 9:00; The angle measures at these times are 90°, which is the measure of a right angle.
    b. Sample answer: 4:00, 5:00, 7:00, 8:00; The angle measures at these times are greater than 90, or obtuse angles.
    c. Sample answer: 1:00, 2:06, 11:00; The angle measures at these times are all equal to each other or congruent and less than 90, or acute angles.

    ANSWER:

    a. Sample answer: 3:00, 6:16, 9:00; The angle measures at these times are 90°, which is the measure of a right angle.
    b. Sample answer: 4:00, 5:00, 7:00, 8:00; The angle measures at these times are greater than 90, or obtuse angles.
    c. Sample answer: 1:00, 2:06, 11:00; The angle measures at these times are all equal to each other or congruent and less than 90, or acute angles.

ALGEBRA In the figure, $\overrightarrow{BA}$ and $\overrightarrow{BC}$ are opposite rays. $\overrightarrow{BH}$ bisects $\angle EBC$.



37. If $m\angle ABE = 2n + 7$ and $m\angle EBF = 4n - 13$, find $m\angle ABE$.

SOLUTION:

Since $\angle ABE \cong \angle EBF$, $m\angle ABE = m\angle EBF$.
So, $2n + 7 = 4n - 13$.

$$
\begin{aligned}
m\angle ABE &= m\angle EBF && \\
2n + 7 &= 4n - 13 && \text{Substitution.} \\
2n - 4n + 7 &= 4n - 4n - 13 && \text{Subtract } 4n \text{ from each side.} \\
-2n + 7 &= -13 && \text{Simplify.} \\
-2n + 7 - 7 &= -13 - 7 && \text{Subtract 7 from each side.} \\
-2n &= -20 && \text{Simplify.} \\
\frac{-2n}{-2} &= \frac{-20}{-2} && \text{Divide each side by } -2. \\
n &= 10 && \text{Simplify.}
\end{aligned}
$$

Substitute.

$$
\begin{aligned}
m\angle ABE &= 2n + 7 \\
&= 2(10) + 7 \\
&= 27
\end{aligned}
$$

ANSWER:

27

38. If $m\angle EBH = 6x + 12$ and $m\angle HBC = 8x - 10$, find $m\angle EBH$.

SOLUTION:

Since $\overline{BH}$ bisects $\angle EBC$, $m\angle EBH = m\angle HBC$.
Then,

$$\begin{aligned}
m\angle EBH &= m\angle HBC && \\
6x + 12 &= 8x - 10 && \text{Substitution.} \\
6x - 8x + 12 &= 8x - 8x - 10 && \text{Subtract } 8x \text{ from each side.} \\
-2x + 12 &= -10 && \text{Simplify.} \\
-2x + 12 - 12 &= -10 - 12 && \text{Subtract 12 from each side.} \\
-2x &= -22 && \text{Simplify.} \\
\frac{-2x}{-2} &= \frac{-22}{-2} && \text{Divide each side by } -2. \\
x &= 11 && \text{Simplify.}
\end{aligned}$$

Substitute.

$$\begin{aligned}
m\angle EBH &= 6x + 12 \\
&= 6(11) + 12 \\
&= 78
\end{aligned}$$

ANSWER:

78

39. If $m\angle ABF = 7b - 24$ and $m\angle ABE = 2b$, find $m\angle EBF$.

SOLUTION:

Since $m\angle ABE = m\angle EBF, \overrightarrow{EB}$ is an angle bisector. So, $m\angle ABE = \frac{1}{2}m\angle ABF$.

Substitute.

$$m\angle ABE = \frac{1}{2}m\angle ABF$$

$$2b = \frac{1}{2}(7b - 24) \quad \text{Substitution.}$$

$$2(2b) = 2\left(\frac{1}{2}(7b - 24)\right) \quad \times \text{ each side by 2.}$$

$$4b = 7b - 24 \quad \text{Simplify.}$$

$$4b - 7b = 7b - 7b - 24 \quad -7b \text{ from each side.}$$

$$-3b = -24 \quad \text{Simplify.}$$

$$\frac{-3b}{-3} = \frac{-24}{-3} \quad \div \text{ each side by } -3.$$

$$b = 8 \quad \text{Simplify.}$$

Substitute.

$$m\angle EBF = m\angle ABE$$
$$= 2b$$
$$= 2(8)$$
$$= 16$$

ANSWER:

16

40. If $m\angle EBC = 31a - 2$ and $m\angle EBH = 4a + 45$, find $m\angle HBC$.

SOLUTION:

Since $\overline{BH}$ is an angle bisector $m\angle HBC = m\angle EBH = \frac{1}{2}m\angle EBC$.

Substitute.

$$
\begin{aligned}
m\angle EBH &= \frac{1}{2}m\angle EBC && \\
4a + 45 &= \frac{1}{2}(31a - 2) && \text{Substitution.} \\
2(4a + 45) &= 2\left(\frac{1}{2}(31a - 2)\right) && \times \text{ each side by 2.} \\
8a + 90 &= 31a - 2 && \text{Simplify.} \\
8a - 31a + 90 &= 31a - 31a - 2 && -31b \text{ from each side.} \\
-23a + 90 &= -2 && \text{Simplify.} \\
-23a + 90 - 90 &= -2 - 90 && \text{Simplify.} \\
\frac{-23a}{-23} &= \frac{-92}{-23} && \div \text{ each side by } -23. \\
a &= 4 && \text{Simplify.}
\end{aligned}
$$

Substitute.

$$
\begin{aligned}
m\angle EBH &= 4a + 45 \\
&= 4(4) + 45 \\
&= 61
\end{aligned}
$$

Since $m\angle HBC = m\angle EBH$, $m\angle HBC = 61$.

ANSWER:

61

41. If $m\angle ABF = 8s - 6$ and $m\angle ABE = 2(s + 11)$, find $m\angle EBF$.

SOLUTION:

Since $\angle ABE \cong \angle EBF, m\angle ABE = m\angle EBF$.

So,

$$\begin{aligned} m\angle ABF &= m\angle ABE + m\angle EBF && \text{Definition of bisecting} \\ &= m\angle ABE + m\angle ABE && \text{Substitution.} \\ &= 2(m\angle ABE). && \text{Simplify.} \end{aligned}$$

Substitute.

$$\begin{aligned} m\angle ABF &= 2m\angle ABE \\ 8s - 6 &= 2(2(s + 11)) && \text{Substitution.} \\ 8s - 6 &= 4s + 44 && \text{Simplify.} \\ 8s - 4s - 6 &= 4s - 4s + 44 && \text{Subtract 4s from each side.} \\ 4s - 6 &= 44 && \text{Simplify.} \\ 4s - 6 + 6 &= 44 + 6 && \text{Add 6 to each side.} \\ 4s &= 50 && \text{Simplify.} \\ \frac{4s}{4} &= \frac{50}{4} && \text{Divide each side by 4.} \\ s &= 12.5 && \text{Simplify.} \end{aligned}$$

Substitute.

$$\begin{aligned} m\angle EBF &= m\angle ABE \\ &= 2(s + 11) \\ &= 2(12.5 + 11) \\ &= 47 \end{aligned}$$

ANSWER:

47

42. If $m\angle EBC = 3r + 10$ and $m\angle ABE = 2r - 20$, find $m\angle EBF$.

SOLUTION:

The angles $\angle ABE$ and $\angle EBC$ form a linear pair.
So, $m\angle ABE + m\angle EBC = 180$.

$$
\begin{aligned}
m\angle ABE + m\angle AEBC &= 180 && \text{Definition of Linear Pair} \\
2r - 20 + 3r + 10 &= 180 && \text{Substitution.} \\
5r - 10 &= 180 && \text{Simplify.} \\
5r - 10 + 10 &= 180 + 10 && \text{Add 10 to each side.} \\
5r &= 190 && \text{Simplify.} \\
\frac{5r}{5} &= \frac{190}{5} && \text{Divide each side by 5.} \\
r &= 38 && \text{Simplify.}
\end{aligned}
$$

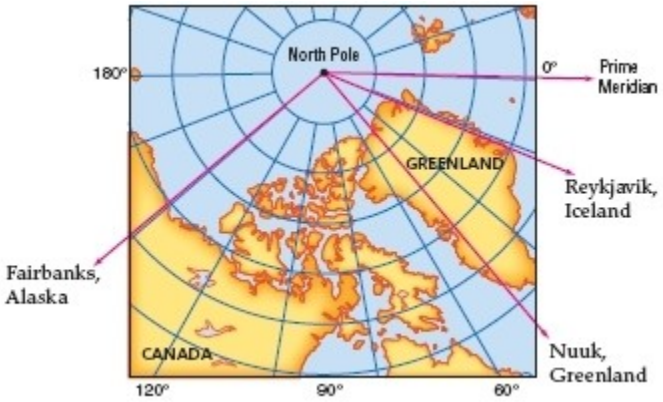
Substitute.
Since $m\angle EBF = m\angle ABE$,

$$
\begin{aligned}
m\angle EBF &= 2r - 20 \\
&= 2(38) - 20 \\
&= 76 - 20 \\
&= 56.
\end{aligned}
$$

ANSWER:

56

43. **MAPS** Estimate the measure of the angle formed by each city or location listed, the North Pole, and the Prime Meridian.



a. Nuuk, Greenland
b. Fairbanks, Alaska
c. Reykjavik, Iceland
d. Prime Meridian

SOLUTION:

a. The measure of the angle formed by the Prime meridian, North Pole, and Nuuk, Greenland is about 50.
b. The measure of the angle formed by the Prime meridian, North Pole, and Fairbanks, Alaska is about 140
c. The measure of the angle formed by the Prime meridian, North Pole, and Reykjavik, Iceland is about 20.
d.The measure of the angle formed by the Prime meridian, North Pole, and Prime Meridian us 0.

ANSWER:

a. about 50
b. about 140
c. about 20
d. 0

44. **CCSS TOOLS** A compass rose is a design on a map that shows directions. In addition to the directions of north, south, east, and west, a compass rose can have as many as 32 markings.
Refer to Page 42.
a. With the center of the compass as its vertex, what is the measure of the angle between due west and due north?
b. What is the measure of the angle between due north and north-west?
c. How does the north-west ray relate to the angle in part **a**?

SOLUTION:

a. The arrows for due west and due north are perpendicular to each other since they are part of a horizontal and vertical line. Perpendicular rays will form an angle with a measure of 90.

b. The ray for north-west will be halfway between the arrows for due north and due west. So, the measure of the angle between north-west and due north is half of 90 or 45.

c. The ray for north-west divides the angle between due north and due west into two congruent angles with measures of 45. Therefore, it is the angle bisector.

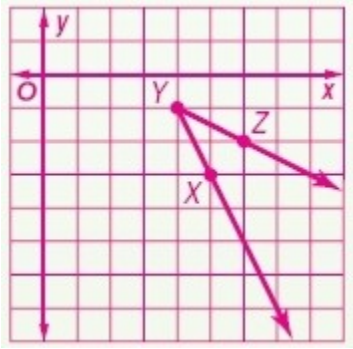
ANSWER:

a. 90
b. 45
c. It is the angle bisector.

Plot the points in a coordinate plane and sketch $\angle XYZ$. Then classify it as *right, acute,* or *obtuse*.

45. $X(5, -3)$, $Y(4, -1)$, $Z(6, -2)$

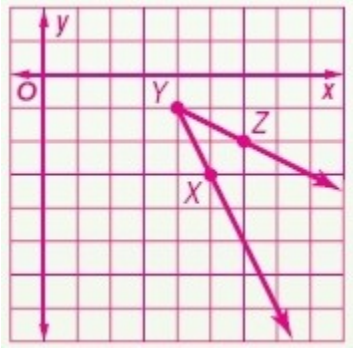
SOLUTION:



The angle measures less than 90°, so it is acute.

ANSWER:

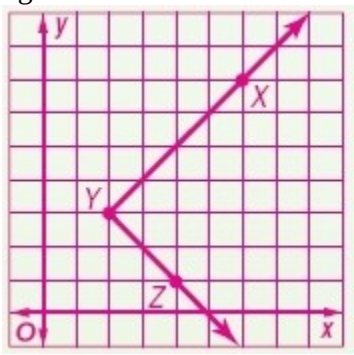
acute



46. $X(6, 7)$, $Y(2, 3)$, $Z(4, 1)$

SOLUTION:

right



The angle formed appears to be a right angle. In Algebra 1, you learned that the product of the slopes of perpendicular lines is –1.

$$m = \frac{y_2 - y_1}{x_2 - x_1} \quad \text{Slope formula}$$

$$m_{\overleftrightarrow{XY}} = \frac{3-7}{2-6} \qquad (x_1, y_1) = (6, 7);\ (x_2, y_2) = (2, 3)$$

$$m_{\overleftrightarrow{XY}} = \frac{-4}{-4} \text{ or } 1 \quad \text{Simplify.}$$

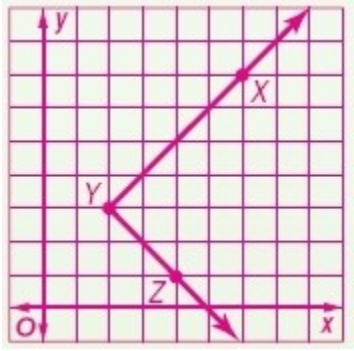
$$m_{\overleftrightarrow{YZ}} = \frac{1-3}{4-2} \qquad (x_1, y_1) = (2, 3);\ (x_2, y_2) = (4, 1)$$

$$m_{\overleftrightarrow{YZ}} = \frac{-2}{2} \text{ or } -1 \quad \text{Simplify.}$$

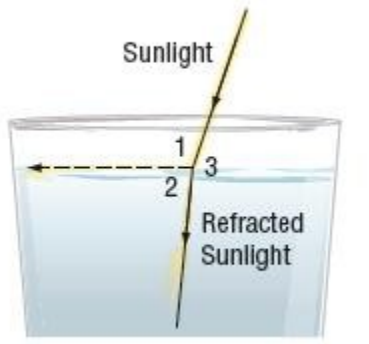
Since 1(–1) = –1, the lines are perpendicular. Thus, the angle formed is a right angle and has a measure of 90.

ANSWER:

right



47. **PHYSICS** When you look at a pencil in water, it looks bent. This illusion is due to *refraction*, or the bending of light when it moves from one substance to the next.
Refer to Page 43.



a. What is $m\angle 1$? Classify this angle as *acute*, *right*, or *obtuse*.
b. What is $m\angle 2$? Classify this angle as *acute*, *right*, or *obtuse*.
c. Without measuring, determine how many degrees the path of the light changes after it enters the water. Explain your reasoning.

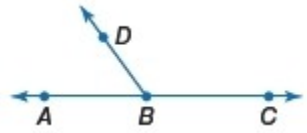
SOLUTION:

a. Use a protractor to measure $\angle 1$ in the diagram. The measure should be about 110; Since the measure is greater than 90 but less than 180, the angle is obtuse.
b. Use a protractor to measure $\angle 2$ in the diagram. The measure should be about 85; Since the measure is less than 90, the angle is acute.
c. about 15; If the original path of the light is extended, the measure of the angle the original path makes with the refracted path represents the number of degrees the path of the light changed. The sum of the measure of this angle and the $m\angle 3$ is 180. The measure of $\angle 3$ is 360 – (110 + 85) or 165, so the measure of the angle the original path makes with the refracted path is 180 – 165 or 15.

ANSWER:

a. about 110; obtuse
b. about 85; acute
c. about 15; If the original path of the light is extended, the measure of the angle the original path makes with the refracted path represents the number of degrees the path of the light changed. The sum of the measure of this angle and the $m\angle 3$ is 180. The measure of $\angle 3$ is 360 – (110 + 85) or 165, so the measure of the angle the original path makes with the refracted path is 180 – 165 or 15.

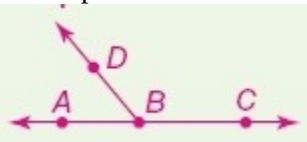
48. **MULTIPLE REPRESENTATIONS** In this problem, you will explore the relationship of angles that compose opposite rays.



a. GEOMETRIC Draw four lines, each with points *A*, *B*, and *C*. Draw $\overrightarrow{BD}$ for each line, varying the placement of point *D*. Use a protractor to measure $\angle ABD$ and $\angle DBC$ for each figure.
b. TABULAR Organize the measures for each figure into a table. Include a row in your table to record the sum of these measures.
c. VERBAL Make a conjecture about the sum of the measures of the two angles. Explain your reasoning.
d. ALGEBRAIC If *x* is the measure of $\angle ABD$ and *y* is the measure of $\angle DBC$, write an equation that relates the two angle measures.

SOLUTION:

a. Sample answer:



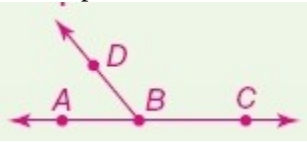
b. Sample answer:

Angle	Measure
$\angle ABD$	50
$\angle DBC$	130
Sum	180

c. Sample answer: The sum of the measures of the two angles will always be 180. Point *B* determines opposite rays $\overrightarrow{AB}$ and $\overrightarrow{BC}$, which form a line. Angles that form a straight line measure 180.
d. $x + y = 180$

ANSWER:

a. Sample answer:



b. Sample answer:

Angle	Measure
$\angle ABD$	50
$\angle DBC$	130
Sum	180

c. Sample answer: The sum of the measures of the two angles will always be 180. Point *B* determines opposite rays $\overrightarrow{AB}$ and $\overrightarrow{BC}$, which form a line. Angles that form a straight line measure 180.
d. $x + y = 180$

49. **OPEN ENDED** Draw an obtuse angle named ABC. Measure $\angle ABC$. Construct an angle bisector $\overrightarrow{BD}$ of $\angle ABC$. Explain the steps in your construction and justify each step. Classify the two angles formed by the angle bisector.

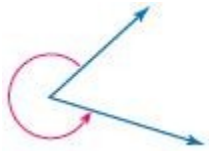
SOLUTION:

Sample answer:
Step 1: Draw an obtuse angle. Label the vertex as B. Measure the angle with a protractor to be sure the measure of the angle is greater than 90. Put the compass on point B and using any compass setting draw an arc that intersects both sides of the angle. Label the two points of intersection of the arc with the sides as A and C.(This will determine points on each side that are equal distance from the vertex.)
Step 2: Choose a setting for the compass that is greater than half the length AC. With the compass at point A, draw an arc in the interior of the angle. With the same compass setting, draw an arc from point C that intersects the arc from point A. (The point of intersection is equidistant from the sides of the angle.) From the vertex, draw $\overrightarrow{BD}$. The points on $\overrightarrow{BD}$ are all equidistant from the $\overrightarrow{BA}$ and $\overrightarrow{BC}$. The measurement of $\angle ABD$ and $\angle DBC$ are the same. Therefore, $\overrightarrow{BD}$ bisects $\angle ABC$. (Verify by using a protractor to measure $\angle ABD$ and $\angle CBD$.)
Half a number between 90 and 180 is less than 90, so the two angles formed by the angle bisector are acute.

ANSWER:

The two angles formed are acute angles.
Sample answer: With my compass at point B, I drew an arc that intersects both sides of the angle. I labeled the two points of intersection of the arc with the sides as A and C. With my compass at point A, I drew an arc in the interior of the angle. With the same compass setting, I drew an arc from point C that intersected the arc from point A. From the vertex, I drew $\overrightarrow{BD}$. I used the same compass setting to draw the intersecting arcs, so $\overrightarrow{BD}$ divides $\angle ABC$ so that the measurement of $\angle ABD$ and $\angle DBC$ are equal. Therefore, $\overrightarrow{BD}$ bisects $\angle ABC$.

50. **CHALLENGE** Describe how you would use a protractor to measure the angle shown.



SOLUTION:

Sample answer: Measure the acute angle and subtract this measure from 360, a full circle.

ANSWER:

Sample answer: Measure the acute angle and subtract this measure from 360, a full circle.

51. **CCSS ARGUMENTS** The sum of two acute angles is *sometimes*, *always*, or *never* an obtuse angle. Explain.

SOLUTION:

Sometimes; sample answer: For example, if you add an angle measure of 4 and an angle measure of 6, you will have an angle measure of 10, which is still acute. But if you add angles with measure of 50 and 60, you will have an obtuse angle with a measure of 110.

ANSWER:

Sometimes; sample answer: For example, if you add an angle measure of 4 and an angle measure of 6, you will have an angle measure of 10, which is still acute. But if you add angles with measure of 50 and 60, you will have an obtuse angle with a measure of 110.

52. **CHALLENGE** $\overrightarrow{MP}$ bisects $\angle LMN$, $\overrightarrow{MQ}$ bisects $\angle LMP$, and $\overrightarrow{MR}$ bisects $\angle QMP$. If $m\angle RMP = 21$, find $m\angle LMN$. Explain your reasoning.

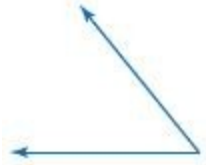
SOLUTION:

If $m\angle RMP = 21$ and $\overrightarrow{MR}$ bisects $\angle QMP$, then $m\angle QMP = 2(21)$ or 42.
If $m\angle QMP = 42$ and $\overrightarrow{MQ}$ bisects $\angle LMP$, then $m\angle LMP = 2(42)$ or 84.
If $m\angle LMP = 84$ and $\overrightarrow{MP}$ bisects $\angle LMN$, then $m\angle LMN = 2(84)$ or 168.

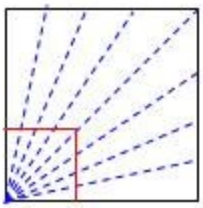
ANSWER:

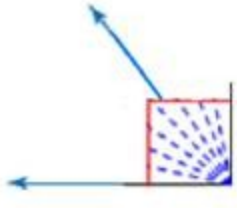
168; If $m\angle RMP = 21$ and $\overrightarrow{MR}$ bisects $\angle QMP$, then $m\angle QMP = 2(21)$ or 42. If $m\angle QMP = 42$ and $\overrightarrow{MQ}$ bisects $\angle LMP$, then $m\angle LMP = 2(42)$ or 84. If $m\angle LMP = 84$ and $\overrightarrow{MP}$ bisects $\angle LMN$, then $m\angle LMN = 2(84)$ or 168.

53. **WRITING IN MATH** Rashid says that he can estimate the measure of an acute angle using a piece of paper to within six degrees of accuracy. Explain how this would be possible. Then use this method to estimate the measure of the angle shown.



SOLUTION:





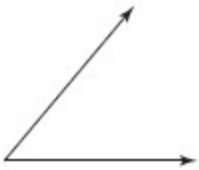
Sample answer: To measure an acute angle, you can fold the corner of the paper so that the edges meet. This would bisect the angle, allowing you to determine whether the angle was between 0° and 45° or between 45° and 90°. If the paper is folded two more times in the same manner and cut off this corner of the paper, the fold lines would form the increments of a homemade protractor that start at 0° on one side and progress in 90 ÷ 8 or 11.25° increments, ending at the adjacent side, which would indicate 90°. You can estimate halfway between each fold line, which would give you an accuracy of 11.25° ÷ 2 or about 6°. The actual measure of the angle shown is 52°. An estimate between 46° and 58° would be acceptable.

ANSWER:

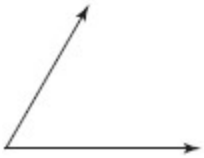
Sample answer: To measure an acute angle, you can fold the corner of the paper so that the edges meet. This would bisect the angle, allowing you to determine whether the angle was between 0° and 45° or between 45° and 90°. If the paper is folded two more times in the same manner and cut off this corner of the paper, the fold lines would form the increments of a homemade protractor that start at 0° on one side and progress in 90 ÷ 8 or 11.25° increments, ending at the adjacent side, which would indicate 90°. You can estimate halfway between each fold line, which would give you an accuracy of 11.25° ÷ 2 or about 6°. The actual measure of the angle shown is 52°. An estimate between 46° and 58° would be acceptable.

54. Which of the following angles measures closest to 60°?

A

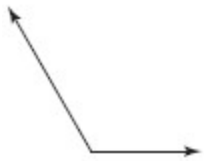


B



C

D



SOLUTION:

$\angle D$ is obtuse, so the measure of $\angle D$ is greater than 90. $\angle C$ is almost a right angle, so the measure of $\angle C$ is close to 90. $\angle A$ appears to be the angle bisector of a right angle, so the measure of $\angle A$ would be close to 45. $\angle B$ between $\angle A$ and $\angle C$ in size, so the measure of $\angle B$ is between 45 and 90. Therefore, the correct choice is B.

ANSWER:

B

55. **SHORT RESPONSE** Leticia surveyed 50 English majors at a university to see if the school should play jazz music in the cafeteria during lunch. The school has 75 different majors and a total of 2000 students. Explain why the results of Leticia's survey are or are not representative of the entire student body.

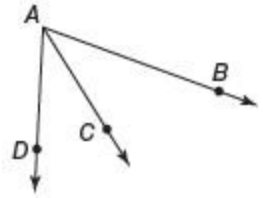
SOLUTION:

Sample answer: Leticia's survey does not represent the entire student body because she did not take a random sample; she only took a sample of students from one major.

ANSWER:

Sample answer: Leticia's survey does not represent the entire student body because she did not take a random sample; she only took a sample of students from one major.

56. In the figure below, if $m\angle BAC = 38$, what must be the measure of $\angle BAD$ in order for $\overrightarrow{AC}$ to be an angle bisector?



F 142
G 76
H 52
J 38

SOLUTION:

If AC is a angle bisector then. $\angle ACD + \angle CAB \cong \angle DAB$.

Then $\angle DAB$ must be twice as much as much as $\angle CAB$ or $\angle DAB \cong 2\angle CAB$

$$m\angle DAB = 2m\angle CAB$$
$$= 2(38) \quad \text{Subsitution.}$$
$$= 76 \quad \text{Simplify.}$$

So, $\angle BAD$ should measure 76°. The correct choice is G.

ANSWER:

G

57. **SAT/ACT** If n is divisible by 2, 5, and 14, which of the following is also divisible by these numbers?

A $n + 7$
B $n + 10$
C $n + 14$
D $n + 40$
E $n + 70$

SOLUTION:

7 is not divisible by either of 2, 5, or 14.
10 is divisible by 2 and 5, but not by 14.
14 is divisible by 2 and 14 not by 5.
70 is divisible by 2, 5 and 14.
So, $n + 70$ is divisible by 2, 5, and 14.
The correct choice is E.

ANSWER:

E

Find the distance between each pair of points. Round to the nearest hundredth.

58. $A(-1, -8)$, $B(3, 4)$

SOLUTION:

Use the Distance Formula.

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \quad \text{Distance Formula}$$
$$= \sqrt{(3 - (-1))^2 + (4 - (-8))^2} \quad \text{Substitution.}$$
$$= \sqrt{4^2 + 12^2} \quad \text{Subtraction.}$$
$$= \sqrt{16 + 144} \quad \text{Square terms.}$$
$$= \sqrt{160} \quad \text{Addition.}$$
$$\approx 12.65 \quad \text{Evaluate square root.}$$

ANSWER:

12.65

59. $C(0, 1)$, $D(-2, 9)$

SOLUTION:

Use the Distance Formula.

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \quad \text{Distance Formula}$$
$$= \sqrt{(-2 - 0)^2 + (9 - 1)^2} \quad \text{Substitution.}$$
$$= \sqrt{(-2)^2 + 8^2} \quad \text{Subtraction.}$$
$$= \sqrt{4 + 64} \quad \text{Square terms.}$$
$$= \sqrt{68} \quad \text{Addition.}$$
$$\approx 8.25 \quad \text{Evaluate square root.}$$

ANSWER:

8.25

60. $E(-3, -12)$, $F(5, 4)$

SOLUTION:

Use the Distance Formula.

$$\begin{aligned} d &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} && \text{Distance Formula} \\ &= \sqrt{(5 - (-3))^2 + (4 - (-12))^2} && \text{Substitution.} \\ &= \sqrt{8^2 + 16^2} && \text{Subtraction.} \\ &= \sqrt{64 + 256} && \text{Square terms.} \\ &= \sqrt{320} && \text{Addition.} \\ &\approx 17.89 && \text{Evaluate square root.} \end{aligned}$$

ANSWER:

17.89

61. $G(4, -10)$, $H(9, -25)$

SOLUTION:

Use the Distance Formula.

$$\begin{aligned} d &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} && \text{Distance Formula} \\ &= \sqrt{(9 - 4)^2 + (-25 - (-10))^2} && \text{Substitution.} \\ &= \sqrt{5^2 + (-15)^2} && \text{Subtraction.} \\ &= \sqrt{25 + 225} && \text{Square terms.} \\ &= \sqrt{250} && \text{Addition.} \\ &\approx 15.81 && \text{Evaluate square root.} \end{aligned}$$

ANSWER:

15.81

62. $J\left(1,\frac{1}{4}\right), K\left(-3,\frac{7}{4}\right)$

SOLUTION:

Use the Distance Formula.

$$d=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2} \quad \text{Distance Formula}$$

$$=\sqrt{(-3-1)^2+\left(\frac{7}{4}-\frac{1}{4}\right)^2} \quad \text{Substitution.}$$

$$=\sqrt{(-4)^2+\left(\frac{6}{4}\right)^2} \quad \text{Subtraction.}$$

$$=\sqrt{16+\frac{9}{4}} \quad \text{Square terms.}$$

$$=\sqrt{18.25} \quad \text{Addition.}$$

$$\approx 4.27 \quad \text{Evaluate square root.}$$

ANSWER:

4.27

63. $L\left(-5,\frac{8}{5}\right), M\left(5,\frac{2}{5}\right)$

SOLUTION:

Use the Distance Formula.

$$d=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2} \quad \text{Distance Formula}$$

$$=\sqrt{(5-(-5))^2+\left(\frac{2}{5}-\frac{8}{5}\right)^2} \quad \text{Substitution.}$$

$$=\sqrt{10^2+\left(-\frac{6}{5}\right)^2} \quad \text{Subtraction.}$$

$$=\sqrt{100+\frac{36}{25}} \quad \text{Square terms.}$$

$$=\sqrt{101.44} \quad \text{Addition.}$$

$$\approx 10.07 \quad \text{Evaluate square root.}$$

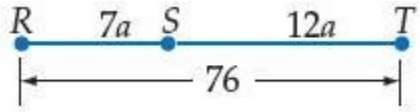
ANSWER:

10.07

Find the value of the variable and *ST* if *S* is between *R* and *T*.

64. $RS = 7a$, $ST = 12a$, $RT = 76$

SOLUTION:



$$RS + ST = RT \quad \text{Segment Addition Postulate}$$
$$7a + 12a = 76 \quad \text{Substitution.}$$
$$19a = 76 \quad \text{Simplify.}$$
$$\frac{19a}{19} = \frac{76}{19} \quad \text{Divide each side by 19.}$$
$$a = 4 \quad \text{Simplify.}$$

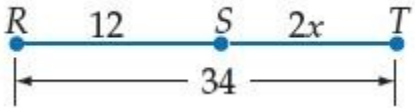
Substitute the value of a to find the value of ST.
$ST = 12a = 12(4) = 48$

ANSWER:

$a = 4$; $ST = 48$

65. $RS = 12$, $ST = 2x$, $RT = 34$

SOLUTION:



$$RS + ST = RT \quad \text{Segment Addition Postulate}$$
$$12 + 2x = 34 \quad \text{Substitution.}$$
$$12 - 12 + 2x = 34 - 12 \quad \text{Subtract 12 from each side.}$$
$$2x = 22 \quad \text{Simplify.}$$
$$\frac{2x}{2} = \frac{22}{2} \quad \text{Divide each side by 2.}$$
$$a = 11 \quad \text{Simplify.}$$

Substitute the value of x to find the value of ST.
$ST = 2a = 2(11) = 22$

ANSWER:

$x = 11$; $ST = 22$

66. **PHOTOGRAPHY** Photographers often place their cameras on tripods. In the diagram, the tripod is placed on an inclined surface, and the length of each leg is adjusted so that the camera remains level with the horizon. Are the feet of the tripod coplanar? Explain your reasoning.

SOLUTION:

Yes; the feet of the tripod model three points that are noncollinear. Because three noncollinear points are used to name exactly one plane, the feet of the tripod are coplanar.

ANSWER:

Yes; the feet of the tripod model three points that are noncollinear. Because three noncollinear points are used to name exactly one plane, the feet of the tripod are coplanar.

Complete each sentence.

67. 54 in. = ? ft

SOLUTION:

54 in. = ? ft smaller unit→larger unit
= 54 ÷ 12 1 ft = 12 in., ÷ by 12
= 4.5 ft

ANSWER:

4.5

68. 275 mm = ? m

SOLUTION:

275 mm = ? cm smaller unit→larger unit
= 275 ÷ 10 10 mm = 1 cm, ÷ by 10
= 27.5 cm Simplify.

27.5 cm = ? m smaller unit→larger unit
= 27.5 ÷ 100 100 cm = 1 m, ÷ by 100
= 0.275 m Simplify.

ANSWER:

0.275

69. 7 gal = ? pt

SOLUTION:

7 gal = ? qt **larger unit→smaller unit**
= 7 × 4 **4 qt = 1 gal, × by 4**
= 28 qt **Simplify.**

28 qt = ? pt **larger unit→smaller unit**
= 28 × 2 **1 qt = 2 pt, × by 2**
= 56 pt **Simplify.**

ANSWER:

56

Solve each equation.

70. $(90 - x) - x = 18$

SOLUTION:

$(90 - x) - x = 18$ Original equation

$90 - 2x = 18$ Simplify.

$90 - 90 - 2x = 18 - 90$ Subtract 90 from each side.

$-2x = -72$ Simplify.

$\frac{-2x}{-2} = \frac{-72}{-2}$ Divide each side by −2.

$x = 36$ Simplify.

ANSWER:

36

71. $(5x + 3) + 7x = 180$

SOLUTION:

$(5x + 3) + 7x = 180$ Original equation

$12x + 3 = 180$ Simplify.

$12x + 3 - 3 = 180 - 3$ Subtract 3 from each side.

$12x = 177$ Simplify.

$\frac{12x}{12} = \frac{177}{12}$ Divide each side by 12.

$x = 14.75$ Simplify.

ANSWER:

14.75

72. $(13x+10)+2x=90$

SOLUTION:

$$\begin{aligned}
(13x+10)+2x &= 90 && \text{Original equation}\\
15x+10 &= 90 && \text{Simplify.}\\
15x+10-10 &= 90-10 && \text{Subtract 10 from each side.}\\
15x &= 80 && \text{Simplify.}\\
\frac{15x}{15} &= \frac{80}{15} && \text{Divide each side by 15.}\\
x &= 5\frac{5}{15} && \text{Simplify.}\\
x &= 5\frac{1}{3}
\end{aligned}$$

ANSWER:

$5\frac{1}{3}$

73. $(180-x)-4x=56$

SOLUTION:

$$\begin{aligned}
(180-x)-4x &= 56 && \text{Original equation}\\
180-5x &= 56 && \text{Simplify.}\\
180-180-5x &= 56-180 && \text{Subtract 180 from each side.}\\
-5x &= -124 && \text{Simplify.}\\
\frac{-5x}{-5} &= \frac{-124}{-5} && \text{Divide each side by } -5.\\
x &= 24.8 && \text{Simplify.}
\end{aligned}$$

ANSWER:

24.8

74. $(4n+17)+(n-2)=180$

SOLUTION:

$$\begin{aligned}
(4n+17)+(n-2) &= 180 && \text{Original equation}\\
5n+15 &= 180 && \text{Simplify.}\\
5n+15-15 &= 180-15 && \text{Subtract 15 from each side.}\\
5n &= 165 && \text{Simplify.}\\
\frac{5n}{5} &= \frac{165}{5} && \text{Divide each side by 5.}\\
n &= 33 && \text{Simplify.}
\end{aligned}$$

ANSWER:

33

75. $(8a-23)+(9-2a)=90$

SOLUTION:

$$(8a-23)+(9-2a)=90 \quad \text{Original equation}$$
$$6a-14=90 \quad \text{Simplify.}$$
$$6a-14+14=90+14 \quad \text{Add 14 to each side.}$$
$$6a=104 \quad \text{Simplify.}$$
$$\frac{6a}{6}=\frac{104}{6} \quad \text{Divide each side by 6.}$$
$$a=17\frac{2}{6} \quad \text{Simplify.}$$
$$a=17\frac{1}{3}$$

ANSWER:

$17\frac{1}{3}$