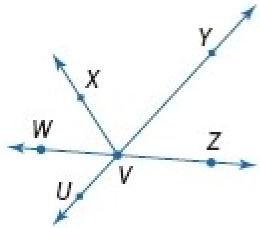
1-5 Angle Relationships

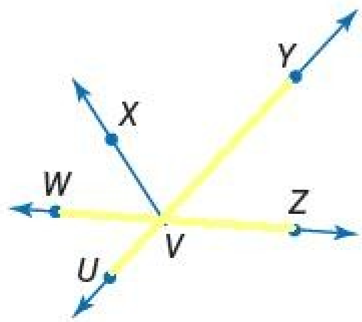
Name an angle pair that satisfies each condition.



1. two acute vertical angles

SOLUTION:

Vertical angles are two nonadjacent angles formed by two intersecting lines.



You can use the corner of a piece of paper to see that $\angle ZVY$ and $\angle WVU$ are less than right angles. Therefore, $\angle ZVY$ and $\angle WVU$ are acute vertical angles.

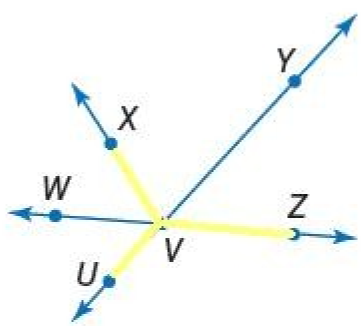
ANSWER:

$\angle ZVY$, $\angle WVU$

2. two obtuse adjacent angles

SOLUTION:

Adjacent angles are two angles that lie in the same plane and have a common vertex and a common side. Since $\angle UVZ$ and $\angle XVZ$ share vertex V and side $\overrightarrow{VZ}$, they are adjacent angles.

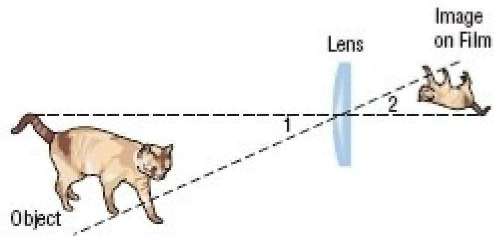


You can use the corner of a piece of paper to see that $\angle UVZ$ and $\angle XVZ$ are each larger than a right angle. Therefore, $\angle UVZ$ and $\angle XVZ$ are obtuse adjacent angles.

ANSWER:

$\angle UVZ$, $\angle XVZ$

3. **CAMERAS** Cameras use lenses and light to capture images.



a. What type of angles are formed by the object and its image?
b. If the measure of $\angle 2$ is 15, what is the measure of $\angle 1$?

SOLUTION:
a. The object and its image are two nonadjacent angles formed by two intersecting lines. So they are vertical angles.
b. Vertical angles are congruent. So $m\angle 1 = m\angle 2$.
Substitute $m\angle 2 = 15$.
$m\angle 1 = 15$
Therefore, the measure of $\angle 1$ is also 15.

ANSWER:
a. vertical
b. 15

4. **ALGEBRA** The measures of two complementary angles are $7x + 17$ and $3x - 20$. Find the measures of the angles.

SOLUTION:
Complementary angles are two angles with measures that have a sum of 90.

$$\begin{aligned}(7x+17)+(3x-20)&=90 && \text{Original equation}\\ 10x-3&=90 && \text{Simplify.}\\ 10x-3+3&=90+3 && \text{Add 3 to each side.}\\ 10x&=93 && \text{Simplify.}\\ \frac{10x}{10}&=\frac{93}{10} && \text{Divide each side by 10.}\\ x&=9.3 && \text{Simplify.}\end{aligned}$$

Substitute $x = 9.3$ in $7x + 17$.
$$\begin{aligned}7x+17&=7(9.3)+17\\ &=82.1\end{aligned}$$

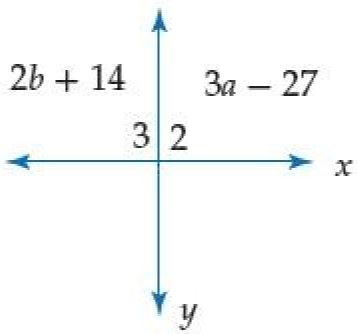
Substitute $x = 9.3$ in $3x - 20$.
$$\begin{aligned}3x-20&=3(9.3)-20\\ &=7.9\end{aligned}$$

The measures of two complementary angles are 82.1 and 7.9.

ANSWER:
82.1, 7.9

5. **ALGEBRA** Lines x and y intersect to form adjacent angles 2 and 3. If $m\angle 2 = 3a - 27$ and $m\angle 3 = 2b + 14$, find the values of a and b so that x is perpendicular to y.

SOLUTION:



Line x is perpendicular to y. So, $m\angle 2 = 90$ and $m\angle 3 = 90$.

Substitute $m\angle 2 = 3a - 27$ in $m\angle 2 = 90$.

$m\angle 2 = 90$	Definition of Right Angles
$3a - 27 = 90$	Substitution.
$3a - 27 + 27 = 90 + 27$	Add 27 to each side.
$3a = 117$	Simplify.
$\frac{3a}{3} = \frac{117}{3}$	Divide each side by 3.
$a = 39$	Simplify.

Substitute $m\angle 3 = 2b + 14$ in $m\angle 3 = 90$.

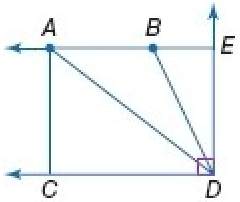
$m\angle 3 = 90$	Definition of Right Angle
$2b + 14 = 90$	Substitution.
$2b + 14 - 14 = 90 - 14$	Subtract 14 from each side.
$2b = 76$	Simplify.
$\frac{2b}{2} = \frac{76}{2}$	Divide each side by 2.
$b = 38$	Simplify.

So, a is 39 and b is 38.

ANSWER:

$a = 39$; $b = 38$

Determine whether each statement can be assumed from the figure. Explain.



6. $\angle CAD$ and $\angle DAB$ are complementary.

SOLUTION:

Complementary angles are two angles with measures that have a sum of 90.
While $\angle CAB$ appears to be a right angle, no information verifies this. So, $\angle CAD$ and $\angle DAB$ may not be complementary. The answer is "No".

ANSWER:

No; while $\angle CAB$ appears to be a right angle, no information verifies this.

7. $\angle EDB$ and $\angle BDA$ are adjacent, but they are neither complementary nor supplementary.

SOLUTION:

Adjacent angles are two angles that lie in the same plane and have a common vertex and a common side.

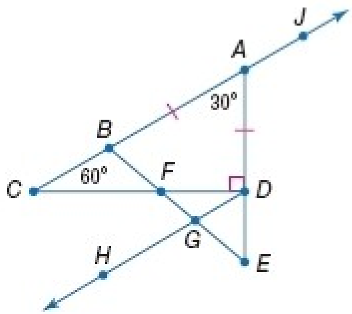
$\angle EDB$ and $\angle BDA$ share a common side and vertex, so they are adjacent.

Since $m\angle EDB + m\angle BDA + m\angle ADC = 90$, $\angle EDB$ and $\angle BDA$ cannot be complementary or supplementary. So, the answer is "Yes".

ANSWER:

Yes: they share a common side and vertex, so they are adjacent. Since $m\angle EDB + m\angle BDA + m\angle ADC = 90$, $\angle EDB$ and $\angle BDA$ cannot be complementary or supplementary.

Name an angle or angle pair that satisfies each condition.



8. two adjacent angles

SOLUTION:

Sample answer: Adjacent angles are two angles that lie in the same plane and have a common vertex and a common side. There are many adjacent angles in the figure.
$\angle CBF$ and $\angle ABF$ are adjacent angles, $\angle HGE$ and $\angle DGE$ are adjacent angles.

ANSWER:

Sample answer: $\angle HGE$, $\angle DGE$

9. two acute vertical angles

SOLUTION:

Sample answer: Vertical angles are two nonadjacent angles formed by two intersecting lines. There are many acute vertical angles in the figure. $\angle BFC$ and $\angle DFE$ are acute vertical angles.

ANSWER:

Sample answer: $\angle BFC$, $\angle DFE$

10. two obtuse vertical angles

SOLUTION:

Sample answer: Vertical angles are two nonadjacent angles formed by two intersecting lines. Nonadjacent angles *HGE* and *FGD* are formed by $\overleftrightarrow{HD}$ and $\overleftrightarrow{BE}$ intersecting at *G*. Each angle is greater than a right angle. Therefore, $\angle HGE$ and $\angle FGD$ are obtuse vertical angles.

ANSWER:

Sample answer: $\angle HGE$, $\angle FGD$

11. two complementary adjacent angles

SOLUTION:

If the sum of the measures of two adjacent angles is 90, then they are complementary adjacent angles.
$\angle FDG$ and $\angle GDE$ share a common side and vertex, also $m\angle FDG + m\angle GDE = 90$.
So, $\angle FDG$ and $\angle GDE$ are complementary adjacent angles.

ANSWER:

$\angle FDG$, $\angle GDE$

12. two complementary nonadjacent angles

SOLUTION:
If the sum of the measures of two nonadjacent angles is 90, then they are complementary nonadjacent angles.
$\angle BCF$ and $\angle BAD$ are nonadjacent angles, and $m\angle BCF + m\angle BAD = 90$.
So, $\angle BCF$ and $\angle BAD$ are complementary nonadjacent angles.

ANSWER:
$\angle BCF$, $\angle BAD$

13. two supplementary adjacent angles

SOLUTION:
Sample answer: If the sum of the measures of two adjacent angles is 180, then they are supplementary adjacent angles. There are many supplementary adjacent angles in the figure.
$\angle CBF$ and $\angle ABF$ share a common side and vertex, also $m\angle CBF + m\angle ABF = 180$.
So, $\angle CBF$ and $\angle ABF$ are supplementary adjacent angles.

ANSWER:
Sample answer: $\angle CBF$, $\angle ABF$

14. a linear pair whose vertex is *F*

SOLUTION:
Sample answer: A linear pair is a pair of adjacent angles with noncommon sides that are opposite rays.
$\angle BFC$ and $\angle BFD$ are linear pair with vertex *F*, $\angle GFD$ and $\angle GFC$ are linear pair with vertex *F*.

ANSWER:
Sample answer: $\angle BFC$, $\angle BFD$

15. an angle complementary to $\angle FDG$

SOLUTION:
Complementary angles are two angles with measures that have a sum of 90.
Since $m\angle FDG + m\angle GDE = 90$,
$\angle GDE$ is complementary to $\angle FDG$.

ANSWER:
$\angle GDE$

16. an angle supplementary to $\angle CBF$

SOLUTION:
Sample answer: Supplementary angles are two angles with measures that have a sum of 180.
Since $m\angle CBF + m\angle JBF = 180$, $\angle JBF$ is supplementary to $\angle CBF$.

ANSWER:
Sample answer: $\angle JBF$

17. an angle supplementary to $\angle JAE$

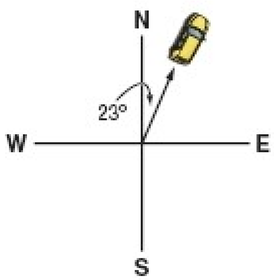
SOLUTION:

Supplementary angles are two angles with measures that have a sum of 180.
Since $m\angle JAE + m\angle CAE = 180$, $\angle CAE$ is supplementary to $\angle JAE$.

ANSWER:

$\angle CAE$

18. **CCSS REASONING** You are using a compass to drive 23° east of north. Express your direction in another way using an acute angle and two of the four directions: north, south, east, and west. Explain your reasoning.



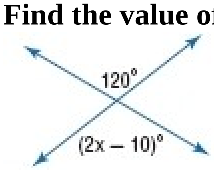
SOLUTION:

Since the measure of the angle between north and east is 90, you can use the complement of $23°$ (the original angle) and describe the direction as north of east instead of east of north. The complement of $23°$ is $67°$. So, the answer is $67°$ north of east.

ANSWER:

Sample answer: 67° north of east; since the measure of the angle between north and east is 90, you can use the complement of the original angle and describe the direction as north of east instead of east of north.

Find the value of each variable.

19. 

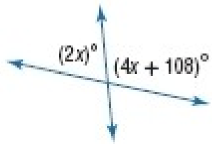
SOLUTION:

In the figure, the $120°$ angle and the $(2x-10)°$ angle are vertical angles.
Vertical angles are congruent.

$$\begin{aligned} 120 &= 2x - 10 && \text{Vertical angles} \\ 120 + 10 &= 2x - 10 + 10 && \text{Add 10 to each side.} \\ 130 &= 2x && \text{Simplify.} \\ \frac{130}{2} &= \frac{2x}{2} && \text{Divide each side by 2.} \\ 65 &= x && \text{Simplify.} \end{aligned}$$

ANSWER:

65

20. 

SOLUTION:

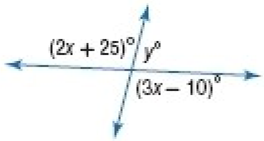
The angles in a linear pair are supplementary.
So, $(2x)° + (4x + 108)° = 180°$.

$$\begin{aligned} 2x + 4x + 108 &= 180 && \text{Def. of Linear Pair} \\ 6x + 108 &= 180 && \text{Simplify.} \\ 6x + 108 - 108 &= 180 - 108 && -108 \text{ from each side.} \\ 6x &= 72 && \text{Simplify.} \\ \frac{6x}{6} &= \frac{72}{6} && \div \text{ each side by 6.} \\ x &= 12 && \text{Simplify.} \end{aligned}$$

So, x is 12.

ANSWER:

x=12

21. 

SOLUTION:

Since $(2x + 25)°$ and $(3x – 10)°$ are vertical angles, they are congruent.

$$\begin{aligned} 2x+25&=3x-10 && \text{Vertical angles} \\ 2x-3x+25&=3x-3x-10 && -3x \text{ from each side.} \\ -x+25&=-10 && \text{Simplify.} \\ -x+25-25&=-10-25 && -25 \text{ from each side.} \\ -x&=-35 && \text{Simplify.} \\ -1(-x)&=-1(-35) && \times \text{ each side by } -1. \\ x&=35 && \text{Simplify.} \end{aligned}$$

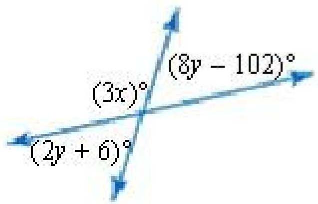
$$\begin{aligned} 2x+25&=2\cdot 35+25 && \text{Replace } x \text{ with } 35. \\ &=70+25 && \text{Multiply.} \\ &=95 && \text{Addition.} \end{aligned}$$

Solve for y.

$$\begin{aligned} (2x+25)+y&=180 && \text{Definition of Supplementary Angles.} \\ 95+y&=180 && \text{Replace } (2x+25) \text{ with } 95. \\ 95-95+y&=180-95 && \text{Subtract } 95 \text{ from each side.} \\ y&=85 && \text{Simplify.} \end{aligned}$$

ANSWER:

$x = 35$; $y = 85$



22.

SOLUTION:

In the figure, $(8y-102)°$ angle and $(2y+6)°$ angle are vertical angles.
Vertical angles are congruent. So, $(8y-102)° = (2y+6)°$.

$$\begin{aligned} 8y-102 &= 2y+6 && \text{Def. of Vertical Angles.} \\ 8y-2y-102 &= 2y-2y+6 && -2y \text{ from each side.} \\ 6y-102 &= 6 && \text{Simplify.} \\ 6y-102+102 &= 6+102 && +102 \text{ to each side.} \\ 6y &= 108 && \text{Simplify.} \\ \frac{6y}{6} &= \frac{108}{6} && \div \text{ each side by 6.} \\ y &= 18 && \text{Simplify.} \end{aligned}$$

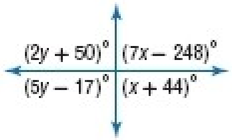
The angles in a linear pair are supplementary.
So, $(8y-102)° + (3x)° = 180°$.

$$\begin{aligned} (8y-102)+(3x) &= 180 && \text{Def.of Supplementary Angles.} \\ 8(18)-102+3x &= 180 && \text{Replace } y \text{ with 18.} \\ 144-102+3x &= 180 && \text{Multiply.} \\ 42+3x &= 180 && \text{Subtraction.} \\ 42-42+3x &= 180-42 && \text{Subtract 42 from each side.} \\ 3x &= 138 && \text{Simplify.} \\ \frac{3x}{3} &= \frac{138}{3} && \text{Divide each side by 3.} \\ x &= 46 && \text{Simplify.} \end{aligned}$$

So, the values of the variables are $x = 46$ and $y = 18$.

ANSWER:

$x = 46$; $y = 18$

23. 

SOLUTION:

Supplementary angles have measures that sum to 180. So, $(2y+50)°+(5y-17)°=180°$ and $(x+44)°+(7x-248)°=180°$.

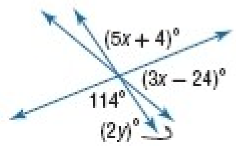
Consider $(2y+50)°+(5y-17)°=180°$.

$$
\begin{aligned}
2y+50+5y-17 &= 180 && \text{Def. of Supplementary Angles.}\\
7y+33 &= 180 && \text{Simplify.}\\
7y+33-33 &= 180-33 && -33 \text{ from each side.}\\
7y &= 147 && \text{Simplify.}\\
\frac{7y}{7} &= \frac{147}{7} && \div \text{ each side by 7.}\\
y &= 21 && \text{Simplify.}
\end{aligned}
$$

$$
\begin{aligned}
7x-248+x+44 &= 180 && \text{Def. of Supplementary Angles.}\\
8x-204 &= 180 && \text{Simplify.}\\
8x-204+204 &= 180+204 && \text{Add 204 to each side.}\\
8x &= 384 && \text{Simplify.}\\
\frac{8x}{8} &= \frac{384}{8} && \text{Divide each side by 8.}\\
x &= 48 && \text{Simplify.}
\end{aligned}
$$

ANSWER:

$x = 48$; $y = 21$

24. 

SOLUTION:

In the figure, $(5x+4)°$ angle and $(114)°$ angle are vertical angles.
Vertical angles are congruent. So, $(5x+4)° = 114°$.

$$\begin{aligned} 5x+4 &= 114 && \text{Def. of Vertical Angles.} \\ 5x+4-4 &= 114-4 && -4 \text{ from each side.} \\ 5x &= 110 && \text{Simplify.} \\ \frac{5x}{5} &= \frac{110}{5} && \div \text{ each side by 5.} \\ x &= 22 && \text{Simplify.} \end{aligned}$$

In the figure, $114° + (3x-24)° + (2y)° = 180°$.

$$\begin{aligned} 114+3x-24+2y &= 180 && \text{Def.of Supplementary Angles.} \\ 3x+2y+90 &= 180 && \text{Subtraction.} \\ 3(22)+2y+90 &= 180 && \text{Replace } x \text{ with 22.} \\ 66+2y+90 &= 180 && \text{Mutliply.} \\ 2y+156 &= 180 && \text{Addition.} \\ 2y+156-156 &= 180-156 && \text{Subtract 156 from each side.} \\ 2y &= 24 && \text{Simplify.} \\ \frac{2y}{2} &= \frac{24}{2} && \text{Divide each side by 2.} \\ y &= 12 && \text{Simplify.} \end{aligned}$$

ANSWER:

$x = 22$; $y = 12$

25. **ALGEBRA** $\angle E$ and $\angle F$ are supplementary. The measure of $\angle E$ is 54 more than the measure of $\angle F$. Find the measures of each angle.

SOLUTION:

Supplementary angles are two angles with measures that have a sum of 180. Then, $m\angle E + m\angle F = 180$. It is given that $m\angle E = m\angle F + 54$.

Substitute.

$$\begin{aligned} m\angle E + m\angle F &= 180 && \text{Def. of Supplementary Angles.} \\ (54 + m\angle F) + m\angle F &= 180 && \text{Replace } m\angle E \text{ with } 54 + m\angle F. \\ 54 + 2(m\angle F) &= 180 && \text{Simplify.} \\ 54 - 54 + 2(m\angle F) &= 180 - 54 && \text{Subtract 54 from each side.} \\ 2(m\angle F) &= 126 && \text{Simplify.} \\ \frac{2(m\angle F)}{2} &= \frac{126}{2} && \text{Divide each side by 2.} \\ m\angle F &= 63 && \text{Simplify.} \end{aligned}$$

Substitute $m\angle F = 63$ in $m\angle E = m\angle F + 54$.

$$\begin{aligned} m\angle E &= 63 + 54 \\ &= 117 \end{aligned}$$

ANSWER:

$m\angle F = 63$; $m\angle E = 117$

26. **ALGEBRA** The measure of an angle's supplement is 76 less than the measure of the angle. Find the measure of the angle and its supplement.

SOLUTION:

Supplementary angles are two angles with measures that have a sum of 180.
Let x and y be the angle and its supplement respectively.
$y = x - 76$

By the definition of supplementary angles, $x + y = 180$.

$$\begin{aligned} x + y &= 180 && \text{Def. of Supplementary Angles.} \\ x + (x - 76) &= 180 && \text{Replace } y \text{ with } (x - 76). \\ 2x - 76 &= 180 && \text{Simplify.} \\ 2x - 76 + 76 &= 180 + 76 && \text{Add 76 to each side.} \\ 2x &= 256 && \text{Simplify.} \\ \frac{2x}{2} &= \frac{256}{2} && \text{Divide each side by 2.} \\ x &= 128 && \text{Simplify.} \end{aligned}$$

Substitute $x = 128$ in $y = x - 76$.
$y = 128 - 76$
$= 52$

The measure of the angle and its supplement are 128 and 52 respectively.

ANSWER:
128; 52

27. **ALGEBRA** The measure of the supplement of an angle is 40 more than two times the measure of the complement of the angle. Find the measure of the angle.

SOLUTION:

Let x be the measure of an angle.
The measure of an angle which is complementary to $x°$ angle is $90 - x$.
The measure of an angle which is supplementary to $x°$ angle is $180 - x$.

$$
\begin{aligned}
180 - x &= 40 + 2(90 - x) && \text{Supplemntary angle} \\
180 - x &= 40 + 180 - 2x && \text{Distributive Property} \\
180 - x &= 220 - 2x && \text{Addition.} \\
180 - x + 2x &= 220 - 2x + 2x && \text{Add } 2x \text{ to each side.} \\
180 + x &= 220 && \text{Simplify.} \\
180 - 180 + x &= 220 - 180 && \text{Subtract 180 from each side.} \\
x &= 40 && \text{Simplify.}
\end{aligned}
$$

The measure of an angle is 40.

ANSWER:

40

28. **ALGEBRA** $\angle 3$ and $\angle 4$ form a linear pair. The measure of $\angle 3$ is four more than three times the measure of $\angle 4$. Find the measure of each angle.

SOLUTION:

The angles in a linear pair are supplementary. So, $m\angle 3 + m\angle 4 = 180$. It is given that $m\angle 3 = 3m\angle 4 + 4$.

$$
\begin{aligned}
m\angle 3 + m\angle 4 &= 180 && \text{Def. of Supplementary Angles.} \\
(3m\angle 4 + 4) + m\angle 4 &= 180 && \text{Subsitute } (3m\angle 4 + 4) \text{ for } m\angle 3. \\
4m\angle 4 + 4 &= 180 && \text{Simplify.} \\
4m\angle 4 + 4 - 4 &= 180 - 4 && \text{Subtract 4 from each side.} \\
4m\angle 4 &= 176 && \text{Simplify.} \\
\frac{4m\angle 4}{4} &= \frac{176}{4} && \text{Divide each side by 4.} \\
m\angle 4 &= 44 && \text{Simplify.}
\end{aligned}
$$

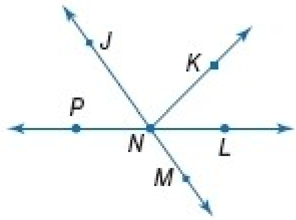
Substitute $m\angle 4 = 44$ in $m\angle 3 = 3m\angle 4 + 4$.

$$
\begin{aligned}
m\angle 3 &= 3(m\angle 4) + 4 \\
&= 3(44) + 4 \\
&= 132 + 4 \\
&= 136
\end{aligned}
$$

ANSWER:

$m\angle 3 = 136$; $m\angle 4 = 44$

ALGEBRA Use the figure below.



29. If $m\angle KNL = 6x - 4$ and $m\angle LNM = 4x + 24$, find the value of x so that $\angle KNM$ is a right angle.

SOLUTION:

In the figure, $m\angle KNL + m\angle LNM = m\angle KNM$.
Since $\angle KNM$ is a right angle, $m\angle KNM = 90$.

$$\begin{aligned} m\angle KNL + m\angle LNM &= m\angle KNM && \text{Def. of Right Angle.} \\ (6x-4)+(4x+24) &= 90 && \text{Substitution.} \\ 10x+20 &= 90 && \text{Addition.} \\ 10x+20-20 &= 90-20 && -20 \text{ from each side.} \\ 10x &= 70 && \text{Simplify.} \\ \frac{10x}{10} &= \frac{70}{10} && \div \text{ each side by 10.} \\ x &= 7 && \text{Simplify.} \end{aligned}$$

ANSWER:

7

30. If $m\angle JNP = 3x - 15$ and $m\angle JNL = 5x + 59$, find the value of x so that $\angle JNP$ and $\angle JNL$ are supplements of each other.

SOLUTION:

Supplementary angles are two angles with measures that have a sum of 180. Then, $m\angle JNP + m\angle JNL = 180$.

$$\begin{aligned} m\angle JNP + m\angle JNL &= 180 && \text{Def. of Supplementary Angles.} \\ (3x-15)+(5x+59) &= 180 && \text{Substitution.} \\ 8x+44 &= 180 && \text{Simplify.} \\ 8x+44-44 &= 180-44 && \text{Subtract 44 from each side.} \\ 8x &= 136 && \text{Simplify.} \\ \frac{8x}{8} &= \frac{136}{8} && \text{Divide each side by 8.} \\ x &= 17 && \text{Simplify.} \end{aligned}$$

ANSWER:

17

31. If $m\angle LNM = 8x + 12$ and $m\angle JNL = 12x - 32$, find $m\angle JNP$.

SOLUTION:

The angles in a linear pair are supplementary. So, $m\angle LNM + m\angle JNL = 180$.

$$
\begin{aligned}
m\angle LNM + m\angle JNL &= 180 && \text{Def. of Linear Pair.}\\
(8x+12)+(12x-32) &= 180 && \text{Substitution.}\\
20x - 20 &= 180 && \text{Simplify.}\\
20x - 20 + 20 &= 180 + 20 && \text{+20 to each side.}\\
20x &= 200 && \text{Simplify.}\\
\frac{20x}{20} &= \frac{200}{20} && \div \text{ each side by 20.}\\
x &= 10 && \text{Simplify.}
\end{aligned}
$$

$\angle LNM$ and $\angle JNP$ are vertical angles. Since the vertical angles are congruent, $m\angle LNM = m\angle JNP$.

Substitute $x = 10$ in $m\angle LNM = 8x + 12.$

$$
\begin{aligned}
m\angle LNM &= 8(10) + 12\\
&= 92
\end{aligned}
$$

So, $\angle JNP = 92.$

ANSWER:

92

32. If $m\angle JNP = 2x + 3$, $m\angle KNL = 3x - 17$, and $m\angle KNJ = 3x + 34$, find the measure of each angle.

SOLUTION:

In the figure, $m\angle KNL + m\angle JNP + m\angle KNJ = 180$.

$$\begin{aligned} m\angle KNL + m\angle JNP + m\angle KNJ &= 180 && \text{Def. of Supplementary Angles.} \\ (3x-17)+(2x+3)+(3x+34) &= 180 && \text{Substitution.} \\ 8x+20 &= 180 && \text{Simplify.} \\ 8x+20-20 &= 180-20 && \text{Subtract 20 from each side.} \\ 8x &= 160 && \text{Simplify.} \\ \frac{8x}{8} &= \frac{160}{8} && \text{Divide each side by 8.} \\ x &= 20 && \text{Simplify.} \end{aligned}$$

Find $m\angle JNP$.

Substitute $x = 20$ in $m\angle JNP = 2x + 3$.

$$\begin{aligned} m\angle JNP &= 2(20)+3 \\ &= 43 \end{aligned}$$

Find $m\angle KNL$.

Substitute $x = 20$ in $m\angle KNL = 3x - 17$.

$$\begin{aligned} m\angle KNL &= 3(20)-17 \\ &= 43 \end{aligned}$$

Find $m\angle KNJ$.

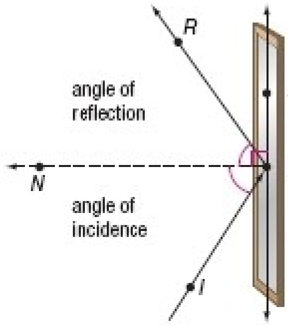
Substitute $x = 20$ in $m\angle KNJ = 3x + 34$.

$$\begin{aligned} m\angle KNJ &= 3(20)+34 \\ &= 94 \end{aligned}$$

ANSWER:

$m\angle JNP = 43$; $m\angle KNL = 43$; $m\angle KNJ = 94$

33. **PHYSICS** As a ray of light meets a mirror, the light is reflected. The angle at which the light strikes the mirror is the *angle of incidence*. The angle at which the light is reflected is the *angle of reflection*. The angle of incidence and the angle of reflection are congruent. In the diagram below, if $m\angle RMI = 106$, find the angle of reflection and $m\angle RMJ$.



SOLUTION:

The angle of reflection and the angle of incidence are congruent.
So, $m\angle RMN = m\angle NMI$.

In the figure, $m\angle RMN + m\angle NMI = m\angle RMI$.

Substitute.

$$\begin{aligned} m\angle RMN + m\angle NMI &= m\angle RMI && \text{Def. of Adjacent } \angle\text{'s.} \\ m\angle RMN + m\angle RMN &= 106 && \text{Substitution.} \\ 2m\angle RMN &= 106 && \text{Simplify.} \\ \frac{2m\angle RMN}{2} &= \frac{106}{2} && \div \text{ each side by 2.} \\ m\angle RMN &= 53 && \text{Simplify.} \end{aligned}$$

The angle of reflection measures 53°.

In the figure, $m\angle RMN + m\angle RMJ = 90.$

$$\begin{aligned} m\angle RMN + m\angle RMJ &= 90 && \text{Def. of Complementary } \angle\text{'s.} \\ 53 + m\angle RMJ &= 90 && \text{Substitution.} \\ 53 - 53 + m\angle RMJ &= 90 - 53 && -53 \text{ from each side.} \\ m\angle RMJ &= 37 && \text{Simplify.} \end{aligned}$$

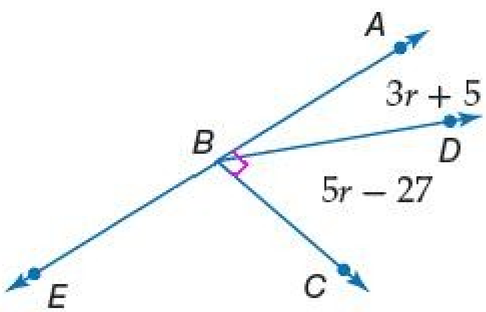
ANSWER:

53; 37

34. **ALGEBRA** Rays *AB* and *BC* are perpendicular. Point *D* lies in the interior of $\angle ABC$. If $m\angle ABD = 3r + 5$ and $m\angle DBC = 5r - 27$, find $m\angle ABD$ and $m\angle DBC$.

SOLUTION:

$m\angle ABD + m\angle DBC = m\angle ABC$. Here, $m\angle ABC = 90$.



$$
\begin{aligned}
m\angle ABD + m\angle DBC &= m\angle ABC && \text{Def. of Adjacent Angles.}\\
(3r+5)+(5r-27) &= 90 && \text{Substitution.}\\
8r - 22 &= 90 && \text{Simplify.}\\
8r - 22 + 22 &= 90 + 22 && \text{Add 22 to each side.}\\
8r &= 112 && \text{Simplify.}\\
\frac{8r}{8} &= \frac{112}{8} && \text{Divide each side by 8.}\\
r &= 14 && \text{Simplify.}
\end{aligned}
$$

Find $m\angle ABD$.

Substitute $r = 14$ in $m\angle ABD = 3r + 5$.

$$
\begin{aligned}
m\angle ABD &= 3(14) + 5\\
&= 42 + 5\\
&= 47
\end{aligned}
$$

Find $m\angle DBC$.

Substitute $r = 14$ in $m\angle DBC = 5r - 27$.

$$
\begin{aligned}
m\angle DBC &= 5r - 27\\
&= 5(14) - 27\\
&= 70 - 27\\
&= 43
\end{aligned}
$$

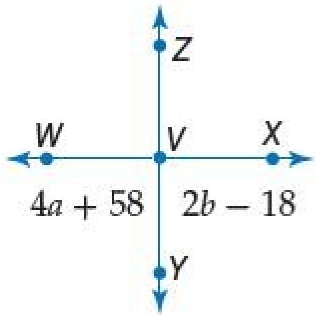
ANSWER:

$m\angle ABD = 47$; $m\angle DBC = 43$

35. **ALGEBRA** $\overleftrightarrow{WX}$ and $\overleftrightarrow{YZ}$ intersect at point V. If $m\angle WVY = 4a + 58$ and $m\angle XVY = 2b - 18$, find the values of a and b so that $\overleftrightarrow{WX}$ is perpendicular to $\overleftrightarrow{YZ}$.

SOLUTION:

Since $\overleftrightarrow{WX}$ and $\overleftrightarrow{YZ}$ intersect at point V and $\overleftrightarrow{WX}$ is perpendicular to $\overleftrightarrow{YZ}$, $m\angle WVY = 90$ and $m\angle XVY = 90$.



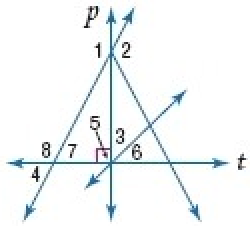
$m\angle WVY = 90$	Def. of Right Angle.
$4a + 58 = 90$	Substitution.
$4a + 58 - 58 = 90 - 58$	−58 from each side.
$4a = 32$	Simplify.
$\frac{4a}{4} = \frac{32}{4}$	÷ each side by 4.
$a = 8$	Simplify.
$m\angle XVY = 90$	Def. of Right Angle.
$2b - 18 = 90$	Substitution.
$2b - 18 + 18 = 90 + 18$	+18 to each side.
$2b = 108$	Simplify.
$\frac{2b}{2} = \frac{108}{2}$	÷ each side by 2.
$b = 54$	Simplify.

So, a is 8 and b is 54.

ANSWER:

$a = 8$; $b = 54$

Determine whether each statement can be assumed from the figure. Explain.



36. $\angle 4$ and $\angle 7$ are vertical angles.

SOLUTION:

$\angle 4$ and $\angle 7$ are nonadjacent angles and formed by two intersecting lines.
So, $\angle 4$ and $\angle 7$ are vertical angles.
The answer is "Yes".

ANSWER:

Yes; the angles are nonadjacent and are formed by two intersecting lines.

37. $\angle 4$ and $\angle 8$ are supplementary.

SOLUTION:

Since $\angle 4$ and $\angle 8$ form a linear pair, they are supplementary.
The answer is "Yes".

ANSWER:

Yes; the angles form a linear pair.

38. $p \perp t$

SOLUTION:

Since the intersection of the lines p and t is a right angle, they are perpendicular.
The answer is "Yes".

ANSWER:

Yes; the intersection of the two lines is a right angle.

39. $\angle 3 \cong \angle 6$

SOLUTION:

From the figure, $\angle 3$ and $\angle 6$ are adjacent. Since $\angle 5$ is a right angle, $\angle 3$ and $\angle 6$ will be complementary. This determines that both angles are acute. However, unless we know that the larger angle was bisected to form $\angle 3$ and $\angle 6$, the measures of $\angle 3$ and $\angle 6$ are unknown. So, we cannot say $\angle 3 \cong \angle 6$.
The answer is "No".

ANSWER:

No; the measures of each angle are unknown.

40. $\angle 5 \cong \angle 3 + \angle 6$

SOLUTION:

In the figure, $m\angle 3 + m\angle 6 = 90$ and $m\angle 5 = 90.$
$m\angle 5 = m\angle 3 + m\angle 6.$
So, $\angle 5 \cong \angle 3 + \angle 6$.
The answer is "Yes".

ANSWER:

Yes; $m\angle 5 = 90$ since it is a right angle, and $m\angle 3 + m\angle 6 = 90$ since it is a right angle.

41. $\angle 5$ and $\angle 7$ form a linear pair.

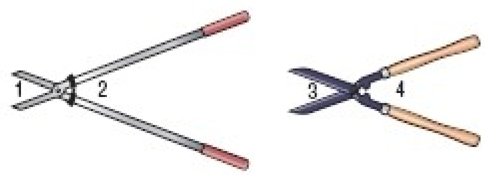
SOLUTION:

A linear pair is a pair of adjacent angles with non-common sides that are opposite rays.
$\angle 5$ and $\angle 7$ do not form a linear pair, since they are not adjacent angles.

ANSWER:

No; the angles are not adjacent.

42. **CCSS ARGUMENTS** In the diagram of the pruning shears shown, $m\angle 1 = m\angle 3$. What conclusion can you reach about the relationship between $\angle 4$ and $\angle 2$? Explain.



SOLUTION:

Vertical angles are two nonadjacent angles formed by two intersecting lines. Here, $\angle 1$ and $\angle 2$ are vertical angles, and $\angle 3$ and $\angle 4$ are vertical angles.
So, $\angle 1 \cong \angle 2$ and $\angle 3 \cong \angle 4$.

We are given that $m\angle 1 = m\angle 3$.
So, $\angle 1 \cong \angle 3$.

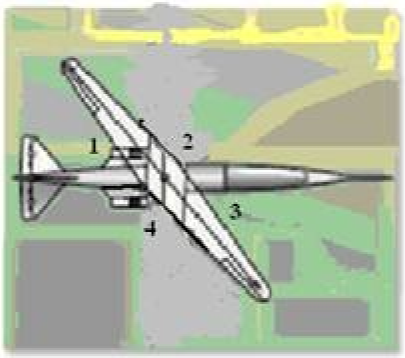
Substitute $\angle 1 \cong \angle 3$ in $\angle 1 \cong \angle 2$.
$\angle 3 \cong \angle 2$

Now, substitute $\angle 3 \cong \angle 2$ in $\angle 3 \cong \angle 4$.
$\angle 2 \cong \angle 4$
So, $m\angle 2 = m\angle 4.$

ANSWER:

$m\angle 2 = m\angle 4$; We are given that $m\angle 1 = m\angle 3$. Since $\angle 1$ and $\angle 2$ are vertical angles, $\angle 1 \cong \angle 2$. So $m\angle 1 = m\angle 2$. Since $\angle 3$ and $\angle 4$ are vertical angles, $\angle 3 \cong \angle 4$. So $m\angle 3 = m\angle 4$. Since $m\angle 1 = m\angle 2$ and $m\angle 1 = m\angle 3$, we can say that $m\angle 2 = m\angle 3$. Since $m\angle 3 = m\angle 4$ and $m\angle 2 = m\angle 3$, we can say that $m\angle 2 = m\angle 4$.

FLIGHT The wings of the aircraft shown can pivot up to 60º in either direction from the perpendicular position.



43. Identify a pair of vertical angles.

SOLUTION:

Vertical angles are two nonadjacent angles formed by two intersecting lines.
$\angle 1$ and $\angle 3$ are vertical angles. So are $\angle 2$ and $\angle 4$.

ANSWER:

Sample answer: $\angle 1$ and $\angle 3$

44. Identify two pairs of supplementary angles.

SOLUTION:

Sample answer: $\angle 1$ and $\angle 2$ form a linear pair, so the angles are supplementary.
$\angle 3$ and $\angle 4$ form a linear pair, so the angles are supplementary.
So, two pairs of supplementary angles are $\angle 1$ and $\angle 2$; $\angle 3$ and $\angle 4$.

ANSWER:

Sample answer: $\angle 1$ and $\angle 2$; $\angle 3$ and $\angle 4$

45. If $m\angle 1 = 110$, what is $m\angle 3$? $m\angle 4$?

SOLUTION:

In the figure, $\angle 1$ and $\angle 3$ are vertical angles. $\angle 1 \cong \angle 3$, since vertical angles are congruent.

By the definition of congruent angles, $m\angle 1 = m\angle 3$.

Given that $m\angle 1 = 110$, then $m\angle 3 = 110$.

Since $\angle 1$ and $\angle 4$ form a linear pair, $\angle 1$ and $\angle 4$ are supplementary angles.

$$\begin{aligned} m\angle 1 + m\angle 4 &= 180 && \text{Def. of Supplementary Angles} \\ 110 + m\angle 4 &= 180 && \text{Substitution.} \\ 110 - 110 + m\angle 4 &= 180 - 110 && \text{Subtract 110 from each side.} \\ m\angle 4 &= 70 && \text{Simplify.} \end{aligned}$$

ANSWER:

110; 70

46. What is the minimum possible value for $m\angle 2$? the maximum?

SOLUTION:

When the wing is in its normal position, it is perpendicular to the body of the plane.

The wing can be pivoted up to 60° in either direction.
If the wing to the left side of the plane is pivoted 60º forward, then $m\angle 1 = 90 + 60$ or 150 and $m\angle 2 = 180 - 150$ or 30.
If the wing to the left side of the plane is pivoted 60º backwards, then $m\angle 1 = 90 - 60$ or 30 and $m\angle 2 = 180 - 30$ or 150.

Therefore, the minimum possible value for $m\angle 2$ is 30 and the maximum possible value is 150.

ANSWER:

30; 150

47. Is there a wing position in which none of the angles are obtuse? Explain.

SOLUTION:

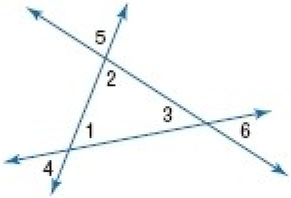
Sample answer: If the wing is not rotated at all, then the wing is perpendicular to the body of the plane. So, all of the angles are right angles, which are neither acute nor obtuse. So, the answer is “Yes”.

ANSWER:

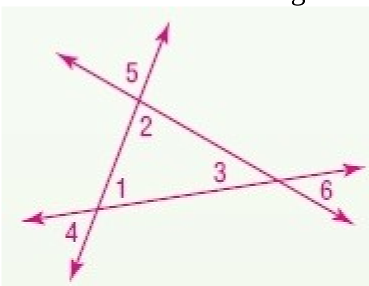
Sample answer: Yes; if the wing is not rotated at all, then all of the angles are right angles, which are neither acute nor obtuse.

48. **MULTIPLE REPRESENTATIONS** In this problem, you will explore the relationship between the sum of the interior angles of a triangle and the angles vertical to them.
a. GEOMETRIC Draw three sets of three intersecting lines and label each as shown.
b. TABULAR For each set of lines, measure and
record $m\angle 1$, $m\angle 2$, and $m\angle 3$ in a table. Record $m\angle 1 + m\angle 2 + m\angle 3$ in a separate column.

c. VERBAL Explain how you can find $m\angle 4$, $m\angle 5$, and $m\angle 6$ when you know $m\angle 1$, $m\angle 2$, and $m\angle 3$.
d. ALGEBRAIC Write an equation that relates $m\angle 1 + m\angle 2 + m\angle 3$ to $m\angle 4 + m\angle 5 + m\angle 6$. Then use substitution to write an equation that relates $m\angle 4 + m\angle 5 + m\angle 6$ to an integer.



SOLUTION:
Sample answers:
a. Draw three intersecting lines and label them.



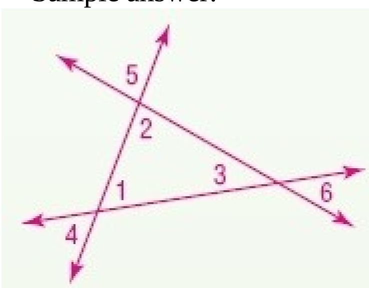
b. Assume the measures for $\angle 1$, $\angle 2$, and $\angle 3$, then find $m\angle 1 + m\angle 2 + m\angle 3$. Record those values in a table.

Set	$m\angle 1$	$m\angle 2$	$m\angle 3$	$m\angle 1 + m\angle 2 + m\angle 3$
1	30	60	90	180
2	45	55	80	180
3	20	60	100	180

c. Vertical angles are two nonadjacent angles formed by two intersecting lines.
$m\angle 4 = m\angle 1$, $m\angle 5 = m\angle 2$, and $m\angle 6 = m\angle 3$, since they are pairs of vertical angles.

d. The sum of the measures of angles in a triangle is 180. So, $m\angle 1 + m\angle 2 + m\angle 3 = 180.$
Refer part **c**.
$m\angle 1 + m\angle 2 + m\angle 3 = m\angle 4 + m\angle 5 + m\angle 6$
Therefore, $m\angle 4 + m\angle 5 + m\angle 6 = 180.$

ANSWER:
a. Sample answer:



b. Sample answer:

Set	$m\angle 1$	$m\angle 2$	$m\angle 3$	$m\angle 1 + m\angle 2 + m\angle 3$
1	30	60	90	180
2	45	55	80	180
3	20	60	100	180

c. Sample answer: Explain how you can find $m\angle 4 = m\angle 1$, $m\angle 5 = m\angle 2$, and $m\angle 6 = m\angle 3$ because they are pairs of vertical angles.

d. Sample answer; $m\angle 1 + m\angle 2 + m\angle 3 = m\angle 4 + m\angle 5 + m\angle 6$; $m\angle 4 + m\angle 5 + m\angle 6 = 180$.

49. **CCSS REASONING** Are there angles that do not have a complement? Explain.

SOLUTION:

Complementary angles are two angles with measures that have a sum of 90. By definition, the measure of an angle must be greater than 0. So, each angle must have a measure less than 90. Thus, each angle in a complementary pair is an acute angle.

Angles that have a measure greater than or equal to 90 can not have a complement, since the addition of any other angle measure will produce a sum greater than 90. Therefore, right angles and obtuse angles do not have a complement.

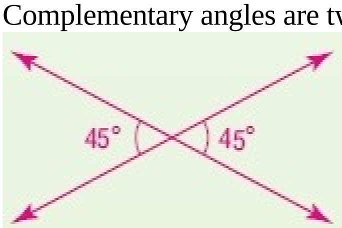
ANSWER:

Yes; angles that are right or obtuse do not have complements because
their measures are greater than or equal to 90.

50. **OPEN ENDED** Draw a pair of intersecting lines that forms a pair of complementary angles. Explain your reasoning.

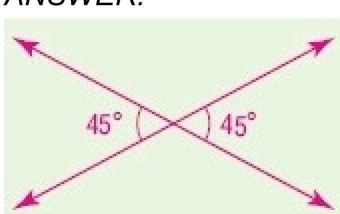
SOLUTION:

Complementary angles are two angles with measures that have a sum of 90.



Two $45°$ angles are complementary, since $45 + 45 = 90$.

ANSWER:



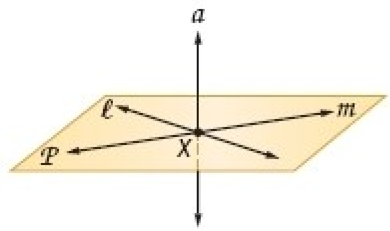
Two 45° angles are complementary.

51. **CHALLENGE** If a line, line segment, or ray is perpendicular to a plane, it is perpendicular to every line, line segment, or ray in the plane that intersects it.

a. If a line is perpendicular to each of two intersecting lines at their point of intersection, then the line is perpendicular to the plane determined by them. If line a is perpendicular to line ℓ and line m at point X, what must also be true?

b. If a line is perpendicular to a plane, then any line perpendicular to the given line at the point of intersection with the given plane is in the given plane. If line a is perpendicular to plane P and line m at point X, what must also be true?

c. If a line is perpendicular to a plane, then every plane containing the line is perpendicular to the given plane. If line a is perpendicular to plane P, what must also be true?



SOLUTION:

a. Since line ℓ and line m are contained in plane P, line a is perpendicular to plane P.

b. Since line m is perpendicular to line a at point X, line m is in plane P.

c. Since line a is perpendicular to plane P, any plane containing line a is perpendicular to plane P.

ANSWER:

a. Line a is perpendicular to plane P.

b. Line m is in plane P.

c. Any plane containing line a is perpendicular to plane P.

52. **WRITING IN MATH** Describe three different ways you can determine that an angle is a right angle.

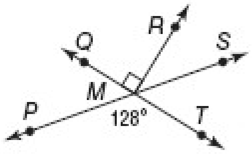
SOLUTION:

Sample answer: We can determine a right angle using three different ways. You can determine if an angle is right if it is marked with a right angle symbol, if the angle is a vertical pair with a right angle, or if the angle forms a linear pair with a right angle.

ANSWER:

Sample answer: You can determine if an angle is right if it is marked with a right angle symbol, if the angle is a vertical pair with a right angle, or if the angle forms a linear pair with a right angle.

53. What is $m\angle RMS$ in the figure below?



A 26
B 38
C 52
D 128

SOLUTION:

$\angle PMT$ and $\angle SMT$ are supplementary, since $m\angle PMT + m\angle SMT = 180$.
Given that $m\angle PMT = 128$.

$$\begin{aligned} m\angle PMT + m\angle SMT &= 180 && \text{Def. of Supplementary Angles.} \\ 128 + m\angle SMT &= 180 && \text{Substitution.} \\ 128 - 128 + m\angle SMT &= 180 - 128 && -128 \text{ from each side.} \\ m\angle SMT &= 52 && \text{Simplify.} \end{aligned}$$

In the figure, $m\angle QMR + m\angle RMS + m\angle SMT = 180$.

Substitute $m\angle SMT = 52$ and $m\angle QMR = 90$.

$$\begin{aligned} m\angle QMR + m\angle RMS + m\angle SMT &= 180 && \text{Def.of Supplementary Angles} \\ 90 + m\angle RMS + 52 &= 180 && \text{Substitution.} \\ m\angle RMS + 142 &= 180 && \text{Addition.} \\ m\angle RMS + 142 - 142 &= 180 - 142 && \text{Subtract 142 from each side.} \\ m\angle RMS &= 38 && \text{Subtraction.} \end{aligned}$$

So, the correct option is B.

ANSWER:
B

54. **EXTENDED RESPONSE** For a fundraiser, a theater club is making 400 cookies. They want to make twice as many chocolate chip as peanut butter cookies and three times as many peanut butter as oatmeal raisin cookies. Determine how many of each type of cookie the theater club will make. Show your work.

SOLUTION:

Let *x* be the number of oatmeal raisin cookies. Then the number of peanut butter cookies is $3x,$ and the number of chocolate chip cookies is $2(3x) = 6x$.

The total number of cookies is $x + 3x + 6x = 400.$

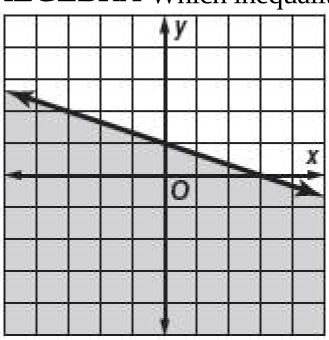
Solve for *x*.
$10x = 400$
Divide each side by 10.
$x = 40$

So, they need 40 oatmeal raisin, 3(40) or 120 peanut butter and 6(40) or 240 chocolate chip cookies.

ANSWER:

240 chocolate chip; 120 peanut butter; 40 oatmeal raisin; Sample answer: Let x = the number of oatmeal raisin cookies. The number of peanut butter cookies equals $3x$. The number of chocolate chip cookies equals $2(3x) = 6x$. The total number of cookies is $x + 3x + 6x = 400$. Solving for x, you get $x = 40$. So they need 40 oatmeal raisin, 3(40) or 120 peanut butter and 6(40) or 240 chocolate chip cookies.

55. **ALGEBRA** Which inequality is graphed below?



F $y > -\frac{1}{3}x + 1$

G $y < -\frac{1}{3}x + 1$

H $y \geq -\frac{1}{3}x + 1$

J $y \leq -\frac{1}{3}x + 1$

SOLUTION:

The graph has a solid line, so the inequality should have either the $\geq$ or $\leq$ symbol, which rules out options F and G. The inequality symbol in the equation should be $\leq$, since the graph has been shaded below the y-axis. So, the correct option is J.

ANSWER:

J

56. **SAT/ACT** One third of a number is three more than one fourth the same number. What is the number?
A 3
B 12
C 36
D 42
E 48

SOLUTION:
Let *x* be the number.

$$\frac{1}{3}x = 3 + \frac{1}{4}x \quad \text{Original equation}$$

$$\frac{1}{3}x - \frac{1}{4}x = 3 + \frac{1}{4}x - \frac{1}{4}x \quad \text{Subtract } \frac{1}{4}x \text{ from each side.}$$

$$\frac{4}{12}x - \frac{3}{12}x = 3 \quad \text{Rewrite fractions with common denominators.}$$

$$\frac{1}{12}x = 3 \quad \text{Simplify.}$$

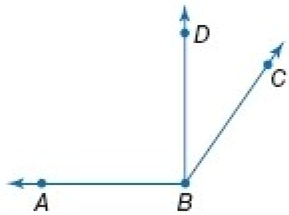
$$12\left(\frac{1}{12}x\right) = 12(3) \quad \text{Multiply each side by 12.}$$

$$x = 36 \quad \text{Simplify.}$$

The correct option is C.

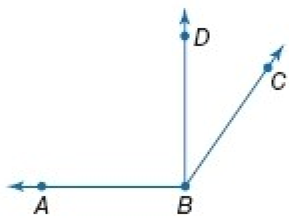
ANSWER:
C

Copy the diagram shown and extend each ray. Classify each angle as *right*, *acute*, or *obtuse*. Then use a protractor to measure the angle to the nearest degree.



57. $\angle ABC$

SOLUTION:

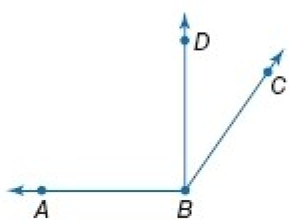


Using the corner of a sheet of paper, $\angle ABD$ in the figure appears to be a right angle, so $m\angle ABD = 90.$
Point *C* on angle $\angle ABC$ lies on the exterior angle of right angle $\angle ABD$, so $\angle ABC$ is an obtuse angle. Using a protractor, you will find that $m\angle ABC = 125$.

ANSWER:
125, obtuse

58. $\angle DBC$

SOLUTION:



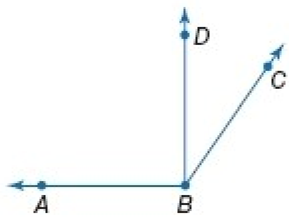
If the left-hand corner of a sheet of paper in set on *B* and the edge of the paper is aligned with $\overrightarrow{BD}$, then point *C* is in the interior of the right angle formed by the paper. So, the measure of $\angle DBC$ must be less than 90. Therefore, $\angle DBC$ is an acute angle. Use a protractor to find that $m\angle DBC = 35$.

ANSWER:
35, acute

1-5 Angle Relationships

59. $\angle ABD$

SOLUTION:



If the corner of a sheet of paper is placed at *B* and one edge of the paper is aligned with $\overrightarrow{AB}$, the other edge of the paper appears to be aligned with $\overrightarrow{BD}$. This would mean that $\angle ABD$ is probably a right angle Using a protactor, $m\angle ABD = 90$.

ANSWER:
90, right

Find the coordinates of the midpoint of a segment with the given endpoints.

60. *P*(3, –7), *Q*(9, 6)

SOLUTION:

If $\overline{PQ}$ has endpoints at $P(x_1, y_1)$ and $Q(x_2, y_2)$ in the coordinate plane, then the midpoint *M* of $\overline{PQ}$ is $M = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$.

Substitute $x_1 = 3$, $y_1 = -7$, $x_2 = 9$, and $y_2 = 6$ in $M = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$.

$$\begin{aligned} M &= \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right) && \text{Midpoint Formula} \\ &= \left(\frac{3 + 9}{2}, \frac{-7 + 6}{2}\right) && \text{Substitution.} \\ &= \left(\frac{12}{2}, \frac{-1}{2}\right) && \text{Addition.} \\ &= \left(6, -\frac{1}{2}\right) && \text{Division.} \end{aligned}$$

The midpoint of $\overline{PQ}$ is $\left(6, -\frac{1}{2}\right)$.

ANSWER:

$\left(6, -\frac{1}{2}\right)$

1-5 Angle Relationships

61. $A(-8, -5)$, $B(1, 7)$

SOLUTION:

If $\overline{AB}$ has endpoints at $A(x_1, y_1)$ and $B(x_2, y_2)$ in the coordinate plane, then the midpoint M of $\overline{AB}$ is $M = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$.

Substitute $x_1 = -8$, $y_1 = -5$, $x_2 = 1$, and $y_2 = 7$ in $M = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$.

$$\begin{aligned} M &= \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right) && \text{Midpoint Formula} \\ &= \left(\frac{-8 + 1}{2}, \frac{-5 + 7}{2}\right) && \text{Substitution.} \\ &= \left(\frac{-7}{2}, \frac{2}{2}\right) && \text{Addition.} \\ &= \left(-3\frac{1}{2}, 1\right) && \text{Division.} \end{aligned}$$

The midpoint of $\overline{AB}$ is $\left(-3\frac{1}{2}, 1\right)$.

ANSWER:

$\left(-3\frac{1}{2}, 1\right)$

62. $J(-7, 4)$, $K(3, 1)$

SOLUTION:

If $\overline{JK}$ has endpoints at $J(x_1, y_1)$ and $K(x_2, y_2)$ in the coordinate plane, then the midpoint M of $\overline{JK}$ is

$M = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$.

Substitute $x_1 = -7$, $y_1 = 4$, $x_2 = 3$, and $y_2 = 1$ in $M = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$.

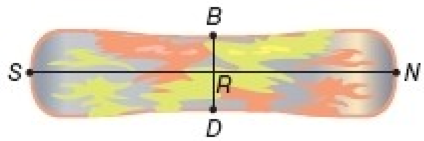
$$\begin{aligned} M &= \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right) && \text{Midpoint Formula} \\ &= \left(\frac{-7 + 3}{2}, \frac{4 + 1}{2}\right) && \text{Substitution.} \\ &= \left(\frac{4}{2}, \frac{5}{2}\right) && \text{Addition.} \\ &= \left(2, 2\frac{1}{2}\right) && \text{Division.} \end{aligned}$$

The midpoint of $\overline{JK}$ is $\left(2, 2\frac{1}{2}\right)$.

ANSWER:

$\left(-2, 2\frac{1}{2}\right)$

63. **SNOWBOARDING** In the design on the snowboard shown, $\overline{BD}$ bisects $\overline{SN}$ at *R*. If *SN* = 163 centimeters, find *RN*.



SOLUTION:

Since $\overline{BD}$ bisects $\overline{SN}$ at *R*, $SR = RN$.

In the figure, $SN = SR + RN.$

Given that $SN = 163.$

Substitute.

$163 = RN + RN$

$163 = 2RN$

Divide each side by 2.

$\frac{163}{2} = \frac{2RN}{2}$

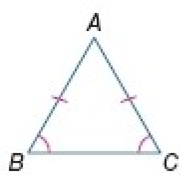
$81.5 = RN$

So, $RN = 81.5$ centimeters.

ANSWER:

81.5 cm

Name the congruent sides and angles in each figure.

64. 

SOLUTION:

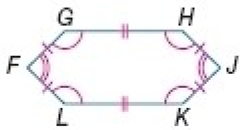
Segments that have the same measure are called congruent segments. Angles that have the same angle measures are called congruent angles.
Here the segments marked with the same symbol are congruent to each other. Similarly, the angles marked with the same symbol are congruent to each other.

Congruent segments: $\overline{AB} \cong \overline{AC}$
Congruent angles: $\angle B \cong \angle C$

ANSWER:

$\overline{AB} \cong \overline{AC};\ \angle B \cong \angle C$

1-5 Angle Relationships

65. 

SOLUTION:

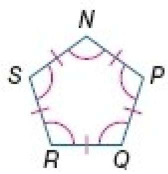
Segments that have the same measure are called congruent segments. Angles that have the same angle measures are called congruent angles.
Here the segments marked with the same symbol are congruent to each other. Similarly, the angles marked with the same symbol are congruent to each other.

Congruent segments: $\overline{FG} \cong \overline{HJ} \cong \overline{JK} \cong \overline{FL}, \overline{GH} \cong \overline{LK}$

Congruent angles: $\angle F \cong \angle J, \angle G \cong \angle H \cong \angle K \cong \angle L$

ANSWER:

$\overline{FG} \cong \overline{HJ} \cong \overline{JK} \cong \overline{FL}, \overline{GH} \cong \overline{LK};$
$\angle F \cong \angle J, \angle G \cong \angle H \cong \angle K \cong \angle L$

66. 

SOLUTION:

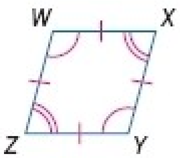
Segments that have the same measure are called congruent segments. Angles that have the same angle measures are called congruent angles.
Here the segments marked with the same symbol are congruent to each other. Similarly, the angles marked with the same symbol are congruent to each other.

Congruent segments: $\overline{NP} \cong \overline{PQ} \cong \overline{QR} \cong \overline{RS} \cong \overline{SN}$

Congruent angles: $\angle N \cong \angle P \cong \angle Q \cong \angle R \cong \angle S$

ANSWER:

$\overline{NP} \cong \overline{PQ} \cong \overline{QR} \cong \overline{RS} \cong \overline{SN};$
$\angle N \cong \angle P \cong \angle Q \cong \angle R \cong \angle S$

67. 

SOLUTION:

Segments that have the same measure are called congruent segments. Angles that have the same angle measures are called congruent angles.
Here the segments marked with the same symbol are congruent to each other. Similarly, the angles marked with the same symbol are congruent to each other.
Congruent segments: $\overline{WX} \cong \overline{XY} \cong \overline{YZ} \cong \overline{ZW}$
Congruent angles: $\angle W \cong \angle Y$, $\angle X \cong \angle Z$

ANSWER:

$\overline{WX} \cong \overline{XY} \cong \overline{YZ} \cong \overline{ZW}$;
$\angle W \cong \angle Y$, $\angle X \cong \angle Z$