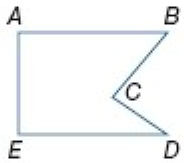
<u>**1-6 Two-Dimensional Figures**</u>

Name each polygon by its number of sides. Then classify it as *convex* or *concave* and *regular* or *irregular*.

1. 

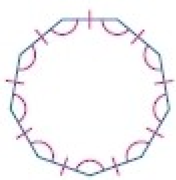
SOLUTION:

The polygon has 5 sides. A polygon with 5 sides is a pentagon.

If you draw the line containing the side $\overline{BC}$ it will contain some points in the interior of the polygon. So, the polygon is concave. Since it is concave, it is irregular.

ANSWER:

pentagon; concave; irregular

2. 

SOLUTION:

The polygon has 9 sides. A polygon with 9 sides is a nonagon.
None of the lines containing the sides will have points in the interior of the polygon. So, the polygon is convex.
All sides of the polygon are congruent and all angles are congruent. So it is regular.

ANSWER:

nonagon; convex; regular

SIGNS Identify the shape of each traffic sign and classify it as *regular* or *irregular*.

3. stop

SOLUTION:

Stop signs are constructed in the shape of a polygon with 8 sides of equal length. The polygon has 8 sides. A polygon with 8 sides is an octagon. All sides of the polygon are congruent and all angles are also congruent. So it is regular.

ANSWER:

octagon; regular

4. caution or warning

SOLUTION:

Caution and warning signs are constructed in the shape of a polygon with 4 sides of equal length.
The polygon has 4 sides. A polygon with 4 sides is a quadrilateral.
All sides of the polygon are congruent. So it is regular.

ANSWER:

quadrilateral; regular

5. slow moving vehicle

SOLUTION:

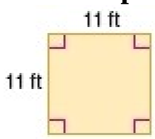
Slow moving vehicle signs are constructed in the shape of a polygon with 6 sides of alternating length. The polygon has 6 sides. A polygon with 6 sides is a hexagon. All sides of the polygon are not congruent. So it is irregular.

ANSWER:

hexagon; irregular

1-6 Two-Dimensional Figures

Find the perimeter or circumference and area of each figure. Round to the nearest tenth.

6. 

SOLUTION:

The perimeter of a square with side *s* is given by $P = 4s$.

Substitute 11 for *s*.

$$\begin{aligned} P &= 4s && \text{Perimeter Formula for a square} \\ &= 4(11) && \text{Substitution.} \\ &= 44 && \text{Simplify.} \end{aligned}$$

The perimeter of the figure is 44 ft.

The area of a square with side *s* is given by $A = s^2$.

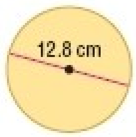
Substitute 11 for *s*.

$$\begin{aligned} A &= s^2 && \text{Area Formula for a square} \\ &= (11)^2 && \text{Substitution.} \\ &= 121 && \text{Simplify.} \end{aligned}$$

The area of the square is 121 ft^2.

ANSWER:

44 ft; 121 ft^2

7. 

SOLUTION:

The circumference of a circle with radius r is given by $C = 2\pi r$.
The diameter of the circle is 12.8 cm.

$$radius = \frac{diameter}{2} \quad \text{Radius is } \frac{1}{2} \text{ of the Diameter.}$$
$$r = \frac{12.8}{2} \quad \text{Substitution.}$$
$$= 6.4 \quad \text{Simplify.}$$

Substitute 6.4 for r.

$$C = 2\pi r \quad \text{Circumference Formula}$$
$$= 2\pi(6.4) \quad \text{Substitution.}$$
$$\approx 40.2 \quad \text{Simplify.}$$

The circumference of the circle is about 40.2 cm.
The area of a circle with radius r is given by $A = \pi r^2$.

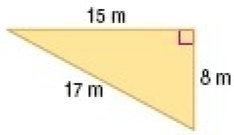
Substitute 6.4 for r.

$$A = \pi r^2 \quad \text{Area Formula}$$
$$= \pi(6.4)^2 \quad \text{Substitution.}$$
$$\approx 128.7 \quad \text{Simplify.}$$

The area of the circle is about 128.7 cm^2.

ANSWER:

$\approx 40.2\ cm; \approx 128.7\ cm^2$

8. 

SOLUTION:

Add all the sides to find the perimeter of a triangle.

$$\begin{aligned} P &= b + c + d && \text{Perimeter Formula} \\ &= 17 + 15 + 8 && \text{Substitution.} \\ &= 40 && \text{Addition.} \end{aligned}$$

The perimeter of the triangle is 40 m.

The area of a triangle with base *b* and height *h* is given by $A = \frac{1}{2}bh$.

Here the base is 8 and height is 15.

$$\begin{aligned} A &= \frac{1}{2}bh && \text{Area Formula} \\ &= \frac{1}{2} \cdot 8 \cdot 15 && \text{Substitution.} \\ &= 60 && \text{Multiply.} \end{aligned}$$

The area of the triangle is 60 m^2.

ANSWER:

40 m; 60 m^2

9. **MULTIPLE CHOICE** Vanesa is making a banner for the game. She has 20 square feet of fabric. What shape will use *most* or all of the fabric?
A a square with a side length of 4 feet
B a rectangle with a length of 4 feet and a width of 3.5 feet
C a circle with a radius of about 2.5 feet
D a right triangle with legs of about 5 feet

SOLUTION:
The area of a square with side length 4 ft is

$A = s^2$ Area Formula
$= 4^2$ Substitution.
$= 16$ Simplify.

Area of square is 16 ft^2.

The area of a rectangle with a length of 4 feet and a width of 3.5 feet is

$A = \ell w$ Area Formula
$= 4 \cdot 3.5$ Substitution.
$= 14$ Simplify.

Area of rectangle is 14 ft^2.

The area of a circle with radius 2.5 ft is

$A = \pi r^2$ Area Formula
$= \pi(2.5)^2$ Substitution.
≈ 19.6 Simplify.

The area of the circle is about 19.6 ft^2.

The area of a right triangle with legs of about 5 ft is

$A = \frac{1}{2}bh$ Area Formula
$= \frac{1}{2}5 \cdot 5$ Substitution.
$= 12.5$ Multiply.

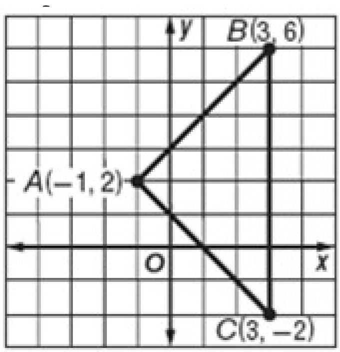
The area of the triangle is 12.5 ft^2.

So, the shape which uses the most of the fabric is the circle.
The correct answer is C.

ANSWER:
C

10. **CCSS REASONING** Find the perimeter and area of ΔABC with vertices $A(-1, 2)$, $B(3, 6)$, and $C(3, -2)$.

SOLUTION:
Graph ΔABC.



To find the perimeter of ΔABC, first find the lengths of each side.

Use the Distance Formula to find the lengths of $\overline{AB}$, $\overline{BC}$, and $\overline{AC}$.

$\overline{AB}$ has end points $A(–1, 2)$ and $B(3, 6)$.

$$\begin{aligned} AB &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} && \text{Distance Formula} \\ &= \sqrt{(3-(-1))^2 + (6-2)^2} && \text{Substitution.} \\ &= \sqrt{4^2 + 4^2} && \text{Subtraction.} \\ &= \sqrt{16+16} && \text{Square terms.} \\ &= \sqrt{32} && \text{Addition.} \end{aligned}$$

$\overline{BC}$ has endpoints $B(3, 6)$ and $C(3, –2)$.

$$\begin{aligned} BC &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} && \text{Distance Formula} \\ &= \sqrt{(3-3)^2 + (6-(-2))^2} && \text{Substitution.} \\ &= \sqrt{0^2 + 8^2} && \text{Subtraction.} \\ &= 8 && \text{Simplify.} \end{aligned}$$

$\overline{AC}$ has endpoints $A(–1, 2)$ and $C(3, –2)$.

$$\begin{aligned} AC &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} && \text{Distance Formula} \\ &= \sqrt{(3-(-1))^2 + (-2-2)^2} && \text{Substitution.} \\ &= \sqrt{4^2 + (-4)^2} && \text{Subtraction.} \\ &= \sqrt{16+16} && \text{Square terms.} \\ &= \sqrt{32} && \text{addition.} \end{aligned}$$

The perimeter of ΔABC is $8 + \sqrt{32} + \sqrt{32}$ or $8 + 2\sqrt{32} \approx 19.3$ units.

Find the area of ΔABC.

To find the area of ΔABC, find the lengths of the height and base. The height of the triangle is the horizontal distance from A to $\overline{BC}$.

Counting the squares on the graph, the height is 4 units. The length of $\overline{BC}$ is 8 units.

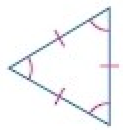
$$A = \frac{1}{2}bh \quad \text{Area Formula for a Triangle}$$
$$= \frac{1}{2} \cdot 8 \cdot 4 \quad \text{Substitution.}$$
$$= 16 \quad \text{Simplify.}$$

The area of ΔABC is 16 square units.

ANSWER:

$P = 8 + 2\sqrt{32}$ or about 19.3 units; $A = 16$ units^2

Name each polygon by its number of sides. Then classify it as *convex* or *concave* and *regular* or *irregular*.

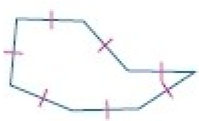
11. 

SOLUTION:

The polygon has 3 sides, so it is a triangle.
None of the lines containing the sides have points in the interior of the polygon. So, the polygon is convex. (All triangles are convex.)
All sides of the polygon are congruent and all angles are also congruent. So it is regular.

ANSWER:

triangle; convex; regular

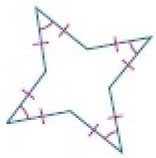
12. 

SOLUTION:

The polygon has 7 sides, so it is a heptagon.
Some of the lines containing the sides will have points in the interior of the polygon. So, the polygon is concave.
Since the polygon is concave, it is irregular.

ANSWER:

heptagon; concave; irregular

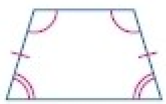
13. 

SOLUTION:

The polygon has 8 sides, so it is a octagon.
At least one of the lines containing the sides will have points in the interior of the polygon. So, the polygon is concave.
Since it is concave, it is irregular.

ANSWER:

octagon; concave; irregular

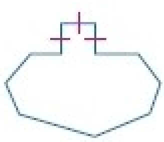
14. 

SOLUTION:

The polygon has 4 sides, so it is a quadrilateral.
None of the lines containing the sides will have points in the interior of the polygon. So, the polygon is convex.
All sides of the polygon are not congruent. So it is irregular.

ANSWER:

quadrilateral; convex; irregular

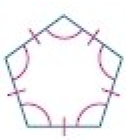
15. 

SOLUTION:

The polygon has 11 sides, so it is an hendecagon.
At lease one of the lines containing the sides will have points in the interior of the polygon. So, the polygon is concave. Since it is concave, it is irregular.

ANSWER:

hendecagon; concave; irregular

16. 

SOLUTION:

The polygon has 5 sides, so it is a pentagon.
None of the lines containing the sides will have points in the interior of the polygon. So, the polygon is convex.
All sides of the polygon are congruent and all angles are also congruent. So it is regular.

ANSWER:

pentagon; convex; regular

1-6 Two-Dimensional Figures

Find the perimeter or circumference and area of each figure. Round to the nearest tenth.

17. 

SOLUTION:

Use the formula for perimeter of a rectangle.

$$\begin{aligned} P &= 2\ell + 2w && \text{Perimeter Formula for a Rectangle} \\ &= 2(2.8) + 2(1.1) && \text{Substitution.} \\ &= 5.6 + 2.2 && \text{Multiply.} \\ &= 7.8 && \text{Addition.} \end{aligned}$$

The perimeter of the rectangle is 7.8 m.

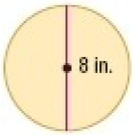
Use the formula for area of a rectangle.

$$\begin{aligned} A &= \ell w && \text{Area Formula for a Rectangle} \\ &= 2.8 \cdot 1.1 && \text{Substitution.} \\ &= 3.08 && \text{Multiply.} \\ &\approx 3.1 \end{aligned}$$

The area of the rectangle is about 3.1 m^2.

ANSWER:

7.8 m; ≈ 3.1 m^2

18. 

SOLUTION:

Use the formula for circumference of a circle.
The diameter of the circle is 8 in. So, the radius of the circle is 4 in.

$$\begin{aligned} C &= 2\pi r && \text{Circumference Formula} \\ &= 2\pi(4) && \text{Substitution.} \\ &= 8\pi && \text{Multiply.} \\ &\approx 25.1 && \text{Multiply.} \end{aligned}$$

The circumference of the circle is about 25.1 in.

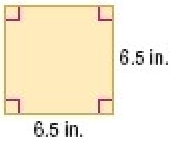
Use the formula for area of a circle.

$$\begin{aligned} A &= \pi r^2 && \text{Area Formula for a Circle} \\ &= \pi(4)^2 && \text{Substitution.} \\ &= 16\pi && \text{Square 4.} \\ &\approx 50.3 && \text{Multiply.} \end{aligned}$$

The area of the circle is about 50.3 in^2.

ANSWER:

8π in. or ≈ 25.1 in.; 16π in^2 or ≈ 50.3 in^2

19. 

SOLUTION:

Use the formula for perimeter of a square.

$$\begin{aligned} P &= 4s && \text{Perimeter Formula for a Square} \\ &= 4(6.5) && \text{Substitution.} \\ &= 26 && \text{Multiply.} \end{aligned}$$

The perimeter of the square is 26 in.

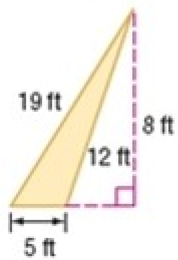
Use the formula for area of a square.

$$\begin{aligned} A &= s^2 && \text{Area Formula for a Square} \\ &= 6.5^2 && \text{Substitution.} \\ &= 42.25 && \text{Square term.} \\ &\approx 42.3 \end{aligned}$$

The area of the square is about 42.3 in^2.

ANSWER:

26 in.; 42.3 in^2

20. 

SOLUTION:

Add all the sides to find the perimeter of a triangle.

$$\begin{aligned} P &= b + c + d && \text{Perimeter Formula for a Triangle} \\ &= 19 + 12 + 5 && \text{Substitution.} \\ &= 36 && \text{Addition.} \end{aligned}$$

The perimeter of the triangle is 36 ft.

The area of a triangle with base *b* and height *h* is given by $A = \frac{1}{2}bh$.

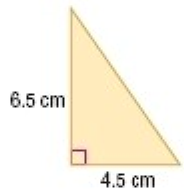
Here the base is 5 ft and height is 8 ft.

$$\begin{aligned} A &= \frac{1}{2}bh && \text{Area Formula for a Triangle} \\ &= \frac{1}{2} \cdot 8 \cdot 5 && \text{Substitution.} \\ &= 20 && \text{Multiply.} \end{aligned}$$

The area of the triangle is 20 ft^2.

ANSWER:

36 ft; 20 ft^2

21. 

SOLUTION:

To find the missing length, use the Pythagorean Theorem.

Let *c* be the missing length.
Then,

$$\begin{aligned} c &= \sqrt{a^2 + b^2} && \text{Pythagorean Theorem.} \\ &= \sqrt{(6.5)^2 + (4.5)^2} && \text{Substitution.} \\ &= \sqrt{42.25 + 20.25} && \text{Square terms.} \\ &= \sqrt{62.50} && \text{Addition.} \\ &\approx 7.9. \end{aligned}$$

Add all the sides to find the perimeter of a triangle.

$$\begin{aligned} P &= a + b + c && \text{Perimeter Formula for a Triangle} \\ &\approx 6.5 + 4.5 + 7.9 && \text{Substitution.} \\ &\approx 18.9 && \text{Addition.} \end{aligned}$$

The perimeter of the triangle is about 18.9 cm.

The area of a triangle with base *b* and height *h* is given by $A = \frac{1}{2}bh$.

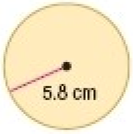
Here the base is 4.5 ft and height is 6.5 ft.

$$\begin{aligned} A &= \tfrac{1}{2}bh && \text{Area Formula for a Triangle} \\ &= \tfrac{1}{2} \cdot 6.5 \cdot 4.5 && \text{Substitution.} \\ &= 14.625 && \text{Multiply.} \\ &\approx 14.6 \end{aligned}$$

The area of the triangle is about 14.6 cm^2.

ANSWER:

≈18.9 cm; ≈14.6 cm^2

22. 

SOLUTION:

Use the formula for circumference of a circle.

$$
\begin{aligned}
C &= 2\pi r && \text{Circumference Formula} \\
&= 2\pi(5.8) && \text{Substitution.} \\
&\approx 36.4 && \text{Mutliply.}
\end{aligned}
$$

The circumference of the circle is about 36.4 cm.

Use the formula for area of a circle.

$$
\begin{aligned}
A &= \pi r^2 && \text{Area Formula} \\
&= \pi(5.8)^2 && \text{Substitution.} \\
&\approx 105.7 && \text{Square and Multiply.}
\end{aligned}
$$

The area of the circle is about 105.7 cm^2.

ANSWER:

≈36.4 cm; ≈105.7 cm^2

23. **CRAFTS** Joy has a square picture that is 4 inches on each side. The picture is framed with a length of ribbon. She wants to use the same piece of ribbon to frame a circular picture. What is the maximum radius of the circular frame?

SOLUTION:

Find the perimeter of the square picture.

Use the formula for the perimeter of a square with side *s*.

$$\begin{aligned} P &= 4s && \text{Perimeter Formula} \\ &= 4(4) && \text{Substitution.} \\ &= 16 && \text{Multiply.} \end{aligned}$$

The perimeter of the square picture is 16 inches.
So, the length of the ribbon is 16 inches.

If the picture is circular, then its circumference is 16 inches.
Use the circumference formula to solve for *r*.

$$\begin{aligned} C &= 2\pi r && \text{Circumference Formula} \\ 16 &= 2\pi r && \text{Substitution.} \\ \frac{16}{2\pi} &= \frac{2\pi r}{2\pi} && \text{Divide each side by } 2\pi. \\ \frac{8}{\pi} &= r && \text{Simplify.} \end{aligned}$$

$2.55 \approx r$

The maximum radius of circular frame should be about 2.55 in.

ANSWER:

≈2.55 in.

24. **LANDSCAPING** Mr. Jackson has a circular garden with a diameter of 10 feet surrounded by edging. Using the same length of edging, he is going to create a square garden. What is the maximum side length of the square?

SOLUTION:

The diameter of the garden is 10 feet. So the radius is 5 feet.

To find the length of the edge, find the circumference of the circular garden.

$C = 2\pi r$ Circumference Formula

$C = 2\pi(5)$ Substitution.

≈ 31.4 Multiply.

The length of the edging is about 31.4 ft.

Now, 31.4 is the perimeter of the square garden.
Equate it to 4*s* and solve for *s*.

$P = 4s$ Perimeter Formula

$31.4 = 4s$ Substitution.

$\frac{31.4}{4} = \frac{4s}{4}$ Divide each side by 4.

$7.85 = s$ Simplify.

The maximum side length of the square is about 7.85 ft.

ANSWER:

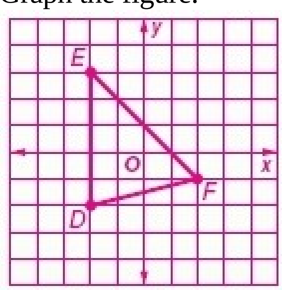
≈7.85 ft

CCSS REASONING Graph each figure with the given vertices and identify the figure. Then find the perimeter and area of the figure.

25. *D*(–2, –2), *E*(–2, 3), *F*(2, –1)

SOLUTION:

Graph the figure.



The polygon has 3 sides. So, it is a triangle.

To find the perimeter of ΔDEF, first find the lengths of each side. Counting the squares on the grid, we find that *ED* = 5.

Use the Distance Formula to find the lengths of $\overline{EF}$ and $\overline{DF}$.

$\overline{EF}$ has end points *E*(–2, 3) and *F*(2, –1).

$EF = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

Substitute.

$$\begin{aligned} EF &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} && \text{Distance Formula} \\ &= \sqrt{(2 - (-2))^2 + (-1 - 3)^2} && \text{Substitution.} \\ &= \sqrt{4^2 + (-4)^2} && \text{Subtraction.} \\ &= \sqrt{16 + 16} && \text{Sqaure terms.} \\ &= \sqrt{32} && \text{Addition.} \end{aligned}$$

$\overline{DF}$ has end points $D(-2, -2)$ and $F(2, -1)$.

$$\begin{aligned} DF &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} && \text{Distance Formula} \\ &= \sqrt{(2 - (-2))^2 + (-1 - (-2))^2} && \text{Substitution.} \\ &= \sqrt{4^2 + 1^2} && \text{Subtraction.} \\ &= \sqrt{16 + 1} && \text{Square terms.} \\ &= \sqrt{17} && \text{Addition.} \end{aligned}$$

The perimeter of ΔDEF is

$$\begin{aligned} P &= ED + EF + DF && \text{Perimeter Formula} \\ &= 5 + \sqrt{32} + \sqrt{17} && \text{Substitution.} \\ &\approx 14.8 && \text{Simplify.} \end{aligned}$$

Find the area of ΔDEF.

To find the area of ΔDEF, find the lengths of the height and base. The height of the triangle is the horizontal distance from F to $\overline{ED}$. Counting the squares on the graph, the height is 4 units. The length of $\overline{ED}$ is 5 units.

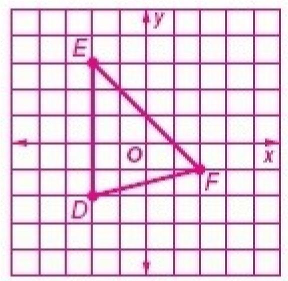
$$\begin{aligned} A &= \frac{1}{2}bh && \text{Area Formula} \\ &= \frac{1}{2} \cdot 5 \cdot 4 && \text{Substitution.} \\ &= 10 && \text{Multiply.} \end{aligned}$$

The area of ΔDEF is 10 square units.

ANSWER:

triangle;

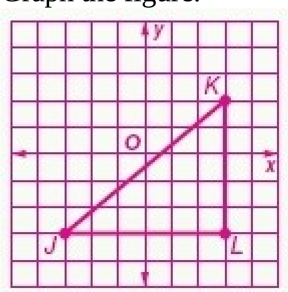
$P = 5 + \sqrt{32} + \sqrt{17}$ **units.** or about 14.78 units; A = 10 units^2



26. $J(–3, –3)$, $K(3, 2)$, $L(3, –3)$

SOLUTION:

Graph the figure.



The polygon has 3 sides. So, it is a triangle.

To find the perimeter of ΔJKL, first find the lengths of each side. Counting the squares on the grid, we find that $JL = 6$ and $KL = 5$.

Use the Distance Formula to find the length of $\overline{JK}$.

$\overline{JK}$ has end points $J(–3, –3)$ and $K(3, 2)$.

$$\begin{aligned} JK &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} && \text{Distance Formula} \\ &= \sqrt{(3 - (-3))^2 + (2 - (-3))^2} && \text{Substitution.} \\ &= \sqrt{6^2 + 5^2} && \text{Subtraction.} \\ &= \sqrt{36 + 25} && \text{Square terms.} \\ &= \sqrt{61} && \text{Addition.} \\ &\approx 7.8 \end{aligned}$$

The perimeter of ΔJKL is $KL + JL + JK$.

$$\begin{aligned} P &= KL + JL + JK && \text{Perimeter Formula} \\ &= 5 + 6 + \sqrt{61} && \text{Substitution.} \\ &= 11 + \sqrt{61} && \text{Addition.} \\ &\approx 18.8 \end{aligned}$$

Find the area of ΔJKL.

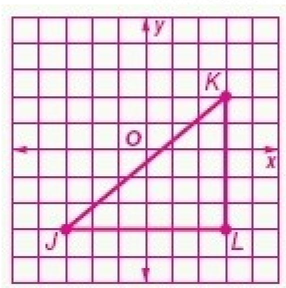
Here the base is 6 and the height is 5.

$$A = \frac{1}{2}bh \quad \text{Area Formula for a Triangle}$$
$$= \frac{1}{2} \cdot 5 \cdot 6 \quad \text{Substitution.}$$
$$= 15 \quad \text{Multiply.}$$

The area of ΔJKL is 15 square units.

ANSWER:

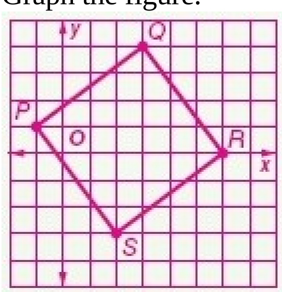
triangle; $P = 11 + \sqrt{61} \approx 18.8$ units.; $A = 15$ units^2



27. $P(-1, 1)$, $Q(3, 4)$, $R(6, 0)$, $S(2, -3)$

SOLUTION:

Graph the figure.



The polygon has 4 sides. So it is a quadrilateral.

To find the perimeter of the quadrilateral, find the length of each side.

Use the Distance Formula to find the lengths of each side.

$\overline{PQ}$ has end points $P(-1, 1)$ and $Q(3, 4)$.

$$PQ = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \quad \text{Distance Formula}$$
$$= \sqrt{(3 - (-1))^2 + (4 - 1)^2} \quad \text{Substitution.}$$
$$= \sqrt{4^2 + 3^2} \quad \text{Subtraction.}$$
$$= \sqrt{16 + 9} \quad \text{Square terms.}$$
$$= \sqrt{25} \quad \text{Addition.}$$
$$= 5 \quad \text{Simplify.}$$

$\overline{QR}$ has end points $Q(3, 4)$ and $R(6, 0)$.

$$QR = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \quad \text{Distance Formula}$$
$$= \sqrt{(6-3)^2 + (0-4)^2} \quad \text{Substitution.}$$
$$= \sqrt{3^2 + (-4)^2} \quad \text{Subtraction.}$$
$$= \sqrt{9+16} \quad \text{Square terms.}$$
$$= \sqrt{25} \quad \text{Addition.}$$
$$= 5 \quad \text{Simplify.}$$

$\overline{RS}$ has end points R(6, 0) and S(2, –3).

$$RS = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \quad \text{Distance Formula}$$
$$= \sqrt{(2-6)^2 + (-3-0)^2} \quad \text{Substitution.}$$
$$= \sqrt{(-4)^2 + (-3)^2} \quad \text{Subtraction.}$$
$$= \sqrt{16+9} \quad \text{Square terms.}$$
$$= \sqrt{25} \quad \text{Addition.}$$
$$= 5 \quad \text{Simplify.}$$

$\overline{PS}$ has end points P(–1, 1) and S(2, –3).

$$PS = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \quad \text{Distance Formula}$$
$$= \sqrt{(2-(-1))^2 + (-3-1)^2} \quad \text{Substitution.}$$
$$= \sqrt{3^2 + (-4)^2} \quad \text{Subtraction.}$$
$$= \sqrt{9+16} \quad \text{Square terms.}$$
$$= \sqrt{25} \quad \text{Addition.}$$
$$= 5 \quad \text{Simplify.}$$

Note that all the sides are congruent. Using a protractor, all four angles of the quadrilateral are right angles. So, it is a square.

Use the formula for the perimeter of a square with sides of length *s*.

$$P = 4s \quad \text{Perimeter Formula}$$
$$= 4(5) \quad \text{Substitution.}$$
$$= 20 \quad \text{Multiply.}$$

The perimeter of the square is 20 units.

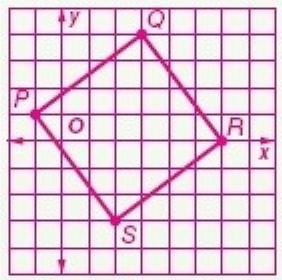
Use the area formula for a square with sides of length *s*.

$$\begin{aligned} A &= s^2 && \text{Area Formula} \\ &= (5)^2 && \text{Substitution.} \\ &= 25 && \text{Square term.} \end{aligned}$$

The area of the square is 25 square units.

ANSWER:

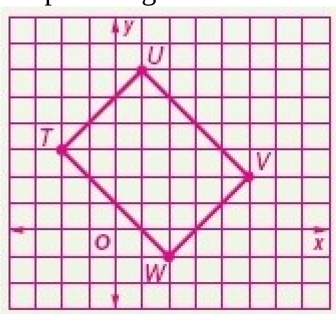
quadrilateral or square; P = 20 units; A = 25 units^2



28. $T(-2, 3)$, $U(1, 6)$, $V(5, 2)$, $W(2, -1)$

SOLUTION:

Graph the figure.



The polygon has 4 sides. So it is a quadrilateral.

To find the perimeter of the quadrilateral, find the length of each side.

Use the Distance Formula to find the lengths of each side.

$\overline{TU}$ has end points $T(-2, 3)$ and $U(1, 6)$.

$$\begin{aligned} TU &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} && \text{Distance Formula} \\ &= \sqrt{(1 - (-2))^2 + (6 - 3)^2} && \text{Substitution.} \\ &= \sqrt{3^2 + 3^2} && \text{Subtraction.} \\ &= \sqrt{9 + 9} && \text{Square terms.} \\ &= \sqrt{18} && \text{Addition.} \end{aligned}$$

$\overline{UV}$ has end points $U(1, 6)$ and $V(5, 2)$.

$$UV = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$ Distance Formula
$$= \sqrt{(5-1)^2 + (2-6)^2}$$ Substitution.
$$= \sqrt{4^2 + (-4)^2}$$ Subtraction.
$$= \sqrt{16+16}$$ Square terms.
$$= \sqrt{32}$$ Addition.

$\overline{VW}$ has end points $V(5, 2)$ and $W(2, -1)$.

$$VW = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$ Distance Formula
$$= \sqrt{(2-5)^2 + (-1-2)^2}$$ Substitution.
$$= \sqrt{(-3)^2 + (-3)^2}$$ Subtraction.
$$= \sqrt{9+9}$$ Square terms.
$$= \sqrt{18}$$ Addition.

$\overline{TW}$ has end points $T(-2, 3)$ and $W(2, -1)$.

$$TW = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$ Distance Formula
$$= \sqrt{(2-(-2))^2 + (-1-3)^2}$$ Substitution.
$$= \sqrt{4^2 + (-4)^2}$$ Subtraction.
$$= \sqrt{16+16}$$ Square terms.
$$= \sqrt{32}$$ Addition.

Note that opposite sides are congruent. Using a protractor, all the angles are right angles. So, the quadrilateral is a rectangle.

Use the formula for the perimeter of a rectangle with length ℓ and width w.

$$P = 2\ell + 2w$$ Perimeter Formula
$$= 2\sqrt{18} + 2\sqrt{32}$$ Substitution.
$$\approx 19.8$$ Simplify.

The perimeter of the rectangle is about 19.8 units.

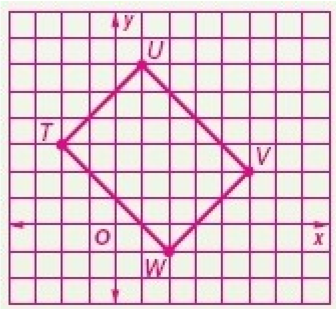
Use the area formula for a rectangle with length ℓ and width w.

$$A = \ell w$$ Area Formula
$$= \sqrt{32} \cdot \sqrt{18}$$ Substitution.
$$= \sqrt{576}$$ Multiply.
$$= 24$$ Simplify.

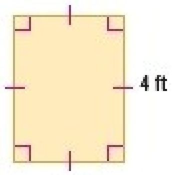
The area of the rectangle is 24 square units.

ANSWER:

quadrilateral or rectangle; $P = 2\sqrt{18} + 2\sqrt{32}$ or about 19.8 units; A = 24 units^2



29. **CHANGING DIMENSIONS** Use the rectangle below.



a. Find the perimeter of the rectangle.
b. Find the area of the rectangle.
c. Suppose the length and width of the rectangle are doubled. What effect would this have on the perimeter? the area? Justify your answer.
d. Suppose the length and width of the rectangle are halved. What effect does this have on the perimeter? the area? Justify your answer.

SOLUTION:

a. Use the formula for the perimeter of a rectangle with length ℓ and w.

$$\begin{aligned} P &= 2\ell + 2w && \text{Perimeter Formula} \\ &= 2(3) + 2(4) && \text{Substitution.} \\ &= 6 + 8 && \text{Multiply.} \\ &= 14 && \text{Addition.} \end{aligned}$$

The perimeter of the rectangle is 14 ft.

b. Use the formula for the area of a rectangle with length ℓ and width w.

$$\begin{aligned} A &= \ell w && \text{Area Formula} \\ &= 3 \cdot 4 && \text{Substitution.} \\ &= 12 && \text{Multiply.} \end{aligned}$$

The area of the rectangle is 12 ft^2.

c. If the length and width of the rectangle are doubled, then the dimensions of the rectangle are 6 ft and 8 ft. The perimeter of a rectangle with dimensions 6 ft and 8 ft is 2(6+8) or 28 ft, which is twice the perimeter of the original figure since $2 \cdot 14 = 28$ So, if the length and width of the rectangle are doubled, then the perimeter is also doubled. The area of the rectangle with dimensions 6 ft and 8 ft is 48 ft^2, which is 4 times the area of the original figure since $4 \cdot 12 = 48$. So, the area quadruples.

d. If the length and width of the rectangle are halved, then the dimensions of the rectangle are 1.5 ft and 2 ft. The perimeter of a rectangle with dimensions 1.5 ft and 2 ft is 2(1.5 + 2) or 7 ft, which is half the perimeter of the original figure since $\left(\frac{1}{2}\right) \cdot 14 = 7$. So, if the length and width of the rectangle are halved, then the perimeter also halved. The area of the rectangle with dimensions 1.5 ft and 2 ft is 3 ft^2, which is $\frac{1}{4}$ times the area of the original figure since $\frac{1}{4} \cdot 12 = 3$.

So, the area is divided by 4.

ANSWER:

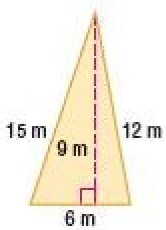
a. 14 ft

b. 12 ft^2

c. The perimeter doubles; the area quadruples. The perimeter of a rectangle with dimensions 6 ft and 8 ft is 28 ft, which is twice the perimeter of the original figure since 2 • 14 ft = 28 ft. The area of a rectangle with dimensions 6 ft and 8 ft is 48 ft^2, which is four times the area of the original figure since 4 • 12 ft^2 = 48 ft^2.

d. The perimeter is halved; the area is divided by 4. The perimeter of a rectangle with dimensions 1.5 ft and 2 ft is 7 ft, which is half the perimeter of the original figure since $\frac{1}{2} \cdot 14 \text{ ft} = 7 \text{ ft}$. The area of a rectangle with dimensions 1.5 ft and 2 ft is 3 ft^2, which is $\frac{1}{4}$ the area of the original figure since $\frac{1}{4} \cdot 12 \text{ ft}^2 = 3 \text{ ft}^2$.

30. **CHANGING DIMENSIONS** Use the triangle below.



a. Find the perimeter of the triangle.

b. Find the area of the triangle.

c. Suppose the side lengths and height of the triangle were doubled. What effect would this have on the perimeter? the area? Justify your answer.

d. Suppose the side lengths and height of the triangle were divided by three. What effect would this have on the perimeter? the area? Justify your answer.

SOLUTION:

a.

$$
\begin{aligned}
P &= b + c + d && \text{Area Formula} \\
&= 12 + 6 + 15 && \text{Substitution.} \\
&= 33 && \text{Addition.}
\end{aligned}
$$

The perimeter of the triangle is 33 m.

b. The area of the rectangle is:

$A = \frac{1}{2}bh$ Area Formula

$A = \frac{1}{2} \cdot 6 \cdot 9$ Substitution.

$= 27$ Multiply.

The area of the rectangle is 27 m^2.

c. If the sides and height are doubled, then the sides of the triangle are 24, 12, and 30 and the height is 18. Its perimeter is 24 + 12 + 30 or 66 m. This is twice the perimeter of the original figure since 2(33) = 66.
So, the perimeter also doubles.

Its area is $\frac{1}{2} \cdot 12 \cdot 18$ or 108 m^2. This is four times the area of the original figure since 4(27) = 108. The area of the triangle quadruples.

d. If the side lengths and height were divided by 3, then the side lengths are 4, 2, and 5 and the height is 3. The perimeter is 4 + 2 + 5 or 11 m. This is $\frac{1}{3}$ of the perimeter of the original figure since $\frac{1}{3} \cdot 33 = 11$. So, the perimeter is divided by 3. The area is $\frac{1}{2} \cdot 2 \cdot 3$ or 3 m^2. This is $\frac{1}{9}$ of the area of the original figure since $\frac{1}{9} \cdot 27 = 3$. So, the area is divided by 9.

ANSWER:

a. 33 m

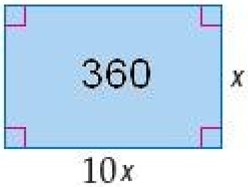
b. 27 m^2

c. The perimeter doubles; the area quadruples. The perimeter of a triangle with side lengths 30 m, 24 m, and 12 m is 66 m, which is twice the perimeter of the original figure since 2 • 33 m = 66 m. The area of a triangle with base 12 m and height 18 m is 108 m^2, which is four times the area of the original figure since 4 • 27 m^2 = 108 m^2.

d. The perimeter is divided by 3; the area is divided by 9. The perimeter of a triangle with side lengths 5 m, 4 m, and 2 m is 11 m, which is $\frac{1}{3}$ the perimeter of the original figure since $\frac{1}{3} \cdot 33 \text{ m} = 11 \text{ m}$. The area of a triangle with base 2 m and height 3 m is 3 m^2, which is $\frac{1}{9}$ the area of the original figure since $\frac{1}{9} \cdot 27 \text{ m}^2 = 3 \text{ m}^2$.

31. **ALGEBRA** A rectangle of area 360 square yards is 10 times as long as it is wide. Find its length and width.

SOLUTION:



Let x be the width. Then the length is $10x$.

Use the area formula for a rectangle.

$$A = \ell w \quad \text{Area Formula}$$
$$360 = (10x)x \quad \text{Substitution.}$$
$$360 = 10x^2 \quad \text{Simplify.}$$
$$\frac{360}{10} = \frac{10x^2}{10} \quad \text{Divide each side by 10.}$$
$$36 = x^2 \quad \text{Simplify.}$$
$$\sqrt{36} = \sqrt{x^2} \quad \text{Square root of each side.}$$
$$\pm 6 = x \quad \text{Simplify.}$$

Since width can never be negative, $x = 6$.

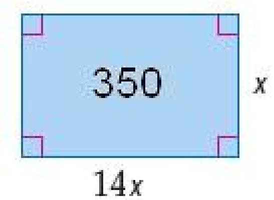
The length of the rectangle is $10x = 10(6) = 60$ yards and the width of the rectangle is 6 yards.

ANSWER:

60 yd, 6 yd

32. **ALGEBRA** A rectangle of area 350 square feet is 14 times as wide as it is long. Find its length and width.

SOLUTION:



Let x be the length. Then the width is $14x$.

Use the area formula for a rectangle. Equate it to 350 and solve for x

$$A = \ell w \quad \text{Area Formula}$$
$$350 = (x)14x \quad \text{Substitution.}$$
$$350 = 14x^2 \quad \text{Simplify.}$$
$$\frac{350}{14} = \frac{14x^2}{14} \quad \text{Divide each side by 14.}$$
$$25 = x^2 \quad \text{Simplify.}$$
$$\sqrt{25} = \sqrt{x^2} \quad \text{Square root of each side.}$$
$$\pm 5 = x \quad \text{Simplify.}$$

Since length can never be negative, $x = 5$.

The length of the rectangle is 5 ft and the width is $14x = 14(5) = 70$ ft.

ANSWER:
5 ft, 70 ft

33. **DISC GOLF** The diameter of the most popular brand of flying disc used in disc golf measures between 8 and 10 inches. Find the range of possible circumferences and areas for these flying discs to the nearest tenth.

SOLUTION:

The circumference is minimized when the diameter is 8 inches.

$$\begin{aligned} C &= \pi d && \text{Circumference Formula} \\ &= \pi(8) && \text{Substitution.} \\ &\approx 25.1 && \text{Simplify.} \end{aligned}$$

The minimum circumference is about 25.1 in.

The circumference is maximized when the diameter is 10 in.

$$\begin{aligned} C &= \pi d && \text{Circumference Formula} \\ &= \pi(10) && \text{Substitution.} \\ &\approx 31.4 && \text{Simplify.} \end{aligned}$$

The maximum circumference is about 31.4 in.

The area is minimum when the radius is 4 inches.

$$\begin{aligned} A &= \pi r^2 && \text{Area Formula} \\ &= \pi(4)^2 && \text{Substitution.} \\ &\approx 50.3 && \text{Simplify.} \end{aligned}$$

The minimum area is about 50.3 in^2.

The area is a maximum when the radius is 5 in.

$$\begin{aligned} A &= \pi r^2 && \text{Area Formula} \\ &= \pi(5)^2 && \text{Substitution.} \\ &\approx 78.5 && \text{Simplify.} \end{aligned}$$

The maximum area is about 78.5 in^2.

ANSWER:

25.1 in.; 31.4 in.; 50.3 in^2 to 78.5 in^2

ALGEBRA Find the perimeter or circumference for each figure described.

34. The area of a square is 36 square units.

SOLUTION:

Find the length of the side.

Use the formula for the area of a square with side *s*.

$$\begin{aligned} A &= s^2 && \text{Area Formula} \\ 36 &= s^2 && \text{Substitution.} \\ \sqrt{36} &= \sqrt{s^2} && \text{Square root of each side.} \\ \pm 6 &= s && \text{Simplify.} \end{aligned}$$

Since the length can never be negative, $s = 6$.

Use the formula for perimeter of the square with side *s*.

$$\begin{aligned} P &= 4s && \text{Perimeter Formula} \\ &= 4(6) && \text{Substitution.} \\ &= 24 && \text{Multiply.} \end{aligned}$$

The perimeter of the square is 24 units.

ANSWER:

24 units

35. The length of a rectangle is half the width. The area is 25 square meters.

SOLUTION:

Let *w* be the width. So, the length of the rectangle is $\frac{w}{2}$.

Use the area formula for a the rectangle.

$$\begin{aligned} A &= \ell w && \text{Area Formula} \\ 25 &= \left(\frac{w}{2}\right)w && \text{Substitution.} \\ 25 &= \frac{w^2}{2} && \text{Multiply.} \\ 2(25) &= 2\left(\frac{w^2}{2}\right) && \text{Multipy each side by 2.} \\ 50 &= w^2 && \text{Simplify.} \\ \sqrt{50} &= \sqrt{w^2} && \text{Square root of each side.} \\ 7.1 &\approx w && \text{Simplify.} \end{aligned}$$

Therefore, the length is $\frac{7.1}{2}$ or 3.5.

Use the formula for perimeter of a rectangle.

$$\begin{aligned} P &= 2\ell + 2w && \text{Perimeter Formula} \\ &\approx 2(3.5) + 2(7.1) && \text{Substitution.} \\ &\approx 7 + 14.2 && \text{Multiply.} \\ &\approx 21.2 && \text{Addition.} \end{aligned}$$

The perimeter of the rectangle is about 21.2 m.

ANSWER:

21.2 m

36. The area of a circle is 25π square units.

SOLUTION:

Use the area formula for a circle with radius *r*.

$$
\begin{aligned}
A &= \pi r^2 && \text{Area Formula} \\
25\pi &= \pi r^2 && \text{Substitution.} \\
\frac{25\pi}{\pi} &= \frac{\pi r^2}{\pi} && \text{Divide each side by } \pi. \\
25 &= r^2 && \text{Simplify.} \\
\sqrt{25} &= \sqrt{r^2} && \text{Square root of each side.} \\
\pm 5 &= r && \text{Simplify.}
\end{aligned}
$$

The radius of the circle is 5 units.

Find the circumference.

Use the formula for the circumference of a circle with radius *r*.

$$
\begin{aligned}
C &= 2\pi r && \text{Circumference Formula} \\
&= 2\pi(5) && \text{Substitution.} \\
&= 31.4 && \text{Simplify.}
\end{aligned}
$$

The circumference of the circle is 10π or about 31.4 units.

ANSWER:

10π or about 31.4 units

37. The area of a circle is 32π square units.

SOLUTION:

Use the formula for the area of a circle with radius *r*.

$$A = \pi r^2 \quad \text{Area Formula}$$
$$32\pi = \pi r^2 \quad \text{Subsitution.}$$
$$\frac{32\pi}{\pi} = \frac{\pi r^2}{\pi} \quad \div \text{ each side by } \pi.$$
$$32 = r^2 \quad \text{Simplify.}$$
$$\sqrt{32} = \sqrt{r^2} \quad \text{Square root}$$
$$\sqrt{32} = r \quad \text{Simplify.}$$

The radius of the circle is about $\sqrt{32}$ units.

Find the circumference.

Use the formula for the circumference of a circle with radius *r*.

$$C = 2\pi r \quad \text{Circumference Formula}$$
$$= 2\pi\sqrt{32} \quad \text{Substitution.}$$
$$\approx 35.5 \quad \text{Simplify.}$$

The circumference of the circle is $2\pi\sqrt{32}$ or about 35.5 units.

ANSWER:

$2\pi\sqrt{32}$ or about 35.5 units

38. A rectangle's length is 3 times its width. The area is 27 square inches.

SOLUTION:

Let *w* be the width. So, the length of the rectangle is 3*w*.

Use the formula for the area of a rectangle.

$$\begin{aligned} A &= \ell w && \text{Area Formula} \\ 27 &= (3w)w && \text{Substitution.} \\ 27 &= 3w^2 && \text{Simplify.} \\ \frac{27}{3} &= \frac{3w^2}{3} && \div \text{ each side by 3.} \\ 9 &= w^2 && \text{Simplify.} \\ \sqrt{9} &= \sqrt{w^2} && \text{Square root} \\ \pm 3 &= w && \text{Simplify.} \end{aligned}$$

Therefore, the length is 3(3) or 9 in.

Substitute in the formula for perimeter.

$$\begin{aligned} P &= 2\ell + 2w && \text{Perimeter formula} \\ &= 2(9) + 2(3) && \text{Substitution.} \\ &= 18 + 6 && \text{Mutliply.} \\ &= 24 && \text{Addition.} \end{aligned}$$

The perimeter of the rectangle is 24 in.

ANSWER:

24 in.

39. A rectangle's length is twice its width. The area is 48 square inches.

SOLUTION:

Let *w* be the width. So, the length of the rectangle is 2*w*.

Use the formula for the area of the rectangle.

$$A = \ell w \quad \text{Area Formula}$$
$$48 = (2w)w \quad \text{Substitution.}$$
$$48 = 2w^2 \quad \text{Simplify.}$$
$$\frac{48}{2} = \frac{2w^2}{2} \quad \div \text{ each side by 2.}$$
$$24 = w^2 \quad \text{Simplify.}$$
$$\sqrt{24} = \sqrt{w^2} \quad \text{Square root}$$
$$2\sqrt{6} = w \quad \text{Simplify.}$$

Therefore, the length is $2\left(2\sqrt{6}\right)$ or $4\sqrt{6}$ in.

Substitute in the formula for perimeter.

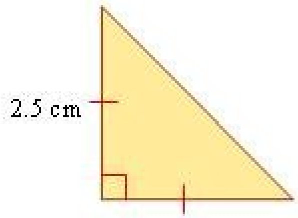
$$P = 2\ell + 2w \quad \text{Perimeter Formula}$$
$$= 2\left(4\sqrt{6}\right) + 2\left(2\sqrt{6}\right) \quad \text{Substitution.}$$
$$= 8\sqrt{6} + 4\sqrt{6} \quad \text{Multiply.}$$
$$= 12\sqrt{6} \quad \text{Addition.}$$

The perimeter of the rectangle is $12\sqrt{6}$ or 29.4 in.

ANSWER:

$12\sqrt{6}$ or about 29.4 in.

CCSS PRECISION Find the perimeter and area of each figure in inches. Round to the nearest hundredth, if necessary.



40.

SOLUTION:

Before finding the perimeter and area, you must find the lengths of the two missing sides of the right triangle. In the diagram, it is indicated that the base is congruent to the height, so $b = 2.5$ cm.

Use the Pythagorean Theorem to find the length of the hypotenuse.

$$\begin{aligned} c^2 &= a^2 + b^2 && \text{Pythagorean Theorem} \\ c^2 &= 2.5^2 + 2.5^2 && \text{Substitution} \\ c^2 &= 12.5 && \text{Simplify.} \\ c &\approx 3.42 && \text{Positive square root} \end{aligned}$$

The perimeter of the triangle is the sum of the sides.

$$\begin{aligned} P &= a + b + c && \text{Perimeter Formula.} \\ &= 2.5 + 2.5 + 3.42 && \text{Substitution.} \\ &= 8.54 \text{ cm} && \text{Addition.} \end{aligned}$$

Use dimensional analysis to change centimeters to inches.

$$8.54 \text{ cm} \times \frac{0.4 \text{ in.}}{1 \text{ cm}} \approx 3.42 \text{ in.}$$

The area of the triangle is half the product of the base and the height.

$$\begin{aligned} A &= \frac{1}{2}bh && \text{Area formula} \\ &= \frac{1}{2}(2.5)(2.5) && \text{Substitution} \\ &= 3.125 \text{ cm}^2 && \text{Multiply.} \end{aligned}$$

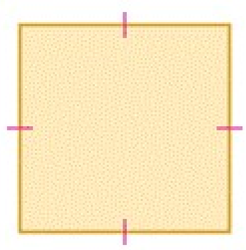
Use dimensional analysis to change cm^2 to in^2.

$$3.125 \text{ cm}^2 \times \frac{0.4 \text{ in.}}{1 \text{ cm}} \times \frac{0.4 \text{ in.}}{1 \text{ cm}} = 0.5 \text{ in}^2$$

So, the perimeter is about 3.42 in. and the area is 0.5 in^2.

ANSWER:

3.42 in.; 0.5 in^2

41. 

SOLUTION:

Use the formulas to find the perimeter and area of the square.

$$\begin{aligned} P &= 4s && \text{Perimeter formula} \\ &= 4(0.75) && \text{Substitution} \\ &= 3\text{yd} && \text{Multiply.} \end{aligned}$$

Use dimensional analysis to change from yards to inches.

$$3\ \text{yd} \times \frac{3\ \text{ft}}{1\ \text{yd}} \times \frac{12\ \text{in.}}{1\ \text{ft}} = 108\ \text{in.}$$

$$\begin{aligned} A &= s^2 && \text{Area formula} \\ &= (0.75)^2 && \text{Substitution} \\ &= 0.5625\text{yd}^2 && \text{Multiply.} \end{aligned}$$

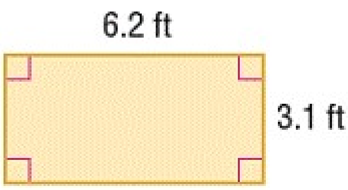
Use dimensional analysis to change yd^2 to in^2.

$$0.5625\ \text{yd}^2 \times \frac{3\ \text{ft}}{1\ \text{yd}} \times \frac{3\ \text{ft}}{1\ \text{yd}} \times \frac{12\ \text{in.}}{1\ \text{ft}} \times \frac{12\ \text{in.}}{1\ \text{ft}} = 729\text{in}^2$$

So, the perimeter is 108 in. and the area is 729 in^2.

ANSWER:

108 in.; 729 in^2

42. 

SOLUTION:

Use the formulas to find the perimeter and area of the rectangle.

$$\begin{aligned} P &= 2\ell + 2w && \text{Perimeter formula} \\ &= 2(6.2) + 2(3.1) && \text{Substitution.} \\ &= 12.4 + 6.2 && \text{Multiply.} \\ &= 18.6\text{ ft} && \text{Simplify.} \end{aligned}$$

Use dimensional analysis to change feet to inches.

$$18.6\text{ ft} \times \frac{12\text{ in.}}{1\text{ ft}} = 223.2\text{ in.}$$

$$\begin{aligned} A &= \ell w && \text{Area formula} \\ &= (6.2)(3.1) && \text{Substitution} \\ &= 19.22\text{ ft}^2 && \text{Multiply.} \end{aligned}$$

Use dimensional analysis to change ft^2 to in^2.

$$19.22\ \cancel{\text{ft}}^2 \times \frac{12\text{ in.}}{1\ \cancel{\text{ft}}} \times \frac{12\text{ in.}}{1\ \cancel{\text{ft}}} = 2767.68\text{in}^2$$

So, the perimeter is 223.2 in. and the area is 2767.68 in^2.

ANSWER:

223.2 in.; 2767.68 in^2

43. **Multiple Representations** Collect and measure the diameter and circumference of ten round objects using a millimeter measuring tape.

a. Tabular Record the measures in a table as shown.

Object	d	C	$\frac{C}{d}$
1			
2			
3			
⋮			
10			

b. Algebraic Compute the value $\frac{C}{d}$ to the nearest hundredth for each object and record the result.

c. Graphical Make a scatter plot of the data with d-values on the horizontal axis and C-values on the vertical axis.

d. Verbal Find an equation for a line of best fit for the data. What does this equation represent? What does the slope of the line represent?

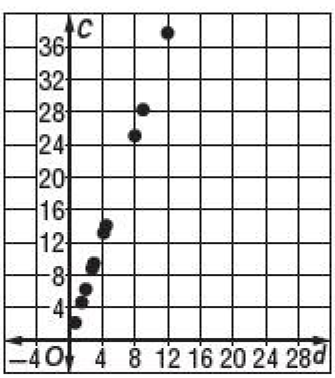
SOLUTION:

a-b. Sample answer: Make a copy of the table given for the problem. Find ten circular objects, then measure the diameter and circumference of each object to the nearest tenth of a centimeter, and record the results in the table. Divide the circumference by the diameter for each object and record the result in the table. For example, an object has a diameter of 3 cm and a circumference of 9.4 cm.

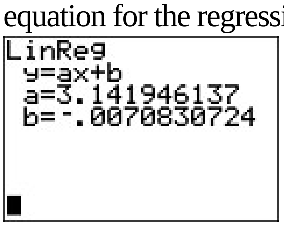
$\frac{C}{d} = \frac{9.4}{3}$ or about 3.14.

Object	d (cm)	C (cm)	$\frac{C}{d}$
1	3	9.4	3.13
2	9	28.3	3.14
3	4.2	13.2	3.14
4	12	37.7	3.14
5	4.5	14.1	3.13
6	2	6.3	3.15
7	8	25.1	3.14
8	0.7	2.2	3.14
9	1.5	4.7	3.13
10	2.8	8.8	3.14

c. Choose a scale for your horizontal and vertical axis that will contain all your diameters and circumferences. Plot the ten points determined by each pair of diameter and circumference measures.



d. Sample answer: Enter your data into a graphing calculator. In the **STAT** menu, enter the diameters into **L1** and the circumferences into **L2.** Then in the **STAT** menu choose CALC and the LinReg($ax + b$) function to find the equation for the regression line.



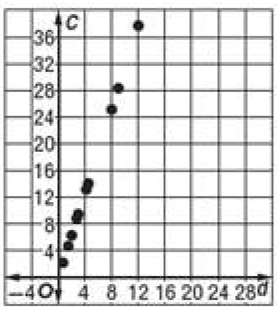
Here, $a \approx 3.14$ and $b \approx 0$, so an equation for a line of best fit would be $C = 3.14d$; the equation represents a formula for approximating the circumference of a circle. The slope represents an approximation for pi.

ANSWER:

a-b. Sample answer:

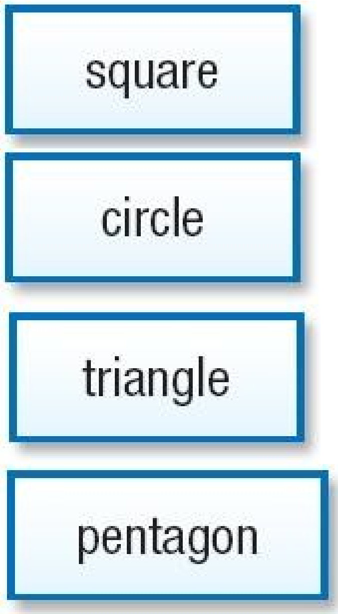
Object	d (cm)	C (cm)	$\frac{C}{d}$
1	3	9.4	3.13
2	9	28.3	3.14
3	4.2	13.2	3.14
4	12	37.7	3.14
5	4.5	14.1	3.13
6	2	6.3	3.15
7	8	25.1	3.14
8	0.7	2.2	3.14
9	1.5	4.7	3.13
10	2.8	8.8	3.14

c.



d. Sample answer: $C = 3.14d$; the equation represents a formula for approximating the circumference of a circle. The slope represents an approximation for pi.

44. **WHICH ONE DOESN'T BELONG?** Identify the term that does not belong with the other three. Explain your reasoning.



SOLUTION:

Circle; The other shapes are polygons.

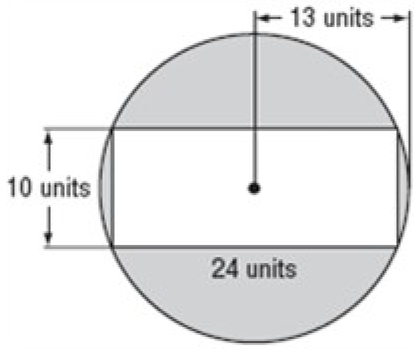
ANSWER:

Circle; the other shapes are polygons.

45. **CHALLENGE** The vertices of a rectangle with side lengths of 10 and 24 are on a circle of radius 13 units. Find the area between the figures.

SOLUTION:

Start by drawing the figure.



The shaded region of the drawing represents the area between the figures. Next, find the area of each figure.

Use the formula to find the area of the rectangle (Area of a rectangle = *lw*).

$A_{\text{rectangle}} = 10 \times 24 = 240$ units^2

Use the formula to find the area of the circle (Area of a circle $= \pi r^2$).

$A_{\text{circle}} = \pi \cdot 13^2 \approx 530.93$ units^2

Then subtract the area of the rectangle from the area of the circle in order to find the area of the shaded region.

$$\begin{aligned} A_{\text{Shaded region}} &= A_{\text{circle}} - A_{\text{rectangle}} \\ &= 530.93 - 240 \\ &= 290.93 \end{aligned}$$

Therefore, the area between the figures is about 290.93 units^2.

ANSWER:

$\approx$290.93 units^2

46. **REASONING** Name a polygon that is always regular and a polygon that is sometimes regular. Explain your reasoning.

SOLUTION:

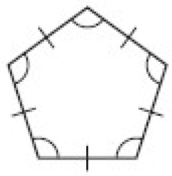
Square; by definition, all sides of a square are congruent and all angles measure 90°, so therefore are congruent. Triangle; triangles can have all sides and angles congruent, just two sides and angle pairs congruent, or no sides or angles congruent.

ANSWER:

Square; by definition, all sides of a square are congruent and all angles measure 90°, so therefore are congruent. Triangle; triangles can have all sides and angles congruent, just two sides and angle pairs congruent, or no sides or angles congruent.

47. **OPEN ENDED** Draw a pentagon. Is your pentagon convex or concave? Is your pentagon regular or irregular? Justify your answers.

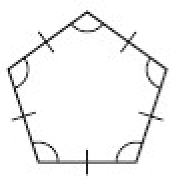
SOLUTION:



Sample answer: The pentagon is convex, since no points of the lines drawn on the edges are in the interior. The pentagon is regular since all of the angles and sides were constructed with the same measurement, making them congruent to each other.

ANSWER:

Sample answer: The pentagon is convex, since no points of the lines drawn on the edges are in the interior. The pentagon is regular since all of the angles and sides were constructed with the same measurement, making them congruent to each other.



48. **CHALLENGE** A rectangular room measures 20 feet by 12.5 feet. How many 5-inch square tiles will it take to cover the floor of this room? Explain.

SOLUTION:

Convert the dimensions from feet to inches.
The length of the room is 20 × 12 or 240 inches and the width of the room is 12.5 × 12 or 150 inches. It needs 240 ÷ 5 = 48 columns of tiles and 150 ÷ 5 = 30 rows of tiles to cover this space. So the number of tiles needed is 48 × 30 or 1440 tiles.

ANSWER:

1440 square tiles; Sample answer: the space is 20 × 12 or 240 inches long and 12.5 × 12 or 150 inches wide. It will take exactly 240 in. ÷ 5 in./tile or 48 columns of tiles and 150 in. ÷ 5 in./tile or 30 rows of tiles to cover this space. So the number of tiles needed is 48 × 30 or 1440 tiles.

49. **WRITING IN MATH** Describe two possible ways that a polygon can be equiangular but not a regular polygon.

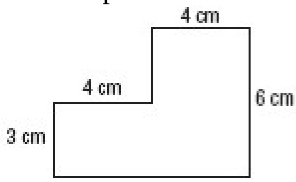
SOLUTION:

Sample answer: If a convex polygon is equiangular but not also equilateral, then it is not a regular polygon. Likewise, if a polygon is equiangular and equilateral, but not convex, then it is not a regular polygon.

ANSWER:

Sample answer: If a convex polygon is equiangular but not also equilateral, then it is not a regular polygon. Likewise, if a polygon is equiangular and equilateral, but not convex, then it is not a regular polygon.

50. Find the perimeter of the figure.



A 17 cm
B 25 cm
C 28 cm
D 31 cm

SOLUTION:

The lengths of two sides are unknown.
The length of the base is 4 + 4 or 8 cm.

To find the length of the unknown vertical side, subtract 3 from 6.
6 – 3 = 3

Add all the sides to find the perimeter.

$$\begin{aligned} P &= a + b + c + d + d + e && \text{Perimeter Formula} \\ &= 8 + 6 + 4 + 4 + 3 + 3 && \text{Substitution.} \\ &= 28 && \text{Addition.} \end{aligned}$$

The perimeter of the figure is 28 cm
The correct choice is C.

ANSWER:

C

51. **PROBABILITY** In three successive rolls of a fair number cube, Matt rolls a 6. What is the probability of Matt rolling a 6 if the number cube is rolled a fourth time?

F $\frac{1}{6}$

G $\frac{1}{4}$

H $\frac{1}{3}$

J 1

SOLUTION:

Probability is defined as
$\frac{\text{Number of favorable outcomes}}{\text{Total number of outcomes}}$.
The number of favorable outcomes is 1, and the total number of outcomes is 6.

So, the probability of rolling a 6 = $\frac{1}{6}$.

The correct choice is F.

ANSWER:

F

52. **SHORT RESPONSE** Miguel is planning a party for 80 guests. According to the pattern in the table, how many gallons of ice cream should Miguel buy?

Number of Guests	Gallons of Ice Cream
8	2
16	4
24	6
32	8

SOLUTION:

From the pattern we see that for every 8 guests Miguel needs 2 gallons of ice cream.
So, for 8 × 10 or 80 guests Miguel needs 2 × 10 or 20 gallons of ice cream.

ANSWER:

20 gal.

53. **SAT/ACT** A frame 2 inches wide surrounds a painting that is 18 inches wide and 14 inches tall. What is the area of the frame?

A 68 in^2

B 84 in^2

C 144 in^2

D 252 in^2

E 396 in^2

SOLUTION:

With 2 inch wide frame, the dimensions of painting with the frame becomes 22 inches by 18 inches.

Find the area of the painting with frame.

$A = 22 \cdot 18$

$= 396 \text{ in}^2$

Find the area of the painting with out the frame.

$A = 18 \cdot 14$

$= 252 \text{ in}^2$

To find the area of the frame, subtract the area of the paint from the area of the paint with the frame.

396 – 252 = 144

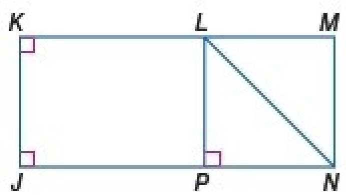
The area of the frame is 144 in^2.

The correct choice is C.

ANSWER:

C

Determine whether each statement can be assumed from the figure. Explain.



54. $\angle KJN$ is a right angle.

SOLUTION:

Yes; the symbol denotes that $\angle KJN$ is a right angle.

ANSWER:

Yes; the symbol denotes that $\angle KJN$ is a right angle.

55. $\angle PLN \cong \angle NLM$

SOLUTION:

It might appear that the angles are congruent but there are no marks on the angles in the diagram to indicate this.

There is also nothing in the diagram that would provide information to know that $\overrightarrow{LN}$ bisects $\angle PLM$. Because we do not know anything about the measures of these two angles, we can not assume that $\angle PLN \cong \angle NLM$.

ANSWER:

No; we do not know anything about these measures.

56. $\angle PNL$ and $\angle MNL$ are complementary.

SOLUTION:

No; we do not know whether $\angle MNP$ is a right angle.

ANSWER:

No; we do not know whether $\angle MNP$ is a right angle.

57. $\angle KLN$ and $\angle MLN$ are supplementary.

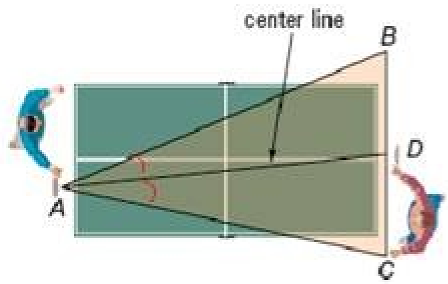
SOLUTION:

Yes; they form a linear pair.

ANSWER:

Yes; they form a linear pair.

58. **TABLE TENNIS** The diagram shows the angle of play for a table tennis player. If a right-handed player has a strong forehand, he should stand to the left of the center line of his opponent's angle of play.



a. What geometric term describes the center line?
b. If the angle of play shown in the diagram measures 43°, what is $m\angle BAD$?

SOLUTION:

a. The center line divides the angle into two congruent angles. So, it is an angle bisector.
b. Since the center line is an angle bisector,

$$\begin{aligned} m\angle BAD &= \frac{m\angle A}{2} \\ &= \frac{43}{2} \\ &= 21.5 \end{aligned}$$

ANSWER:

a. angle bisector
b. 21.5

Name an appropriate method to solve each system of equations. Then solve the system.

59. $-5x + 2y = 13$
$2x + 3y = -9$

SOLUTION:

The appropriate method to solve this system is the elimination method, since neither of the equations have variables with coefficients of 1 and equations can not be simplified.

Multiply the first equation by 2 and the second equation by 5

$$\begin{aligned} -10x + 4y &= 26 && \text{Multiply equation 1 by 2.} \\ \underline{10x + 15y} &\underline{= -45} && \text{Multiply equation 2 by 5.} \\ 19y &= -19 && \text{Add equations.} \\ \frac{19y}{19} &= \frac{-19}{19} && \text{Divide each side by 19.} \\ y &= -1 && \text{Simplify.} \end{aligned}$$

Substitute the value of y in one of the given equations.

$$\begin{aligned} 2x + 3y &= -9 && \text{Original equation.} \\ 2x + 3(-1) &= -9 && \text{Replace } y \text{ with } -1. \\ 2x - 3 &= -9 && \text{Simplify.} \\ 2x - 3 + 3 &= -9 + 3 && \text{Add 3 to each side.} \\ 2x &= -6 && \text{Simplify.} \\ \frac{2x}{2} &= \frac{-6}{2} && \text{Divide each side by 2.} \\ x &= -3 && \text{Simplify.} \end{aligned}$$

The solution of the system is: $x = -3, y = -1$

ANSWER:

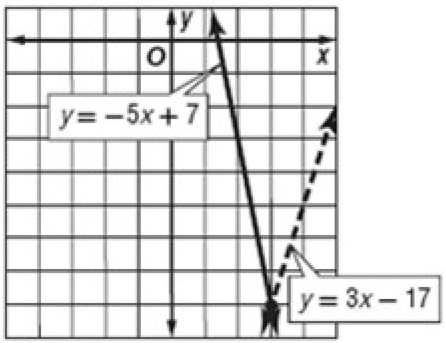
elimination; $x = -3, y = -1$

60. $y = -5x + 7$
$y = 3x - 17$

SOLUTION:

The appropriate method to solve this system is graphing since both equations are in slope-intercept form and can be easily graphed.

Graph the lines on a coordinate grid.



The graphs appear to intersect at (3, –8).
So, the solution of the system of equations is $x = 3, y = -8$.

ANSWER:

graphing; $x = 3, y = -8$

61. $x - 8y = 16$
$7x - 4y = -18$

SOLUTION:

The appropriate method to solve this system is substitution, since equation 1 has a variable with a coefficient of 1.

Solve the first equation for *x*.

$$\begin{aligned} x - 8y &= 16 && \text{Original equation 1.} \\ x - 8y - 8y &= 16 - 8y && +8y \text{ to each side.} \\ x &= 16 + 8y && \text{Simplify.} \end{aligned}$$

Substitute this in the second equation.

$$\begin{aligned} 7x - 4y &= -18 && \text{Original equation 2.} \\ 7(16 - 8y) - 4y &= -18 && \text{Replace } x \text{ with } (16 - 8y). \\ 112 + 56y - 4y &= -18 && \text{Multiply.} \\ 122 + 52y &= -18 && \text{Simplify.} \\ 112 - 112 + 52y &= -18 - 112 && \text{Subtract 112 from each side.} \\ 52y &= -130 && \text{Simplify.} \\ \frac{52y}{52} &= \frac{-130}{52} && \text{Divide each side by 52.} \\ y &= -2.5 && \text{Simplify.} \end{aligned}$$

Substitute the value of y in one of the given equation.

$$\begin{aligned} x - 8y &= 16 && \text{Original equation 1.} \\ x - 8(-2.5) &= 16 && \text{Replace } y \text{ with } -2.5. \\ x + 20 &= 16 && \text{Simplify.} \\ x + 20 - 20 &= 16 - 20 && \text{Subtract 20 from each side.} \\ x &= -4 && \text{Simplify.} \end{aligned}$$

The solution to the system is $x = -4, y = -2.5$

ANSWER:

substitution; $x = -4, y = -2.5$

Evaluate each expression if *P* = 10, *B* = 12, *h* = 6, *r* = 3, and ℓ = 5. Round to the nearest tenth, if necessary.

62. $\frac{1}{2}P\ell + B$

SOLUTION:

Substitute.

$$\begin{aligned}\frac{1}{2}P\ell + B &= \frac{1}{2}(10)(5) + 12 && \text{Substitution}\\ &= \frac{1}{2}(50) + 12 && \text{Multiply.}\\ &= 25 + 12 && \text{Multiply.}\\ &= 37 && \text{Addition.}\end{aligned}$$

ANSWER:

37

63. $\frac{1}{3}Bh$

SOLUTION:

Substitute.

$$\begin{aligned}\frac{1}{3}Bh &= \frac{1}{3}(12)(6) && \text{Substitution}\\ &= \frac{1}{3}(72) && \text{Multiply.}\\ &= 24 && \text{Multiply.}\end{aligned}$$

ANSWER:

24

64. $\frac{1}{3}\pi r^2 h$

SOLUTION:

Substitute.

$$\begin{aligned}\frac{1}{3}\pi r^2 h &= \frac{1}{3}\pi(3)^2(6) && \text{Substitution}\\ &= \frac{1}{3}\pi(9)(6) && \text{Square 3.}\\ &= 18\pi && \text{Multiply.}\\ &\approx 56.5\end{aligned}$$

ANSWER:

56.5

65. $2\pi rh + 2\pi r^2$

SOLUTION:

Substitute.

$$\begin{aligned} 2\pi rh + 2\pi r^2 &= 2\pi(3)(6) + 2\pi(3^2) && \text{Substitution} \\ &= 36\pi + 18\pi && \text{Multiply/Square.} \\ &= 54\pi && \text{Addition.} \\ &\approx 169.6 \end{aligned}$$

ANSWER:

169.6