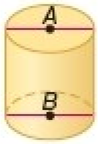
Determine whether the solid is a polyhedron. Then identify the solid. If it is a polyhedron, name the bases, faces, edges, and vertices.

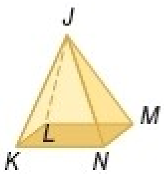
1. 

SOLUTION:

A polyhedron is a solid made from flat surfaces that enclose a single region of space. This solid has curved surfaces, so it is not a polyhedron. The given figure is a solid with congruent parallel circular bases connected by a curved surface. Therefore, it is a cylinder.

ANSWER:

not a polyhedron; cylinder

2. 

SOLUTION:

The solid is formed by polygonal faces, so it is a polyhedron. It has a rectangular base and three or more triangular faces that meet at a common vertex. So, it is a rectangular pyramid.

Base: $\square KLMN$

Faces: $\square KLMN, \Delta JNK, \Delta JNM, \Delta JML, \Delta JLK$

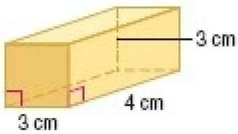
Edges: $\overline{KN}, \overline{NM}, \overline{ML}, \overline{LK}, \overline{JK}, \overline{JN}, \overline{JM}, \overline{JL}$

Vertices, K, L, M, N, J

ANSWER:

a polyhedron; rectangular pyramid; base: $\square KLMN$; faces $\square KLMN, \Delta JNK, \Delta JNM, \Delta JML, \Delta JLK$; edges: $\overline{KN}, \overline{NM}, \overline{ML}, \overline{LK}, \overline{JK}, \overline{JN}, \overline{JM}, \overline{JL}$; vertices: K, L, M, N, J

Find the surface area and volume of each solid to the nearest tenth.

3. 

SOLUTION:

The formulas for finding the volume and surface areas of a prism are $V = Bh$ and $S = Ph + 2B$, where S = total surface area, V = volume, h = height of a solid, B = area of the base, P = perimeter of the base.

Since the base of the prism is a rectangle, the perimeter P of the base is $2(4 + 3)$ or 14 centimeters. The area of the base B is
4×3 or 12 square centimeters. The height is 3 centimeters.

$$\begin{aligned} S &= Ph + 2B && \text{Surface Area Formula} \\ &= (14 \cdot 3) + 2(12) && \text{Substitution.} \\ &= 42 + 24 && \text{Multiply.} \\ &= 66 && \text{Addition.} \end{aligned}$$

The surface area of the prism is 66 square centimeters.

$$\begin{aligned} V &= Bh && \text{Volume Formula} \\ &= 12 \cdot 3 && \text{Substitution.} \\ &= 36 && \text{Multiply.} \end{aligned}$$

The volume of the prism is 36 cubic centimeters.

ANSWER:

66 cm^2 ; 36 cm^3

4. 

SOLUTION:

The formulas for finding the volume and surface area of a sphere are $V = \frac{4}{3}\pi r^3$ and $S = 4\pi r^2$, where S = total surface area, V = volume, and r = radius.

Here, $r = 6$ in.

$$\begin{aligned} V &= \frac{4}{3}\pi r^3 && \text{Volume Formula} \\ &= \frac{4}{3}\pi(6)^3 && \text{Substitution.} \\ &= \frac{4}{3}\pi(216) && \text{Cube 6.} \\ &= 288\pi && \text{Multiply.} \\ &\approx 904.8 && \text{Use a calculator.} \end{aligned}$$

The volume of the sphere is 288π or about 904.8 cubic inches.

$$\begin{aligned} S &= 4\pi r^2 && \text{Surface Area formula} \\ &= 4\pi(6)^2 && \text{Replace } r \text{ with 6.} \\ &= 144\pi && \text{Simplify.} \\ &\approx 452.4 && \text{Use a calculator.} \end{aligned}$$

The surface area of the sphere is 144π or about 452.4 square inches.

ANSWER:

144π or about 452.4 in^2 ; 288π or about 904.8 in^3

5. **PARTY FAVORS** Lawana is making cone-shaped hats 4 inches in diameter, 6.5 inches tall, with a slant height of approximately 6.8 inches for party favors. Find each measure to the nearest tenth.
a. the volume of candy that will fill each cone
b. the area of material needed to make each hat assuming there is no overlap of material

SOLUTION:

The formulas for finding the volume and surface area of a cone are $V = \frac{1}{3}\pi r^2 h$ and $S = \pi r\ell + \pi r^2$, where S = total surface area, V = volume, r = radius, ℓ = slant height, and h = height.

a. Here, the diameter of the cone shaped hat is 4 inches, so the radius is 2 inches. $h = 6.5$ inches and $\ell = 6.8$ inches.

$$\begin{aligned} V &= \frac{1}{3}\pi r^2 h && \text{Volume Formula for a Cone.} \\ &= \frac{1}{3}\pi(2)^2(6.5) && \text{Substitution.} \\ &= \frac{1}{3}\pi(4)(6.5) && \text{Square 2.} \\ &= \frac{26}{3}\pi && \text{Multiply.} \\ &\approx 27.2 && \text{Use a calculator.} \end{aligned}$$

The volume of the hat is about 27.2 cubic inches.

b. Find area of cone not including the base.

$$\begin{aligned} A &= \pi r\ell && \text{Formula for the area.} \\ &= \pi(2)(6.8) && \text{Substitution.} \\ &= 13.6\pi && \text{Multiply.} \\ &\approx 42.7 && \text{Use a calculator.} \end{aligned}$$

The surface area of the hat is 13.6π or about 42.7 square inches.

ANSWER:

a. ≈ 27.2 in^3
b. 13.6π or about 42.7 in^2

Identify the solid modeled by each object. State whether the solid modeled is a polyhedron.

6. Refer to Page 71.

SOLUTION:

This object models a solid with a circular base connected by a curved surface to a single vertex. So it is a cone. A solid with all flat surfaces that enclose a single region of space is called a polyhedron. This solid has a curved surface, so it is not a polyhedron.

ANSWER:

cone; not a polyhedron

7. Refer to Page 71.

SOLUTION:

This object models a solid that has two visible triangular faces that meet at a common vertex. So, it is a pyramid. The type of pyramid will be determined by its base, which is not visible. The base will also determine the total number of triangular faces. The solid is formed by polygonal faces, so it is a polyhedron.

ANSWER:

pyramid; a polyhedron

8. Refer to Page 71.

SOLUTION:

This object models a solid has parallel triangular bases connected by three rectangular faces. So, it is a triangular prism. It is formed by polygonal faces, so it is a polyhedron.

ANSWER:

triangular prism; a polyhedron

9. Refer to Page 71.

SOLUTION:

This object models a solid that has two parallel congruent rectangular bases connected by four rectangular faces, so it is a rectangular prism. The solid is formed by polygonal faces, so it is a polyhedron.

ANSWER:

rectangular prism; a polyhedron

10. Refer to Page 71.

SOLUTION:

This object models a solid that is a set of points in space that are the same distance from a given point. So, it is a sphere. A sphere has no faces, edges, or vertices, so it is not a polyhedron.

ANSWER:

sphere; not a polyhedron

11. Refer to Page 71.

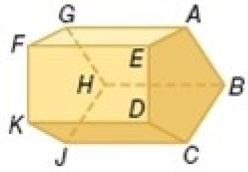
SOLUTION:

This object models a solid with congruent parallel circular bases connected by a curved surface. Therefore, it is a cylinder. This solid has a curved surface, so it is not a polyhedron.

ANSWER:

cylinder; not a polyhedron

CCSS STRUCTURE Determine whether the solid is a polyhedron. Then identify the solid. If it is a polyhedron, name the bases, faces, edges, and vertices.

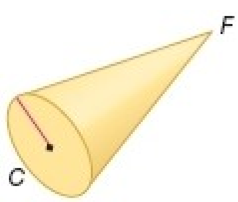
12. 

SOLUTION:

The solid is formed by polygonal faces, so it is a polyhedron. This solid has two congruent pentagonal bases, so it is a pentagonal prism.
Face: Each flat surface is called face.
Edges: The line segments where the faces intersect are called edges.
Vertex: The point where three or more edges intersect is called a vertex.
Bases: *ABCDE*, *FGHJK*
Faces: *ABCDE*, *FGHJK* $\square ABHG, \square BCJH, \square DCJK, \square EDKF, \square AEFG;$
Edges: $\overline{AB}, \overline{BC}, \overline{CD}, \overline{DE}, \overline{EA}, \overline{AG}, \overline{BH}, \overline{CJ}, \overline{DK}, \overline{EF}, \overline{GH}, \overline{HJ}, \overline{JK}, \overline{KF}, \overline{FG}$
Vertices: $A, B, C, D, E, G, G, H, J, K$

ANSWER:

a polyhedron; pentagonal prism; bases: *ABCDE*, *FGHJK*; faces: *ABCDE*, *FGHJK*;
$\square ABHG, \square BCJH, \square DCJK, \square EDKF, \square AEFG;$ edges:
$\overline{AB}, \overline{BC}, \overline{CD}, \overline{DE}, \overline{EA}, \overline{AG}, \overline{BH}, \overline{CJ}, \overline{DK}, \overline{EF}, \overline{GH}, \overline{HJ}, \overline{JK}, \overline{KF}, \overline{FG}$
vertices: *A*, *B*, *C*, *D*, *E*, *F*, *G*, *H*, *J*, *K*

13. 

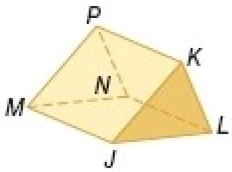
SOLUTION:

A solid with all flat surfaces that enclose a single region of space is called a polyhedron. This solid has a curved surface, so it is not a polyhedron. The given figure is a solid with a circular base connected by a curved surface to a single vertex. So it is a cone.

ANSWER:

not a polyhedron; cone

1-7 Three-Dimensional Figures

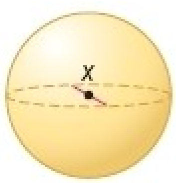
14. 

SOLUTION:

This solid is formed by polygonal faces, so it is a polyhedron. It has triangular bases. So, it is a triangular prism.
Face: Each flat surface is called face.
Edges: The line segments where the faces intersect are called edges.
Vertex: The point where three or more edges intersect is called a vertex.
Bases: $\Delta JLK, \Delta MNP$
Faces: $\boldsymbol{\Delta JKL, \Delta MNP, \square MPKJ, \square PKLN, \square MNLJ;}$
Edges: $\overline{MP}, \overline{PK}, \overline{KJ}, \overline{JM}, \overline{JL}, \overrightarrow{KL}, \overline{LN}, \overline{MN}, \overline{PN}$
Vertices: M, P, L, J, N, K

ANSWER:

a polyhedron; triangular prism; bases: $\boldsymbol{\Delta JLK, \Delta MNP;}$ faces: $\boldsymbol{\Delta JKL, \Delta MNP, \square MPKJ, \square PKLN, \square MNLJ;}$ edges $\overline{MP}, \overline{PK}, \overline{KJ}, \overline{JM}, \overline{JL}, \overrightarrow{KL}, \overline{LN}, \overline{MN}, \overline{PN}$; vertices: *M, P, L, J, N, K*

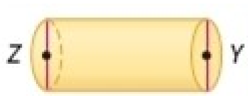
15. 

SOLUTION:

This solid has no faces, edges, or vertices, so it is not a polyhedron. It is a set of points in space that are the same distance from a given point. So, it is a sphere.

ANSWER:

not a polyhedron; sphere

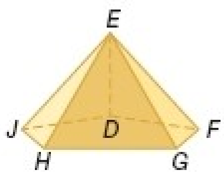
16. 

SOLUTION:

A solid with all flat surfaces that enclose a single region of space is called a polyhedron. The solid has a curved surface, so it is not a polyhedron. The given figure is a solid with congruent parallel circular bases connected by a curved surface. Therefore, it is a cylinder.

ANSWER:

not a polyhedron; cylinder

17. 

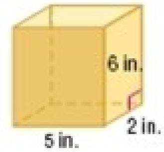
SOLUTION:

The solid is formed by polygonal faces, so it is a polyhedron. The given pyramid has a pentagonal base, so it is a pentagonal pyramid.
Faces: Each flat surface is called face.
Edges: The line segments where the faces intersect are called edges.
Vertex: The point where three or more edges intersect is called a vertex.
Base: *JHGFD*
Faces: $JHGFD$, ΔJEH, ΔHEG, ΔGEF, ΔFED, ΔEDJ
Edges: $\overline{HG}, \overline{GF}, \overline{FD}, \overline{DJ}, \overline{JH}, \overline{EJ}, \overline{EH}, \overline{EG}, \overline{EF}, \overline{ED}$
Vertices: J, H, G, F, D, E

ANSWER:

a polyhedron; pentagonal pyramid; base: *JHGFD*; faces: *JHGFD*, ΔJEH, ΔHEG, ΔGEF, ΔFED, ΔEDJ; edges: $\overline{HG}, \overline{GF}, \overline{FD}, \overline{DJ}, \overline{JH}, \overline{EJ}, \overline{EH}, \overline{EG}, \overline{EF}, \overline{ED}$ vertices: *J, H, G, F, D, E*

Find the surface area and volume of each solid to the nearest tenth.

18. 

SOLUTION:

The formulas for finding the volume and surface area of of a prism are $V = Bh$ and $S = Ph + 2B$, where *S* = total surface area, *V* = volume, *h* = height of a solid, *B* = area of the base, and *P* = perimeter of the base.

Since the base of the prism is a rectangle, the perimeter *P* of the base is $2(5) + 2(2)$ or 14 inches. The area of the base *B* is 5×2 or 10
square inches. The height is 6 inches.

$$\begin{aligned} S &= Ph + 2B && \text{Surface Area formula} \\ &= (14 \cdot 6) + 2(10) && \text{Substitution.} \\ &= 84 + 20 && \text{Multiply.} \\ &= 104 && \text{Addition.} \end{aligned}$$

The surface area of the prism is 104 in^2.

$$\begin{aligned} V &= Bh && \text{Volume Formula} \\ &= 10 \cdot 6 && \text{Substitution.} \\ &= 60 && \text{Multiply.} \end{aligned}$$

The volume of the prism is 60 in^3.

ANSWER:

104 in^2 ; 60 in^3

19. 

SOLUTION:

The formulas for finding the volume and surface area of a prism are $V = Bh$ and $S = Ph + 2B$, where S = total surface area, V = volume, h = height of a solid, B = area of the base, and P = perimeter of the base.

Since the base of the prism is a square, the perimeter P of the base is $4(4.5)$ or 18 meters. The area of the base B is 4.5×4.5 or 20.25 square meters. The height is 4.5 meters.

$$\begin{aligned} S &= Ph + 2B && \text{Surface Area formula} \\ &= (18 \cdot 4.5) + 2(20.25) && \text{Substitution.} \\ &= 81 + 40.5 && \text{Multiply.} \\ &= 121.5 && \text{Addition.} \end{aligned}$$

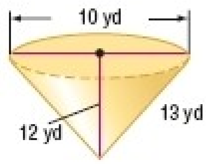
The surface area of the prism is 121.5 square meters.

$$\begin{aligned} V &= Bh && \text{Volume Formula} \\ &= (20.25)(4.5) && \text{Substitution.} \\ &\approx 91.1 && \text{Multiply.} \end{aligned}$$

The volume of the prism is 91.1 cubic meters.

ANSWER:

121.5 m^2 ; 91.1 m^3

20. 

SOLUTION:

The formulas for finding the volume and surface area of a cone are $V = \frac{1}{3}\pi r^2 h$ and $S = \pi r\ell + \pi r^2$, where S = total surface area, V = volume, r = radius, ℓ = slant height, and h = height.

Here, the diameter of the cone is 10 yards, so the radius is 5 yards. $h = 12$ yards and $\ell = 13$ yards

$$\begin{aligned} S &= \pi(5)(13) + \pi(5)^2 && \text{Surface Area Formula} \\ &= \pi(5)(13) + \pi(5)^2 && \text{Substitution.} \\ &= 65\pi + 25\pi && \text{Simplify.} \\ &= 90\pi && \text{Addition.} \\ &\approx 282.7 && \text{Use a calculator.} \end{aligned}$$

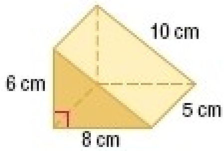
The surface area of the cone is 90π or about 282.7 square yards.

$$\begin{aligned} V &= \frac{1}{3}\pi r^2 h && \text{Volume Formula} \\ &= \frac{1}{3}\pi(5)^2(12) && \text{Substitution.} \\ &= \frac{1}{3}\pi(25)(12) && \text{Square 5.} \\ &= 100\pi && \text{Multiply.} \\ &\approx 314.2 && \text{Use a calculator.} \end{aligned}$$

The volume of the cone is about 314.2 cubic yards.

ANSWER:

90π or about 282.7 yd^2 ; 100π or about 314.2 yd^3

21. 

SOLUTION:

The formulas for finding the volume and surface area of a prism are $V = Bh$ and $S = Ph + 2B$, where S = total surface area, V = volume, h = height, B = area of the base, and P = perimeter of the base.
Since the base of the prism is a triangle, the perimeter P of the base is $8 + 6 + 10$ or 24 centimeters. The area of the base B is $\frac{1}{2}(8 \times 6)$ or 24 square centimeters. The height of the prism is 5 centimeters.

$$\begin{aligned} S &= Ph + 2B && \text{Surface Area formula.} \\ &= (24 \cdot 5) + 2(24) && \text{Substitution.} \\ &= 120 + 48 && \text{Multiply.} \\ &= 168 && \text{Addition.} \end{aligned}$$

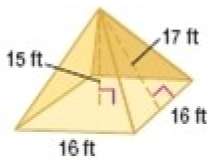
The surface area of the triangular prism is 168 square centimeters.

$$\begin{aligned} V &= Bh && \text{Volume Formula} \\ &= 24 \cdot 5 && \text{Substitution.} \\ &= 120 && \text{Multiply.} \end{aligned}$$

The volume of the prism is 120 cubic centimeters.

ANSWER:

168 cm^2 ; 120 cm^3

22. 

SOLUTION:

The formulas for finding the volume and total surface area of a pyramid are $V = \frac{1}{3}Bh$ and $S = \frac{1}{2}P\ell + B$, where *S* = total surface area, *V* = volume, *h* = height, *B* = area of the base, *P* = perimeter of the base, and ℓ = slant height.

Since the base of the pyramid is a square, the perimeter *P* of the base is 4×16 or 64 feet. The area of the base *B* is 16×16 or 256 square feet.

Here, $h = 15$ ft and $\ell = 17$ ft.

$$\begin{aligned} S &= \frac{1}{2}P\ell + B && \text{Surface Area formula} \\ &= \frac{1}{2}(64 \cdot 17) + 256 && \text{Substitution.} \\ &= 544 + 256 && \text{Multiply.} \\ &= 800 && \text{Addition.} \end{aligned}$$

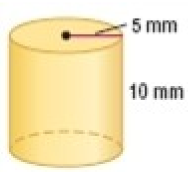
The surface area of the triangular prism is 800 square feet.

$$\begin{aligned} V &= \frac{1}{3}Bh && \text{Volume Formula} \\ &= \frac{1}{3}(256)(15) && \text{Substitution.} \\ &= 1280 && \text{Multiply.} \end{aligned}$$

The volume of the prism is 1280 cubic feet.

ANSWER:

800 ft^2 ; 1280 ft^3

23. 

SOLUTION:

The formulas for finding the volume and surface area of a cylinder are $V = \pi r^2 h$ and $S = 2\pi rh + 2\pi r^2$, where *S* = total surface area, *V* = volume, *r* = radius, and *h* = height.

Here, $r = 5$ mm and $h = 10$ mm.

$$\begin{aligned} S &= 2\pi rh + 2\pi r^2 && \text{Surface Area formula} \\ &= 2\pi(5)(10) + 2\pi(5)^2 && \text{Substitution.} \\ &= 100\pi + 50\pi && \text{Simplify.} \\ &= 150\pi && \text{Addition.} \\ &\approx 471.2 && \text{Use a calculator.} \end{aligned}$$

The surface area of the cylinder is 150π or about 471.2 mm^2.

$$\begin{aligned} V &= \pi r^2 h && \text{Volume Formula} \\ &= \pi(5)^2(10) && \text{Substitution.} \\ &= 250\pi && \text{Simplify.} \\ &\approx 785.4 && \text{Use a calculator.} \end{aligned}$$

The volume of the cylinder is 250π or about 785.4 mm^3.

ANSWER:

150π or about 471.2 mm^2 ; 250π or about 785.4 mm^3

24. **SANDBOX** A rectangular sandbox is 3 feet by 4 feet. The depth of the box is 8 inches, but the depth of the sand is $\frac{3}{4}$ of the depth of the box. Find each measure to the nearest tenth.

a. the surface area of the sandbox assuming there is no lid
b. the volume of sand in the sandbox

SOLUTION:

a. The formula for finding the surface area of a prism is $S = Ph + 2B$.
But here we don't have the lid. So, the surface area is given by $S = Ph + B$, where S = total surface area, h = height, B = area of the base, and P = perimeter of the base.

Since the base of the prism is a rectangle, the perimeter P of the base is $2(3) + 2(4)$ or 14 ft. The area of the base B is
3×4 or 12 ft^2. The height is 8 in or $\frac{2}{3}$ ft.

$$\begin{aligned} S &= Ph + 2B && \text{Surface Area formula} \\ &= \left(14 \cdot \frac{2}{3}\right) + 12 && \text{Substitution.} \\ &= \frac{28}{3} + 12 && \text{Multiply.} \\ &\approx 9.3 + 12 && \text{Division.} \\ &\approx 21.3 && \text{Addition.} \end{aligned}$$

The surface area of the box is about 21.3 ft^2.

b. The formula for finding the volume of a prism is $V = Bh$.

The depth of the sand is $\frac{3}{4}$ of the depth of the box.

$$\frac{3}{4} \cdot \frac{2}{3} = \frac{1}{2}$$

The depth of the sand is $\frac{1}{2}$ ft.

Substitute.

$$\begin{aligned} V &= Bh && \text{Volume formula} \\ &= 12 \cdot \frac{1}{2} && \text{Substitution.} \\ &= 6 && \text{Multiply.} \end{aligned}$$

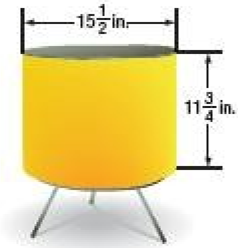
The volume of the sand in the sandbox is 6 ft^3.

ANSWER:

a. 21.3 ft^2
b. 6 ft^3

25. **ART** Fernando and Humberto Campana designed the Inflating Table shown below. The diameter of the table is

$15\frac{1}{2}$ inches. Suppose the height of the cylinder is $11\frac{3}{4}$ inches. Find each measure to the nearest tenth. Assume that the sides of the table are perpendicular to the bases of the table.



a. the volume of air that will fully inflate the table
b. the surface area of the table when fully inflated

SOLUTION:

The formulas for finding the volume and surface area of a cylinder are $V = \pi r^2 h$ and $S = 2\pi rh + 2\pi r^2$, where *S* = total surface area, *V* = volume, *r* = radius, and *h* = height.

$15\frac{1}{2} = \frac{31}{2}$

$11\frac{3}{4} = \frac{47}{4}$

a. The diameter of the cylinder is $\frac{31}{2}$, so the radius is $\frac{31}{4}$. The height $h = \frac{47}{4}$.

$$\begin{aligned} V &= \pi r^2 h && \text{Volume formula} \\ &= \pi\left(\frac{31}{4}\right)^2\left(\frac{47}{4}\right) && \text{Substitution.} \\ &= \pi(60.0625)(11.75) && \text{Simplify.} \\ &= (705.73\ldots)\pi && \text{Multiply.} \\ &\approx 2217.1 && \text{Use a calculator.} \end{aligned}$$

The volume of the air is about 2217.1 in^3.

b. Here, $r = 5$ mm and $h = 10$ mm.

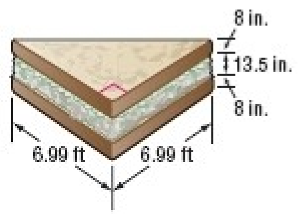
$$\begin{aligned} S &= 2\pi rh + 2\pi r^2 && \text{Surface Area Formula} \\ &= 2\pi\left(\frac{31}{4}\right)\left(\frac{47}{4}\right) + 2\pi\left(\frac{31}{4}\right)^2 && \text{Substitution.} \\ &= 182.125\pi + 120.125\pi && \text{Simplify.} \\ &= 302.25\pi && \text{Addition.} \\ &\approx 949.5 && \text{Use a calculator.} \end{aligned}$$

The area of the table is 302.25π or about 949.5 in^2.

ANSWER:

a. 2217.1 in^3
b. 949.5 in^2

26. **CCSS SENSE-MAKING** In 1999, Marks & Spencer, a British department store, created the biggest sandwich ever made. The tuna and cucumber sandwich was in the form of a triangular prism. Suppose each slice of bread was 8 inches thick. Find each measure to the nearest tenth.
a. the surface area in square feet of the sandwich when filled
b. the volume of filling in cubic feet to the nearest tenth



SOLUTION:

a. Use the Pythagorean Theorem to find the length of the third side of the triangle.

Let *x* be the length of the third side.
Then

$$\begin{aligned} d &= \sqrt{a^2 + b^2} && \text{Pythagorean Theorem} \\ &= \sqrt{6.99^2 + 6.99^2} && \text{Substitution.} \\ &= \sqrt{48.8601 + 48.8601} && \text{Square terms.} \\ &= \sqrt{97.7202} && \text{Addition.} \\ &\approx 9.89 && \text{Use a calculator.} \end{aligned}$$

The length of the third side of the triangle is about 9.89 ft.

Find the perimeter of the triangle.
The perimeter of the triangle is 6.99 + 6.99 + 9.89 or 23.87 ft.

Find the base area of the sandwich.
The base is a triangle. So, its area is given by,

$$\begin{aligned} A &= \frac{1}{2} \cdot 6.99 \cdot 6.99 \\ &\approx 24.43 \end{aligned}$$

The height of the sandwich is 8 + 13.5 + 8 or 29.5 in or $\frac{29.5}{12}$ ft

The formula for finding the surface area of a prism is $S = Ph + 2B$.

Substitute.

$$\begin{aligned} S &= Ph + 2B && \text{Surface Area formula} \\ &= 23.87 \cdot \frac{29.5}{12} + 2(24.43) && \text{Substitution.} \\ &\approx 58.68 + 48.86 && \text{Multiply.} \\ &\approx 107.5 && \text{Addition.} \end{aligned}$$

b. The formula for finding the volume of a prism is $V = Bh$.

Substitute.

$$\begin{aligned} V &= Bh && \text{Volume formula} \\ &= 24.43 \cdot \frac{13.5}{12} && \text{Substitution.} \\ &\approx 27.5 && \text{Multiply.} \end{aligned}$$

The volume of the filling is about 27.5 cubic feet.

ANSWER:

a. 107.5 ft^2
b. 27.5 ft^3

27. **ALGEBRA** The surface area of a cube is 54 square inches. Find the length of each edge.

SOLUTION:

There are six congruent sides in a cube. Each side is in the shape of a square. To find the surface area of the cube, find the sum of the area of each side of a cube.

Let *a* be the length of each side of a cube. So, the surface area of the cube is $6a^2$.

$$\begin{aligned} S &= 6a^2 && \text{Surface Area formula} \\ 54 &= 6a^2 && \text{Substitution.} \\ \frac{54}{6} &= \frac{6a^2}{6} && \text{Divide each side by 6.} \\ 9 &= a^2 && \text{Simplify.} \\ \sqrt{9} &= \sqrt{a^2} && \text{Square roots} \\ \pm 3 &= a && \text{Simplify.} \end{aligned}$$

The length must be positive. So, $a = 3$.
The length of each edge is 3 inches.

ANSWER:

3 in.

28. **ALGEBRA** The volume of a cube is 729 cubic centimeters. Find the length of each edge.

SOLUTION:

The formula for finding the volume of the prism is $V = Bh$.

The base of the cube is a square, so the area of the base is a^2. The length of height is equal to the length of the side, since all the sides are congruent in a cube.

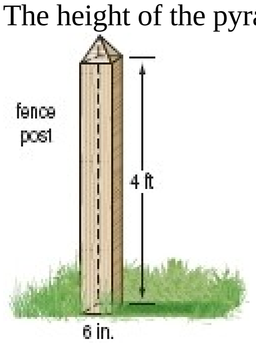
$$\begin{aligned} V &= a^3 && \text{Surface Area formula} \\ 729 &= a^3 && \text{Substitution.} \\ \sqrt[3]{729} &= \sqrt[3]{a^3} && \text{Square root} \\ 9 &= a && \text{Simplify.} \end{aligned}$$

The length of each edge is 9 cm.

ANSWER:

9 cm

29. **PAINTING** Tara is painting her family's fence. Each post is composed of a square prism and a square pyramid. The height of the pyramid is 4 inches. Determine the surface area and volume of each post.



SOLUTION:

Since the base of both the pyramid and the prism is a square, the perimeter P of the base is 4×6 or 24 inches. The area of the base B is 6×6 or 36 square feet. Here, height of the prism = 4 ft or 48 inches and the height of the pyramid = 4 inches.

Find the slant height. Use the Pythagorean Theorem to find the slant height.
The length of the side of the pyramid is 6 in. If you draw a slant height, it will form a right triangle with base 3 and height 4.

The diagonal is the slant height.
So,

$$\begin{aligned} \ell &= \sqrt{a^2 + b^2} && \text{Pythagorean Theorem} \\ &= \sqrt{4^2 + 3^2} && \text{Substitution.} \\ &= \sqrt{16 + 9} && \text{Square terms.} \\ &= \sqrt{25} && \text{Addition.} \\ &= 5 && \text{Simplify.} \end{aligned}$$

The total surface area of a square pyramid is $\frac{1}{2}P\ell + B$.

Here the base is attached with the prism. So, there is no need to add the base area.

$$\begin{aligned} S &= \frac{1}{2}P\ell && \text{Surface Area Formula} \\ &= \frac{1}{2}(24)(5) && \text{Substitution.} \\ &= 60 && \text{Multiply.} \end{aligned}$$

The total surface area of a square prism $= Ph + 2B$.

Here the top base is attached to the pyramid and the bottom is in the ground. So, there is no need to add the area of the two bases.

$$\begin{aligned} Ph &= (24)(48) \\ &= 1152 \end{aligned}$$

To find the surface area of the post, find the sum of the area of the prism and the pyramid.

Surface area of the post = 1152 + 60 = 1212 in^2

Volume of the square pyramid = $\frac{1}{3}Bh$

$$\begin{aligned} V &= \frac{1}{3}Bh && \text{Volume Formula} \\ &= \frac{1}{3}(36)(4) && \text{Substitution.} \\ &= 48 && \text{Multiply.} \end{aligned}$$

Volume of the square prism

$$\begin{aligned} V &= Bh && \text{Volume Formula} \\ &= (36)(4) && \text{Substitution.} \\ &= 1728 && \text{Multiply.} \end{aligned}$$

To find the volume of the post, find the sum of the volume of the prism and the pyramid.

Volume of the post = 1728 + 48 = 1776 in^3

ANSWER:

1212 in^2 ; 1776 in^3

30. **COLLECT DATA** Use a ruler or tape measure and what you have learned in this lesson to find the surface area and volume of a soup can.

SOLUTION:

A can of soup may be 3 inches in diameter and 4 inches high. Use a radius of 1.5 inches and the formulas to find the surface area and volume of the can.

$$
\begin{aligned}
T &= 2\pi rh + 2\pi r^2 && \text{Surface Area formula} \\
&= 2\pi(1.5)(4) + 2\pi(1.5)^2 && \text{Substitution} \\
&= 12\pi + 4.5\pi && \text{Multiply.} \\
&= 16.5\pi && \text{Simplify.} \\
&\approx 51.84 && \text{Use a calculator.}
\end{aligned}
$$

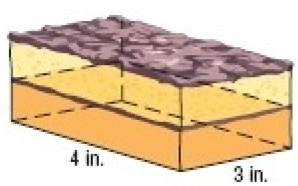
$$
\begin{aligned}
V &= \pi r^2 h && \text{Volume Formula} \\
&= \pi(1.5)^2(4) && \text{Substitution} \\
&= 9\pi && \text{Simplify.} \\
&\approx 28.27 && \text{Use a calculator.}
\end{aligned}
$$

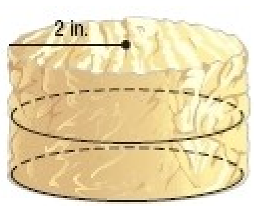
This can would have a surface area of about 51.84 in^2 and a volume of 28.27 in^3.
See students' work as measurements for soup cans will vary.

ANSWER:

See students' work.

31. **CAKES** Cakes come in many shapes and sizes. Often they are stacked in two or more layers, like those in the diagrams shown below.





a. If each layer of the rectangular prism cake is 3 inches high, calculate the area of the cake that will be frosted assuming there is no frosting between layers.
b. Calculate the area of the cylindrical cake that will be frosted, if each layer is 4 inches in height.
c. If one can of frosting will cover 50 square inches of cake, how many cans of frosting will be needed for each cake?
d. If the height of each layer of cake is 5 inches, what does the radius of the cylindrical cake need to be, so the same amount of frosting is used for both cakes? Explain your reasoning.

SOLUTION:

a. The formula for finding the surface area of a prism is $S = Ph + 2B$, where S = total surface area, h = height, B = area of the base, and P = perimeter of the base

Since the base of the prism is a rectangle, the perimeter P of the base is $2(3) + 2(4)$ or 14 inches. The area of the base B is
4 × 3 or 12 square inches. Each cake is 3 inches high. So, the height is 6 inches.

The top is not going to be frosted. So, the area to be frosted is given by $S = Ph + B$.

Substitute.

$S = Ph + B$	Surface Area formula
$= (14)(6) + 12$	Substitution.
$= 84 + 12$	Multiply.
$= 96$	Addition.

The area of the cake to be frosted is 96 in^2.

b. The formula for finding the surface area of a cylinder is $S = 2\pi rh + 2\pi r^2$, where S = total surface area, r = radius, and h = height.

Here, r = 2. The height of each cylindrical cake is 4 in. So, the total height is 8 in.
Since the top is not going to be frosted, the area to be frosted is given by $S = 2\pi rh + \pi r^2$.

$S = 2\pi rh + \pi r^2$	Surface Area formula
$= 2\pi(2)(8) + \pi(2)^2$	Substitution.
$= 32\pi + 4\pi$	Simplify.
$= 36\pi$	Addition.
≈ 113.1	Use a calcualtor.

The area of the cylindrical cake to be frosted is about 113.1 in^2.

c. Divide the area to be frosted by 50.

$\frac{96}{50} = 1.92$

So, 2 cans of frosting are needed for the rectangular prism cake.

$\frac{113.1}{50} = 2.262$

So, 3 cans of frosting are needed for the cylindrical cake.

d. Find the surface area of the rectangular cake if the height of the each layer 5 in.

$S = Ph + B$	Surface Area Formula
$= (14)(10) + 12$	Substitution.
$= 140 + 12$	Multiply.
$= 152$	Addition.

The surface area of the rectangular cake is 152 in^2.

To find the radius of a cylindrical cake with the same height, solve the equation $152 = \pi r^2 + 20\pi r$.
Solving the equation using the quadratic formula gives $r = –22.18$ and $r = 2.18$.

Since the radius can never be negative, $r = 2.18$.

The same amount of frosting will be needed if the radius of the cake is 2.18 in.

ANSWER:

a. 96 in^2
b. 113.1 in^2
c. prism: 2 cans; cylinder: 3 cans
d. 2.18 in.; if the height is 10 in., then the surface area of the rectangular cake is 152 in^2. To find the radius of a cylindrical cake with the same height, solve the equation $152 = \pi r^2 + 20\pi r$. The solutions are $r = –22.18$ or $r = 2.18$. Using a radius of 2.18 in. gives surface area of about 152 in^2.

32. **CHANGING UNITS** A gift box has a surface area of 6.25 square feet. What is the surface area of the box in square inches?

SOLUTION:

Surface area of the gift box = 6.25 ft^2
1 foot = 12 inches
Surface area of the gift box = 6.25(12 inches)^2
$= 6.25 \times 12 \times 12$ in^2
= 900 in^2

ANSWER:

900 in^2

33. **CHANGING UNITS** A square pyramid has a volume of 4320 cubic inches. What is the volume of this pyramid in cubic feet?

SOLUTION:

Volume of the pyramid = 4320 in^3
1 foot = 12 inches

So, 1 inch = $\frac{1}{12}$ foot.
Volume of a pyramid

$= 4320 \times \left(\frac{1}{12}\text{ft}\right)^3$

$= 4320 \times \left(\frac{1}{1728}\right)\text{ft}^3$

= 2.5 ft^3

ANSWER:

2.5 ft^3

34. **EULER'S FORMULA** The number of faces *F*, vertices *V*, and edges *E* of a polyhedron are related by Euler's (OY luhrz) Formula: $F + V = E + 2$. Determine whether Euler's Formula is true for each of the figures in Exercises 18–23.

SOLUTION:

Use Euler's formula: F + $V = E + 2$
Exercise 18:
Rectangular Prism: 6 + 8 = 12 + 2
So, Euler's formula is true. The answer is "Yes".

Exercise 19:
Square Prism: 6 + 8 = 12 + 2
So, Euler's formula is true. The answer is "Yes".

Exercise 20:
This figure isa cone and not a polyhedron, so Euler's Formula does not apply. So, the answer is "No".

Exercise 21:
Triangular Prism: 5 + 6 = 9 + 2
So, Euler's formula is true. The answer is "Yes".

Exercise 22:
Square Pyramid: 5 +5 = 8 +2
So, Euler's formula is true. The answer is "Yes".

Exercise 23:
This figure is a cylinder and not a polyhedron, so Euler's Formula does not apply. So, the answer is "No".

ANSWER:

Exercise 18: yes, 6 + 8 = 12 + 2; Exercise 19: yes, 6 + 8 = 12 + 2;Exercise 20: no, this figure is not a polyhedron, so Euler's Formula does not apply; Exercise 21: yes, 5 + 6 = 9 + 2; Exercise 22: yes, 5 + 5 = 8 + 2; Exercise 23: no, this figure is not a polyhedron, so Euler's Formula does not apply.

35. **CHANGING DIMENSIONS** A rectangular prism has a length of 12 centimeters, width of 18 centimeters, and height of 22 centimeters. Describe the effect on the volume of a rectangular prism when each dimension is doubled.

SOLUTION:

The formula for finding the volume of a prism is $V = Bh$, where V = volume, h = height, and B = area of the base. Since the base of the prism is a rectangle, the area of the base B is 12×18 or 216 square centimeters. Here, height of the prism = 22 cm.

$$\begin{aligned} V &= Bh && \text{Volume formula} \\ &= (216)(22) && \text{Substitution.} \\ &= 4752 && \text{Multiply.} \end{aligned}$$

The volume of the original prism is 4752 cm^3.

Double the dimensions and find the volume.

Volume of the new prism

$$\begin{aligned} V &= \ell wh && \text{Volume Formula} \\ &= (2 \cdot 12)(2 \cdot 18)(2 \cdot 22) && \text{Substitution.} \\ & && \text{Double the dimensions.} \\ &= (2 \cdot 2 \cdot 2)(12 \cdot 18 \cdot 22) && \text{Associative Property} \\ &= 8 \cdot 4752 && \text{Multiply.} \\ &= 38,016 && \text{Multiply.} \end{aligned}$$

The volume increased by a factor of 8 when each dimension was doubled.

ANSWER:

The volume of the original prism is 4752 cm^3. The volume of the new prism is 38,016 cm^3. The volume increased by a factor of 8 when each dimension was doubled.

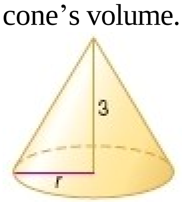
36. **MULTIPLE REPRESENTATIONS** In this problem, you will investigate how changing the length of the radius of a cone affects the cone's volume.

a. TABULAR Create a table showing the volume of a cone when doubling the radius. Use radius values between 1 and 8.

b. GRAPHICAL Use the values from your table to create a graph of radius versus volume.

c. VERBAL Make a conjecture about the effect of doubling the radius of a cone on the volume. Explain your reasoning.

d. ALGEBRAIC If r is the radius of a cone, write an expression showing the effect doubling the radius has on the cone's volume.



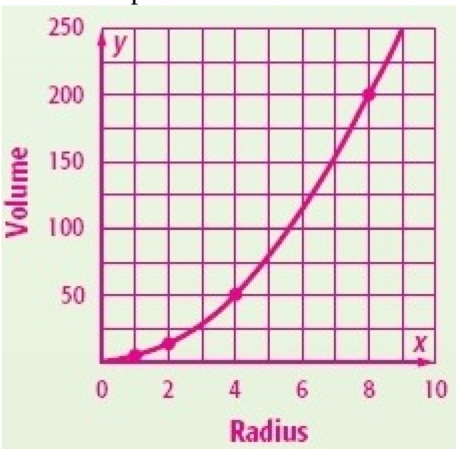
SOLUTION:

a. The formula for finding the volume of a cone is $V = \frac{1}{3}\pi r^2 h$, where V = volume, r = radius, and h = height.

Choose radius values between 1 and 8 and find the volume of the cone, then tabulate the results.

Radius	Volume
1	π
2	4π
4	16π
8	64π

b. Plot the points and draw a curve through the points on the coordinate plane.



c. Double the radius values and calculate the volume of the cone, then tabulate the results.

Double Radius	Volume
2	4π
4	16π
8	64π
16	256π

The volume of a cone is increased by a factor of 4 when the radius is doubled.

d. The formula for finding the volume of a cone is $V = \frac{1}{3}\pi r^2 h$.

Double the radius, that is, $2r$.

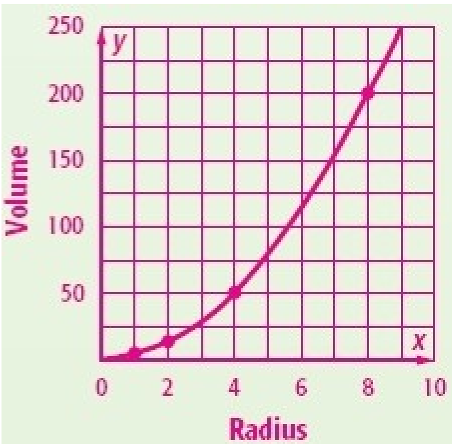
$$V = \frac{1}{3}\pi(2r)^2 h$$
$$= \frac{4}{3}\pi r^2 h$$

ANSWER:

a.

Radius	Volume
1	π
2	4π
4	16π
8	64π

b.



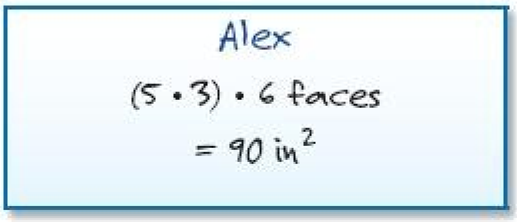
c.

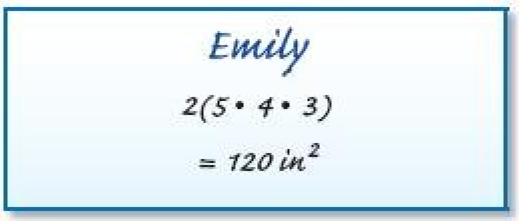
Double Radius	Volume
2	4π
4	16π
8	64π
16	256π

Doubling the radius results in an increase in the volume by a factor of 4.

d. $V = \frac{1}{3}\pi(2r)^2 h = \frac{4}{3}\pi r^2 h$

37. **CCSS CRITIQUE** Alex and Emily are calculating the surface area of the rectangular prism shown. Is either of them correct? Explain your reasoning.





SOLUTION:

Sample answer: The formula for finding the surface area of a prism is $S = Ph + 2B$, where S = total surface area, h = height, B = area of the base, and P = perimeter of the base.

Since the base of the prism is a rectangle, the perimeter P of the base is $2(5) + 2(4)$ or 18 inches. The area of the base B is 5×4 or 20 square inches.

Here, height of the prism = 3 ft.

$$\begin{aligned} S &= Ph + 2B && \text{Surface Area formula} \\ &= 18(3) + 2(20) && \text{Substitution.} \\ &= 54 + 40 && \text{Multiply.} \\ &= 94 && \text{Addition.} \end{aligned}$$

The total surface area of the prism is 94 in^2.
So, both answers are incorrect.

ANSWER:

Neither; sample answer: the surface area is twice the sum of the areas of the top, front, and left side of the prism or $2(5 \cdot 3 + 5 \cdot 4 + 3 \cdot 4)$, which is 94 in^2.

38. **REASONING** Is a cube a regular polyhedron? Explain.

SOLUTION:

In a cube, all of the faces are regular congruent squares and all of the edges are congruent. So, it is a regular polyhedron.
The answer is “Yes”.

ANSWER:

Yes; all of the faces are regular congruent squares and all of the edges are congruent.

39. **CHALLENGE** Describe the solid that results if the number of sides of each base increases infinitely. The bases of each solid are regular polygons inscribed in a circle.

a. pyramid
b. prism

SOLUTION:

a. A pyramid is a polyhedron that has a polygonal base and three or more triangular faces that meet at a common vertex. As the number of sides for the base increases towards infinity, the polygon for the base will approach the shape of the circle in which it is inscribed, and the triangular faces will become more of a curved surface. A cone is a solid with a circular base connected by a curved surface to a single vertex. So, as the number of sides of the base increases infinitely, the solid becomes a cone.
b. A prism is a polyhedron that has two parallel congruent polygonal bases connected by parallelogram faces. As the number of sides for the bases increases towards infinity, the polygon for the base will approach the shape of the circle in which it in inscribed and the parallelogram faces will become more of a curved surface. A cylinder is a solid with congruent parallel circular bases connected by a curved surface. So, as the number of sides of the base increases infinitely, the solid becomes a cylinder.

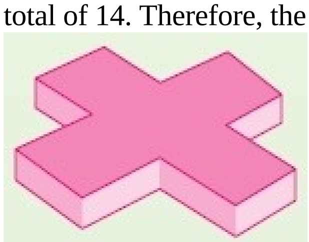
ANSWER:

a. cone
b. cylinder

40. **OPEN ENDED** Draw an irregular 14-sided polyhedron which has two congruent bases.

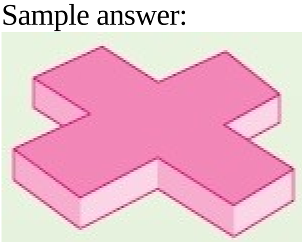
SOLUTION:

Sample answer: If the bases are congruent are congruent, there must be 12 faces connecting the bases to make a total of 14. Therefore, the two bases must be congruent 12-sided polygons.



ANSWER:

Sample answer:



41. **CHALLENGE** Find the volume of a cube that has a total surface area of 54 square millimeters.

SOLUTION:

There are six congruent sides in a cube. Each side is in the shape of a square. To find the surface area of the cube, find the sum of the area of each side of a cube.

Let *a* be the length of each side of a cube. So, the surface area of the cube is $6a^2$.

$$6a^2 = 54$$

$$\frac{6a^2}{6} = \frac{54}{6}$$

$$a^2 = 9$$

Take square root of each side.

$$a = \pm 3$$

The length must be positive. So, $a = 3$.

The length of each edge is 3 mm.

The formula for finding the volume of the prism is $V = Bh$.

The base of the cube is a square, so the area of the base is a^2. The length of height is equal to the length of the side, since all the sides are congruent in a cube.

$$V = a^3$$

Substitute $a = 3$.

$$V = 3^3$$

$$= 27$$

The volume of the cube is 27 mm^3.

ANSWER:

27 mm^3

42. **WRITING IN MATH** A reference sheet listed the formula for the surface area of a prism as $SA = Bh + 2B$. Use units of measure to explain why there must be a typographical error in the formula.

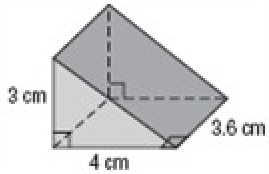
SOLUTION:

Sample answer: A prism has a rectangular base 5 inches long and 3 inches wide. The area of the base is $B = 5$ in. × 3 in. or 15 in^2. If the prism is 4 inches high, then $Bh = (15\ \text{in}^2)(4\ \text{in.})$ or 60 in^3. Twice the area of the base is $2B = 2(15\ \text{in}^2)$ or 30 in^2. The formula for the surface area then yields the expression $SA = 60\ \text{in}^3 + 30\ \text{in}^2$. The expression *Bh* is measured in cubic units and the expression 2*B* is measured in square units. Different units cannot be added, and surface area is measured in square units.

ANSWER:

Sample answer: The expression *Bh* is measured in cubic units and the expression 2*B* is measured in square units. Different units cannot be added, and surface area is measured in square units.

43. **GRIDDED RESPONSE** What is the surface area of the triangular prism in square centimeters?



SOLUTION:

The formula for finding the surface area of a prism is $S = Ph + 2B$, where S = total surface area, h = height, B = area of the base, and P = perimeter of the base.

Since the base of the prism is a triangle, the perimeter P of the base is $3+4+5$ or 12 cm. The area of the base B is $\frac{1}{2}(4\times 3)$ or 6 square centimeters.

Here, height of the prism = 3.6 cm.

$$\begin{aligned} S &= Ph + 2B && \text{Surface Area formula} \\ &= 12(3.6) + 2(6) && \text{Substitution.} \\ &= 43.2 + 12 && \text{Multiply.} \\ &= 55.2 && \text{Addition.} \end{aligned}$$

The total surface area of the triangular prism is 55.2 in cubic centimeters.

ANSWER:

55.2

44. **ALGEBRA** What is the value of $(–0.8)^2 + (–0.3)^3$?

A 0.627
B 0.613
C 0.370
D 0.327

SOLUTION:

$$\begin{aligned} (-0.8)^2 + (-0.3)^3 &= (0.64) + (-0.027) \\ &= 0.613 \end{aligned}$$

The correct choice is B.

ANSWER:

B

45. The length of each side of a cube is multiplied by 5. What is the change in the volume of the cube?
F The volume is 125 times the original volume.
G The volume is 25 times the original volume.
H The volume is 10 times the original volume.
J The volume is 5 times the original volume.

SOLUTION:

Let *a* be the length of each side of a cube. The base of the cube is a square, so the area of the base is a^2. The length of height is equal to the length of the side, since all the sides are congruent in a cube.

Volume of the original cube $= a^3$
The length of each side of a cube is multiplied by 5.
Volume of a new cube $= (5a)(5a)(5a)$
$V = 125a^3$
$V = 125 \times$ volume of the original cube

So, the correct option is F.

ANSWER:
F

46. **SAT/ACT** What is the difference in surface area between a cube with an edge length of 7 inches and a cube with edge length of 4 inches?

A 18 in^2

B 33 in^2

C 66 in^2

D 99 in^2

E 198 in^2

SOLUTION:

There are six congruent sides in a cube. Each side is in the shape of a square. To find the surface area of the cube, find the sum of the area of each side of a cube.

Let *a* be the length of each side of a cube. So, the surface area of the cube is $6a^2$.

Area of a cube with edge length of 4 = $6(4)^2$

= 6×16

= 96 in^2

Area of a cube with edge length of 7 = $6(7)^2$

= 6×49

= 294 in^2

Difference = 294 – 96

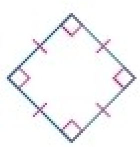
= 198 in^2

So, the correct option is E.

ANSWER:

E

Name each polygon by its number of sides. Then classify it as *convex* or *concave* and *regular* or *irregular*.

47. 

SOLUTION:

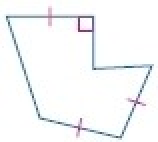
The polygon has 4 sides. A polygon with 4 sides is a quadrilateral.

None of the lines containing the sides will have points in the interior of the polygon. So, the polygon is convex.

All sides of the polygon are congruent and all angles are congruent. So it is regular.

ANSWER:

quadrilateral; convex; regular

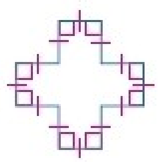
48. 

SOLUTION:

The polygon has 6 sides. A polygon with 6 sides is a hexagon.
If two of the sides are extended to make lines, they will pass through the interior of the hexagon, so it is concave. Since it is concave, it cannot be regular. So it is irregular.

ANSWER:

hexagon; concave; irregular

49. 

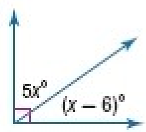
SOLUTION:

The polygon has 12 sides. A polygon with 12 sides is a dodecagon.
If two of the sides are extended to make lines, they will pass through the interior of the dodecagon, so it is concave. Since it is concave, it cannot be regular. So it is irregular.

ANSWER:

dodecagon; concave; irregular

Find the value of each variable.

50. 

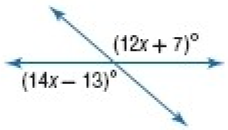
SOLUTION:

In the figure, the angles are complementary. Complementary angles have measures that sum to 90.

$$\begin{aligned} 5x + x - 6 &= 90 && \text{Def. of Complementary Angles} \\ 6x - 6 &= 90 && \text{Simplify.} \\ 6x - 6 + 6 &= 90 + 6 && \text{Add 6 to each side.} \\ 6x &= 96 && \text{Simplify.} \\ \frac{6x}{6} &= \frac{96}{6} && \text{Divide each side by 6.} \\ x &= 16 && \text{Simplify.} \end{aligned}$$

ANSWER:

16

51. 

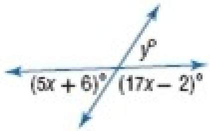
SOLUTION:

In the figure, angles $(14x-13)°$ and $(12x+7)°$ are vertical angles. Vertical angles are congruent.

$$\begin{aligned}
14x-13 &= 12x+7 && \text{Def. of Vertical Angles}\\
14x-12x-13 &= 12x-12x+7 && -12x \text{ from each side.}\\
2x-13 &= 7 && \text{Simplify.}\\
2x-13+13 &= 7+13 && +13 \text{ to each side.}\\
2x &= 20 && \text{Simplify.}\\
\frac{2x}{2} &= \frac{20}{2} && \div \text{ each side by 2.}\\
x &= 10 && \text{Simplify.}
\end{aligned}$$

ANSWER:

10

52. 

SOLUTION:

The angles in a linear pair are supplementary.
So, $(5x+6)°+(17x-2)°=180°$ and $y°+(17x-2)°=180°$.

Find x.

$$\begin{aligned} (5x+6)+(17x-2) &= 180 && \text{Def. of Linear Pair} \\ 22x+4 &= 180 && \text{Simplify.} \\ 22x+4-4 &= 180-4 && -4 \text{ from each side.} \\ 22x &= 176 && \text{Simplify.} \\ \frac{22x}{22} &= \frac{176}{22} && \div \text{ each side by 22.} \\ x &= 8 && \text{Simplify.} \end{aligned}$$

Substitute $x = 8$.

$$\begin{aligned} y+(17x-2) &= 180 && \text{Def. of Linear Pair.} \\ y+(17(8)-2) &= 180 && \text{Substitution.} \\ y+134 &= 180 && \text{Simplify.} \\ y+134-134 &= 180-134 && +134 \text{ to each side.} \\ y &= 46 && \text{Simplify.} \end{aligned}$$

ANSWER:

$x = 8$; $y = 46$

GAMES What type of geometric intersection is modeled in each photograph?

53. Refer to Page 74.

SOLUTION:

The wheel models a plane, and the arrow models a line. The intersection of a plane and a line not in the plane is a point.

ANSWER:

The intersection of a plane and a line not in the plane is a point.

54. Refer to Page 74.

SOLUTION:

The sides of the folder represent two planes that intersect at the crease. The crease has the shape of a line. So, this photograph represents that two planes can intersect in a line.

ANSWER:

Two planes can intersect in a line.

55. Refer to Page 74.

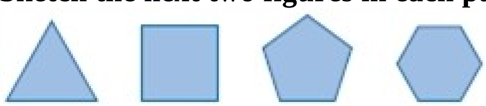
SOLUTION:

This photograph displays a set of black and red lines that run horizontally and vertically on a yellow plane. Each horizontal line intersects each vertical line in exactly one point. Therefore, this photograph represents that two lines always intersect in one point.

ANSWER:

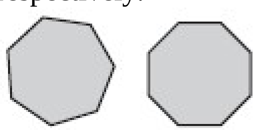
Two lines intersect in one point.

Sketch the next two figures in each pattern.

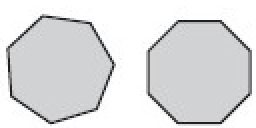
56. 

SOLUTION:

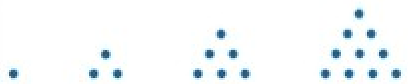
The number of sides increases by 1 with each figure. The first four figures are a regular triangle, square, pentagon, and hexagon. Therefore, the fifth figure and sixth figure should be a regular heptagon and a regular octagon, respectively.



ANSWER:

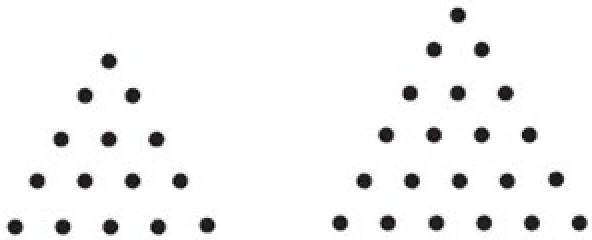


1-7 Three-Dimensional Figures

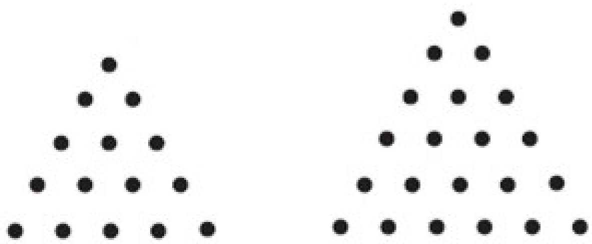
57. 

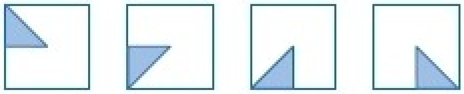
SOLUTION:

Each successive "triangle" has one more dot on each side.



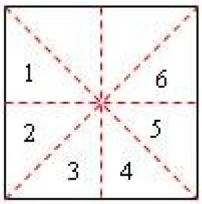
ANSWER:



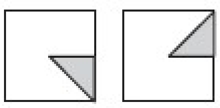
58. 

SOLUTION:

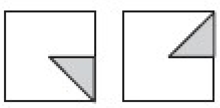
Divide the figure into eighths. Starting in the upper left corner, one of the eight sections is shaded each time rotating in a counterclockwise direction. The next two figures in the pattern should have the pieces shaded that are labeled 5 and 6 in the diagram.

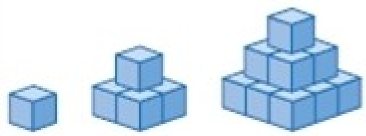


So, the next two figures in the pattern are:



ANSWER:

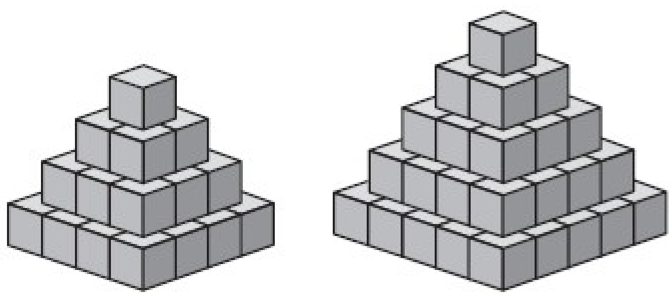


59. 

SOLUTION:

The figures resemble a pyramid with one more layer added to the base each time. The base starts as a square composed of one block, then a square of 4 blocks, and then a square of 9 blocks. Since $1 = 1^2$, $4 = 2^2$, and $9 = 3^2$, the next two figures would have a base with $4^2 = 16$ blocks and $5^2 = 25$ blocks, respectively.

So, the next two figures in the pattern are:



ANSWER: