2-1 Inductive Reasoning and Conjecture

Write a conjecture that describes the pattern in each sequence. Then use your conjecture to find the next item in the sequence.

1. Costs: $4.50, $6.75, $9.00 . . .

   SOLUTION:

   $6.75 = $4.50 + $2.25
   $9 = $7.75 + $2.25

   Each cost is $2.25 more than the previous cost.
   $9.00 + $2.25 = $11.25

   The next item in the sequence is $11.25.

   ANSWER:

   Each cost is $2.25 more than the previous cost; $11.25.

2. Appointment times: 10:15 a.m., 11:00 a.m., 11:45 a.m., . .

   SOLUTION:

   11:00 a.m. = 10:15 a.m. + 0:45
   11:45 a.m. =11:00 a.m. + 0:45

   Each time is 45 minutes later than the previous time.

   Then, 11:45 a.m. + 0:45 = 12:30 p.m.

   The next item in the sequence is 12:30 p.m.

   ANSWER:

   Each time is 45 minutes later than the previous time; 12:30 P.M.

3. 

   SOLUTION:

   In each figure, the shading moves to the next point clockwise.
   The next figure in the sequence is:

    .

   ANSWER:

   In each figure, the shading moves to the next point clockwise.

4.

SOLUTION:

Each figure in the pattern has an additional circle around the outside. The first figure is 1 circle, then 2 circles, and then 3 circles.
The next figure in the sequence is:

.

It has 4 circles.

ANSWER:

Each figure in the pattern has an additional circle around the outside.

5. 3, 3, 6, 9, 15, . . .

SOLUTION:

$6 = 3 + 3$
$9 = 3 + 6$
$15 = 6 + 9$

Each element in the pattern is sum of the previous two elements.

$9 + 15 = 24$

The next item in the sequence is 24.

ANSWER:

Each element in the pattern is the sum of the previous two elements; 24.

2-1 Inductive Reasoning and Conjecture

6. 2, 6, 14, 30, 62, . . .

SOLUTION:

Consider the difference in the terms.
$6 - 2 = 4$
$14 - 6 = 8$
$30 - 14 = 16$
$62 - 30 = 32$

Since the difference are not the same, the sequence is not arithmetic. However, each line, the answer is a factor of two more than the previous one.

$6 = 2(2) + x$
$14 = 2(6) + x$
$30 = 2(14) + x$
$62 = 2(30) + x$

Find the ? that make each statement true?

$$6 = 2(2) + x$$
$$6 = 4 + x$$
$$6 - 4 = 4 - 4 + x$$
$$6 - 4 = x$$

Then,
$6 = 2 + 2(2)$
$14 = 2 + 2(6)$
$30 = 2 + 2(14)$
$62 = 2 + 2(30)$.

Each element is two more than two times the previous element.

$2 + 2(62) = 126$

The next item in the sequence is 126.

ANSWER:

Each element is two more than two times the previous element; 126.

Make a conjecture about each value or geometric relationship.

7. the product of two even numbers

SOLUTION:

The product of two even numbers is an even number.
Examples:
$2 \times 2 = 4$
$2 \times 6 = 12$
$2 \times 8 = 16$ …

ANSWER:

The product of two even numbers is an even number.

8. the relationship between a and b if $a + b = 0$

   SOLUTION:

   If $a + b = 0$, then a and b are additive inverses.

   Examples:
   $3 + (-3) = 0$, 3 and –3 are additive inverses
   $4 + (-4) = 0$, 4 and –4 are additive inverses
   $1000 + (-1000) = 0$, 1000 and –1000 are additive inverses…

   ANSWER:

   a and b are additive inverses.

9. the relationship between the set of points in a plane equidistant from point A

   SOLUTION:

   The set of points in a plane equidistant from point A is a circle.

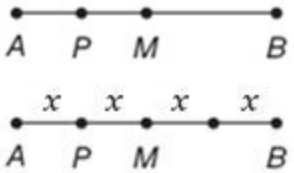
   ANSWER:

   The set of points in a plane equidistant from point A is a circle.

10. the relationship between $\overline{AP}$ and $\overline{PB}$ if M is the midpoint of $\overline{AB}$ and P is the midpoint of $\overline{AM}$

    SOLUTION:

    Given: M is the midpoint of $\overline{AB}$ and P is the midpoint of $\overline{AM}$

    Let the distance from A to P be x. Then the distance from A to M is $2x$ and from M to B is $2x$.
    The midpoint of M and B would be x away from either endpoint.
    So, $\overline{PB}$ is three times as long as $\overline{AP}$.

    ANSWER:

    $\overline{PB}$ is three times as long as $\overline{AP}$.

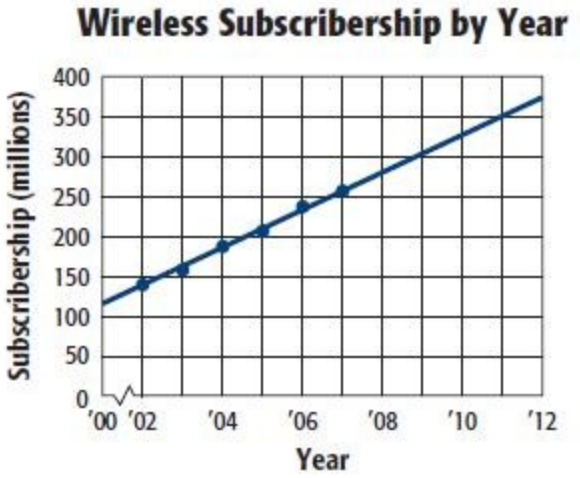
11. **CELL PHONES** Refer to the table of the number of wireless subscriptions in the United States by year.
    a. Make a graph that shows U.S. wireless use from 2002 to 2007.
    b. Make a conjecture about U.S. wireless use in 2012.

U.S. Wireless Subscribership	
Year	Subscribers (Millions)
2002	140.8
2003	158.7
2004	182.1
2005	207.9
2006	233.0
2007	255.4

Source: Cellular Telecommunications and Internet Association

SOLUTION:

a. Plot the points on the coordinate plane.



b. Sample answer: Use the first and last data points to find the slope.

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$
$$= \frac{255.4 - 140.8}{2007 - 2002}$$
$$= 22.92$$

Use the point-slope formula to find the equation of the line that models the graph.

$$y - y_1 = m(x - x_1) \quad \text{Point-Slope Form}$$
$$y - 140.8 = 22.92(x - 2002) \quad \text{Substitution.}$$
$$y - 140.8 = 22.92x - 45,885.84 \quad \text{Distributive Property}$$
$$y = 22.92x - 45,745.04 \quad \text{Simplify.}$$

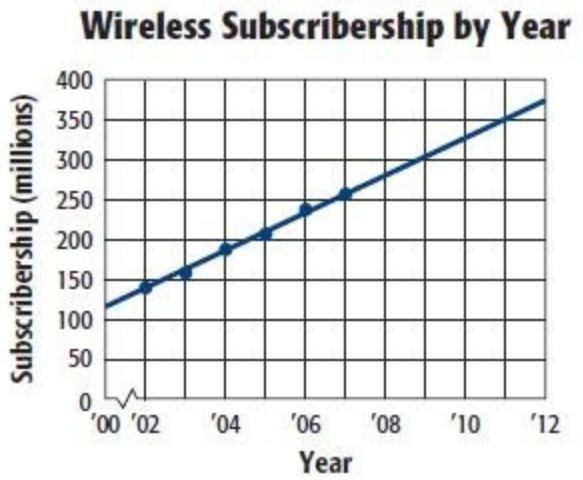
Substitute 2012 for x in this equation to find the number of subscribers in millions.

$$\begin{aligned} y &= 22.92x - 45,745.04 \\ &= 22.92(2012) - 45,745.04 && \text{Substitution.} \\ &= 46,115.04 - 45,745.04 && \text{Simplify.} \\ &= 370 && \text{Simplify.} \end{aligned}$$

So, in 2012 about 370,000,000 Americans will have wireless subscriptions.

ANSWER:

a.



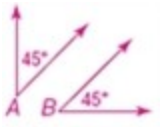
b. Sample answer: About 370,000,000 Americans will have wireless subscriptions in 2012.

CCSS CRITIQUE Find a counterexample to show that each conjecture is false.

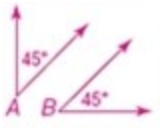
12. If $\angle A$ and $\angle B$ are complementary angles, then they share a common side.

SOLUTION:

Draw two angles that are not connected, but still sum to 90 degrees.



ANSWER:

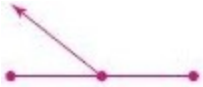


13. If a ray intersects a segment at its midpoint, then the ray is perpendicular to the segment.

SOLUTION:

Draw a ray that intersects a segment, but not at a 90 degree angle.

ANSWER:



Write a conjecture that describes the pattern in each sequence. Then use your conjecture to find the next item in the sequence.

14. 0, 2, 4, 6, 8

SOLUTION:

$2 = 0 + 2$
$4 = 2 + 2$
$6 = 2 + 4$
$8 = 2 + 6$
Each element in the pattern is two more than the previous element.
$2 + 8 = 10$
The next element in the sequence is 10.

ANSWER:

Each element in the pattern is two more than the previous element; 10.

15. 3, 6, 9, 12, 15

SOLUTION:

$6 = 3 + 3$
$9 = 3 + 6$
$12 = 3 + 9$
$15 = 3 + 12$
Each element in the pattern is three more than previous element.
$3 + 15 = 18$
The next element in the sequence is 18.

ANSWER:

Each element in the pattern is three more than the previous element; 18.

16. 4, 8, 12, 16, 20

SOLUTION:

$8 = 4 + 4$
$12 = 4 + 8$
$16 = 4 + 12$
$20 = 4 + 16$
Each element in the pattern is four more than the previous element.
$4 + 20 = 24$
The next element in the sequence is 24.

ANSWER:

Each element in the pattern is four more than the previous element; 24.

17. 2, 22, 222, 2222

SOLUTION:

Each element has an additional two as part of the number.
$22 = 2 + 2 \cdot 10$
$222 = 2 + 22 \cdot 10$
$2222 = 2 + 222 \cdot 10$
Thus the next term is $2 + 2222 \cdot 10$ or 22222.

ANSWER:

Each element has an additional two as part of the number; 22222.

18. 1, 4, 9, 16

SOLUTION:

$1 = 1^2$
$4 = 2^2$
$9 = 3^2$
$16 = 4^2$
Each element is the square of increasing natural numbers.
$5^2 = 25$
The next element in the sequence is 25.

ANSWER:

Each element is the square of increasing natural numbers; 25.

19. $1, \frac{1}{2}, \frac{1}{4}, \frac{1}{8}$

SOLUTION:

$\frac{1}{2} = 1\left(\frac{1}{2}\right)$

$\frac{1}{4} = \frac{1}{2}\left(\frac{1}{2}\right)$

$\frac{1}{8} = \frac{1}{4}\left(\frac{1}{2}\right)$

Each element is one half the previous element.

$\frac{1}{8}\left(\frac{1}{2}\right) = \frac{1}{16}$

The next element in the sequence is $\frac{1}{16}$.

ANSWER:

Each element is one half the previous element; $\frac{1}{16}$.

20. Arrival times: 3:00 P.M., 12:30 P.M., 10:00 A.M., . . .

SOLUTION:

12:30 P.M.= 3:00 P.M., – 2:30
10:00 A.M.= 12:30 P.M. – 2:30

Each arrival time is 2 hours and 30 minutes prior to the previous arrival time.

10:00 A.M. – 2:30 = 7:30 A.M.

The next arrival time in the sequence is 7:30 A.M.

ANSWER:

Each arrival time is 2 hours and 30 minutes prior to the previous arrival time; 7:30 A.M.

21. Percent humidity: 100%, 93%, 86%, . . .

SOLUTION:

93% = 100% – 7%
86% = 93% – 7%

Each percentage is 7% less than the previous percentage.
86% – 7% = 79%

The next percentage in the sequence is 79%.

ANSWER:

Each percentage is 7% less than the previous percentage; 79%.

22. Work-out days: Sunday, Tuesday, Thursday, . . .

SOLUTION:

Sample answer:
Tuesday is two days after Sunday.
Thursday is two days after Tuesday.

Each work out day is two days after the previous day.

So, the next work out day in the sequence is Saturday.

ANSWER:

Sample answer: each work out day is two days after the previous day; Saturday.

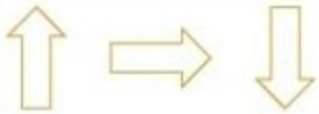
23. Club meetings: January, March, May, . . .

SOLUTION:

March is two month after January and May is two months after March.
Each meeting is two months after the previous meeting. The next month is in the sequence is July.

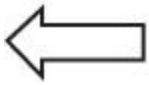
ANSWER:

Each meeting is two months after the previous meeting; July.

24. 

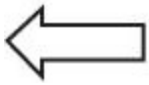
SOLUTION:

The direction of the arrow in the pattern rotates clockwise from one figure to the next.



ANSWER:

The direction of the arrow in the pattern rotates clockwise from one figure to the next.

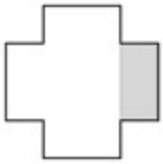


2-1 Inductive Reasoning and Conjecture

25. 

SOLUTION:

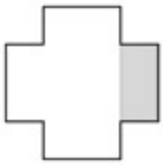
In each figure, the shading moves to the next area of the figure counter clockwise. The next figure in the sequence is



.

ANSWER:

In each figure, the shading moves to the next area of the figure counter clockwise.



26. 

SOLUTION:

The first polygon has 3 sides, then 4 sides, and then 5 sides.Each figure in the pattern is the next largest regular polygon. The next figure in the sequence will have 6 sizes and is

.

ANSWER:

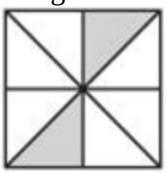
Each figure in the pattern is the next largest regular polygon.

2-1 Inductive Reasoning and Conjecture

27. 

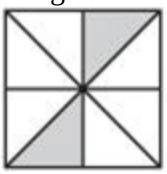
SOLUTION:

The shading of the lower triangle in the upper right quadrant of the first figure moves clockwise through each set of triangles from one figure to the next. The next figure in the sequence is

 .

ANSWER:

The shading of the lower triangle in the upper right quadrant of the first figure moves clockwise through each set of triangles from one figure to the next.



28. **FITNESS** Gabriel started training with the track team two weeks ago. During the first week, he ran 0.5 mile at each practice. The next three weeks he ran 0.75 mile, 1 mile, and 1.25 miles at each practice. If he continues this pattern, how many miles will he be running at each practice during the 7th week?

SOLUTION:

Form a sequence using the given information.
Running speed at each practice: 0.5, 0.75, 1, 1.25, …

Each element in the pattern is 0.25 more than previous element.

$1 \text{ week} \rightarrow 0.5$

$2 \text{ week} \rightarrow 0.25 + 0.5 = 0.75$

$3 \text{ week} \rightarrow 0.25 + 0.75 = 1$

$4 \text{ week} \rightarrow 0.25 + 1 = 1.25$

$5 \text{ week} \rightarrow 0.25 + 1.25 = 1.5$

$6 \text{ week} \rightarrow 0.25 + 1.5 = 1.75$

$7 \text{ week} \rightarrow 0.25 + 1.75 = 2$

He will run 2 miles during the 7th week.

ANSWER:

2 mi

29. **CONSERVATION** When there is a shortage of water, some municipalities limit the amount of water each household is allowed to consume. Most cities that experience water restrictions are in the western and southern parts of the United States. Make a conjecture about why water restrictions occur in these areas.

SOLUTION:

Dry, hot conditions can be caused by a lack of rainfall. When there is little to no rainfall, water stores cannot be refilled. There is increased demand for water for lawns and gardens. It is drier in the west and hotter in the south than other parts of the country, so less water would be readily available.

ANSWER:

Sample answer: It is drier in the west and hotter in the south than other parts of the country, so less water would be readily available.

30. **VOLUNTEERING** Carrie collected canned food for a homeless shelter in her area each day for one week. On day one, she collected 7 cans of food. On day two, she collected 8 cans. On day three she collected 10 cans. On day four, she collected 13 cans. If Carrie wanted to give at least 100 cans of food to the shelter and this pattern of can collecting continued, did she meet her goal?

SOLUTION:

First day : 7
Second day: 7 + 1 = 8
Third day: 8 + 2 = 10
Fourth day: 10 + 3 = 13
Fifth day: 13 + 4 = 17
Sixth day: 17 + 5 = 22
Seventh day: 22 + 6 = 28

7 + 8 + 10 + 13 + 17 + 22 + 28 = 105

Yes. She collected 105 can at the end of one week.

ANSWER:

Yes; she collected 105 cans.

Make a conjecture about each value or geometric relationship.

31. the product of two odd numbers

SOLUTION:

The product of two odd numbers is an odd number.
Examples:

$$3 \times 3 = 9$$
$$101 \times 3 = 303$$
$$99 \times 9 = 891$$

ANSWER:

The product is an odd number.

32. the product of two and a number, plus one

SOLUTION:

The product of two and a number, plus one is always an odd number.
Examples:

$$2(3)+1=7$$
$$2(5)+1=11$$
$$2(1000)+1=2001$$

ANSWER:

The result is odd.

33. the relationship between a and c if $ab = bc$, $b \neq 0$

SOLUTION:

If $ab = bc, b \neq 0$, then a and c are equal.
Examples:

$$2 \times 3 = 3 \times 2$$
$$100 \times 5 = 5 \times 100$$
$$9999 \times 59 = 59 \times 9999$$

ANSWER:

They are equal.

34. the relationship between a and b if $ab = 1$

SOLUTION:

If $ab = 1$, then a and b are reciprocals.
Examples:

$$2 \times \frac{1}{2} = 1$$
$$100 \times \frac{1}{100} = 1$$
$$9999 \times \frac{1}{9999} = 1$$

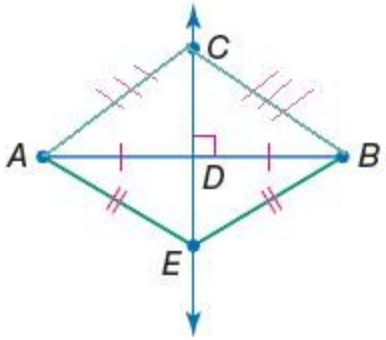
ANSWER:

They are reciprocals.

35. the relationship between $\overline{AB}$ and the set of points equidistant from A and B

SOLUTION:

The points equidistant from A and B form the perpendicular bisector of $\overline{AB}$.



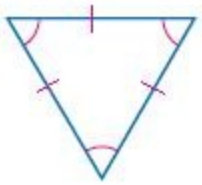
ANSWER:

The points equidistant from A and B form the perpendicular bisector of $\overline{AB}$.

36. the relationship between the angles of a triangle with all sides congruent

SOLUTION:

If all the sides are congruent, then the angles are all congruent.



ANSWER:

The angles are all congruent.

37. the relationship between the areas of a square with side x and a rectangle with sides x and $2x$

SOLUTION:

Area of the square = x^2

Area of the rectangle = $2x^2$

The area of the rectangle is two times the area of the square.

ANSWER:

The area of the rectangle is two times the area of the square.

38. the relationship between the volume of a prism and a pyramid with the same base

SOLUTION:

Volume of a prism = Bh

Volume of a pyramid = $\frac{1}{3}Bh$

The volume of a prism is three times the volume of a pyramid.

ANSWER:

The volume of the prism is three times the volume of the pyramid.

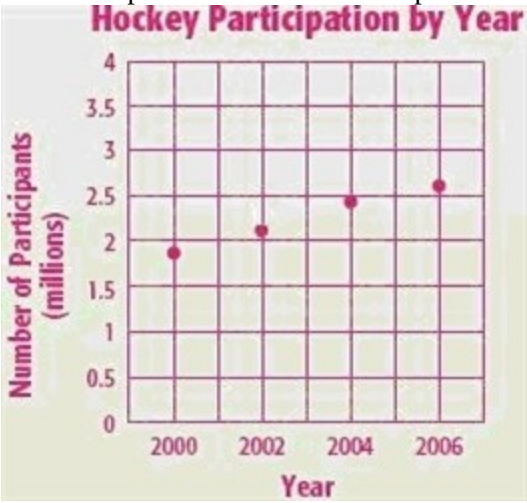
39. **SPORTS** Refer to the table of Americans over the age of 7 that played hockey.

Year	Number of Participants (millions)
2000	1.9
2002	2.1
2004	2.4
2006	2.6

a. Construct a statistical graph that best displays the data.
b. Make a conjecture based on the data and explain how this conjecture is supported by your graph.

SOLUTION:

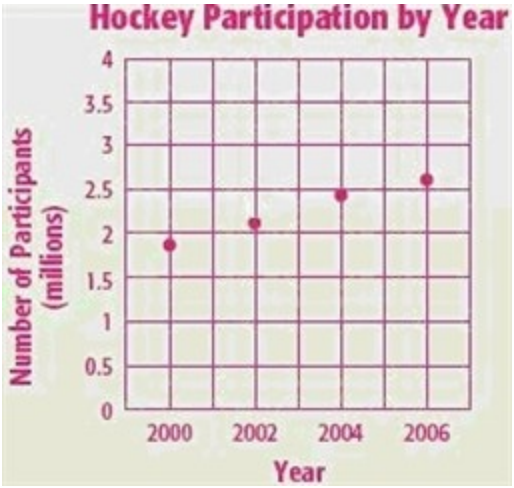
a. Plot the points on the coordinate plane.



b. Sample answer: More people over the age of 7 will play hockey in the future. The number of people playing hockey increases each year, so the graph suggests that even more people will play hockey in subsequent years.

ANSWER:

a.



b. Sample answer: More people over the age of 7 will play hockey in the future. The number of people playing hockey increases each year, so the graph suggests that even more people will play hockey in subsequent years.

CCSS CRITIQUE Determine whether each conjecture is *true* or *false* . Give a counterexample for any false conjecture.

40. If n is a prime number, then $n + 1$ is not prime.

SOLUTION:

False

Sample answer: If $n = 2$, then $n + 1 = 3$, a prime number.

ANSWER:

False; Sample answer: If $n = 2$, then $n + 1 = 3$, a prime number.

41. If x is an integer, then $-x$ is positive.

SOLUTION:

False.

Sample answer: Suppose $x = 2$, then $-x = -2$.

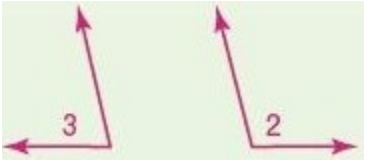
ANSWER:

False; sample answer: Suppose $x = 2$, then $-x = -2$.

42. If $\angle 2$ and $\angle 3$ are supplementary angles, then $\angle 2$ and $\angle 3$ form a linear pair.

SOLUTION:

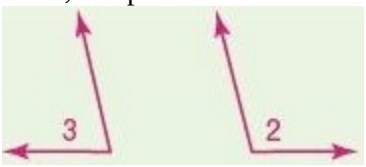
False



Since $\angle 3$ and $\angle 2$ are supplementary $m\angle 3 + m\angle 2 = 180$. However, to be a linear pair, then need to be adjacent angles and have noncommon side that are opposite rays.

ANSWER:

False; sample answer:



43. If you have three points A, B, and C, then A, B, C are noncollinear.

SOLUTION:

False

Sample answer:

ANSWER:

False; sample answer:

44. If in $\Delta ABC, (AB)^2 + (BC)^2 = (AC)^2$, then ΔABC is a right triangle.

SOLUTION:
This follows the Pythagorean Theorem.

true

ANSWER:
true

45. If the area of a rectangle is 20 square meters, then the length is 10 meters and the width is 2 meters.

SOLUTION:
False
Sample answer: The length could be 4 m and the width could be 5 m.

ANSWER:
False; sample answer: The length could be 4 m and the width could be 5 m.

FIGURAL NUMBERS Numbers that can be represented by evenly spaced points arranged to form a geometric shape are called figural numbers. For each figural pattern below,
a. write the first four numbers that are represented,
b. write a conjecture that describes the pattern in the sequence,
c. explain how this numerical pattern is shown in the sequence of figures,
d. find the next two numbers, and draw the next two figures.

46. 

SOLUTION:
a. Each points represents 1. Count the points for each figure.

1, 3, 6, 10

b. A few different methods can be used here.Look for a basic pattern first.

1 + 2 = 3
3 + 3 = 6
6 + 4 = 10

It appears that if we add one more to the number than we did the previous time, we will get the next number. So, the 2nd term equals the 1st term plus 2, the 3rd term equals the 2nd term plus 3, and the 4th term equals the 3rd term plus 4.

Sample answer: Add the position number to the previous number to get the next number in the sequence.

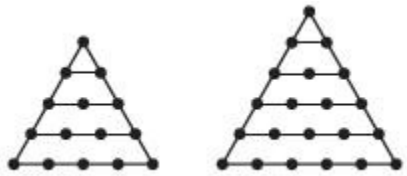
c. An additional row is is placed at the bottom of the figure each time. The number of points in the additional row is equal to the number of the figure.

Sample answer: Each figure is the previous figure with an additional row added that has one more point in it than in the last row.

d. For the 5th term, add 5: 10 + 5 = 15.

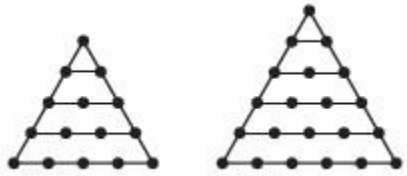
For the 6th term, add 6: 15 + 6 = 21.

Add a row of 5 points, and then add a row of 6 points.



ANSWER:

a. 1, 3, 6, 10
b. Sample answer: Add the position number to the previous number to get the next number in the sequence.
c. Sample answer: Each figure is the previous figure with an additional row added that has one more point in it than in the last row.
d. 15, 21



47.

SOLUTION:

a. Each points represents 1. Count the points for each figure.

1, 4, 9, 16

b. A few different methods can be used here.Look for a basic pattern first.

1, 4, 9, and 16 are all perfect squares, and the figures all look like squares, so this could be the pattern..

Another method could be adding 3 to 1 to get the second number, 4. Continue adding the next odd number to the previous number to get the next number in the sequence.

1 + 3 = 4
4 + 5 = 9
9 + 7 = 16

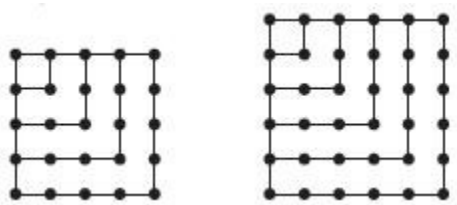
c. It looks like an extra row and column of points is added to the previous square to create a larger square. The number of points on each side of the square is equal to the square's position in the pattern.

Each figure is the previous figure with an additional row and column of points added, which is 2(position number) –

2-1 Inductive Reasoning and Conjecture

1. One is subtracted since 2(position number) counts the corner point twice. 2(position number) – 1 is always an odd number.

d. Add the row and column minus one. (Add 9 points and then 11 points)



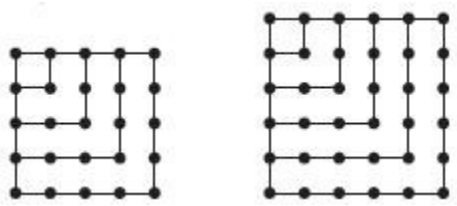
ANSWER:

a. 1, 4, 9, 16

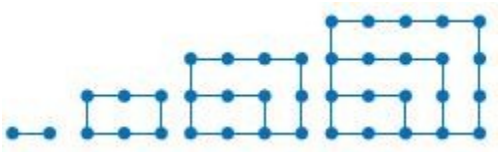
b.Sample answer: Start by adding 3 to 1 to get the second number, 4. Continue adding the next odd number to the previous number to get the next number in the sequence.

c. Sample answer: Each figure is the previous figure with an additional row and column of points added, which is 2 (position number) – 1. One is subtracted since 2(position number) counts the corner point twice. 2(position number) – 1 is always an odd number.

d. 25, 36



2-1 Inductive Reasoning and Conjecture

48. 

SOLUTION:

a.Each point represents one. Count the points.

2, 6, 12, 20

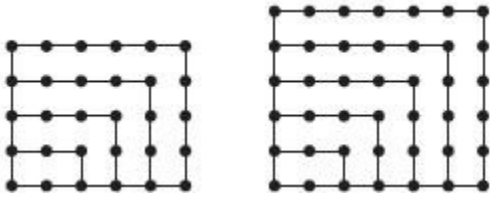
b. Look for a simple pattern first.

2 + 4 = 6
6 + 6 = 12
12 + 8 = 20

It looks like for each term that we add two more than we did previously.

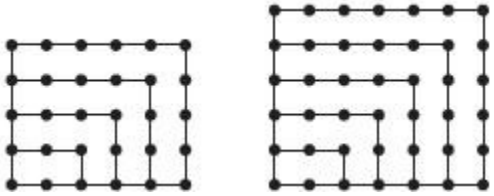
c. Sample answer: The second figure is the previous figure with 4 points added to make a rectangle. The third figure is the previous figure with 6 points added, which is 2 more than the last number of points added. The fourth figure is the previous figure with 8 points added, which is 2 more than the last number of points added.

d. Add 6 points along the top and 4 along the side to make 30. Add 7 points along the top and 5 along the side to make 42.



ANSWER:

a.2, 6, 12, 20
b. Sample answer: Start by adding the even number 4 to the first number to get the second number, 6. To get each of the next numbers, add the next even number to the previous number in the sequence.
c. Sample answer: The second figure is the previous figure with 4 points added to make a rectangle. The third figure is the previous figure with 6 points added, which is 2 more than the last number of points added. The fourth figure is the previous figure with 8 points added, which is 2 more than the last number of points added.
d. 30, 42



2-1 Inductive Reasoning and Conjecture

49. 

SOLUTION:

a. Each point represents one. count the number of points.
1, 5, 12, 22

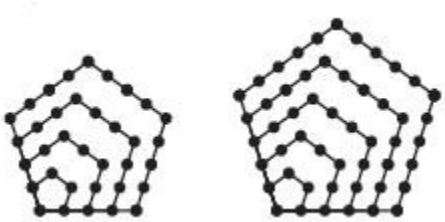
b. Look for a simple pattern first.
1 + 4 = 5
5 + 7 = 12
12 + 10 = 22

It looks like we are adding 3 more each time.

Sample answer: Start by adding 4 to 1 to get the second number, 5. Increase the amount added to the previous number by 3 each time to get the next number in sequence. So, add 4 + 3 or 7 to 5 to get 12, and add 4 + 3 + 3 or 10 to 12 to get 22.

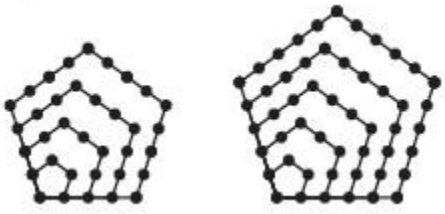
c. Sample answer: The second figure is the previous figure with 4 points added to make a pentagon. The third figure is the previous figure with 7 more points added, which is 3 more than the last number of points added. The fourth figure is the previous figure with 10 points added, which is 3 more than the last number of points added.

d. From left to right, add 5 points up, 4 points down, then 4 points back to complete the pentagon with 35 points. From left to right, add 6 points up, 5 points down, then 5 points back to complete the pentagon with 51 points.



ANSWER:

a. 1, 5, 12, 22
b. Sample answer: Start by adding 4 to 1 to get the second number, 5. Increase the amount added to the previous number by 3 each time to get the next number in sequence. So, add 4 + 3 or 7 to 5 to get 12, and add 4 + 3 + 3 or 10 to 12 to get 22.
c. Sample answer: The second figure is the previous figure with 4 points added to make a pentagon. The third figure is the previous figure with 7 more points added, which is 3 more than the last number of points added. The fourth figure is the previous figure with 10 points added, which is 3 more than the last number of points added.
d. 35, 51



50. The sequence of odd numbers, 1, 3, 5, 7, . . . can also be a sequence of figural numbers. Use a figural pattern to represent this sequence.

SOLUTION:

More than one pattern is possible. The basic pattern would be to make an L -shape with one point at the bottom and an equal number of points up and down. Since we will be adding 2 each time, one point added to the top and one added to the bottom will represent the addition easily.

Sample answer:

ANSWER:

Sample answer:

51. **GOLDBACH'S CONJECTURE** Goldbach's conjecture states that every even number greater than 2 can be written as the sum of two primes. For example, $4 = 2 + 2$, $6 = 3 + 3$, and $8 = 3 + 5$.

a. Show that the conjecture is true for the even numbers from 10 to 20.

b. Given the conjecture *All odd numbers greater than 2 can be written as the sum of two primes,* is the conjecture *true* or *false*? Give a counterexample if the conjecture is false.

SOLUTION:

a. $10 = 5 + 5$, $12 = 5 + 7$, $14 = 7 + 7$, $16 = 5 + 11$, $18 = 7 + 11$, and $20 = 7 + 13$.

b. False; 3 cannot be written as the sum of two primes.

ANSWER:

a. $10 = 5 + 5$, $12 = 5 + 7$, $14 = 7 + 7$, $16 = 5 + 11$, $18 = 7 + 11$, $20 = 7 + 13$

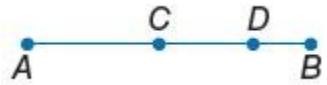
b. False; 3 cannot be written as the sum of two primes.

52. **SEGMENTS** Two collinear points form one segment, as shown for $\overline{AB}$. If a collinear point is added to $\overline{AB}$, the three collinear points form two segments.

a. How many distinct segments are formed by four collinear points? by five collinear points?
b. Make a conjecture about the number of distinct segments formed by *n* collinear points.
c. Test your conjecture by finding the number of distinct segments formed by six points.

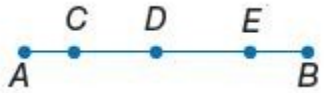
SOLUTION:

a. Six distinct segments are formed by four collinear points.



Segments: *ACDB, ACD, CDB, AC, CD,* and *DB*

Ten distinct segments are formed by five collinear points.



Segments: *ACDEB, ABDE, CDEB, ACD, CDE, DEB, AC, CD, DE, EB*

b. Three distinct segments are formed by 3 collinear points, Six distinct segments are formed by four collinear points and, ten distinct segments are formed by five collinear points.Let n = number collinear points
When $n = 3$, there is one segment of length 2 and two segments of length 1 for a total of 3 segments. Or $3 = 1 + 2$

When $n = 4$, there is one segment of length 3, two segments of length 2, and three segments of length 1, for a total of 5 segments. Or
$6 = 1 + 2 + 3$

When $n = 5$, there is one segment of length 4, two segments of length 3, three segments of length 2, and four segments of length 1, for a total of 10 segments. Or $10 = 1 + 2 + 3 + 4$.

Thus, the number of segments formed is the sum of the whole numbers less than n.
c. Fifteen segments are formed with six collinear points.
Test the conjecture.
$n = 6$
$0 + 1 + 2 + 3 + 4 + 5 = 15$

Segments: *ACDEFB, ACDEF, CDEFB, ACDE, CDEF, DEFB, ACD, CDE, DEF, EFB, AC, CD, DE, EF, FB*
So, the conjecture is correct.

ANSWER:

a. 6; 10
b. The number of segments formed is the sum of the whole numbers less than n.
c. Fifteen segments are formed with six points. The conjecture is correct.

53. **CCSS TOOLS** Using dynamic geometry software, Nora calculates the perimeter P and area A of a regular hexagon with a side length of 2 units. The change to the perimeter and area after three doublings of this side length are listed in the table. Analyze the patterns in the table. Then make a conjecture as to the effects on the perimeter and area of a regular hexagon when the side length is doubled. Explain.

Side (units)	P (units)	A (units^2)
2	12	$6\sqrt{3}$
4	24	$24\sqrt{3}$
8	48	$96\sqrt{3}$
16	96	$384\sqrt{3}$

SOLUTION:

In the sequence of perimeters, each measure is twice the previous measure.

Side	# of sides	P	P
2	6	12	12
4	6	$2 \cdot 12$	24
8	6	$2 \cdot 24$	48
16	6	$2 \cdot 48$	96

Therefore, doubling the side length of a regular hexagon doubles its perimeter.

In the sequence of areas, each measure is four times the previous measure.

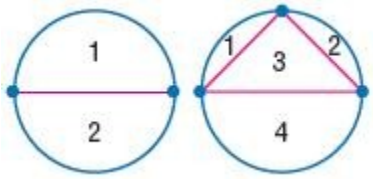
Side	# of sides	Area	Area
2	6	$6\sqrt{3}$	$6\sqrt{3}$
4	6	$4 \cdot 6\sqrt{3}$	$24\sqrt{3}$
8	6	$4 \cdot 24\sqrt{3}$	$48\sqrt{3}$
16	6	$4 \cdot 48\sqrt{3}$	$384\sqrt{3}$

Therefore, doubling the side length of a regular hexagon appears to quadruple its area.

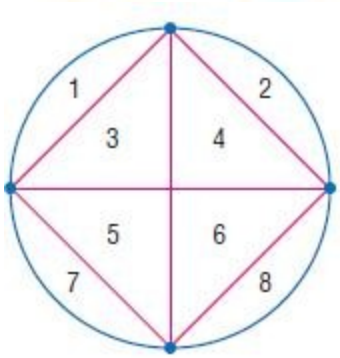
ANSWER:

In the sequence of perimeters, each measure is twice the previous measure. Therefore, doubling the side length of a regular hexagon appears to also double its perimeter. In the sequence of areas, each measure is four times the previous measure. Therefore, doubling the side length of a regular hexagon appears to quadruple its area.

54. **CHALLENGE** If you draw points on a circle and connect every pair of points, the circle is divided into regions. For example, two points form two regions, three points form four regions, and four points form eight regions.



2-1 Inductive Reasoning and Conjecture



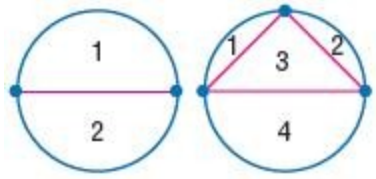
a. Make a conjecture about the relationship between the number of points on a circle and the number of regions formed in the circle

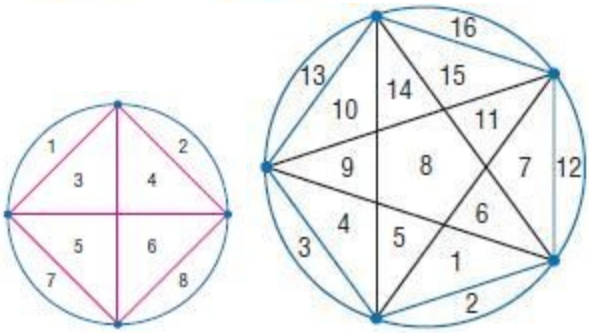
b. Does your conjecture hold true when there are six points? Support your answer with a diagram.

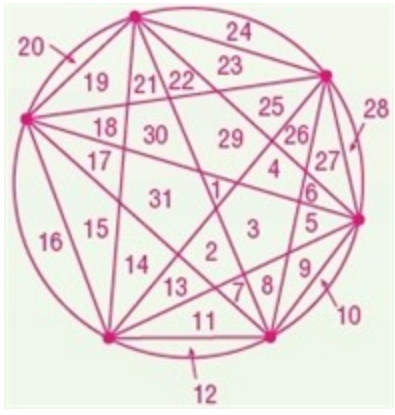
SOLUTION:

a. The number of regions doubles when you add a point on the circle.

b. For 5 points, there are 16 regions. For 6 points, we expect 32 regions. Draw the diagram and count the regions.





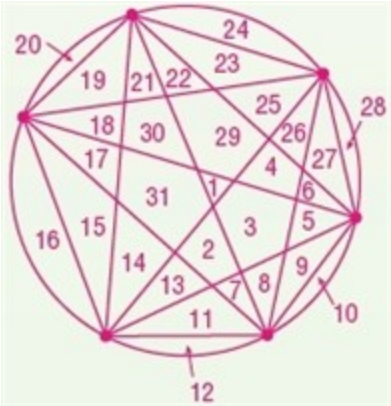


For six points, there should be 32 regions, however only 31 regions are formed. The conjecture is false.

ANSWER:

a. Sample answer: The number of regions doubles when you add a point on the circle.

b. For six points, there should be 32 regions; however only 31 regions are formed. The conjecture is false.



55. **ERROR ANALYSIS** Juan and Jack are discussing prime numbers. Juan states a conjecture that all prime numbers are odd. Jack disagrees with the conjecture and states all prime numbers are not odd. Is either of them correct? Explain.

SOLUTION:

Jack is correct.
2 is an even prime number.

ANSWER:

Jack; 2 is an even prime number.

56. **OPEN ENDED** Write a number sequence that can be generated by two different patterns. Explain your patterns.

SOLUTION:

Sample answer: 2, 4, 8, 16, 32,.... Each number in the sequence can be generated by adding each number to itself to form the next number. Each number in the sequence is 2^n, where $n \geq 1$.

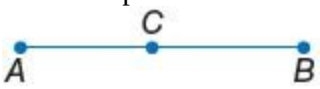
ANSWER:

Sample answer: 2, 4, 8, 16, 32,.... Each number in the sequence can be generated by adding each number to itself to form the next number. Each number in the sequence is 2^n, where $n \geq 1$.

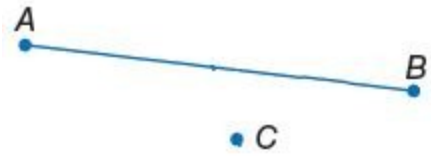
57. **REASONING** Consider the conjecture *If two points are equidistant from a third point, then the three points are collinear.* Is the conjecture *true* or *false*? If false, give a counterexample.

SOLUTION:

The conjecture is sometimes true.
If the two points create a straight angle that includes the third point, then the conjecture is true.



If the two points do not create a straight angle with the third point, then the conjecture is false.



ANSWER:

Sample answer: Sometimes; if the two points create a straight angle that includes the third point, then the conjecture is true. If the two points do not create a straight angle with the third point, then the conjecture is false.

58. **WRITING IN MATH** Suppose you are conducting a survey. Choose a topic and write three questions you would include in your survey. How would you use inductive reasoning with your responses?

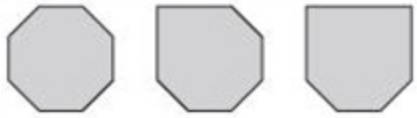
SOLUTION:

Sample answer: I would conduct a survey to find out the types of activities that people participate in on the weekends. I would ask the following questions: What is your age? What is your favorite weekend activity? How often do you participate in the activity? I would then use inductive reasoning to find patterns in the responses to determine if people who are the same age like to participate in the same types of activities.

ANSWER:

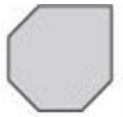
Sample answer: I would conduct a survey to find out the types of activities that people participate in on the weekends. I would ask the following questions: What is your age? What is your favorite weekend activity? How often do you participate in the activity? I would then use inductive reasoning to find patterns in the responses to determine if people who are the same age like to participate in the same types of activities.

59. Look at the pattern below.



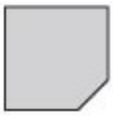
If the pattern continues, what will be the next shape?

A



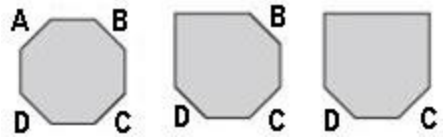
B

C



D

SOLUTION:



The first figure is a octagon. In figure 2, the edge ***A*** on the top left is removed and top becomes a corner. In figure 3, the same is done to the top right side on edge **B**. The pattern is going clockwise.

Thus the next pattern would be


.

So, the correct option is B.

ANSWER:

B

60. **GRIDDED RESPONSE** What is the value of the expression below if $a = 10$ and $b = 1$?

$2b + ab \div (a + b)$

SOLUTION:

Substitute $a = 10$ and $b = 1$.

$$\begin{aligned} 2b + ab \div (a + b) &= 2 \times 1 + 10 \times 1 \div (10 + 1) \\ &= 2 \times 1 + 10 \times 1 \div 11 \\ &= 2 \times 1 + 10 \times 1 \div 11 \\ &= 2 + 10 \div 11 \\ &= 2 + \frac{10}{11} \\ &= \frac{32}{11} \end{aligned}$$

ANSWER:

$\frac{32}{11}$

61. **ALGEBRA** A chemistry student mixed some 30% copper sulfate with some 40% copper sulfate solution to obtain 100 mL of a 32% copper sulfate solution. How much of the 30% copper sulfate solution did the student use in the mixture?
F 90 mL
G 80 mL
H 60 mL
J 20 mL

SOLUTION:
For this mixture problem, write two equations: One involving the total amount of solution, and another involving the total amount of copper sulfate.

Let x be the amount of 30% copper sulfate solution and y be the amount of 40% copper sulfate solution.

$$x + y = 100$$
$$0.3x + 0.4y = 32$$

Solve the first equation for y.

$$y = 100 - x$$

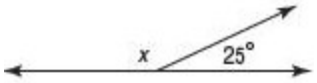
Substitute $y = 100 - x$ in $0.3x + 0.4y = 32$.

$$\begin{aligned} 0.3x + 0.4(100 - x) &= 32 \\ 0.3x + 40 - 0.4x &= 32 \\ 40 - 0.1x &= 32 \\ 40 - 32 &= 0.1x \\ 8 &= 0.1x \\ x &= 80 \end{aligned}$$

Therefore, the correct option is G.

ANSWER:
G

62. **SAT/ACT** Which of the following is equal to $2x$?



A 50°
B 78°
C 155°
D 310°
E 360°

SOLUTION:
The angles in a linear pair are supplementary.
Then, $x + 25 = 180$.

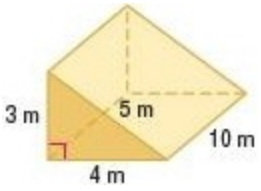
$$x = 155$$

So, $2x = 2(155) = 310$.

The correct choice is D.

ANSWER:
D

Find the surface area and volume of each solid.

63. 

SOLUTION:

The formula for finding the volume of a prism is $V = Bh$.
The formula for finding the surface area of a prism is $S = Ph + 2B$.
S = total surface area, V = volume, h = height of a solid, B = area of the base, P = perimeter of the base

Since the base of the prism is a triangle, the perimeter P of the base is $3+4+5$ or 12 meters. The area of the base B is $\frac{1}{2}(4\times 3)$ or 6 square meters. The height of the prism is 10 meters.

$$\begin{aligned} S &= Ph + 2B \\ &= (12\times 10) + 2(6) \\ &= 120 + 12 \\ &= 132 \end{aligned}$$

The surface area of the triangular prism is 132 square meters.

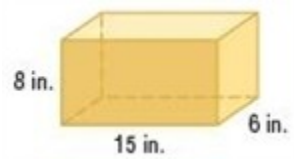
$$\begin{aligned} V &= Bh \\ &= 6\times 10 \\ &= 60 \end{aligned}$$

The volume of the prism is 60 cubic meters.

ANSWER:

132 m^2; 60m^3

2-1 Inductive Reasoning and Conjecture

64. 

SOLUTION:

The formula for finding the volume of a prism is $V = Bh$. The formula for finding the surface area of a prism is $T = Ph + 2B$.

T = total surface area, V = volume, h = height of a solid, B = area of the base, P = perimeter of the base

Since the base of the prism is a rectangle, the perimeter P of the base is $2(15 + 6)$ or 42 inches. The area of the base B is

15×6 or 90 square inches. The height is 8 inches.

$$\begin{aligned} T &= Ph + 2B \\ &= (42 \times 8) + 2(90) \\ &= 336 + 180 \\ &= 516 \end{aligned}$$

The surface area of the prism is 516 in^2.

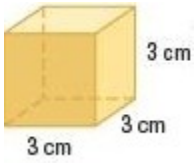
$$\begin{aligned} V &= Bh \\ &= 90 \times 8 \\ &= 720 \end{aligned}$$

The volume of the prism is 720 in^3.

ANSWER:

516 in^2; 720 in^3

2-1 Inductive Reasoning and Conjecture

65. 

SOLUTION:

The formula for finding the volume of a prism is $V = Bh$.
The formula for finding the surface area of a prism is $T = Ph + 2B$.
T = total surface area, V = volume, h = height of a solid, B = area of the base, P = perimeter of the base
Since the base of the prism is a square, the perimeter P of the base is $4(3)$ or 12 centimeters. The area of the base B is 3×3 or 9 square centimeters. The height is 3 centimeters.

$$\begin{aligned} T &= Ph + 2B \\ &= (12 \times 3) + 2(9) \\ &= 36 + 18 \\ &= 54 \end{aligned}$$

The surface area of the prism is 54 square centimeters.

$$\begin{aligned} V &= Bh \\ &= 9 \times 3 \\ &= 27 \end{aligned}$$

The volume of the prism is 27 cubic centimeters.

ANSWER:

54cm^2; 27cm^3

Find the perimeter of ΔABC to the nearest hundredth, given the coordinates of its vertices.

66. $A(1, 6)$, $B(1, 2)$, $C(3, 2)$

SOLUTION:

Use the Distance Formula.

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Substitute.

$$\begin{aligned} AB &= \sqrt{(1-1)^2 + (2-6)^2} \\ &= \sqrt{0+16} \\ &= \sqrt{16} \\ &= 4 \end{aligned}$$

$$\begin{aligned} BC &= \sqrt{(3-1)^2 + (2-2)^2} \\ &= \sqrt{4+0} \\ &= \sqrt{4} \\ &= 2 \end{aligned}$$

$$\begin{aligned} AC &= \sqrt{(3-1)^2 + (2-6)^2} \\ &= \sqrt{4+16} \\ &= \sqrt{20} \\ &\approx 4.47 \end{aligned}$$

To find the perimeter of the triangle, add the sides of the triangle.

$$\begin{aligned} \text{Perimeter} &= AB + BC + AC \\ &\approx 4 + 2 + 4.47 \\ &\approx 10.47 \end{aligned}$$

ANSWER:

10.47

67. $A(-3, 2)$, $B(2, -9)$, $C(0, -10)$

SOLUTION:

Use the Distance Formula.

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Substitute.

$$\begin{aligned} AB &= \sqrt{(2-(-3))^2 + (-9-2)^2} \\ &= \sqrt{25+121} \\ &= \sqrt{146} \\ &\approx 12.08 \end{aligned}$$

$$\begin{aligned} BC &= \sqrt{(0-2)^2 + (-10-(-9))^2} \\ &= \sqrt{4+1} \\ &= \sqrt{5} \\ &\approx 2.24 \end{aligned}$$

$$\begin{aligned} AC &= \sqrt{(0-(-3))^2 + (-10-2)^2} \\ &= \sqrt{9+144} \\ &= \sqrt{153} \\ &\approx 12.37 \end{aligned}$$

To find the perimeter of the triangle, add the sides of the triangle.

Perimeter = $AB + BC + AC$
$\approx 12.08 + 2.24 + 12.37$
≈ 26.69

ANSWER:

26.69

68. **ALGEBRA** The measures of two complementary angles are $16z - 9$ and $4z + 3$. Find the measures of the angles.

SOLUTION:

If the sum of the measures of two adjacent angles is 90, then they are complementary adjacent angles.

$$\begin{aligned} 16z - 9 + 4z + 3 &= 90 \\ 20z - 6 &= 90 \\ 20z &= 96 \\ z &= 4.8 \end{aligned}$$

Substitute.

$16z - 9 = 16(4.8) - 9$
$= 67.8$
$4z + 3 = 4(4.8) + 3$
$= 22.2$

The measures of the complementary angles are 67.8 and 22.2.

ANSWER:

67.8; 22.2

69. **FLAGS** The Wyoming state flag is shown at the right. Name the geometric term modeled by this flag: point, line, or plane.

SOLUTION:

A plane is a flat surface made up of points that extends infinitely in all directions. The given figure is a plane.

ANSWER:

Plane

70. **ALGEBRA** Evaluate $5|x+y|-3|2-z|$ if $x=3, y=-4$, and $z=-5$

SOLUTION:

Substitute $x=3, y=-4$, and $z=-5$.

$$\begin{aligned}5|x+y|-3|2-z| &= 5|3-4|-3|2-(-5)| \\ &= 5|-1|-3|7| \\ &= 5(1)-3(7) \\ &= 5-21 \\ &= -16\end{aligned}$$

ANSWER:

–16

ALGEBRA Determine which values in the replacement set make the inequality true.

71. $x - 3 > 12$

{6, 10, 14, 18}

SOLUTION:

$$\begin{aligned}x-3 &> 12 \\ x-3+3 &> 12+3 \\ x &> 15\end{aligned}$$

So, the value 18 makes the inequality true.

ANSWER:

18

72. $6 + x > 9$
{8, 6, 4, 2}

SOLUTION:

$$\begin{aligned} 6+x &> 9 \\ 6+x-6 &> 9-6 \\ x &> 3 \end{aligned}$$

So, the values 8, 6, and 4 make the inequality true.

ANSWER:

8, 6, 4

73. $2x - 4 > 10$
{5, 6, 7, 8}

SOLUTION:

$$\begin{aligned} 2x-4 &> 10 \\ 2x &> 14 \\ x &> 7 \end{aligned}$$

So, only the value 8 makes the inequality true.

ANSWER:

8