

2-2 Logic

Use the following statements to write a compound statement for each conjunction or disjunction. Then find its truth value. Explain your reasoning.

p : A week has seven days.

q : There are 20 hours in a day.

r : There are 60 minutes in an hour.

1. p and r

SOLUTION:

p and r is a conjunction. A conjunction is true only when both statements that form it are true. p is true since a week has seven day. r is true since there are 60 minutes in an hour. Then p and r is true, because both p and r are true.

ANSWER:

A week has seven days, and there are 60 minutes in an hour. p and r is true, because p is true and r is true.

2. $p \wedge q$

SOLUTION:

$p \wedge q$ is a conjunction. A conjunction is true only when both statements that form it are true. p is true since a week has seven day. q is false since there are 24 hours in a day, not 20 hours in a day. Thus, $p \wedge q$ is false, because p is true and q is false.

ANSWER:

A week has seven days, and there are 20 hours in a day. $p \wedge q$ is false, because p is true and q is false.

3. $q \vee r$

SOLUTION:

$q \vee r$ is a disjunction. A disjunction is true if at least one of the statements is true. q is false, since there are 24 hours in a day, not 20 hours. However, r is true since there there are 60 minutes in an hour. Thus, $q \vee r$ is true, since at least one of them is true, r .

ANSWER:

There are 20 hours in a day, or there are 60 minutes in an hour.

$q \vee r$ is true, because q is false and r is true.

4. $\sim p \text{ or } q$

SOLUTION:

$\sim p$ is a negation of statement p , or the opposite of statement p . The **or** in $\sim p$ or q indicates a disjunction. A disjunction is true if at least one of the statements is true.

$\sim p$ would be : A week **does not** have seven days, which is false. q is false since there are 24 hours in a day, not 20 hours in a day. Then $\sim p$ or q is false, because both $\sim p$ and q are false.

ANSWER:

A week does not have seven days, or there are 20 hours in a day. $\sim p$ or q is false, because $\sim p$ is false and q is false.

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5. $p \vee r$

SOLUTION:

$p \vee r$ is a disjunction. A disjunction is true if at least one of the statements is true.

p is true since a week has seven days. r is true, since there are 60 minutes in an hour. Thus, $p \vee r$ is true, because p is true and r is true.

ANSWER:

A week has seven days, or there are 60 minutes in an hour. $p \vee r$ is true, because p is true and r is true.

6. $\sim p \wedge \sim r$

SOLUTION:

$\sim p$ and $\sim r$ indicate the negation of statements p and r . Then find the disjunction $\sim p \wedge \sim r$. A disjunction is true if at least one of the statements is true.

$\sim p$ is : A week **does not** have seven days, which is false. $\sim r$ is : There **are not** 60 minutes in an hour, which is false. Then $\sim p \wedge \sim r$ is false, because both $\sim p$ and $\sim r$ are false.

ANSWER:

A week does not have seven days, and there are not 60 minutes in an hour. $\sim p \wedge \sim r$ is false, because $\sim p$ is false and $\sim r$ is false.

7. Copy and complete the truth table at the right.

p	q	$\sim q$	$p \vee \sim q$
T	T	F	
T	F		
F	T		
F	F		

SOLUTION:

$\sim q$ is the negation of q or the opposite truth value. If q is T, then $\sim q$ is F, and when q is F $\sim q$ is T.

The \vee indicate a disjunction. A disjunction is true if at least one of the statements is true or it will only be false if both statements are false. When p is T and $\sim q$ is F, $p \vee \sim q$ is T, or when p is T and $\sim q$ is T, $p \vee \sim q$ is T, when p is F and $\sim q$ is T, $p \vee \sim q$ is T, or when p is F and $\sim q$ is F, $p \vee \sim q$ is F.

p	q	$\sim q$	$p \vee \sim q$
T	T	F	T
T	F	T	T
F	T	F	F
F	F	T	T

ANSWER:

p	q	$\sim q$	$p \vee \sim q$
T	T	F	T
T	F	T	T
F	T	F	F
F	F	T	T

2-2 Logic

Construct a truth table for each compound statement.

8. $p \wedge q$

SOLUTION:

This statement is interpreted as p **and** q . $p \wedge q$ is a conjunction. A conjunction is true only when both statements that form it are true

p	q	$p \wedge q$
T	T	T
T	F	F
F	T	F
F	F	F

ANSWER:

p	q	$p \wedge q$
T	T	T
T	F	F
F	T	F
F	F	F

9. $\sim p \vee \sim q$

SOLUTION:

This statement is interpreted as: **not** p **or** **not** q .

Negate both p and q . Then find the disjunction. A disjunction is true if at least one of the statements is true.

p	q	$\sim p$	$\sim q$	$\sim p \vee \sim q$
T	T	F	F	F
T	F	F	T	T
F	T	T	F	T
F	F	T	T	T

ANSWER:

p	q	$\sim p$	$\sim q$	$\sim p \vee \sim q$
T	T	F	F	F
T	F	F	T	T
F	T	T	F	T
F	F	T	T	T

2-2 Logic

10. **CLASSES** Refer to the Venn diagram that represents the foreign language classes students selected in high school.
- How many students chose only Spanish?
 - How many students chose Spanish and French?
 - Describe the class(es) the three people in the nonintersecting portion of the French region chose.



SOLUTION:

- The students that chose only Spanish are represented by numbers in the Spanish circle, that are not in the intersection with French. There are 89 students chose only Spanish
- The students that chose both French and Spanish are represented by the intersection of the sets . There are 5 students who chose Spanish and French.
- The students that chose only French are represented by numbers in the French circle, that are not in the intersection with Spanish. There are three students who chose to take only French classes.

ANSWER:

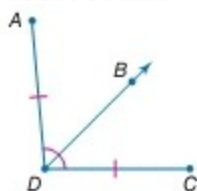
- 89
- 15
- The three students chose to take only French classes.

Use the following statements and figure to write a compound statement for each conjunction or disjunction. Then find its truth value. Explain your reasoning.

p : \overline{DB} is the angle bisector of $\angle ADC$.

q : Points $C, D,$ and B are collinear.

r : $\overline{AD} \cong \overline{DC}$



11. p and r

SOLUTION:

p and r is a conjunction. A conjunction is true only when both statements that form it are true. p is that \overline{DB} is the angle bisector of $\angle ADC$ which is true. r is $\overline{AD} \cong \overline{DC}$, which is true. Thus, p **and** r is true because p is true and r is true.

ANSWER:

\overline{DB} is the angle bisector of $\angle ADC$, and $\overline{AD} \cong \overline{DC}$. p and r is true because p is true and r is true.

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12. q or p

SOLUTION:

q or p is a disjunction. A disjunction is true if at least one of the statements is true. q is false since points C , D , and B are **not** collinear. p is true since \overline{DB} is the angle bisector of $\angle ADC$. Thus, q or p is true because q is false and p is true.

ANSWER:

Points C , D , and B are collinear, or \overline{DB} is the angle bisector of $\angle ADC$. q or p is true because q is false and p is true.

13. r or $\sim p$

SOLUTION:

Negate p , then find the disjunction r or $\sim p$. A disjunction is true if at least one of the statements is true.

r is true since $\overline{AD} \cong \overline{DC}$. The negation of p is \overline{DB} is **not** the angle bisector of $\angle ADC$, which is false. Thus, r or $\sim p$ is true because r is true and $\sim p$ is false.

ANSWER:

$\overline{AD} \cong \overline{DC}$ or \overline{DB} is not the angle bisector of $\angle ADC$. r or $\sim p$ is true because r is true and $\sim p$ is false.

14. r and q

SOLUTION:

r and q is a conjunction. A conjunction is true only when both statements that form it are true. r is true since $\overline{AD} \cong \overline{DC}$. q is false, since points C , D , and B are **not** collinear. Thus r and q is false because r is true and q is false.

ANSWER:

$\overline{AD} \cong \overline{DC}$ and Points C , D , and B are collinear. r and q is false because r is true and q is false.

15. $\sim p$ or $\sim r$

SOLUTION:

Negate both p and r and find the disjunction. A disjunction is true if at least one of the statements is true.

$\sim p$ is \overline{DB} is **not** the angle bisector of $\angle ADC$, which is false. $\sim r$ is \overline{AB} is **not** congruent to \overline{DC} , which is false. Thus, $\sim p$ or $\sim r$ is false because $\sim p$ is false and $\sim r$ is false.

ANSWER:

\overline{DB} is not the angle bisector of $\angle ADC$, or \overline{AB} . $\sim p$ or $\sim r$ is false because $\sim p$ is false and $\sim r$ is false.

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16. $\sim p$ and $\sim r$

SOLUTION:

Negate both p and r and find the conjunction. A conjunction is true only when both statements that form it are true.

$\sim p$ is \overline{DB} is **not** the angle bisector of $\angle ADC$, which is false. $\sim r$ is and \overline{AB} is **not** congruent to \overline{DC} , which is false. Thus, $\sim p$ **and** $\sim r$ is false because $\sim p$ is false and $\sim r$ is false.

ANSWER:

\overline{DB} is not the angle bisector of $\angle ADC$, and \overline{AB} is not congruent to \overline{DC} . $\sim p$ and $\sim r$ is false because $\sim p$ is false and $\sim r$ is false

CCSS REASONING

Use the following statements to write a compound statement for each conjunction or disjunction. Then find its truth value. Explain your reasoning.

p : Springfield is the capital of Illinois.

q : Illinois borders the Atlantic Ocean.

r : Illinois shares a border with Kentucky.

s : Illinois is to the west of Missouri.



17. $p \wedge r$

SOLUTION:

$p \wedge r$ is a conjunction. A conjunction is true only when both statements that form it are true.

p is Springfield is the capital of Illinois, which is true. r is Illinois shares a border with Kentucky, which is true. Then $p \wedge r$ is true because p is true and r is true.

ANSWER:

Springfield is the capital of Illinois, and Illinois shares a border with Kentucky. $p \wedge r$ is true because p is true and r is true.

18. $p \wedge q$

SOLUTION:

$p \wedge q$ is a conjunction. A conjunction is true only when both statements that form it are true.

p is Springfield is the capital of Illinois, which is true. q is Illinois borders the Atlantic Ocean, which is false. Then $p \wedge q$ is false because p is true and q is false.

ANSWER:

Springfield is the capital of Illinois, and Illinois borders the Atlantic Ocean. $p \wedge q$ is false because p is true and q is false.

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19. $\sim r \vee s$

SOLUTION:

Negate r and find the disjunction $\sim r \vee s$. A disjunction is true if at least one of the statements is true.

$\sim r$ is Illinois **does not** share a border with Kentucky, which is false. s is Illinois is to the west of Missouri, which is false. Then $\sim r \vee s$ is false because $\sim r$ is false and s is false.

ANSWER:

Illinois does not share a border with Kentucky, or Illinois is to the west of Missouri. $\sim r \vee s$ is false because $\sim r$ is false and s is false.

20. $r \vee q$

SOLUTION:

$r \vee q$ is a disjunction. A disjunction is true if at least one of the statements is true.

r is Illinois shares a border with Kentucky, which is true. q is Illinois borders the Atlantic Ocean, which is false. Then $r \vee q$ is true because r is true and q is false.

ANSWER:

Illinois shares a border with Kentucky, or Illinois borders the Atlantic Ocean. $r \vee q$ is true because r is true and q is false.

21. $\sim p \wedge \sim r$

SOLUTION:

Negate both p and r and find the conjunction $\sim p \wedge \sim r$. A conjunction is true only when both statements that form it are true.

$\sim p$ is Springfield **is not** the capital of Illinois, which is false, $\sim r$ is and Illinois does not share a border with Kentucky, which is false. Then $\sim p \wedge \sim r$ is false because $\sim p$ is false and $\sim r$ is false

ANSWER:

Springfield is not the capital of Illinois, and Illinois does not share a border with Kentucky. $\sim p \wedge \sim r$ is false because $\sim p$ is false and $\sim r$ is false.

22. $\sim s \vee \sim p$

SOLUTION:

Negate both s and p and find the disjunction $\sim s \vee \sim p$. A disjunction is true if at least one of the statements is true.

$\sim s$ is Illinois is not to the west of Missouri, which is true. $\sim p$ is Springfield **is not** the capital of Illinois, which is false. Then $\sim s \vee \sim p$ is true because $\sim s$ is true and $\sim p$ is false.

ANSWER:

Illinois is not to the west of Missouri, or Springfield is not the capital of Illinois. $\sim s \vee \sim p$ is true because $\sim s$ is true and $\sim p$ is false.

2-2 Logic

Copy and complete each truth table.

p	q	$\sim p$	$\sim p \wedge q$
T		F	
T		F	
F		T	
F		T	

23.

SOLUTION:

Add values to the column for q so that each pair of p and q are distinct.

$\sim p \wedge q$ indicated a conjunction. A conjunction is true only when both statements that form it are true. $\sim p \wedge q$ is only true when $\sim p$ is true and q is true.

p	q	$\sim p$	$\sim p \wedge q$
T	T	F	F
T	F	F	F
F	T	T	T
F	F	T	F

ANSWER:

p	q	$\sim p$	$\sim p \wedge q$
T	T	F	F
T	F	F	F
F	T	T	T
F	F	T	F

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p	q	$\sim p$	$\sim q$	$\sim p \vee \sim q$
T			F	
T			T	
F			F	
F			T	

24.

SOLUTION:

Add values to the column for q so that each pair of p and q are distinct.

Negate both p and q . Negation means the opposite meaning or opposite truth value. If q is true then $\sim q$ is false.

$\sim p \vee \sim q$ indicated a disjunction. A disjunction is true if at least one of the statements is true. $\sim p \vee q$ is true at all times except with $\sim p$ and $\sim q$ are both false.

p	q	$\sim p$	$\sim q$	$\sim p \vee \sim q$
T	T	F	F	F
T	F	F	T	T
F	T	T	F	T
F	F	T	T	T

ANSWER:

p	q	$\sim p$	$\sim q$	$\sim p \vee \sim q$
T	T	F	F	F
T	F	F	T	T
F	T	T	F	T
F	F	T	T	T

Construct a truth table for each compound statement.

25. $p \wedge r$

SOLUTION:

Write the values T and F for p and r so that each pair are distinct.

$p \wedge r$ is a conjunction. A conjunction is true only when both statements that form it are true. Then $p \wedge r$ will only be true with both p and r are true.

p	r	$p \wedge r$
T	T	T
T	F	F
F	T	F
F	F	F

ANSWER:

p	r	$p \wedge r$
T	T	T
T	F	F
F	T	F
F	F	F

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26. $r \wedge q$

SOLUTION:

Add values T and F for r and q so that each pair are distinct.

$r \wedge q$ is a conjunction. A conjunction is true only when both statements that form it are true. $r \wedge q$ will only be true with both r and q are true.

r	q	$r \wedge q$
T	T	T
T	F	F
F	T	F
F	F	F

ANSWER:

r	q	$r \wedge q$
T	T	T
T	F	F
F	T	F
F	F	F

27. $p \vee r$

SOLUTION:

Add values T and F for p and r so that each pair are distinct. $p \vee r$ is a disjunction. A disjunction is true if at least one of the statements is true. $p \vee r$ will be true for all values except when both r and p are false.

p	r	$p \vee r$
T	T	T
T	F	T
F	T	T
F	F	F

ANSWER:

p	r	$p \vee r$
T	T	T
T	F	T
F	T	T
F	F	F

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28. $q \vee r$

SOLUTION:

Add values T and F for q and r so that each pair are distinct. $q \vee r$ is a disjunction. A disjunction is true if at least one of the statements is true. $q \vee r$ will be true for all values except when both q and r are false.

q	r	$q \vee r$
T	T	T
T	F	T
F	T	T
F	F	F

ANSWER:

q	r	$q \vee r$
T	T	T
T	F	T
F	T	T
F	F	F

29. $\sim p \wedge r$

SOLUTION:

Add values T and F for p and r so that each pair are distinct.

Negate p . This means, find the opposite truth value.

$\sim p \wedge r$ is a conjunction. A conjunction is true only when both statements that form it are true. $\sim p \wedge r$ will be true only when both $\sim p$ and r are true.

p	r	$\sim p$	$\sim p \wedge r$
T	T	F	F
T	F	F	F
F	T	T	T
F	F	T	F

ANSWER:

p	r	$\sim p$	$\sim p \wedge r$
T	T	F	F
T	F	F	F
F	T	T	T
F	F	T	F

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30. $\sim q \vee \sim r$

SOLUTION:

Add T and F to the columns for q and r so that each pair distinct. Negate both q and r . To negate, find the opposite truth value. If q is T, then $\sim q$ is F. $\sim q \wedge \sim r$ is a conjunction. A conjunction is true only when both statements that form it are true. Thus, $\sim q \wedge \sim r$ is only true when $\sim q$ and $\sim r$ are true.

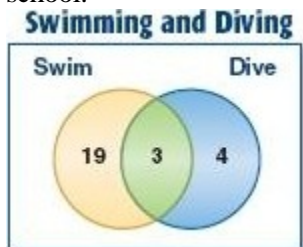
q	r	$\sim q$	$\sim r$	$\sim q \wedge \sim r$
T	T	F	F	F
T	F	F	T	F
F	T	T	F	F
F	F	T	T	T

ANSWER:

q	r	$\sim q$	$\sim r$	$\sim q \wedge \sim r$
T	T	F	F	F
T	F	F	T	F
F	T	T	F	F
F	F	T	T	T

2-2 Logic

31. **WATER SPORTS** Refer to the Venn diagram that represents the number of students who swim and dive at a high school.



- a. How many students dive?
- b. How many students participate in swimming or diving or both?
- c. How many students swim and dive?

SOLUTION:

a. The number of students that dive are represented by the students in the dive circle which is made up of students that do both and only dive. The number of students who participate in both activities is 3 and the number of students who participate only in diving is 4.

So, the number of students participate in diving is $3 + 4$ or 7.

b. To find the students that do either or both, find the union of the sets. The number of students who participate in swimming or diving or both is $19 + 3 + 4$ or 26.

c. To find the number of students that participate in both activities, find the intersection. The number of students who participate in both activities is 3.

ANSWER:

- a. 7
- b. 26
- c. 3

2-2 Logic

32. **CCSS REASONING** Venus has switches at the top and bottom of her stairs to control the light for the stairwell. She notices that when the upstairs switch is up and the downstairs switch is down, the light is turned on.

a. Copy and complete the truth table.

Position of Switch		Light On
Upstairs	Downstairs	
up		
up	down	T

- b. If both the upstairs and downstairs switches are in the up position, will the light be on? Explain your reasoning.
 c. If the upstairs switch is in the down position and the downstairs switch is in the up position, will the light be on?
 d. In general, how should the two switches be positioned so that the light is on?

SOLUTION:

a.

Position of Switch		Light On
Upstairs	Downstairs	
up	up	F
up	down	T
down	up	T
down	down	F

- b. Since the light is on when the upstairs switch is up and the downstairs switch is down, when both switches are up, the value is false in the light on column.
 c. Since the light is on when the upstairs switch is up and the downstairs switch is down when the upstairs switch is down and the downstairs switch is up, the value is true in the light on column.
 d. The light is on when switches are in opposite positions.

ANSWER:

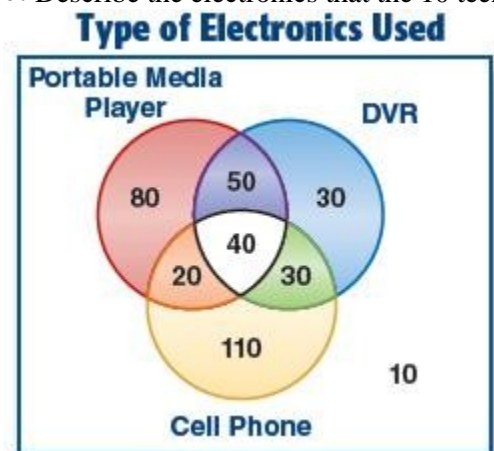
a.

Position of Switch		Light On
Upstairs	Downstairs	
up	up	F
up	down	T
down	up	T
down	down	F

- b. No; when both switches are up, the value is false in the light on column.
 c. Yes; when the upstairs switch is down and the downstairs switch is up, the value is true in the light on column.
 d. The light is on when switches are in opposite positions.

2-2 Logic

33. **ELECTRONICS** A group of 330 teens were surveyed about what type of electronics they used. They chose from a cell phone, a portable media player, and a DVR. The results are shown in the Venn diagram.
- How many teens used only a portable media player and DVR?
 - How many said they used all three types of electronics?
 - How many said they used only a cell phone?
 - How many teens said they used only a portable media player and a cell phone?
 - Describe the electronics that the 10 teens outside of the regions own.



SOLUTION:

- To find the number that only used both a portable media player and DVR, find the intersection of a portable media player and DVR, but no cell phone. The number who used both media player and DVR is 50.
- To find the number that used all three, find the intersection of all three. There are 40 that use all three.
- To find the number that only used cell phones, find the number that are in cell phone but not in a portable media player or DVR. The number who used only a cell phone is 110.
- To find the number that only used both media and cell phone, find the intersection of MP3 and cell phone, but not DVR. The number of teens who used both a portable media player and cell phone is 20.
- The students outside the circles do not use a portable media player, DVR or cell phones.

ANSWER:

- 50
- 40
- 110
- 20
- These teens do not use any of the listed electronics.

2-2 Logic

Construct a truth table for each compound statement. Determine the truth value of each compound statement if the given statements are true.

34. $p \wedge (q \wedge r)$; p, q

SOLUTION:

$p \wedge (q \wedge r)$ is a conjunction. A conjunction is true only when all the statements that form it are true.

First find $q \wedge r$, then find $p \wedge (q \wedge r)$.

p	q	r	$q \wedge r$	$p \wedge (q \wedge r)$
T	T	T	T	T
T	F	T	F	F
T	T	F	F	F
T	F	F	F	F
F	T	T	T	F
F	F	T	F	F
F	T	F	F	F
F	F	F	F	F

If p and q are true, then the statement is true if r is true false if r is false.

ANSWER:

If p and q are true, then the statement is true if r is true and false if r is false.

p	q	r	$q \wedge r$	$p \wedge (q \wedge r)$
T	T	T	T	T
T	F	T	F	F
T	T	F	F	F
T	F	F	F	F
F	T	T	T	F
F	F	T	F	F
F	T	F	F	F
F	F	F	F	F

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35. $p \wedge (\sim q \vee r); p, r$

SOLUTION:

$p \wedge (\sim q \vee r)$ is the conjunction of a disjunction. First negate q . Then find the disjunction with r . It will be true when either $\sim q$ or r are true. Then find the conjunction with p . It will be true with both p and $\sim q \vee r$ is true.

p	q	$\sim q$	r	$\sim q \vee r$	$p \wedge (\sim q \vee r)$
T	T	F	T	T	T
T	F	T	T	T	T
T	T	F	F	F	F
T	F	T	F	T	T
F	T	F	T	T	F
F	F	T	T	T	F
F	T	F	F	F	F
F	F	T	F	T	F

If p and r are true, then the statement is true.

ANSWER:

If p and r are true, and q is true or false, then $p \wedge (\sim q \vee r)$ is true.

p	q	$\sim q$	r	$\sim q \vee r$	$p \wedge (\sim q \vee r)$
T	T	F	T	T	T
T	F	T	T	T	T
T	T	F	F	F	F
T	F	T	F	T	T
F	T	F	T	T	F
F	F	T	T	T	F
F	T	F	F	F	F
F	F	T	F	T	F

2-2 Logic

36. $(\sim p \vee q) \wedge r; q, r$

SOLUTION:

Negate p . Then find its disjunction with q . $\sim p \vee q$ will be true when either $\sim p$ **or** q are true. Then find the conjunction $(\sim p \vee q) \wedge r$. The conjunction is true only if $(\sim p \vee q)$ is true **and** r is true.

p	q	$\sim p$	$\sim p \vee q$	r	$(\sim p \vee q) \wedge r$
T	T	F	T	T	T
T	F	F	F	T	F
T	T	F	T	F	F
T	F	F	F	F	F
F	T	T	T	T	T
F	F	T	T	T	T
F	T	T	T	F	F
F	F	T	T	F	F

If p is true or false, then $(\sim p \vee q) \wedge r$ is true.

ANSWER:

If p is true or false, then $(\sim p \vee q) \wedge r$ is true.

p	q	$\sim p$	$\sim p \vee q$	r	$(\sim p \vee q) \wedge r$
T	T	F	T	T	T
T	F	F	F	T	F
T	T	F	T	F	F
T	F	F	F	F	F
F	T	T	T	T	T
F	F	T	T	T	T
F	T	T	T	F	F
F	F	T	T	F	F

2-2 Logic

37. $p \vee (\sim q \wedge \sim r)$; p, q, r

SOLUTION:

Negate both q and r , finding the opposite truth values. Then find the conjunction $\sim q$ and $\sim r$. The conjunction will be true when both $\sim q$ **and** $\sim r$ are true. Then find the disjunction $p \vee (\sim q \wedge \sim r)$. It will be true if either p **or** $(\sim q \wedge \sim r)$ is true.

p	q	$\sim q$	r	$\sim r$	$\sim q \wedge \sim r$	$p \vee (\sim q \wedge \sim r)$
T	T	F	T	F	F	T
T	F	T	T	F	F	T
T	T	F	F	T	F	T
T	F	T	F	T	T	T
F	T	F	T	F	F	F
F	F	T	T	F	F	F
F	T	F	F	T	F	F
F	F	T	F	T	T	T

If p , q , and r are true, then the given statement is true.

ANSWER:

true

p	q	$\sim q$	r	$\sim r$	$\sim q \wedge \sim r$	$p \vee (\sim q \wedge \sim r)$
T	T	F	T	F	F	T
T	F	T	T	F	F	T
T	T	F	F	T	F	T
T	F	T	F	T	T	T
F	T	F	T	F	F	F
F	F	T	T	F	F	F
F	T	F	F	T	F	F
F	F	T	F	T	T	T

2-2 Logic

38. $\sim p \wedge (\sim q \wedge \sim r); p, q, r$

SOLUTION:

Negate p , q and r finding the opposite truth values. Find the conjunction $(\sim q \wedge \sim r)$. It is true when both $\sim q$ **and** $\sim r$ are true. Then find the conjunct $\sim p \wedge (\sim q \wedge \sim r)$. It is true when both $\sim p$ **and** $(\sim q \wedge \sim r)$ are true.

p	$\sim p$	q	$\sim q$	r	$\sim r$	$\sim q \wedge \sim r$	$p \wedge (\sim q \wedge \sim r)$
T	F	T	F	T	F	F	F
T	F	F	T	T	F	F	F
T	F	T	F	F	T	F	F
T	F	F	T	F	T	T	F
F	T	T	F	T	F	F	F
F	T	F	T	T	F	F	F
F	T	T	F	F	T	F	F
F	T	F	T	F	T	T	T

If p , q , and r are true, then the given statement is false.

ANSWER:

false

p	$\sim p$	q	$\sim q$	r	$\sim r$	$\sim q \wedge \sim r$	$p \wedge (\sim q \wedge \sim r)$
T	F	T	F	T	F	F	F
T	F	F	T	T	F	F	F
T	F	T	F	F	T	F	F
T	F	F	T	F	T	T	F
F	T	T	F	T	F	F	F
F	T	F	T	T	F	F	F
F	T	T	F	F	T	F	F
F	T	F	T	F	T	T	T

2-2 Logic

39. $(\sim p \vee q) \vee \sim r; p, q$

SOLUTION:

Negate p and r finding the opposite truth value. Find the disjunction $(\sim p \vee q)$. It is true when either $\sim p$ **or** q is true. Then find the disjunction $(\sim p \vee q) \vee \sim r$. It is true when $(\sim p \vee q)$ **or** $\sim r$ is true.

p	$\sim p$	q	r	$\sim r$	$(\sim p \vee q)$	$(\sim p \vee q) \vee \sim r$
T	F	T	T	F	T	T
T	F	F	T	F	F	F
T	F	T	F	T	T	T
T	F	F	F	T	F	T
F	T	T	T	F	T	T
F	T	F	T	F	T	T
F	T	T	F	T	T	T
F	T	F	F	T	T	T

If r is true or false, then $(\sim p \vee q) \vee \sim r$ is true.

ANSWER:

If r is true or false, then $(\sim p \vee q) \vee \sim r$ is true.

p	$\sim p$	q	r	$\sim r$	$(\sim p \vee q)$	$(\sim p \vee q) \vee \sim r$
T	F	T	T	F	T	T
T	F	F	T	F	F	F
T	F	T	F	T	T	T
T	F	F	F	T	F	T
F	T	T	T	F	T	T
F	T	F	T	F	T	T
F	T	T	F	T	T	T
F	T	F	F	T	T	T

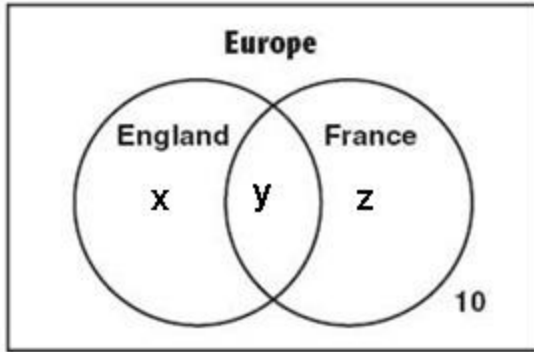
40. **CCSS REASONING** A travel agency surveyed 70 of their clients who had visited Europe about international travel. Of the 70 clients who had visited Europe, 60 had traveled to England, France, or both. Of those 60 clients, 45 had visited England, and 50 had visited France.

- Make a Venn diagram to show the results of the survey.
- If p represents a client who has visited England and q represents a client who has visited France, write a compound statement to represent each area of the Venn diagram. Include the compound statements on your Venn diagram.
- What is the probability that a randomly chosen participant in the survey will have visited both England and France? Explain your reasoning.

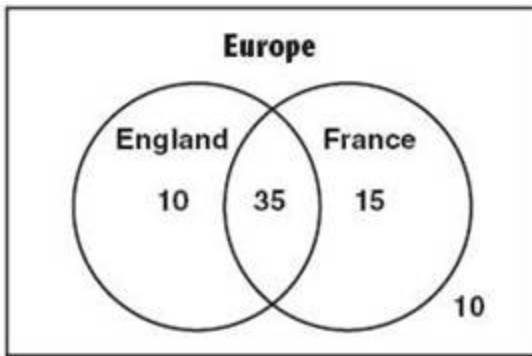
SOLUTION:

- Draw a Venn diagram for Europe. Place two intersecting circles for England and France. Of the total 70 clients surveyed, 60 traveled to either England, France or both. Thus, 10 did not. Place the 10 outside the circles. Solve a system of equations to find the other values.

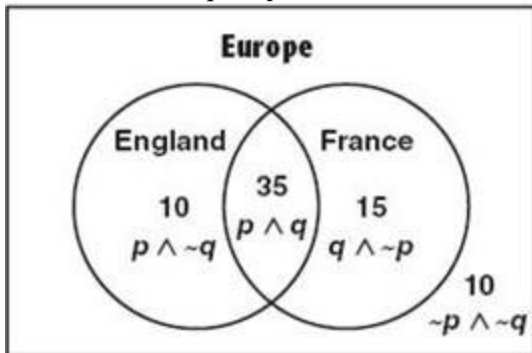
2-2 Logic



$x + y + z = 60$, $x + y = 45$, $y + z = 50$. Then $x = 10$, $y = 35$ and $z = 15$.



b. Since p represents a client who has visited England and q represents a client who has visited France, then the clients that did not visit either country would be $\sim p \wedge \sim q$. The clients that visited both would be $p \wedge q$. Clients that visited England only would have to exclude the clients that went to France or $\sim q$. England only clients would be $p \wedge \sim q$. Clients that visited France only would have to exclude the clients that went to England or $\sim p$. France only clients would be $q \wedge \sim p$.



c. $\frac{1}{2}$

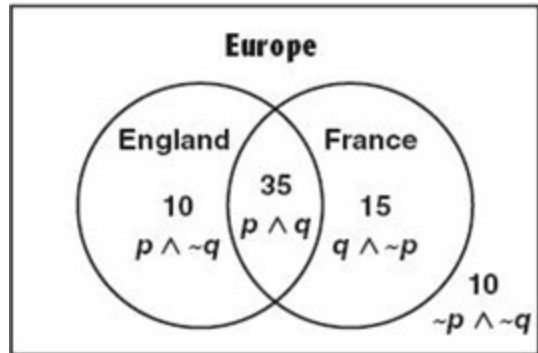
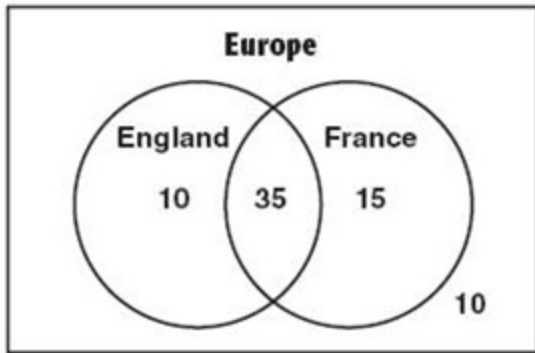
Since 35 of those surveyed have visited both England and France and there are 70 total survey participants, the

probability that a randomly chosen participant would have visited both England and France is $\frac{35}{70}$ or $\frac{1}{2}$.

ANSWER:

a.

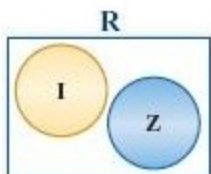
2-2 Logic



b.

c. $\frac{1}{2}$; Sample answer: Since 35 of those surveyed have visited both England and France and there are 70 total survey participants, the probability that a randomly chosen participant would have visited both England and France is $\frac{35}{70}$ or $\frac{1}{2}$.

41. **REASONING** Irrational numbers and integers both belong to the set of real numbers (R). Based upon the Venn diagram, is it *sometimes*, *always*, or *never* true that integers (Z) are irrational numbers (I)? Explain your reasoning.



SOLUTION:

Integers are never irrational numbers. Both belong to the real number, but they never intersect. Integers are rational numbers, not irrational.

ANSWER:

Never; integers are rational numbers, not irrational.

2-2 Logic

CHALLENGE To negate a statement containing the words all or for every, you can use the phrase at least one or there exists. To negate a statement containing the phrase there exists, use the phrase for all or for every.

p : All polygons are convex.

$\sim p$: At least one polygon is not convex.

q : There exists a problem that has no solution. $\sim q$: For every problem, there is a solution.

Sometimes these phrases may be implied. For example, The square of a real number is nonnegative implies the following conditional and its negation.

p : For every real number x , $x^2 \geq 0$.

$\sim p$: There exists a real number x such that $x^2 < 0$.

Use the information given to write the negation of each statement.

42. Every student at Hammond High School has a locker.

SOLUTION:

To negate a statement containing the word "every", use the phrase "at least one".

There exists at least one student at Hammond High School that does not have a locker.

ANSWER:

There exists at least one student at Hammond High School that does not have a locker.

43. All squares are rectangles.

SOLUTION:

To negate a statement containing the word "all", use the phrase "at least one".

There exists at least one square that is not a rectangle.

ANSWER:

There exists at least one square that is not a rectangle.

44. There exists a real number x such that $x^2 = x$

SOLUTION:

To negate a statement containing the phrase "there exists", use the word "every".

For every real number x , $x^2 \neq x$.

ANSWER:

For every real number x , $x^2 \neq x$.

45. There exists a student who has at least one class in C-Wing.

SOLUTION:

To negate a statement containing the phrase "every", use the word "no".

No students have classes in C-Wing.

ANSWER:

No students have classes in C-Wing.

2-2 Logic

46. Every real number has a real square root.

SOLUTION:

To negate a statement containing the word "every", use the phrase "there exists" .
There exists a real number that does not have a real square root.

ANSWER:

There exists a real number that does not have a real square root.

47. There exists a segment that has no midpoint.

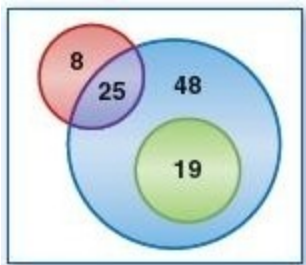
SOLUTION:

To negate a statement containing the phrase "there exists", use the word "every".
Every segment has a midpoint.

ANSWER:

Every segment has a midpoint.

48. **WRITING IN MATH** Describe a situation that might be depicted using the Venn diagram.



SOLUTION:

100 people were surveyed to see if they liked vanilla, strawberry, or chocolate ice cream. There were 8 people who only liked strawberry. There were 25 people who liked both strawberry and vanilla. There were 48 people who only liked vanilla, and there were 19 people who liked both chocolate and vanilla.

ANSWER:

Sample answer: 100 people were surveyed to see if they liked vanilla, strawberry, or chocolate ice cream. There were 8 people who only liked strawberry. There were 25 people who liked both strawberry and vanilla. There were 48 people who only liked vanilla, and there were 19 people who liked both chocolate and vanilla.

49. **OPEN ENDED** Write a compound statement that results in a true conjunction.

SOLUTION:

For a conjunction to be true, both statements must be true.

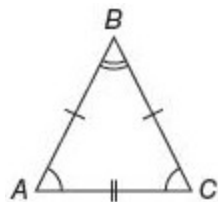
A triangle has three sides, and a square has four sides. Both are true, so the compound statement is true.

ANSWER:

A triangle has three sides, and a square has four sides. Both are true, so the compound statement is true.

2-2 Logic

50. Which statement about $\triangle ABC$ has the same truth value as $AB = BC$?



- A** $m\angle A = m\angle C$
B $m\angle A = m\angle B$
C $AC = BC$
D $AB = AC$

SOLUTION:

The statement $AB = BC$ is true since sides are congruent. We need to identify which statement has a truth value of true.

The statement for **B**, $m\angle A = m\angle B$ has a value of false, since the angles are not equal.

The statement for **C**, $AC = BC$ has a value of false, since the sides do not have the same length.

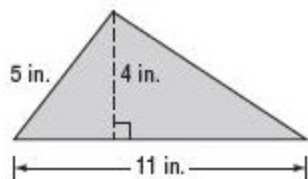
The statement for **D**, $AB = AC$ has a value of false, since the sides do not have the same length.

The statement for **A**, $m\angle A = m\angle C$ has a value of true, since the angles have the same measure. Thus, A is the correct choice.

ANSWER:

A

51. **EXTENDED RESPONSE** What is the area of the triangle shown below? Explain how you found your answer.



SOLUTION:

The area of a triangle is $\frac{1}{2}bh$. The base of the triangle is 11 inches and the height is 4 inches, so the area is

$$\frac{1}{2}(11)(4) \text{ or } 22 \text{ in}^2.$$

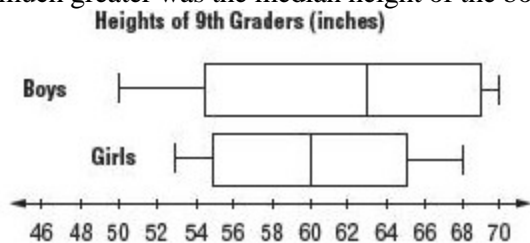
ANSWER:

22 in^2 ; The area of a triangle is $\frac{1}{2}bh$. The base of the triangle is 11 inches and the height is 4 inches, so the area is

$$\frac{1}{2}(11)(4) \text{ or } 22 \text{ in}^2..$$

2-2 Logic

52. **STATISTICS** The box-and-whisker plot below represents the height of 9th graders at a certain high school. How much greater was the median height of the boys than the median height of the girls?



- F 3 inches
- G 4 inches
- H 5 inches
- J 6 inches

SOLUTION:

Median height of the boys is 63 inches.

Median height of the girls is 60 inches.

$$\text{Difference} = 63 - 60 = 3$$

So, the median height of the boys is 3 inches greater than the median height of the girls.

The correct option is F.

ANSWER:

F

53. **SAT/ACT** Heather, Teresa, and Nina went shopping for new clothes. Heather spent twice as much as Teresa, and Nina spent three times what Heather spent. If they spent a total of \$300, how much did Teresa spend?

- A \$33.33
- B \$50.00
- C \$66.33
- D \$100.00
- E \$104.33

SOLUTION:

Let h , t , and n be the amount spent by Heather, Teresa, and Nina respectively.

$$h = 2t$$

$$n = 3h = 3(2t) = 6t$$

$$h + t + n = \$300$$

Substitute.

$$2t + t + 6t = \$300$$

$$9t = \$300$$

$$t \approx \$33.33$$

So, Teresa spent about \$33.33.

The correct option is A.

ANSWER:

A

2-2 Logic

54. **LUNCH** For the past four Tuesdays, Jason's school has served chicken sandwiches for lunch. Jason assumes that chicken sandwiches will be served for lunch on the next Tuesday. What type of reasoning did he use? Explain.

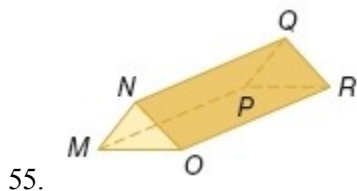
SOLUTION:

Inductive; sample answer: Inductive reasoning is reasoning that uses a number of specific examples to arrive at a conclusion. Jason noticed that chicken sandwiches were served for lunch on Tuesday and assumed that the pattern would continue, therefore he used inductive reasoning.

ANSWER:

Inductive; sample answer: Jason noticed that chicken sandwiches were served for lunch on Tuesday and assumed that the pattern would continue, therefore he used inductive reasoning.

Identify each solid. Name the bases, faces, edges, and vertices.



SOLUTION:

It has two congruent triangular bases. So, it is a triangular prism.

Bases: $\triangle MNO, \triangle PQR$

Faces: $\triangle MNO, \triangle PQR, \text{OMPR}, \text{ONQR}, \text{PQNM}$

Edges: $\overline{MN}, \overline{NO}, \overline{OM}, \overline{PQ}, \overline{QR}, \overline{PR}, \overline{NQ}, \overline{NP}, \overline{OR}$

Vertices: M, N, O, P, Q, R

ANSWER:

Triangular prism

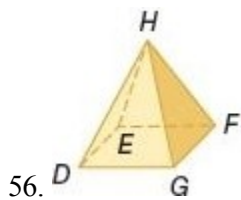
Bases: $\triangle MNO, \triangle PQR$

Faces: $\triangle MNO, \triangle PQR, \text{OMPR}, \text{ONQR}, \text{PQNM}$

Edges: $\overline{MN}, \overline{NO}, \overline{OM}, \overline{PQ}, \overline{QR}, \overline{PR}, \overline{NQ}, \overline{MP}, \overline{OR}$

Vertices: $M, N, O, P, Q,$ and R

2-2 Logic



SOLUTION:

It has a rectangular base and three or more triangular faces that meet at a common vertex. So, it is a rectangular pyramid.

Base: $\square DEFG$

Faces: $\square DEFG, \triangle DHG, \triangle GHF, \triangle FEH, \triangle DHE$

Edges: $\overline{DG}, \overline{GF}, \overline{FE}, \overline{ED}, \overline{DH}, \overline{EH}, \overline{FH}, \overline{GH}$

Vertices: D, E, F, G, H

ANSWER:

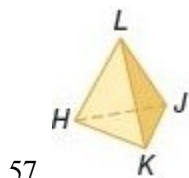
Rectangular pyramid.

Base: $\square DEFG$

Faces: $\square DEFG, \triangle DHG, \triangle GHF, \triangle FEH, \triangle DHE$

Edges: $\overline{DG}, \overline{GF}, \overline{FE}, \overline{ED}, \overline{DH}, \overline{EH}, \overline{FH}, \overline{GH}$

Vertices: D, E, F, G, H



SOLUTION:

It has a triangular base and three or more triangular faces that meet at a common vertex. So, it is a triangular pyramid.

Base: $\triangle HJK$

Faces: $\triangle HJK, \triangle HLK, \triangle KLJ, \triangle HLJ$

Edges: $\overline{HK}, \overline{JL}, \overline{KJ}, \overline{JH}, \overline{KL}, \overline{HL}$

Vertices: H, K, J, L

ANSWER:

Triangular pyramid

Base: $\triangle HJK$

Faces: $\triangle HJK, \triangle HLK, \triangle KLJ, \triangle HLJ$

Edges: $\overline{HK}, \overline{JL}, \overline{KJ}, \overline{JH}, \overline{KL}, \overline{HL}$

Vertices: H, K, J, L

2-2 Logic

ALGEBRA Solve each equation.

58. $\frac{y}{2} - 7 = 5$

SOLUTION:

$$\frac{y}{2} - 7 = 5$$

$$\frac{y}{2} - 7 + 7 = 5 + 7$$

$$\frac{y}{2} = 12$$

$$2\left(\frac{y}{2}\right) = 2(12)$$

$$y = 24$$

ANSWER:

24

59. $3x + 9 = 6$

SOLUTION:

$$3x + 9 = 6$$

$$3x + 9 - 9 = 6 - 9$$

$$3x = -3$$

$$\frac{3x}{3} = \frac{-3}{3}$$

$$x = -1$$

ANSWER:

-1

60. $4(m - 5) = 12$

SOLUTION:

$$4(m - 5) = 12$$

$$4m - 20 = 12$$

$$4m - 20 + 20 = 12 + 20$$

$$4m = 32$$

$$\frac{4m}{4} = \frac{32}{4}$$

$$m = 8$$

ANSWER:

8

2-2 Logic

61. $6(w + 7) = 0$

SOLUTION:

$$6(w + 7) = 0$$

$$6w + 42 = 0$$

$$6w = -42$$

$$\frac{6w}{6} = \frac{-42}{6}$$

$$w = -7$$

ANSWER:

-7

62. $2x - 7 = 11$

SOLUTION:

$$2x - 7 = 11$$

$$2x - 7 + 7 = 11 + 7$$

$$2x = 18$$

$$\frac{2x}{2} = \frac{18}{2}$$

$$x = 9$$

ANSWER:

9

63. $\frac{y}{5} + 4 = 9$

SOLUTION:

$$\frac{y}{5} + 4 = 9$$

$$\frac{y}{5} + 4 - 4 = 9 - 4$$

$$\frac{y}{5} = 5$$

$$5\left(\frac{y}{5}\right) = 5(5)$$

$$y = 25$$

ANSWER:

25

2-2 Logic

ALGEBRA Evaluate each expression for the given values.

64. $2y + 3x$ if $y = 3$ and $x = -1$

SOLUTION:

Substitute $y = 3$ and $x = -1$.

$$\begin{aligned}2y + 3x &= 2(3) + 3(-1) \\ &= 6 - 3 \\ &= 3\end{aligned}$$

ANSWER:

3

65. $4d - c$ if $d = 4$ and $c = 2$

SOLUTION:

Substitute $d = 4$ and $c = 2$.

$$\begin{aligned}4d - c &= 4(4) - 2 \\ &= 16 - 2 \\ &= 14\end{aligned}$$

ANSWER:

14

66. $m^2 + 7n$ if $m = 4$ and $n = -2$

SOLUTION:

Substitute $m = 4$ and $n = -2$.

$$\begin{aligned}m^2 + 7n &= 4^2 + 7(-2) \\ &= 16 - 14 \\ &= 2\end{aligned}$$

ANSWER:

2

67. $ab - 2a$ if $a = -2$ and $b = -3$

SOLUTION:

Substitute $a = -2$ and $b = -3$.

$$\begin{aligned}ab - 2a &= (-2)(-3) - 2(-2) \\ &= 6 + 4 \\ &= 10\end{aligned}$$

ANSWER:

10