2-3 Conditional Statements

Identify the hypothesis and conclusion of each conditional statement.

1. If today is Friday, then tomorrow is Saturday.

SOLUTION:

The hypothesis of a conditional statement is the phrase immediately following the word ***if***. The conclusion of a conditional statement is the phrase immediately following the word ***then***.

Hypothesis: Today is Friday.
Conclusion: Tomorrow is Saturday.

ANSWER:

H: today is Friday; C: tomorrow is Saturday.

2. If $2x + 5 > 7$, then $x > 1$.

SOLUTION:

The hypothesis of a conditional statement is the phrase immediately following the word ***if***. The conclusion of a conditional statement is the phrase immediately following the word ***then***.

Hypothesis: $2x + 5 > 7$
Conclusion: $x > 1$

ANSWER:

H: $2x + 5 > 7$; C: $x > 1$

3. If two angles are supplementary, then the sum of the measures of the angles is 180.

SOLUTION:

The hypothesis of a conditional statement is the phrase immediately following the word ***if***. The conclusion of a conditional statement is the phrase immediately following the word ***then***.

Hypothesis: Two angles are supplementary.
Conclusion: The sum of the measures of the angles is 180.

ANSWER:

H: two angles are supplementary;
C: the sum of the measures of the angles is 180

4. If two lines form right angles, then the lines are perpendicular.

SOLUTION:

The hypothesis of a conditional statement is the phrase immediately following the word ***if***. The conclusion of a conditional statement is the phrase immediately following the word ***then***.

Hypothesis: Two lines form right angles
Conclusion: The lines are perpendicular.

ANSWER:

H: two lines form right angles; C: the lines are perpendicular

2-3 Conditional Statements

Write each statement in if-then form.

5. Sixteen-year-olds are eligible to drive.

SOLUTION:

To write these statements in if-then form, identify the hypothesis and conclusion. The word if is not part of the hypothesis.
The word then is not part of the conclusion.

If you are sixteen years old, then you are eligible to drive.

ANSWER:

If you are sixteen years old, then you are eligible to drive.

6. Cheese contains calcium.

SOLUTION:

To write these statements in if-then form, identify the hypothesis and conclusion. The word if is not part of the hypothesis. The word then is not part of the conclusion.
If it is cheese, then it contains calcium.

ANSWER:

If it is cheese, then it contains calcium.

7. The measure of an acute angle is between 0 and 90.

SOLUTION:

To write these statements in if-then form, identify the hypothesis and conclusion. The word if is not part of the hypothesis. The word then is not part of the conclusion.
If the angle is acute, then its measure is between 0 and 90.

ANSWER:

If the angle is acute, then its measure is between 0 and 90.

8. Equilateral triangles are equiangular.

SOLUTION:

To write these statements in if-then form, identify the hypothesis and conclusion. The word if is not part of the hypothesis. The word then is not part of the conclusion.
If a triangle is equilateral, then it is equiangular.

ANSWER:

If a triangle is equilateral, then it is equiangular.

9. **WEATHER** Various kinds of precipitation form under different conditions. Write the three conditionals below in if-then form.
   a. Moisture in the air condenses and falls to form rain.
   b. Supercooled moisture in cumulonimbus clouds forms hail.
   c. When the temperature is freezing in all or most of the atmosphere, precipitation falls as snow.

   SOLUTION:

   To write these statements in if-then form, identify the hypothesis and conclusion. The word if is not part of the hypothesis. The word then is not part of the conclusion.

   a. If moisture in the air condenses and falls, then it rains.
   b. If a cumulonimbus cloud has supercooled moisture, then hail forms.
   c. If the temperature is freezing in all or most of the atmosphere, then precipitation falls as snow.

   ANSWER:

   a. If moisture in the air condenses and falls, then it rains.
   b. If a cumulonimbus cloud has supercooled moisture, then hail forms.
   c. If the temperature is freezing in all or most of the atmosphere, then precipitation falls as snow.

   Determine the truth value of each conditional statement. If *true,* explain your reasoning. If *false,* give a counterexample.

10. If $x^2 = 16$, then $x = 4$.

    SOLUTION:

    False.

    If $x = -4$, $(-4)^2 = 16$. The hypothesis of the conditional is true, but the conclusion is false. This counterexample shows that the conditional statement is false.

    ANSWER:

    False; if $x = -4$, $(-4)^2 = 16$. The hypothesis of the conditional is true, but the conclusion is false. This counterexample shows that the conditional statement is false

11. If you live in Charlotte, then you live in North Carolina.

    SOLUTION:

    The conditional is false. You could live in Charlotte, Michigan or Charlotte, North Carolina.

    ANSWER:

    False; Charlotte, Michigan; The hypothesis of the conditional is true, but the conclusion is false. This counter example shows that the conditional statement is false.

12. If tomorrow is Friday, then today is Thursday.

    SOLUTION:

    The conditional statement "If tomorrow is Friday, then today is Thursday." is true. When this hypothesis is true, the conclusion is also true, since Friday is the day that follows Thursday. So, the conditional statement is true.

    ANSWER:

    True; when this hypothesis is true, the conclusion is also true, since Friday is the day that follows Thursday. So, the conditional statement is true.

13. If an animal is spotted, then it is a Dalmatian.

    SOLUTION:

    False
    The animal could be a leopard. The hypothesis of the conditional is true, but the conclusion is false. This counterexample shows that the conditional statement is false.

    ANSWER:

    False; the animal could be a leopard. The hypothesis of the conditional is true, but the conclusion is false. This counterexample shows that the conditional statement is false.

14. If the measure of a right angle is 95, then bees are lizards.

    SOLUTION:

    The conditional statement "If the measure of a right angle is 95, then bees are lizards." is true. The hypothesis is false, since the measure of a right angle is 90. A conditional with a false hypothesis is always true, so this conditional statement is true.

    ANSWER:

    True; the hypothesis is false, since the measure of a right angle is 90. A conditional with a false hypothesis is always true, so this conditional statement is true.

15. If pigs can fly, then $2 + 5 = 7$.

    SOLUTION:

    The conditional statement "If pigs can fly, then $2 + 5 = 7$" is true. The hypothesis is false, since pigs cannot fly. A conditional with a false hypothesis is always true, so this conditional statement is true.

    ANSWER:

    True; the hypothesis is false, since pigs cannot fly. A conditional with a false hypothesis is always true, so this conditional statement is true.

CCSS ARGUMENTS Write the converse, inverse, and contrapositive of each true conditional statement.
Determine whether each related conditional is true or false. If a statement is false, find a counterexample.

16. If a number is divisible by 2, then it is divisible by 4.

SOLUTION:

The converse is formed by exchanging the hypothesis and conclusion of the conditional.
Converse: If a number is divisible by 4, then it is divisible by 2; true.

The inverse is formed by negating both the hypothesis and conclusion of the conditional.
Inverse: If a number is not divisible by 2, then it is not divisible by 4; true.

The contrapositive is formed by negating both the hypothesis and the conclusion of the converse of the conditional.
Contrapositive: If a number is not divisible by 4, then it is not divisible by 2; false.
Counterexample: 6 is not divisible by 4 but divisible by 2.

ANSWER:

Converse: if a number is divisible by 4, then it is divisible by 2; true. Inverse: If a number is not divisible by 2, then it is not divisible by 4; true. Contrapositive: If a number is not divisible by 4, then it is not divisible by 2; false. Sample answer: 6 is not divisible by 4 but divisible by 2.

17. All whole numbers are integers.

SOLUTION:

If a number is a whole number, then it is an integer.

The converse is formed by exchanging the hypothesis and conclusion of the conditional.
Converse: If a number is an integer, then it is a whole number. False; Sample answer: –3.

The inverse is formed by negating both the hypothesis and conclusion of the conditional.
Inverse: If a number is not a whole number, then it is not an integer. False: Sample answer: –3.

The contrapositive is formed by negating both the hypothesis and the conclusion of the converse of the conditional.
Contrapositive: If a number is not an integer, then it is not a whole number; true.

ANSWER:

If a number is a whole number, then it is an integer. Converse: If a number is an integer, then it is a whole number. False; Sample answer: –3. Inverse: If a number is not a whole number, then it is not an integer. False: Sample answer: –3. Contrapositive: If a number is not an integer, then it is not a whole number; true.

2-3 Conditional Statements

Identify the hypothesis and conclusion of each conditional statement.

18. If two angles are adjacent, then they have a common side.

SOLUTION:

The hypothesis of a conditional statement is the phrase immediately following the word ***if***. The conclusion of a conditional statement is the phrase immediately following the word ***then***.

Hypothesis: Two angles are adjacent.
Conclusion: They have a common side.

ANSWER:

H: two angles are adjacent; C: they have a common side

19. If you lead, then I will follow.

SOLUTION:

The hypothesis of a conditional statement is the phrase immediately following the word ***if***. The conclusion of a conditional statement is the phrase immediately following the word ***then***.
Hypothesis: You lead
Conclusion: I will follow

ANSWER:

H: you lead; C: I will follow

20. If $3x - 4 = 11$, then $x = 5$.

SOLUTION:

The hypothesis of a conditional statement is the phrase immediately following the word ***if***. The conclusion of a conditional statement is the phrase immediately following the word ***then***.

Hypothesis: $3x - 4 = 11$
Conclusion: $x = 5$

ANSWER:

H: $3x - 4 = 11$; C: $x = 5$

21. If two angles are vertical, then they are congruent.

SOLUTION:

The hypothesis of a conditional statement is the phrase immediately following the word ***if***. The conclusion of a conditional statement is the phrase immediately following the word ***then***.

Hypothesis: Two angles are vertical
Conclusion: They are congruent

ANSWER:

H: two angles are vertical;
C: they are congruent

2-3 Conditional Statements

Identify the hypothesis and conclusion of each conditional statement.

22. If the degree measure of an angle is between 90 and 180, then the angle is obtuse.

SOLUTION:

The hypothesis of a conditional statement is the phrase immediately following the word ***if***. The conclusion of a conditional statement is the phrase immediately following the word ***then***.
Hypothesis: The degree measure of an angle is between 90 and 180
Conclusion: The angle is obtuse

ANSWER:

H: the degree measure of an angle is between 90 and 180;
C: the angle is obtuse

23. "If there is no struggle, there is no progress." (Frederick Douglass)

SOLUTION:

The hypothesis of a conditional statement is the phrase immediately following the word ***if***. The conclusion of a conditional statement is the phrase immediately following the word ***then***.
Hypothesis: There is no struggle
Conclusion: There is no progress

ANSWER:

H: there is no struggle; C: there is no progress

24. If a quadrilateral has four congruent sides, then it is a square.

SOLUTION:

The hypothesis of a conditional statement is the phrase immediately following the word ***if***. The conclusion of a conditional statement is the phrase immediately following the word ***then***.
Hypothesis: A quadrilateral has four congruent sides.
Conclusion: It is a square.

ANSWER:

H: a quadrilateral has four congruent sides; C: it is a square

25. If a convex polygon has five sides, then it is a pentagon.

SOLUTION:

The hypothesis of a conditional statement is the phrase immediately following the word ***if***. The conclusion of a conditional statement is the phrase immediately following the word ***then***.
Hypothesis: A convex polygon has five sides.
Conclusion: It is a pentagon.

ANSWER:

H: a convex polygon has five sides; C: it is a pentagon.

2-3 Conditional Statements

Write each statement in if-then form.

26. Get a free water bottle with a one-year membership.

SOLUTION:

To write these statements in if-then form, identify the hypothesis and conclusion. The word if is not part of the hypothesis. The word then is not part of the conclusion.

If you buy a 1-year membership, then you get a free water bottle.

ANSWER:

If you buy a 1-year membership, then you get a free water bottle.

27. Everybody at the party received a gift.

SOLUTION:

To write these statements in if-then form, identify the hypothesis and conclusion. The word if is not part of the hypothesis. The word then is not part of the conclusion.

If you were at the party, then you received a gift.

ANSWER:

If you were at the party, then you received a gift.

28. The intersection of two planes is a line.

SOLUTION:

To write these statements in if-then form, identify the hypothesis and conclusion. The word if is not part of the hypothesis. The word then is not part of the conclusion.

If two planes intersect, then it is a line.

ANSWER:

If two planes intersect, then it is a line.

29. The area of a circle is πr^2.

SOLUTION:

To write these statements in if-then form, identify the hypothesis and conclusion. The word if is not part of the hypothesis. The word then is not part of the conclusion.

If a figure is a circle, then the area is πr^2.

ANSWER:

If a figure is a circle, then the area is πr^2.

30. Collinear points lie on the same line.

SOLUTION:

To write these statements in if-then form, identify the hypothesis and conclusion. The word if is not part of the hypothesis. The word then is not part of the conclusion.
If points are collinear, then they lie on the same line.

ANSWER:

If points are collinear, then they lie on the same line.

31. A right angle measures 90 degrees.

SOLUTION:

To write these statements in if-then form, identify the hypothesis and conclusion. The word if is not part of the hypothesis. The word then is not part of the conclusion.

If an angle is right, then the angle measures 90 degrees.

ANSWER:

If an angle is right, then the angle measures 90 degrees.

32. **MUSIC** Different instruments are emphasized in different types of music. Write each statement in if-then form.
- Jazz music often incorporates trumpet or saxophone.
- Rock music emphasizes guitar and drums.
- In hip-hop music, the bass is featured.

SOLUTION:

To write these statements in if-then form, identify the hypothesis and conclusion. The word if is not part of the hypothesis. The word then is not part of the conclusion.

If you listen to jazz music, then you will likely hear trumpet or saxophone.
If you listen to rock music, then you will likely hear guitar and drums.
If you listen to hip-hop music, then you will likely hear bass.

ANSWER:

If you listen to jazz music, then you will likely hear trumpet or saxophone. If you listen to rock music, then you will likely hear guitar and drums. If you listen to hip-hop music, then you will likely hear bass.

33. **ART** Write the following statement in if-then form: At the Andy Warhol Museum in Pittsburgh, Pennsylvania, most of the collection is Andy Warhol's artwork.

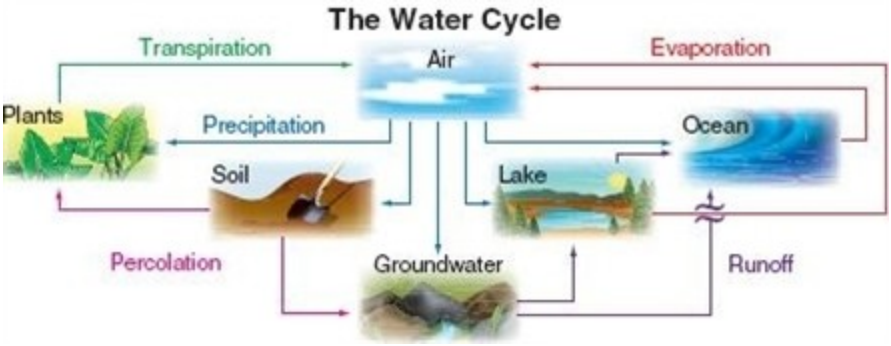
SOLUTION:

To write these statements in if-then form, identify the hypothesis and conclusion. The word if is not part of the hypothesis. The word then is not part of the conclusion.
If the museum is the Andy Warhol Museum, then most of the collection is Andy Warhol's artwork.

ANSWER:

If the museum is the Andy Warhol Museum, then most of the collection is Andy Warhol's artwork.

34. **SCIENCE** The water on Earth is constantly changing through a process called the water cycle. Write the three conditionals below in if-then form.



a. As runoff, water flows into bodies of water.
b. Plants return water to the air through transpiration.
c. Water bodies return water to the air through evaporation.

SOLUTION:

To write these statements in if-then form, identify the hypothesis and conclusion. The word if is not part of the hypothesis. The word then is not part of the conclusion.
a. If water runs off, it flows into bodies of water.
b. If plants return water to the air, they transpire.
c. If water bodies return water to the air, it is through evaporation.

ANSWER:

a. If water runs off, it flows into bodies of water.
b. If plants return water to the air, they transpire.
c. If water bodies return water to the air, it is through evaporation.

CCSS ARGUMENTS Determine the truth value of each conditional statement. If *true*, explain your reasoning. If *false*, give a counterexample.

35. If a number is odd, then it is divisible by 5.

SOLUTION:

False; 9 is an odd number, but not divisible by 5. The hypothesis of the conditional is true, but the conclusion is false. This counterexample shows that the conditional statement is false.

ANSWER:

False; 9 is an odd number, but not divisible by 5. The hypothesis of the conditional is true, but the conclusion is false. This counterexample shows that the conditional statement is false.

36. If a dog is an amphibian, then the season is summer.

SOLUTION:

The conditional statement "If a dog is an amphibian, then the season is summer." is true. The hypothesis is false, since a dog is not an amphibian. A conditional with a false hypothesis is always true, so this conditional statement is true.

ANSWER:

True; the hypothesis is false, since a dog is not an amphibian. A conditional with a false hypothesis is always true, so this conditional statement is true.

37. If an angle is acute, then it has a measure of 45.

SOLUTION:

False; the angle drawn is an acute angle whose measure is not 45. The hypothesis of the conditional is true, but the conclusion is false. This counterexample shows that the conditional statement is false.

ANSWER:

False; the angle drawn is an acute angle whose measure is not 45. The hypothesis of the conditional is true, but the conclusion is false. This counterexample shows that the conditional statement is false.

38. If a polygon has six sides, then it is a regular polygon.

SOLUTION:

False; this polygon has six sides, but is not regular. The hypothesis of the conditional is true, but the conclusion is false. This counterexample shows that the conditional statement is false.

ANSWER:

False; this polygon has six sides, but is not regular. The hypothesis of the conditional is true, but the conclusion is false. This counterexample shows that the conditional statement is false.

Determine the truth value of each conditional statement. If true, explain your reasoning. If false, give a counterexample.

39. If an angle's measure is 25, then the measure of the angle's complement is 65.

SOLUTION:

The conditional statement "If an angle's measure is 25, then the measure of the angle's complement is 65." is true. When this hypothesis is true, the conclusion is also true, since an angle and its complement's sum is 90. So, the conditional statement is true.

ANSWER:

True; when this hypothesis is true, the conclusion is also true, since an angle and its complement's sum is 90. So, the conditional statement is true.

40. If North Carolina is south of Florida, then the capital of Ohio is Columbus.

SOLUTION:

The conditional statement "If North Carolina is south of Florida, then the capital of Ohio is Columbus." is true. The hypothesis is false, since North Carolina is not south of Florida. A conditional with a false hypothesis is always true, so this conditional statement is true.

ANSWER:

True; the hypothesis is false, since North Carolina is not south of Florida. A conditional with a false hypothesis is always true, so this conditional statement is true.

2-3 Conditional Statements

41. If red paint and blue paint mixed together make white paint, then 3 – 2 = 0.

SOLUTION:

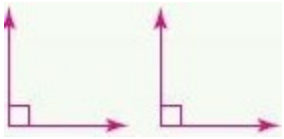
The conditional statement "If red paint and blue paint mixed together make white paint, then 3 – 2 = 0" is true. The hypothesis is false, since red and blue paint make green paint. A conditional with a false hypothesis is always true, so this conditional statement is true.

ANSWER:

True; the hypothesis is false, since red and blue paint make green paint. A conditional with a false hypothesis is always true, so this conditional statement is true.

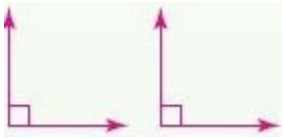
42. If two angles are congruent, then they are vertical angles.

SOLUTION:



False; the angles are congruent, but they are not vertical angles. The hypothesis of the conditional is true, but the conclusion is false.
This counterexample shows that the conditional statement is false.

ANSWER:



False; the angles are congruent, but they are not vertical angles. The hypothesis of the conditional is true, but the conclusion is false. This counterexample shows that the conditional statement is false.

43. If an animal is a bird, then it is an eagle.

SOLUTION:

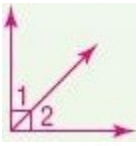
False; the animal could be a falcon. The hypothesis of the conditional is true, but the conclusion is false. This counterexample shows that the conditional statement is false.

ANSWER:

False; the animal could be a falcon. The hypothesis of the conditional is true, but the conclusion is false. This counterexample shows that the conditional statement is false.

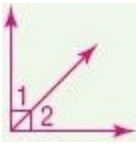
44. If two angles are acute, then they are supplementary.

SOLUTION:



False; $\angle 1$ and $\angle 2$ are acute, but their sum is 90°. The hypothesis of the conditional is true, but the conclusion is false. This counterexample shows that the conditional statement is false.

ANSWER:



False; $\angle 1$ and $\angle 2$ are acute, but their sum is 90°. The hypothesis of the conditional is true, but the conclusion is false. This counterexample shows that the conditional statement is false.

45. If two lines intersect, then they form right angles.

SOLUTION:

False; These lines intersect, but do not form right angles. The hypothesis of the conditional is true, but the conclusion is false. This counterexample shows that the conditional statement is false.

ANSWER:

False; these lines intersect, but do not form right angles. The hypothesis of the conditional is true, but the conclusion is false. This counterexample shows that the conditional statement is false.

46. If a banana is blue, then an apple is a vegetable.

SOLUTION:

The conditional statement "If a banana is blue, then an apple is a vegetable." is true. The hypothesis is false, since a banana is never blue. A conditional with a false hypothesis is always true, so this conditional statement is true.

ANSWER:

True; the hypothesis is false, since a banana is never blue. A conditional with a false hypothesis is always true, so this conditional statement is true.

Write the converse, inverse, and contrapositive of each true conditional statement. Determine whether each related conditional is true or false. If a statement is false, find a counterexample.

47. If you live in Chicago, you live in Illinois.

SOLUTION:

The converse is formed by exchanging the hypothesis and conclusion of the conditional.
Converse: If you live in Illinois, then you live in Chicago. The converse is false.
Counterexample: You can live in Springfield.

The inverse is formed by negating both the hypothesis and conclusion of the conditional.
Inverse: If you do not live in Chicago, then you do not live in Illinois. The inverse is false.
Counterexample: You can live in Springfield.

The contrapositive is formed by negating both the hypothesis and the conclusion of the converse of the conditional.
Contrapositive: If you do not live in Illinois, then you do not live in Chicago. The contrapositive is true.

ANSWER:

Converse: If you live in Illinois, then you live in Chicago. False: You can live in Springfield. Inverse: If you do not live in Chicago, then you do not live in Illinois. False: You can live in Springfield. Contrapositive: If you do not live in Illinois, then you do not live in Chicago; true.

48. If a bird is an ostrich, then it cannot fly.

SOLUTION:

The converse is formed by exchanging the hypothesis and conclusion of the conditional.
Converse: If a bird cannot fly, then it is an ostrich. The converse is false.
Counterexample: The bird could be a penguin.

The inverse is formed by negating both the hypothesis and conclusion of the conditional.
Inverse: If a bird is not an ostrich, then it can fly. The inverse is false.
Counterexample: The bird could be a penguin.

The contrapositive is formed by negating both the hypothesis and the conclusion of the converse of the conditional.
Contrapositive: If a bird can fly, then the bird is not an ostrich; The contrapositive is true.

ANSWER:

Converse: If a bird cannot fly, then it is an ostrich. False; The bird could be a penguin. Inverse: If a bird is not an ostrich, then it can fly. False; The bird could be a penguin. Contrapositive: If a bird can fly, then the bird is not an ostrich; true.

2-3 Conditional Statements

49. If two angles have the same measure, then the angles are congruent.

SOLUTION:

The converse is formed by exchanging the hypothesis and conclusion of the conditional.
Converse: If two angles are congruent, then they have the same measure. The converse is true.

The inverse is formed by negating both the hypothesis and conclusion of the conditional.
Inverse: If two angles do not have the same measure, then the angles are not congruent. The inverse is true.

The contrapositive is formed by negating both the hypothesis and the conclusion of the converse of the conditional.
Contrapositive: If two angles are not congruent, then they do not have the same measure. The contrapositive is true.

ANSWER:

Converse: If two angles are congruent, then they have the same measure; true. Inverse: If two angles do not have the same measure, then the angles are not congruent; true. Contrapositive: If two angles are not congruent, then they do not have the same measure; true.

50. All squares are rectangles.

SOLUTION:

If a figure is a square, then it is a rectangle.

The converse is formed by exchanging the hypothesis and conclusion of the conditional.
Converse: If a figure is a rectangle, then it is a square. The converse is false.
Counterexample: A rectangle does not have to have all sides congruent.

The inverse is formed by negating both the hypothesis and conclusion of the conditional.
Inverse: If a figure is not a square, then it is not a rectangle. The inverse is false.
Counterexample: The figure could be a rectangle, even though it is not a square.

The contrapositive is formed by negating both the hypothesis and the conclusion of the converse of the conditional.
Contrapositive: If a figure is not a rectangle, then it is not a square. The contrapositive is true.

ANSWER:

If a figure is a square, then it is a rectangle. Converse: If a figure is a rectangle, then it is a square. False. A rectangle does not have to have all sides congruent. Inverse: If a figure is not a square, then it is not a rectangle. False. The figure could be a rectangle, even though it is not a square. Contrapositive: If a figure is not a rectangle, then it is not a square; true.

2-3 Conditional Statements

51. All congruent segments have the same length.

SOLUTION:

If segments are congruent, then they have the same length.

The converse is formed by exchanging the hypothesis and conclusion of the conditional.
Converse: If segments have the same length, then they are congruent. The converse is true.

The inverse is formed by negating both the hypothesis and conclusion of the conditional.
Inverse: If segments are not congruent, then they do not have the same length. The inverse is true.

The contrapositive is formed by negating both the hypothesis and the conclusion of the converse of the conditional.
Contrapositive: If segments do not have the same length, then they are not congruent. The contrapositive is true.

ANSWER:

If segments are congruent, then they have the same length. Converse: If segments have the same length, then they are congruent; true. Inverse: If segments are not congruent, then they do not have the same length; true. Contrapositive: If segments do not have the same length, then they are not congruent; true.

52. A right triangle has an angle measure of 90.

SOLUTION:

If a triangle is right, then it has an angle measure of 90.

The converse is formed by exchanging the hypothesis and conclusion of the conditional.
Converse: If a triangle has an angle measure of 90, then it is a right triangle. The converse is true.

The inverse is formed by negating both the hypothesis and conclusion of the conditional.
Inverse: If a triangle is not right, then it does not have an angle measure of 90. The inverse is true.

The contrapositive is formed by negating both the hypothesis and the conclusion of the converse of the conditional.
Contrapositive: If a triangle does not have an angle measure of 90, then it is not a right triangle. The contrapositive is true.

ANSWER:

If a triangle is right, then it has an angle measure of 90. Converse: If a triangle has an angle measure of 90, then it is a right triangle; true. Inverse: If a triangle is not right, then it does not have an angle measure of 90; true. Contrapositive: If a triangle does not have an angle measure of 90, then it is not a right triangle; true.

CCSS ARGUMENTS Write the statement indicated, and use the information at the left to determine the truth value of each statement. If a statement is false, give a counterexample.

Animals with stripes are zebras.

53. conditional

SOLUTION:

To write these statements in conditional form, identify the hypothesis and conclusion. The word if is not part of the hypothesis. The word then is not part of the conclusion.
If an animal has stripes, then it is a zebra. The conditional is false. Counterexample: A tiger has stripes.

ANSWER:

If an animal has stripes, then it is a zebra; false: a tiger has stripes.

54. converse

SOLUTION:

The converse is formed by exchanging the hypothesis and conclusion of the conditional.
If an animal is a zebra, then it has stripes. The converse is true.

ANSWER:

If an animal is a zebra, then it has stripes; true.

55. inverse

SOLUTION:

The inverse is formed by negating both the hypothesis and conclusion of the conditional.
If an animal does not have stripes, then it is not a zebra. The inverse is true.

ANSWER:

If an animal does not have stripes, then it is not a zebra; true.

56. contrapositive

SOLUTION:

The contrapositive is formed by negating both the hypothesis and the conclusion of the converse of the conditional.
If an animal is not a zebra, then it does not have stripes. The contrapositive is false. Counterexample: A tiger has stripes.

ANSWER:

If an animal is not a zebra, then it does not have stripes; false: a tiger has stripes.

57. **SCIENCE** Chemical compounds are grouped and described by the elements that they contain. Acids contain hydrogen (H). Bases contain hydroxide (OH). Hydrocarbons contain only hydrogen (H) and carbon (C).

Compound	Example	Chemical Formula
Acid	Hydrochloric Acid	HCl
Base	Sodium Hydroxide	NaOH
Hydrocarbon	Methane	CH_4

a. Write three conditional statements in if-then form for classifying chemical compounds.

b. Write the converse of the three true conditional statements.

SOLUTION:

a. To write these statements in if-then form, identify the hypothesis and conclusion. The word if is not part of the hypothesis. The word then is not part of the conclusion.
If a compound is an acid, it contains hydrogen. If a compound is a base, it contains hydroxide. If a compound is a hydrocarbon, it contains only hydrogen and carbon.

b. The converse is formed by exchanging the hypothesis and conclusion of the conditional.
If a compound contains hydrogen, it is an acid. The converse is false. Counterexample: A hydrocarbon contains hydrogen.
If a compound contains hydroxide, it is a base. The converse is true.
If a compound contains only hydrogen and carbon, it is a hydrocarbon. The converse is true.

ANSWER:

a. Sample answer: If a compound is an acid, it contains hydrogen.
If a compound is a base, it contains hydroxide. If a compound is a hydrocarbon, it contains only hydrogen and carbon.

b. Sample answer: If a compound contains hydrogen, it is an acid.
False; a hydrocarbon contains hydrogen. If a compound contains hydroxide, it is a base; true. If a compound contains only hydrogen and carbon, it is a hydrocarbon; true.

58. **SPORTS** In football, touchdowns are worth 6 points, extra point conversions are worth 2 points, and safeties are worth 2 points.

a. Write three conditional statements in if-then form for scoring in football.

b. Write the converse of the three true conditional statements. State whether each is *true* or *false*. If a statement is *false*, find a counterexample.

SOLUTION:

a. To write these statements in if-then form, identify the hypothesis and conclusion. The word if is not part of the hypothesis. The word then is not part of the conclusion.
If a football team makes a touchdown, they get 6 points; If a football team makes a two-point conversion, they get 2 points; If a football team makes a safety, they get 2 points.

b. The converse is formed by exchanging the hypothesis and conclusion of the conditional.
If a football team gets 6 points, they made a touchdown. The converse is true.
If a football team gets 2 points, they made a two-point conversion. The converse is false. Counterexample: They could have gotten a safety;
If a football team gets 2 points, they made a safety. the converse is false. Counterexample, They could have gotten a two-point conversion.

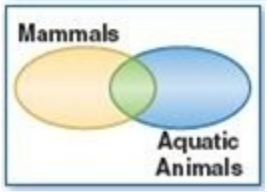
ANSWER:

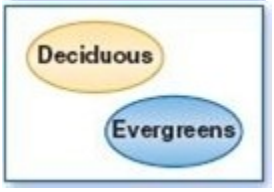
a. Sample answer: If a football team makes a touchdown, they get 6 points; If a football team makes a two-point conversion, they get 2 points; If a football team makes a safety, they get 2 points.

b. Sample answer: If a football team gets 6 points, they made a touchdown. True; If a football team gets 2 points, they made a two-point conversion. False; they could have gotten a safety; If a football team gets 2 points, they made a safety. False; they could have gotten a two-point conversion.

2-3 Conditional Statements

Use the Venn diagrams below to determine the truth value of each conditional. Explain your reasoning.





59. If a function is nonlinear, then it is quadratic.

SOLUTION:

The statement "If a function is nonlinear, then it is quadratic." is false. The blue area of the Venn diagram includes nonlinear functions but not quadratic functions. The diagram shows quadratics are a part of the nonlinear functions, not the other way around.

ANSWER:

False; the blue area of the Venn diagram includes nonlinear functions but not quadratic functions.

60. If an animal is a mammal, then it cannot be aquatic.

SOLUTION:

The statement " If an animal is a mammal, then it cannot be aquatic." is false. The green area of the Venn diagram includes both mammals and aquatic animals. Some mammals are aquatic.

ANSWER:

False; the green area of the Venn diagram includes both mammals and aquatic animals.

61. If a tree is deciduous, then it is not an evergreen.

SOLUTION:

The statement "If a tree is deciduous, then it is not an evergreen." is true. The deciduous area and the evergreen area have no common areas, so a deciduous tree cannot be evergreen.

ANSWER:

True; the deciduous area and the evergreen area have no common areas, so a deciduous tree cannot be evergreen.

2-3 Conditional Statements

62. **MULTIPLE REPRESENTATIONS** In this problem, you will investigate a law of logic by using conditionals.
a. LOGICAL Write three true conditional statements, using each consecutive conclusion as the hypothesis for the next statement.
b. GRAPHICAL Create a Venn diagram to model your series of statements.
c. LOGICAL Write a conditional using the hypothesis of your first conditional and the conclusion of your third conditional. Is the conditional true if the hypothesis is true?
d. VERBAL Given two conditionals *If a, then b* and *If b, then c*, make a conjecture about the truth value of *c* when *a* is true. Explain your reasoning.

SOLUTION:

a.With consecutive conclusion, the conclusion is the hypothesis of the next statement. If *a*, then *b*. If *b*, then *c*, If *c*, then *d*.
If you live in New York City, then you live in New York State; If you live in New York State, then you live in the United States; If you live in the United States, then you live in North America.
b. The figure in the venn diagram is nested.

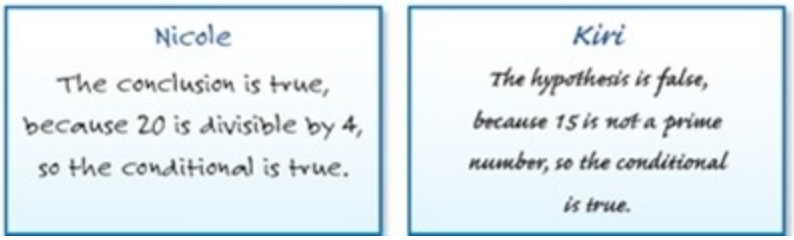
c. The hypothesis of the first statement is "If you live in New York City". The conclusion of the third statement is " you live in North America". Then the new conditional is "If you live in New York City, then you live in North America" . The new conditional statement is true.
d. If *a* is true, then *c* is true. If we know that *a* is true, then we know that *b* is true, and if we know that *b* is true, then we know that *c* is true. Therefore, when *a* is true, *c* is true.

ANSWER:

a. Sample answer: If you live in New York City, then you live in New York State; If you live in New York State, then you live in the United States; If you live in the United States, then you live in North America.
b.

c. If you live in New York City, then you live in North America; yes.
d. Sample answer: If a is true, then *c* is true. If we know that *a* is true, then we know that *b* is true, and if we know that *b* is true, then we know that *c* is true. Therefore, when a is true, *c* is true.

63. **CCSS CRITIQUE** Nicole and Kiri are evaluating the conditional *If 15 is a prime number, then 20 is divisible by 4*. Both think that the conditional is true, but their reasoning differs. Is either of them correct? Explain.



SOLUTION:

Kiri is correct. For the conditional statement "If 15 is a prime number, then 20 is divisible by 4." p is *15* is "a prime number", which is false and q is "20 is divisible by 4" is true. When the hypothesis of a conditional is false, the conditional is always true. Nicole, did not consider the truth value of the hypothesis.

ANSWER:

Sample answer: Kiri; when the hypothesis of a conditional is false, the conditional is always true.

64. **CHALLENGE** You have learned that statements with the same truth value are logically equivalent. Use logical equivalence to create a truth table that summarizes the conditional, converse, inverse, and contrapositive for the statements p and q.

SOLUTION:

The conditional statement is ($p \rightarrow q$). The converse is formed by exchanging the hypothesis and conclusion of the conditional ($q \rightarrow p$). The inverse is formed by negating both the hypothesis and conclusion of the conditional ($\sim p \rightarrow \sim q$). The contrapositive is formed by negating both the hypothesis and the conclusion of the converse of the conditional ($\sim q \rightarrow \sim p$).

p	q	Conditional $p \rightarrow q$	Converse $q \rightarrow p$	Inverse $\sim p \rightarrow \sim q$	Contrapositive $\sim q \rightarrow \sim p$
T	T	T	T	T	T
T	F	F	T	T	F
F	T	T	F	F	T
F	F	T	T	T	T

ANSWER:

p	q	Conditional $p \rightarrow q$	Converse $q \rightarrow p$	Inverse $\sim p \rightarrow \sim q$	Contrapositive $\sim q \rightarrow \sim p$
T	T	T	T	T	T
T	F	F	T	T	F
F	T	T	F	F	T
F	F	T	T	T	T

65. **REASONING** You are evaluating a conditional statement in which the hypothesis is true, but the conclusion is false. Is the inverse of the statement true or false? Explain your reasoning.

SOLUTION:

The inverse of a conditional statement in which the hypothesis is true is true. Since the conclusion is false, the converse of the statement must be true. The converse and inverse are logically equivalent, so the inverse is also true. Consider the following truth table.

		Conditional	Converse	Inverse
p	q	$p \rightarrow q$	$q \rightarrow p$	$\sim p \rightarrow \sim q$
T	F	F	T	T

ANSWER:

True; since the conclusion is false, the converse of the statement must be true. The converse and inverse are logically equivalent, so the inverse is also true.

66. **OPEN ENDED** Write a conditional statement in which the converse, inverse, and contrapositive are all true. Explain your reasoning.

SOLUTION:

If four is divisible by two, then birds have feathers. In order for the converse, inverse, and contrapositive to be true, the hypothesis and the conclusion must both be either true or false.

p = " four is divisible by two" and q = "birds have feathers"

p	q	$\sim p$	$\sim q$	Conditional $p \rightarrow q$	Converse $q \rightarrow p$	Inverse $\sim p \rightarrow \sim q$	Contrapositive $\sim q \rightarrow \sim p$
T	T	F	F	T	T	T	T
T	F	F	T	F	T	T	F
F	T	T	F	T	F	F	T
F	F	T	T	T	T	T	T

ANSWER:

Sample answer: If four is divisible by two, then birds have feathers. In order for the converse, inverse, and contrapositive to be true, the hypothesis and the conclusion must both be either true or false.

67. **CHALLENGE** The inverse of conditional A is given below. Write conditional A, its converse, and its contrapositve. Explain your reasoning.
If I received a detention, then I did not arrive at school on time.

SOLUTION:

The inverse is formed by negating both the hypothesis and conclusion of the conditional.
The hypothesis q of the inverse statement is *I received a detention*. The conclusion p of the inverse statement is *I did not arrive at school on time*.

So the conditional A is $p \rightarrow q$: If I did not arrive at school on time, then I received a detention.

The converse is formed by exchanging the hypothesis and conclusion of the conditional.
So the converse of statement A is $\sim p \rightarrow \sim q$: If I arrived at school on time, then I did not receive a detention.

The contrapositive is formed by negating both the hypothesis and the conclusion of the converse of the conditional.
The contrapositive of Statement A is $\sim q \rightarrow \sim p$: If I did not receive a detention, then I arrived at school on time.

ANSWER:

The hypothesis q of the inverse statement is *I received a detention*. The conclusion p of the inverse statement is *I did not arrive at school on time*. So the conditional A is $p \rightarrow q$: If I did not arrive at school on time, then I received a detention. So the converse of statement A is $\sim p \rightarrow \sim q$: If I arrived at school on time, then I did not receive a detention. The contrapositive of Statement A is $\sim q \rightarrow \sim p$: If I did not receive a detention, then I arrived at school on time.

68. **WRITING IN MATH** Describe the relationship between a conditional**,** its converse**,** its inverse, and its contrapositive.

SOLUTION:

The converse is formed by exchanging the hypothesis and conclusion of the conditional. The inverse is formed by negating both the hypothesis and conclusion of the conditional.The contrapositive is formed by negating both the hypothesis and the conclusion of the converse of the conditional.
Since they are logically equivalent, a conditional and its contrapositive always have the same truth value. The inverse and converse of a conditional are also logically equivalent and have the same truth value. The conditional and its contrapositive can have the same truth value as its inverse and converse, or it can have the opposite truth value of its inverse and converse.

ANSWER:

Sample answer: Since they are logically equivalent, a conditional and its contrapositive always have the same truth value. The inverse and converse of a conditional are also logically equivalent and have the same truth value. The conditional and its contrapositive can have the same truth value as its inverse and converse, or it can have the opposite truth value of its inverse and converse.

2-3 Conditional Statements

69. *If the sum of the measures of two angles is 90, then the angles are complementary angles.* Which of the following is the converse of the conditional above?

A If the angles are complementary angles**,** then the sum of the measures of two angles is 90.
B If the angles are not complementary angles**,** then the sum of the measures of the angles is 90.
C If the angles are complementary angles, then the sum of the measures of the angles is not 90.
D If the angles are not complementary angles**,** then the sum of the measures of two angles is not 90.

SOLUTION:

The converse is formed by exchanging the hypothesis and conclusion of the conditional.

A

ANSWER:

A

70. **ALGEBRA** What is $\frac{10a^2 - 15ab}{4a^2 - 9b^2}$ reduced to lowest terms?

F $\frac{5a}{2a - 2b}$
G $\frac{5a}{2a + 3b}$
H $\frac{a}{2a + 3b}$
J $\frac{a}{2a - 3b}$

SOLUTION:

$$\frac{10a^2 - 15ab}{4a^2 - 9b^2} = \frac{5a(2a - 3b)}{(2a)^2 - (3b)^2}$$

$$= \frac{5a(2a - 3b)}{(2a + 3b)(2a - 3b)}$$

$$= \frac{5a\ \cancel{(2a - 3b)}}{(2a + 3b)\ \cancel{(2a - 3b)}}$$

$$= \frac{5a}{2a + 3b}$$

So, the correct option is G.

ANSWER:

G

2-3 Conditional Statements

71. **SHORT RESPONSE** What is the standard notation for the following expression?

4.62×10^{-3}

SOLUTION:

Since the exponent is negative, move the decimal point to the left. The exponent determines the number of places to be moved. Since the exponent is 3, move the decimal point 3 places to the left.

$4.62 \times 10^{-3} = 0.00462$

ANSWER:

0.00462

72. **SAT/ACT** What is the greatest prime factor of 18 and 33?

A 1
B 2
C 3
D 5
E 11

SOLUTION:

$18 = 2 \cdot 3 \cdot 3$

$33 = 3 \cdot 11$

The greatest common prime factor of 18 and 33 is 3. So, the correct option is C.

ANSWER:

C

Construct a truth table for each compound statement.

73. p and q

SOLUTION:

p and q is a conjunction. A conjunction is true only when both statements that form it are true.

p	q	$p \wedge q$
T	T	T
T	F	F
F	T	F
F	F	F

ANSWER:

p	q	$p \wedge q$
T	T	T
T	F	F
F	T	F
F	F	F

2-3 Conditional Statements

74. p or $\sim q$

SOLUTION:

Negate q finding the opposite truth values. Then find the disjunction p or $\sim q$. A disjunction is true if at least one of the statements is true.

p	q	$\sim q$	$p \vee \sim q$
T	T	F	T
T	F	T	T
F	T	F	F
F	F	T	T

ANSWER:

p	q	$\sim q$	$p \vee \sim q$
T	T	F	T
T	F	T	T
F	T	F	F
F	F	T	T

75. $\sim p \wedge q$

SOLUTION:

Negate p, finding the opposite truth values. Then find the conjunction $\sim p \wedge q$. A conjunction is true only when both statements that form it are true.

p	q	$\sim p$	$\sim p \wedge q$
T	T	F	F
T	F	F	F
F	T	T	T
F	F	T	F

ANSWER:

p	q	$\sim p$	$\sim p \wedge q$
T	T	F	F
T	F	F	F
F	T	T	T
F	F	T	F

76. $\sim p \wedge \sim q$

SOLUTION:

Negate both p and q, finding the opposite truth values. Then find the conjunction $\sim p \wedge \sim q$. A conjunction is true only when both statements that form it are true.

p	q	$\sim p$	$\sim q$	$\sim p \wedge \sim q$
T	T	F	F	F
T	F	F	T	F
F	T	T	F	F
F	F	T	T	T

ANSWER:

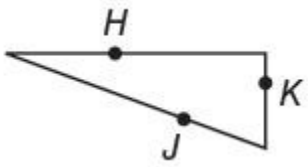
p	q	$\sim p$	$\sim q$	$\sim p \wedge \sim q$
T	T	F	F	F
T	F	F	T	F
F	T	T	F	F
F	F	T	T	T

Make a conjecture based on the given information. Draw a figure to illustrate your conjecture.

77. Points *H*, *J*, and *K* are each located on different sides of a triangle.

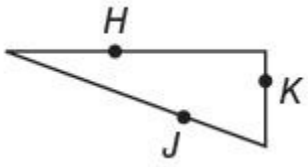
SOLUTION:

Since the points *H*, *J*, and *K* are each located on different sides of a triangle, they are not on the same line. Then *H*, *J*, and *K* are noncollinear.



ANSWER:

H, *J*, and *K* are noncollinear.

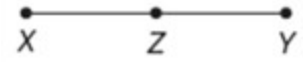


2-3 Conditional Statements

78. Collinear points X, Y, and Z; Z is between X and Y.

SOLUTION:

Since points X, Y, and Z are collinear, they are on the same line. Draw Z on the line between X and Y.



Then by segment addition $XZ + ZY = XY$.

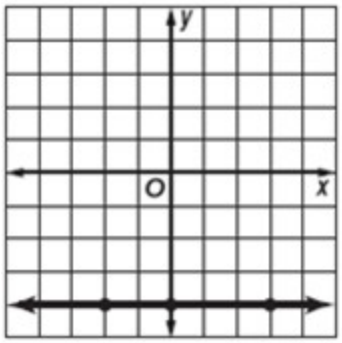
ANSWER:

$XZ + ZY = XY$

79. $R(3, -4)$, $S(-2, -4)$, and $T(0, -4)$

SOLUTION:

Graph points $R(3, -4)$, $S(-2, -4)$, and $T(0, -4)$ on a coordinate grid.
You can draw a line through the three points.



Thus, R, S, and T are collinear.

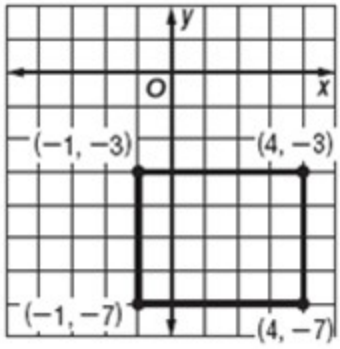
ANSWER:

R, S, and T are collinear.

80. $A(-1, -7)$, B(4, −7), C(4, −3), and D(−1, −3)

SOLUTION:

Plot the points $A(-1, -7)$, B(4, −7), C(4, −3), and D(−1, −3) on a coordinate grid. Connect the points. They form a quadrilateral with right angles and width of 5 units and length of 4.



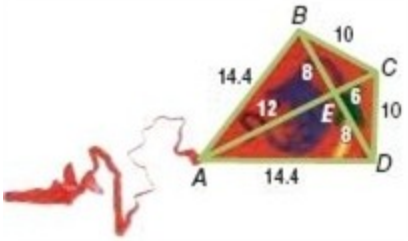
Thus, $ABCD$ is a rectangle.

ANSWER:

$ABCD$ is a rectangle.

2-3 Conditional Statements

81. **KITES** Kite making has become an art form. The kite shown is known as a diamond kite. The measures are in inches. Name all of the congruent segments in the figure.



SOLUTION:

Identify the segments with the same measures.

$\overline{BC} \cong \overline{CD}, \overline{BE} \cong \overline{ED}, \overline{BA} \cong \overline{DA}$

ANSWER:

$\overline{BC} \cong \overline{CD}, \overline{BE} \cong \overline{ED}, \overline{BA} \cong \overline{DA}$

Refer to the conversion charts.

Units of Length	
Customary → Metric	Metric → Customary
1 in. ≈ 2.5 cm	1 cm ≈ 0.4 in.
1 yd ≈ 0.9 m	1 m ≈ 1.1 yd
1 mi ≈ 1.6 km	1 km ≈ 0.6 mi

Units of Capacity	
Customary → Metric	Metric → Customary
1 qt ≈ 0.9 L	1 L ≈ 1.1 qt
1 pt ≈ 0.5 L	1 L ≈ 2.1 pt

Units of Weight/Mass	
Customary → Metric	Metric → Customary
1 oz ≈ 28.3 g	1 g ≈ 0.04 oz
1 lb ≈ 0.5 kg	1 kg ≈ 2.2 lb

82. **RUNNING** Ling is participating in a 5-kilometer charity run next weekend. About how many miles is the race?

SOLUTION:

There are approximately 0.62 miles in a kilometer.

$$5 \text{ km} \approx (5 \times 0.62) \text{ mi}$$
$$\approx 3 \text{ mi}$$

ANSWER:

about 3 mi

2-3 Conditional Statements

83. **NATURE** An African elephant weighs about 9 tons. About how many kilograms is this?

SOLUTION:

1 ton = 2000 lb
1 lb ≈ 0.5 kg
9 ton = 9 × 2000 lb
= 18000 lb
18000 lb ≈ 18000 × 0.5 kg
≈ 9000 kg

ANSWER:

about 9000 kg

84. **SPORTS** A football field is 120 yards long from one end zone to the other. How many feet long is a football field?

SOLUTION:

1 yard = 3 feet
So, 120 yards = 3(120) feet
= 360 ft

ANSWER:

360 ft

ALGEBRA Identify the operation used to change Equation (1) to Equation (2).

85. (**1**) $8(y - 11) = 32$
(**2**) $y - 11 = 4$

SOLUTION:

Divide each side by 8.

ANSWER:

Divide each side by 8.

86. (**1**) $x + 9 = 4 - 3x$
(**2**) $4x + 9 = 4$

SOLUTION:

Add $3x$ to each side.

ANSWER:

Add $3x$ to each side.

87. (**1**) $\frac{1}{3}m = 2$
(**2**) $m = 6$

SOLUTION:

Multiply each side by 3.

ANSWER:

Multiply each side by 3.