

## 2-4 Deductive Reasoning

**Determine whether each conclusion is based on *inductive* or *deductive* reasoning.**

1. Students at Olivia's high school must have a B average in order to participate in sports. Olivia has a B average, so she concludes that she can participate in sports at school.

**SOLUTION:**

The conclusion is based on a fact. So, it is deductive reasoning.

**ANSWER:**

deductive reasoning

2. Holly notices that every Saturday, her neighbor mows his lawn. Today is Saturday. Holly concludes her neighbor will mow his lawn.

**SOLUTION:**

Holly is basing her conclusion on a pattern of observations, so she is using inductive reasoning.

**ANSWER:**

inductive reasoning

**Determine whether the stated conclusion is valid based on the given information. If not, write invalid.**

**Explain your reasoning.**

3. **Given:** If a number is divisible by 4, then the number is divisible by 2. 12 is divisible by four.

**Conclusion:** 12 is divisible by 2.

**SOLUTION:**

If  $p \rightarrow q$  is a true statement and  $p$  is true, then  $q$ . Here, a number divisible 4 is divisible by 2 also and 12 is divisible by 4. So, 12 is divisible by 2 is a valid statement by the Law of Detachment.

**ANSWER:**

valid; Law of Detachment

4. **Given:** If Elan stays up late, he will be tired the next day. Elan is tired.

**Conclusion:** Elan stayed up late.

**SOLUTION:**

If  $p$  is true, then  $q$  is true, but  $q$  being true does not necessarily mean that  $p$  is true. Elan could be tired because he worked out. So, the statement is invalid.

**ANSWER:**

Invalid; Elan could be tired because he worked out.

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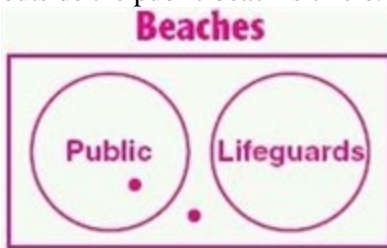
Determine whether the stated conclusion is valid based on the given information. If not, write invalid. Explain your reasoning using a Venn diagram.



5. **Given:** If a beach is public, then it does not have a lifeguard. Bayview does not have a lifeguard.  
**Conclusion:** Bayview is a public beach.

**SOLUTION:**

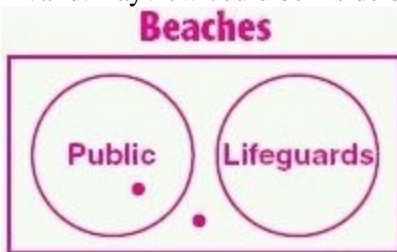
If  $p$  is true, then  $q$  is true, but  $q$  being true does not necessarily mean that  $p$  is true. Bayview could be inside or outside the public beach's circle.



So, the statement is invalid.

**ANSWER:**

Invalid: Bayview could be inside or outside the public beach's circle.



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6. **Given:** If students pass an entrance exam, they will be accepted into college. Latisha passed the entrance exam.  
**Conclusion:** Latisha will be accepted into college.

**SOLUTION:**

Latisha is inside the students who pass the entrance exam circle which puts her inside the students who are accepted into college circle, so Latisha will be accepted into college.



So, it is a valid statement.

**ANSWER:**

Valid; Latisha is inside the students who pass the entrance exam circle which puts her inside the students who are accepted into college circle, so Latisha will be accepted into college.



7. **MULTIPLE CHOICE** Determine which statement follows logically from the given statements.

- (1) If a triangle is a right triangle, then it has an angle that measures 90.  
(2) If a triangle has an angle that measures 90, then its acute angles are complementary.  
**A** If a triangle is not a right triangle, then it has an angle that measures 90.  
**B** If an angle of a triangle measures 90, then its acute angles are not complementary.  
**C** If a triangle is a right triangle, then its acute angles are complementary.  
**D** If a triangle has an angle that measures 90, then it is not a right triangle.

**SOLUTION:**

The statement (1) suggests that the triangle mentioned in statement (2) is a right triangle. So in a right triangle, acute angles are complementary. Therefore, the correct choice is C.

**ANSWER:**

C

## 2-4 Deductive Reasoning

**CCSS ARGUMENTS** Draw a valid conclusion from the given statements, if possible. Then state whether your conclusion was drawn using the Law of Detachment or the Law of Syllogism. If no valid conclusion can be drawn, write no valid conclusion and explain your reasoning.

8. **Given:** If Dalila finishes her chores, she will receive her allowance. If Dalila receives her allowance, she will buy a CD.



**SOLUTION:**

By the Law of Syllogism, if  $p \rightarrow q$  and  $q \rightarrow r$  are true statements, then  $p \rightarrow r$  is a true statement. If Dalila finishes her chores, she will buy a CD.

**ANSWER:**

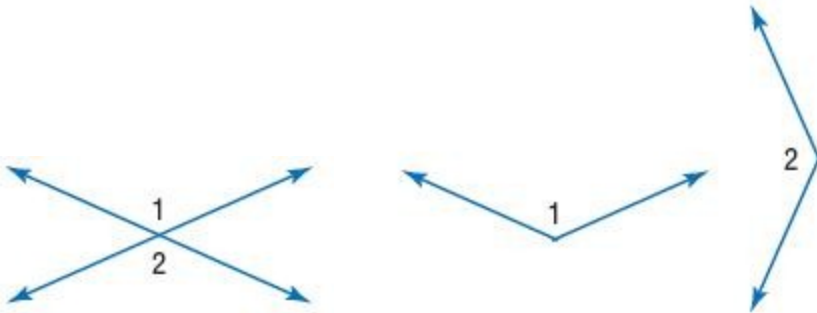
If Dalila finishes her chores, she will buy a CD; Law of Syllogism.

9. **Given:** Vertical angles are congruent.

$$\angle 1 \cong \angle 2$$

**SOLUTION:**

There is no valid conclusion.  $\angle 1$  and  $\angle 2$  do not have to be vertical in order to be congruent.



**ANSWER:**

No valid conclusion;  $\angle 1$  and  $\angle 2$  do not have to be vertical in order to be congruent.

**Determine whether each conclusion is based on inductive or deductive reasoning.**

10. At Fumio's school if you are late five times, you will receive a detention. Fumio has been late to school five times; therefore he will receive a detention.

**SOLUTION:**

The conclusion is based on a fact. So, it is deductive reasoning.

**ANSWER:**

deductive reasoning

## **2-4 Deductive Reasoning**

11. A dental assistant notices a patient has never been on time for an appointment. She concludes the patient will be late for her next appointment.

**SOLUTION:**

The dental assistant is basing her conclusion on a pattern of observations, so she is using inductive reasoning.

**ANSWER:**

inductive reasoning

12. A person must have a membership to work out at a gym. Jesse is working out at a gym. Jesse has a membership to the gym.

**SOLUTION:**

The conclusion is based on a fact. So, it is deductive reasoning.

**ANSWER:**

deductive reasoning

13. If Eduardo decides to go to a concert tonight, he will miss football practice. Tonight, Eduardo went to a concert. Eduardo missed football practice.

**SOLUTION:**

The conclusion is based on a fact. So, it is deductive reasoning.

**ANSWER:**

deductive reasoning

14. Every Wednesday Lucy's mother calls. Today is Wednesday, so Lucy concludes her mother will call.

**SOLUTION:**

Lucy is basing her conclusion on a pattern of observations, so she is using inductive reasoning.

**ANSWER:**

inductive reasoning

15. Whenever Juanita has attended a tutoring session she notices that her grades have improved. Juanita attends a tutoring session and she concludes her grades will improve.

**SOLUTION:**

Juanita is basing her conclusion on a pattern of observations, so she is using inductive reasoning.

**ANSWER:**

inductive reasoning

## 2-4 Deductive Reasoning

**CCSS CRITIQUE** Determine whether the stated conclusion is valid based on the given information. If not, write invalid. Explain your reasoning.

16. **Given:** Right angles are congruent.  $\angle 1$  and  $\angle 2$  are right angles.

**Conclusion:**  $\angle 1 \cong \angle 2$

**SOLUTION:**

If  $p \rightarrow q$  is a true statement and  $p$  is true, then  $q$ . Here, the statement “right angles are congruent” is a true statement and  $\angle 1$  and  $\angle 2$  are right angles. So,  $\angle 1 \cong \angle 2$  is a valid statement by the Law of Detachment.

**ANSWER:**

valid; Law of Detachment

17. **Given:** If a figure is a square, it has four right angles. Figure  $ABCD$  has four right angles.

**Conclusion:** Figure  $ABCD$  is a square.

**SOLUTION:**

If  $p$  is true, then  $q$  is true, but  $q$  being true does not necessarily mean that  $p$  is true. A figure with 4 right angles could be a rectangle. So, the statement is invalid.

**ANSWER:**

Invalid; the figure could be a rectangle.

18. **Given:** An angle bisector divides an angle into two congruent angles.

$\overline{KM}$  is an angle bisector of  $\angle JKL$ .

**Conclusion:**  $\angle JKM \cong \angle MKL$

**SOLUTION:**

If  $p \rightarrow q$  is a true statement and  $p$  is true, then  $q$ . Here, the statement “an angle bisector divides an angle into two congruent angles” is a true statement and  $\overline{KM}$  is an angle bisector of  $\angle JKL$ . So,  $\angle JKM \cong \angle MKL$  is a valid statement by the Law of Detachment.

**ANSWER:**

valid; Law of Detachment

19. **Given:** If you leave your lights on while your car is off, your battery will die. Your battery is dead.

**Conclusion:** You left your lights on while the car was off.

**SOLUTION:**

If  $p$  is true, then  $q$  is true, but  $q$  being true does not necessarily mean that  $p$  is true. The reason for the battery for being dead could be something else also, say it is too old. So, it is an invalid statement.

**ANSWER:**

Invalid; your battery could be dead because it was old.

## 2-4 Deductive Reasoning

20. **Given:** If Dante obtains a part-time job, he can afford a car payment. Dante can afford a car payment.

**Conclusion:** Dante obtained a part-time job.

**SOLUTION:**

If  $p$  is true, then  $q$  is true, but  $q$  being true does not necessarily mean that  $p$  is true.

The reason that he could afford the car payment could be something else also, say he paid off his other bills. So, it is an invalid statement.

**ANSWER:**

Invalid; Dante could afford a car payment because he paid off his other bills.

21. **Given:** If 75% of the prom tickets are sold, the prom will be conducted at the country club. 75% of the prom tickets were sold.

**Conclusion:** The prom will be held at the country club.

**SOLUTION:**

If  $p \rightarrow q$  is a true statement and  $p$  is true, then  $q$ . Here, the statement “if 75% of the prom tickets are sold, the prom will be conducted at the country club” is a true statement and 75% of the prom tickets were sold. So, the prom will be held at the country club is a valid statement by the Law of Detachment.

**ANSWER:**

valid; Law of Detachment

Game Ratings	
Rating	Age
EC	3 and older
E	6 and older
E10+	10 and older
T	13 and older
M	17 and older

22. **COMPUTER GAMES** Refer to the game ratings. Determine whether the stated conclusion is valid based on the given information. If not, write *invalid*. Explain your reasoning.

**Given:** If a title is rated E, then it has content that may be suitable for ages 6 and older. Cesar buys a computer game that he believes is suitable for his little sister, who is 7.

**Conclusion:** The game Cesar purchased has a rating of E.

**SOLUTION:**

If  $p$  is true, then  $q$  is true, but  $q$  being true does not necessarily mean that  $p$  is true.

The game Cesar purchased could be rated EC (Early Childhood), which is suitable for ages 3 and up. So, the conclusion is an invalid statement.

**ANSWER:**

Invalid; the game could be rated EC (Early Childhood), which is suitable for ages 3 and up.

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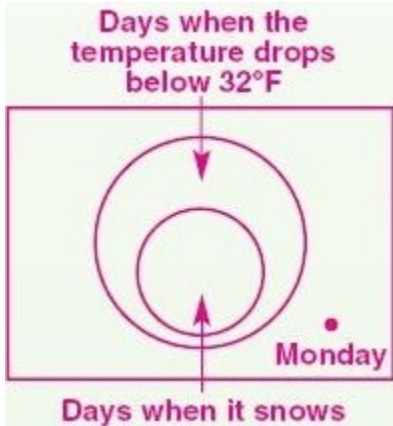
Determine whether the stated conclusion is valid based on the given information. If not, write *invalid*. Explain your reasoning using a Venn diagram.

23. **Given:** If the temperature drops below 32°F, it may snow. The temperature did not drop below 32°F on Monday.

**Conclusion:** It did not snow on Monday.

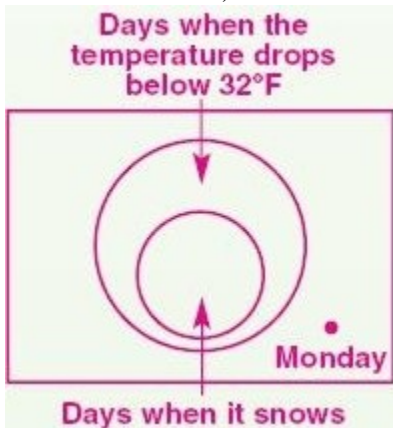
**SOLUTION:**

Monday is outside of the days when the temperature drops below 32°F, so it cannot be inside the days when it snows circle either, so the conclusion is valid.



**ANSWER:**

Valid; Monday is outside of the days when the temperature drops below 32°F, so it cannot be inside the days when it snows circle either, so the conclusion is valid.





## 2-4 Deductive Reasoning

24. **Given:** If a person is a Missouri resident, he or she does not live by a beach. Michelle does not live by the beach.  
**Conclusion:** Michelle is a Missouri resident.

**SOLUTION:**

If  $p$  is true, then  $q$  is true, but  $q$  being true does not necessarily mean that  $p$  is true.

Michelle could be inside the Missouri circle or inside the People who do not live by the beach circle and outside the Missouri Circle. So, the conclusion is invalid.



**ANSWER:**

Invalid; Michelle could be inside the Missouri circle or inside the People who do not live by the beach circle and outside the Missouri Circle.



## 2-4 Deductive Reasoning

25. **Given:** Some nurses wear blue uniforms. Sabrina is a nurse.

**Conclusion:** Sabrina wears a blue uniform.

**SOLUTION:**

If  $p$  is true, then  $q$  is true, but  $q$  being true does not necessarily mean that  $p$  is true.

Sabrina could be inside just the nurses' circle or inside the intersection of the circles, so the conclusion is invalid.



**ANSWER:**

Invalid; Sabrina could be inside just the nurses' circle or inside the intersection of the circles, so the conclusion is invalid.



## 2-4 Deductive Reasoning

26. **Given:** All vegetarians do not eat meat. Theo is a vegetarian.

**Conclusion:** Theo does not eat meat.

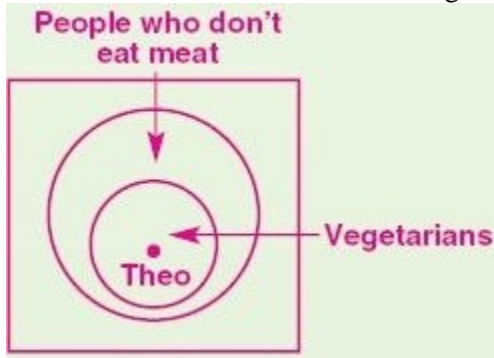
**SOLUTION:**

Theo is inside the small and large circle, so the conclusion is valid.



**ANSWER:**

Valid; Theo is inside the small and large circle, so the conclusion is valid.



## 2-4 Deductive Reasoning

27. **TRANSPORTATION** There are many types of vehicles and they are classified using different sets of criteria. Determine whether the stated conclusion is valid based on the given information. If not, write *invalid*. Explain your reasoning using a Venn diagram.

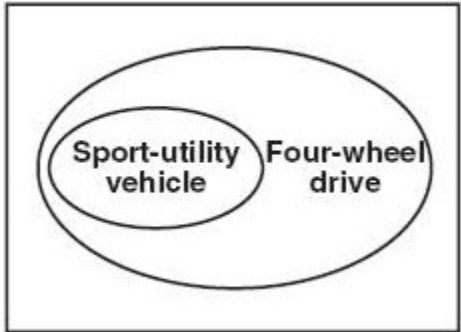
**Given:** If a vehicle is a sport-utility vehicle, then it is a four-wheel-drive car built on a truck chassis. Ms. Rodriguez has just purchased a vehicle that has four-wheel drive.

**Conclusion:** Ms. Rodriguez has just purchased a sport-utility vehicle.

**SOLUTION:**

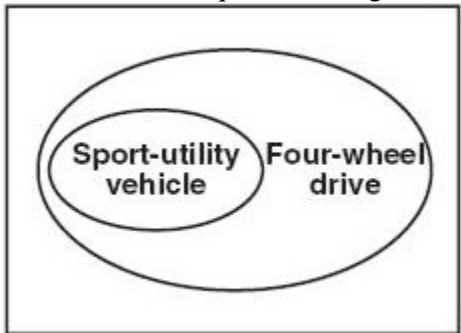
If  $p$  is true, then  $q$  is true, but  $q$  being true does not necessarily mean that  $p$  is true.

Ms. Rodriguez's car might be in the four-wheel-drive section of the diagram but not in the sport-utility vehicle section. So, the conclusion is invalid.



**ANSWER:**

Invalid; Ms. Rodriguez's car might be a four wheel drive vehicle that is not a sport-utility vehicle section.



28. **GOLF** Zach Johnson won the Masters Tournament in 2007. Use the Law of Syllogism to draw a valid conclusion from each set of statements, if possible. If no valid conclusion can be drawn, write *no valid conclusion* and explain your reasoning.

(1) If Zach Johnson's score is lower than the other golfers at the end of the tournament, then he wins the tournament.

(2) If a golfer wins the Masters Tournament, then he gets a green jacket.

**SOLUTION:**

The Law of Syllogism states that if  $p \rightarrow q$  and  $p \rightarrow r$  are true statements, then  $p \rightarrow r$  is a true statement.

Let  $p$  = "Zach Johnson's score is lower than the other golfers at the end of the tournament." Let  $q$  = "He wins the tournament."

Let  $r$  = "He gets a green jacket."

Then by the law of Syllogism "If Zach Johnson's score is lower than the other golfers at the end of the tournament, then he gets a green jacket."

**ANSWER:**

If Zach Johnson's score is lower than the other golfers at the end of the tournament, then he gets a green jacket.

## 2-4 Deductive Reasoning

**CCSS ARGUMENTS** Use the Law of Syllogism to draw a valid conclusion from each set of statements, if possible. If no valid conclusion can be drawn, write no valid conclusion and explain you're reasoning.

29. If you interview for a job, then you wear a suit.  
If you interview for a job, then you will update your resume.

**SOLUTION:**

The Law of Syllogism allows you to draw conclusions from two true conditional statements when the conclusion of one statement is the hypothesis of the other. The hypothesis of one statement is not the conclusion of the other statement, so no valid conclusion can be made from the statements.

**ANSWER:**

no valid conclusion

30. If Tina has a grade point average of 3.0 or greater, she will be on the honor roll.  
If Tina is on the honor roll, then she will have her name in the school paper.

**SOLUTION:**

The Law of Syllogism allows you to draw conclusions from two true conditional statements when the conclusion of one statement is the hypothesis of the other. The hypothesis of one statement *is* the conclusion of the other statement, so a valid conclusion can be made from the statements. If Tina has a grade point average of 3.0 or greater, then she will have her name in the school paper.

**ANSWER:**

If Tina has a grade point average of 3.0 or greater, then she will have her name in the school paper.

31. If two lines are perpendicular, then they intersect to form right angles.  
Lines  $r$  and  $s$  form right angles.

**SOLUTION:**

The Law of Syllogism allows you to draw conclusions from two true conditional statements when the conclusion of one statement is the hypothesis of the other. The hypothesis of one statement is not the conclusion of the other statement, so no valid conclusion can be made from the statements.

**ANSWER:**

no valid conclusion

32. If the measure of an angle is between 90 and 180, then it is obtuse.  
If an angle is obtuse, then it is not acute.

**SOLUTION:**

The Law of Syllogism allows you to draw conclusions from two true conditional statements when the conclusion of one statement is the hypothesis of the other. The hypothesis of one statement *is* the conclusion of the other statement, so a valid conclusion can be made from the statements.

If the measure of an angle is between 90 and 180, then it is not acute.

**ANSWER:**

If the measure of an angle is between 90 and 180, then it is not acute.

## 2-4 Deductive Reasoning

33. If two lines in a plane are not parallel, then they intersect.  
If two lines intersect, then they intersect in a point.

**SOLUTION:**

The Law of Syllogism allows you to draw conclusions from two true conditional statements when the conclusion of one statement is the hypothesis of the other. The hypothesis of one statement *is* the conclusion of the other statement, so a valid conclusion can be made from the statements.  
If two lines are not parallel, then they intersect in a point.

**ANSWER:**

If two lines in a plane are not parallel, then they intersect in a point.

34. If a number ends in 0, then it is divisible by 2.  
If a number ends in 4, then it is divisible by 2.

**SOLUTION:**

The Law of Syllogism allows you to draw conclusions from two true conditional statements when the conclusion of one statement is the hypothesis of the other. The hypothesis of one statement is not the conclusion of the other statement, so no valid conclusion can be made from the statements.

**ANSWER:**

no valid conclusion

**Draw a valid conclusion from the given statements, if possible. Then state whether your conclusion was drawn using the Law of Detachment or the Law of Syllogism. If no valid conclusion can be drawn, write *no valid conclusion* and explain your reasoning.**

35. **Given:** If a figure is a square, then all the sides are congruent.  
Figure  $ABCD$  is a square.

**SOLUTION:**

By the Law of Detachment if  $p \rightarrow q$  is a true statement and  $p$  is true, then  $q$ . Here, the statement “if a figure is a square, then all the sides are congruent” is a true statement and figure  $ABCD$  is a square. So, the figure  $ABCD$  has all sides congruent.

**ANSWER:**

Figure  $ABCD$  has all sides congruent; Law of Detachment.

36. **Given:** If two angles are complementary, the sum of the measures of the angles is 90.  
 $\angle 1$  and  $\angle 2$  are complements of each other.

**SOLUTION:**

By the Law of Detachment, if  $p \rightarrow q$  is a true statement and  $p$  is true, then  $q$ . Here, the statement “if two angles are complementary, the sum of the measures of the angles is 90” is a true statement and  $\angle 1$  and  $\angle 2$  are complements of each other. So, the sum of the measures of  $\angle 1$  and  $\angle 2$  is 90.

**ANSWER:**

The sum of the measures of  $\angle 1$  and  $\angle 2$  is 90; Law of Detachment.

## 2-4 Deductive Reasoning

37. **Given:** Ballet dancers like classical music.

If you like classical music, then you enjoy the opera.

**SOLUTION:**

By the Law of Syllogism, If  $p \rightarrow q$  and  $q \rightarrow r$  are true statements, then  $p \rightarrow r$  is a true statement. So, if you are a ballet dancer, then you enjoy the opera.

**ANSWER:**

If you are a ballet dancer, then you enjoy the opera; Law of Syllogism.

38. **Given:** If you are athletic, then you enjoy sports.

If you are competitive, then you enjoy sports.

**SOLUTION:**

Both of the given statements are true conditionals with the same conclusion so the Law of Detachment does not apply. Since the conclusion of statement (1) is not the hypothesis of statement (2) the Law of Syllogism does not apply. There is no valid conclusion.

**ANSWER:**

No valid conclusion; the conclusion of statement (1) is not the hypothesis of statement (2) so the Law of Syllogism does not apply.

39. **Given:** If a polygon is regular, then all of its sides are congruent.

All sides of polygon  $WXYZ$  are congruent.

**SOLUTION:**

The Law of Detachment states that if  $p \rightarrow q$  is a true statement and  $p$  is true, then  $q$  is true.

From the first statement,  $p$  is a polygon is regular and  $q$  is all of its sides are congruent. The second statement give  $q$  is true. To apply the Law of Detachment, we need to have  $p$  is true to show  $q$  is true. Thus, knowing a conclusion is true does not imply the hypothesis will be true. So, no valid conclusion can be made.

**ANSWER:**

No valid conclusion; knowing a conclusion is true does not imply the hypothesis will be true.

40. **Given:** If Bob completes a course with a grade of C, then he will not receive credit.

If Bob does not receive credit, he will have to take the course again.

**SOLUTION:**

By the Law of Syllogism, if  $p \rightarrow q$  and  $q \rightarrow r$  are true statements, then  $p \rightarrow r$  is a true statement. So, Bob completes a course with a grade of C, then he will have to take the course again.

**ANSWER:**

If Bob completes a course with a grade of C, then he will have to take the course again; Law of Syllogism.

41. **DATA ANALYSIS** The table shows the number of at bats and hits for some of the members of the Florida Marlins in a recent season.

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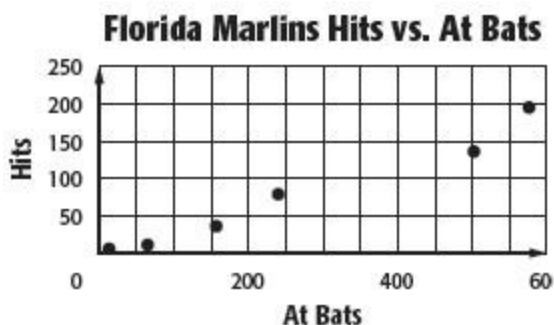
At Bats	Hits
13	6
576	195
240	79
502	139
157	36
64	11

Source: ESPN

- Construct a scatter plot to represent the data.
- Predict the number of hits a player with 300 at bats would get. Identify and explain your reasoning.
- Did the player with 157 at bats or the player with 240 at bats get more hits? What type of reasoning did you use? Explain.

**SOLUTION:**

- Plot “At bats” on the  $x$ -axis and “Hits” on the  $y$ -axis. Plot the points.



- Find the ratio of the hits to the at bats for each pair.

At Bats	Hits	Ratio
13	6	$\frac{6}{13} \approx 0.46$
576	195	$\frac{195}{576} \approx 0.34$
240	79	$\frac{79}{240} \approx 0.33$
502	139	$\frac{139}{502} \approx 0.28$
157	36	$\frac{36}{157} \approx 0.23$
64	11	$\frac{11}{64} \approx 0.17$

The average of the ratios is about 0.30.

That is, about 0.30 times the at bat will be a hit.

So, a player with 300 at bats will get about 91 hits.

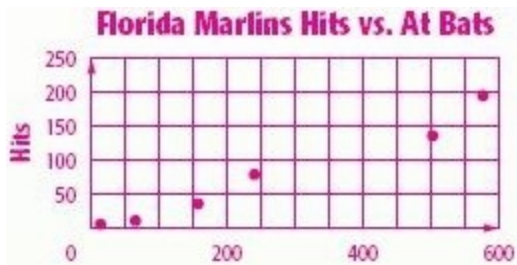
- The player with 240 at bats got more hits than the one with 157 at bats. Here, the facts provided in the table were used to reach the conclusion. So, it is deductive reasoning.

**ANSWER:**

-



## 2-4 Deductive Reasoning



b. Sample answer: About 91; inductive; a pattern was used to reach the conclusion.

c. Sample answer: The player with 240 at bats got more hits; deductive; the facts provided in the table were used to reach the conclusion.

42. **WRITING IN MATH** Explain why the Law of Syllogism cannot be used to draw a conclusion from these conditionals.

*If you wear winter gloves, then you will have warm hands.*

*If you do not have warm hands, then your gloves are too thin.*

**SOLUTION:**

The Law of Syllogism states that if  $p \rightarrow q$  and  $p \rightarrow r$  are true statements, then  $p \rightarrow r$  is a true statement.

From the first statement,  $p$  is "you wear winter gloves" and  $q$  is "you will have warm hands."

From the second statement,  $q$  is "you do not have warm hands". This is  $\sim q$  of the first statement. From the second statement,  $r$  is "your gloves are too thin."

The Law of Syllogism cannot be used, because the hypothesis of the second conditional is the negation of the conclusion of the first conditional. In order to use the Law of Syllogism, the conclusion of one conditional must be the hypothesis of the second conditional.

**ANSWER:**

Sample answer: The Law of Syllogism cannot be used, because the hypothesis of the second conditional is the negation of the conclusion of the first conditional. In order to use the Law of Syllogism, the conclusion of one conditional must be the hypothesis of the second conditional.

43. **CHALLENGE** Use the symbols from Lesson 2-2 for conjunction and disjunction, and the symbol for implies from Lesson 2-3 to represent the Law of Detachment and the Law of Syllogism symbolically. Let  $p$  represent the hypothesis, and let  $q$  represent the conclusion.

**SOLUTION:**

By the Law of Detachment if  $p \rightarrow q$  is a true statement and  $p$  is true, then  $q$ . The symbol  $\rightarrow$  stands for "implies" and the symbol  $\wedge$  stands for "and". That is, the law can be written as  $[(p \rightarrow q) \wedge p] \rightarrow q$ .

By the Law of Syllogism, if  $p \rightarrow q$  and  $q \rightarrow r$  are true statements, then  $p \rightarrow r$  is a true statement. This can be written as,

$$[(p \rightarrow q) \wedge (q \rightarrow r)] \rightarrow (p \rightarrow r).$$

**ANSWER:**

Law of Detachment:  $[(p \rightarrow q) \wedge p] \rightarrow q$ .

Law of Syllogism:  $[(p \rightarrow q) \wedge (q \rightarrow r)] \rightarrow (p \rightarrow r)$ .

## 2-4 Deductive Reasoning

44. **OPEN ENDED** Write a pair of statements in which the Law of Syllogism can be used to reach a valid conclusion. Specify the conclusion that can be reached.

**SOLUTION:**

The Law of Syllogism states that if  $p \rightarrow q$  and  $q \rightarrow r$  are true statements, then  $p \rightarrow r$  is a true statement.

For statement (1) let  $p$  is "a student earns 40 credits" and  $q$  is "he/she will graduate from high school."

For statement (2) let  $q$  is "a student graduates from high school" and  $r$  is "he or she will receive a diploma."

(1) If a student earns 40 credits, then he/she will graduate from high school.

(2) If a student graduates from high school, then he or she will receive a diploma.

Conclusion: If a student earns 40 credits, he or she will receive a diploma.

**ANSWER:**

Sample answer: (1) If a student earns 40 credits, then he/she will graduate from high school. (2) If a student graduates from high school, then he or she will receive a diploma. Conclusion: If a student earns 40 credits, he or she will receive a diploma.

45. **CCSS REASONING** Students in Mr. Kendrick's class are divided into two groups for an activity. Students in group A must always tell the truth. Students in group B must always lie. Jonah and Janeka are in Mr. Kendrick's class. When asked if he and Janeka are in group A or B, Jonah says, "We are both in Group B." To which group does each student belong? Explain your reasoning.

**SOLUTION:**

Jonah statement can be restated as, "Jonah is in group B and Janeka is in group B."

In order for this compound statement to be true, both parts of the statement must be true. If Jonah was in group A, he would not be able to say that he is in group B, since students in group A must always tell the truth. Therefore the statement that Jonah is in group B is true.

Since students in Group B must always lie, the compound statement must be false. For the compound statement to be false, the statement that Janeka is in group B must be false. Therefore, Jonah is in group B and Janeka is in group A.

**ANSWER:**

Jonah statement can be restated as, "Jonah is in group B and Janeka is in group B." In order for this compound statement to be true, both parts of the statement must be true. If Jonah was in group A, he would not be able to say that he is in group B, since students in group A must always tell the truth. Therefore the statement that Jonah is in group B is true. For the compound statement to be false, the statement that Janeka is in group B must be false. Therefore, Jonah is in group B and Janeka is in group A.

## 2-4 Deductive Reasoning

46. **WRITING IN MATH** Compare and contrast inductive and deductive reasoning when making conclusions and proving conjectures.

**SOLUTION:**

Sample answer: Inductive reasoning uses several specific examples to reach a conclusion, while deductive reasoning relies on established facts, rules, definitions, and/or properties to reach a conclusion. One counterexample is enough to disprove a conjecture reached using inductive or deductive reasoning. Deductive reasoning, however, is the only valid method of proving a conjecture. Inductive reasoning cannot be used to prove a conjecture.

**ANSWER:**

Sample answer: Inductive reasoning uses several specific examples to reach a conclusion, while deductive reasoning relies on established facts, rules, definitions, and/or properties to reach a conclusion. One counterexample is enough to disprove a conjecture reached using inductive or deductive reasoning. Deductive reasoning, however, is the only valid method of proving a conjecture. Inductive reasoning cannot be used to prove a conjecture.

47. Determine which statement follows logically from the given statements.

*If you order two burritos, then you also get nachos. Michael ordered two burritos.*

- A Michael ordered one burrito.
- B Michael will order two burritos.
- C Michael ordered nachos.
- D Michael got nachos.

**SOLUTION:**

The Law of Detachment states that if  $p \rightarrow q$  is a true statement and  $p$  is true, then  $q$ . Here, “if you order two burritos, then you also get nachos” is a true statement, and Michael ordered two burritos. So, by the Law of Detachment, Michael will get nachos. Therefore, the correct choice is D.

**ANSWER:**

D

## 2-4 Deductive Reasoning

48. **ALGEBRA** Solve for  $x$ :  $4(x + 2) = x - 1$

**F** -3

**G** -5

**H** -6

**J** -8

**SOLUTION:**

$$4(x + 2) = x - 1$$

$$4(x) + 4(2) = x - 1$$

$$4x + 8 = x - 1$$

$$4x - x = -1 - 8$$

$$3x = -9$$

$$\frac{3x}{3} = \frac{-9}{3}$$

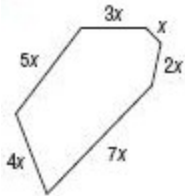
$$x = -3$$

Therefore, the correct choice is F.

**ANSWER:**

F

49. **SHORT RESPONSE** If the perimeter of the figure shown is 52 units, what is the value of  $x$ ?



**SOLUTION:**

The perimeter is the sum of the lengths of the sides. So,

$$x + 2x + 7x + 4x + 5x + 3x = 52.$$

$$22x = 52$$

Divide each side by 22.

$$\begin{aligned} x &= \frac{52}{22} \\ &= \frac{26}{11} \end{aligned}$$

**ANSWER:**

$$\frac{26}{11}$$

## **2-4 Deductive Reasoning**

50. **SAT/ACT** If 30% of  $x$  is 50, then 60% of  $x$  is

- A** 300
- B** 250
- C** 175
- D** 150
- E** 100

**SOLUTION:**

Then 60% of  $x$  will be double the amount that is 30% of  $x$ . Given 30% of  $x$  is 50, it follows that 60% of  $x$  is 100. Therefore, the correct answer is E.

**ANSWER:**

E

51. **TIME** All states in the United States observe daylight savings time except for Arizona and Hawaii.

- a.** Write a true conditional statement in if-then form for daylight savings time.
- b.** Write the converse of the true conditional statement. State whether the statement is true or false. If false, find a counterexample.

**SOLUTION:**

**a.** To write these statements in if-then form, identify the hypothesis and conclusion. The word if is not part of the hypothesis. The word then is not part of the conclusion.

The converse is formed by exchanging the hypothesis and conclusion of the conditional.

If you live in Hawaii or Arizona then you do not observe Daylight Savings Time.

**b.** If you do not observe Daylight Savings Time, then you live in Hawaii or Arizona.

Since all other states in the United States observe daylight savings time the converse is true.

**ANSWER:**

**a.** If you live in Hawaii or Arizona then you do not observe Daylight Savings Time.

**b.** If you do not observe Daylight Savings Time, then you live in Hawaii or Arizona; true.

## 2-4 Deductive Reasoning

Construct a truth table for each compound statement.

52.  $a$  and  $b$

**SOLUTION:**

Find the conjunction of  $a$  and  $b$ . A conjunction is true only when both statements that form it are true.

$a$	$b$	$a$ and $b$
T	T	T
T	F	F
F	T	F
F	F	F

**ANSWER:**

$a$	$b$	$a$ and $b$
T	T	T
T	F	F
F	T	F
F	F	F

53.  $\sim p$  or  $\sim q$

**SOLUTION:**

$\sim p$  and  $\sim q$  is the negation of  $p$  and  $q$  or the opposite truth value. Then find the disjunction  $\sim p$  or  $\sim q$ . A disjunction is true if at least one of the statements is true.

$p$	$\sim p$	$q$	$\sim q$	$\sim p$ or $\sim q$
T	F	T	F	F
T	F	F	T	T
F	T	T	F	T
F	T	F	T	T

**ANSWER:**

$p$	$\sim p$	$q$	$\sim q$	$\sim p$ or $\sim q$
T	F	T	F	F
T	F	F	T	T
F	T	T	F	T
F	T	F	T	T

## 2-4 Deductive Reasoning

54.  $k$  and  $\sim m$

SOLUTION:

$k$	$m$	$\sim m$	$k$ and $\sim m$
T	T	F	F
T	F	T	T
F	T	F	F
F	F	T	F

ANSWER:

$k$	$m$	$\sim m$	$k$ and $\sim m$
T	T	F	F
T	F	T	T
F	T	F	F
F	F	T	F

55.  $\sim y$  or  $z$

SOLUTION:

$\sim y$  is the negation of  $y$  or the opposite truth value. Then find the disjunction  $\sim y$  or  $z$ . A disjunction is true if at least one of the statements is true or it will only be false if both statements are false.

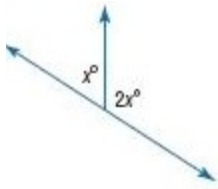
$y$	$\sim y$	$z$	$\sim y$ or $z$
T	F	T	T
T	F	F	F
F	T	T	T
F	T	F	T

ANSWER:

$y$	$\sim y$	$z$	$\sim y$ or $z$
T	F	T	T
T	F	F	F
F	T	T	T
F	T	F	T

## 2-4 Deductive Reasoning

**ALGEBRA Find  $x$ .**



56.

**SOLUTION:**

The two angles form a linear pair. So, their sum is  $180^\circ$ .

$$x^\circ + 2x^\circ \cong 180^\circ \quad \text{Definition of Linear pair}$$

$$x + 2x = 180 \quad \text{Definition of congruent angles}$$

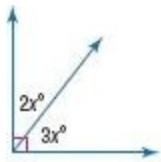
$$3x = 180 \quad \text{Simplify.}$$

$$\frac{3x}{3} = \frac{180}{3} \quad \text{Divide each side by 3}$$

$$x = 60 \quad \text{Simplify.}$$

**ANSWER:**

60



57.

**SOLUTION:**

The two angles are complementary. So, their sum is  $90^\circ$ .

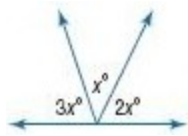
$$2x^\circ + 3x^\circ = 90^\circ.$$

$$5x = 90$$

$$x = 18$$

**ANSWER:**

18



58.

**SOLUTION:**

The three angles form a linear pair. So, their sum is  $180^\circ$ .

$$3x^\circ + x^\circ + 2x^\circ = 180^\circ.$$

$$6x = 180$$

$$x = 30$$

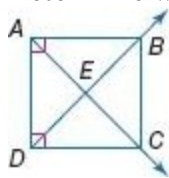
**ANSWER:**

30



## 2-4 Deductive Reasoning

Determine whether each statement can be assumed from the figure. Explain.



59.  $\angle DAB$  is a right angle.

**SOLUTION:**

From the figure,  $\angle DAB$  is a right angle. The right angle symbol at vertex A denotes that  $\angle DAB$  is a right angle.

**ANSWER:**

Yes; the symbol denotes that  $\angle DAB$  is a right angle.

60.  $\angle AEB \cong \angle DEC$

**SOLUTION:**

The angles  $AEB$  and  $DEC$  are vertical angles. So, they are congruent. Therefore,  $\angle AEB \cong \angle DEC$ .

**ANSWER:**

Yes; they are vertical angles.

61.  $\angle ADB$  and  $\angle BDC$  are complementary.

**SOLUTION:**

To be complementary, the angles must sum to 90.

From the figure,  $\angle ADB + \angle BDC = \angle ADC$  and angle  $m\angle ADC$  is a right angle.

Thus, the sum of the measures of  $\angle ADB$  &  $\angle BDC$  is  $m\angle ADC$ , which is 90.

**ANSWER:**

Yes; the sum of their measures is  $m\angle ADC$ , which is 90.

62.  $\angle DAE \cong \angle ADE$

**SOLUTION:**

From the figure,  $\angle DAB \cong \angle CDA$ , since they are both right angles. However, there is no indication of the measures of  $\angle DAE$  &  $\angle ADE$ . So, we cannot say whether they are congruent.

**ANSWER:**

No; there is no indication of the measures of these angles.

63.  $AB \perp BC$

**SOLUTION:**

To be perpendicular, we need to determine that the intersection  $\overline{AB}$  &  $\overline{BC}$  form a right angle at  $\angle ABC$ . However, we do not know the measure of  $\angle ABC$ . So, we cannot say that  $AB \perp BC$ .

**ANSWER:**

No; we do not know the  $m\angle ABC$ .

## **2-4 Deductive Reasoning**

64.  $\angle AEB$  and  $\angle BEC$  are supplementary

**SOLUTION:**

To be supplementary, the measures of  $\angle AEB$  and  $\angle BEC$  must equal 180.

Since  $\angle AEB$  and  $\angle BEC$  are a linear pair, so they are supplementary.

**ANSWER:**

Yes; they are a linear pair, so they are supplementary.