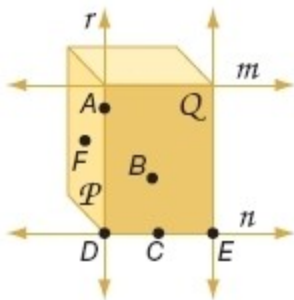## 2-5 Postulates and Paragraph Proofs

Explain how the figure illustrates that each statement is true. Then state the postulate that can be used to show each statement is true.

1. Planes *P* and *Q* intersect in line *r*.

SOLUTION:

Identify planes *P* and *Q* and locate their intersection.
The left side and front side have a common edge line *r*. Planes *P* and *Q* only intersect along line *r*.
Postulate 2.7, which states that if two planes intersect, then their intersection is a line.

ANSWER:

The left side and front side have a common edge line *r*. Planes *P* and *Q* only intersect along line *r*.
Postulate 2.7, which states that if two planes intersect, then their intersection is a line.

2. Lines *r* and *n* intersect at point *D*.

SOLUTION:

Identify lines *r* and *n* and locate their intersection.
The edges of the figure form intersecting lines. Lines *r* and *n* intersect at only one place, point *D*. Postulate 2.6, which states if two lines intersect, their intersection is exactly one point.

ANSWER:

The edges of the figure form intersecting lines. Lines *r* and *n* intersect at only one place, point D. Postulate 2.6, which states if two lines intersect, their intersection is exactly one point.

3. Line *n* contains points *C, D,* and *E*.

SOLUTION:

Identify line *n* and locate the points on it.
The front bottom edge of the figure is line *n* which contains points *D, C,* and *E*. Postulate 2.3, which states a line contains at least two points.

ANSWER:

The front bottom edge of the figure is line *n* which contains points *D, C,* and *E*. Postulate 2.3, which states a line contains at least two points.

4. Plane *P* contains the points *A, F,* and *D*.

   SOLUTION:

   Identify Plane *P* and locate the points on it..
   The left side of the figure or plane *P* contains points *A, F,* and *D*. Postulate 2.4, which states a plane, contains at least three noncollinear points.

   ANSWER:

   The left side of the figure or plane *P* contains points *A, F,* and *D*. Postulate 2.4, which states a plane, contains at least three noncollinear points.

5. Line *n* lies in plane *Q*.

   SOLUTION:

   Identify plane *Q* and locate line *n* .
   Points *D* and *E,* which are on line *n,* lie in plane *Q*. Postulate 2.5, which states that if two points lie in a plane, then the entire line containing those points lies in that plane.

   ANSWER:

   Points *D* and *E,* which are on line *n,* lie in plane *Q*. Postulate 2.5, which states that if two points lie in a plane, then the entire line containing those points lies in that plane.

6. Line *r* is the only line through points *A* and *D*.

   SOLUTION:

   Identify line *r* and the point on it.
   Line *r* contains points *A* and *D*. Postulate 2.1, which states there is exactly one line through two points.

   ANSWER:

   Line *r* contains points *A* and *D*. Postulate 2.1, which states there is exactly one line through two points.

2-5 Postulates and Paragraph Proofs

Determine whether each statement is *always, sometimes,* or *never* true. Explain your reasoning.

7. The intersection of three planes is a line.

SOLUTION:

If three planes intersect, then their intersection may be a line or a point. Postulate 2.7 states that two planes intersect, then their intersection is a line. Therefore, the statement is *sometimes* true.
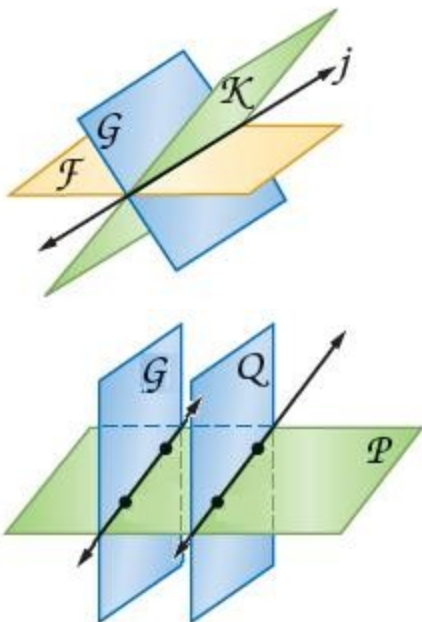
ANSWER:

Sometimes; if three planes intersect, then their intersection may be a line or a point.

8. Line *r* contains only point *P*.

SOLUTION:

The postulate 2.3 states that a line contains at least two points. Therefore, line *r* must include at least one point besides point *P,* and the statement that the line contains *only* point *P* is *never* true.
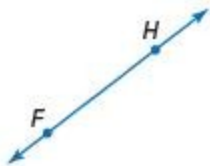
ANSWER:

Never; Postulate 2.3 states that a line contains at least two points.

9. Through two points, there is exactly one line.

SOLUTION:

Postulate 2.1 states that through any two points, there is exactly one line. Therefore, the statement is *always* true.
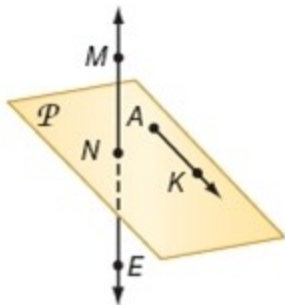

ANSWER:

Always; Postulate 2.1 states that through any two points, there is exactly one line.

In the figure, $\overleftrightarrow{AK}$ is in plane P and M is on $\overleftrightarrow{NE}$. State the postulate that can be used to show each statement is true.

10. *M, K,* and *N* are coplanar.

SOLUTION:

M, K, and N are all points and they are not collinear.
Postulate 2.2 states that through any three noncollinear points, there is exactly one plane. So, there exist a plane through the points *M, K,* and *N*. So, *M, K,* and *N* are coplanar.

ANSWER:

Postulate 2.2; through any three noncollinear points, there is exactly one plane.

11. $\overleftrightarrow{NE}$ contains points *N* and *M*.

SOLUTION:

It is stated in that *M* is on $\overleftrightarrow{NE}$. *N* is a part of the name of $\overleftrightarrow{NE}$, so *N* must also be on $\overleftrightarrow{NE}$.

Postulate 2.3 states that a line contains at least two points. Here, *N* and *M* are on the line $\overleftrightarrow{NE}$. Therefore, $\overleftrightarrow{NE}$ contains the points *N* and *M*.

ANSWER:

Postulate 2.3; a line contains at least two points.

12. *N* and *K* are collinear.

SOLUTION:

N and *K* are two points in the figure. No other relevant information is provided.

Postulate 2.1 states that through any two points, there is exactly one line. So, we can draw a line through the points *N* and *K*. So, they are collinear.

ANSWER:

Postulate 2.1; through any two points, there is exactly one line.

13. Points *N, K,* and *A* are coplanar.

    SOLUTION:

    N, *K*, and *A* are three points in the figure. We do not know for sure that *N* is on plane *P*. No other relevant information is provided.

    Postulate 2.4 states that a plane contains at least three non-collinear points. Here, the points *N, K,* and *A* are on a plane, most likely plane *P*. So, they are coplanar.

    ANSWER:

    Postulate 2.4; a plane contains at least three non-collinear points.

14. **SPORTS** Each year, Jennifer's school hosts a student vs. teacher basketball tournament to raise money for charity. This year, there are eight teams participating in the tournament. During the first round, each team plays all of the other teams.

    a. How many games will be played in the first round?
    b. Draw a diagram to model the number of first round games. Which postulate can be used to justify your diagram?
    c. Find a numerical method that you could use regardless of the number of the teams in the tournament to calculate the number of games in the first round.

    SOLUTION:

    a. The first team will play with the other 7 teams. Then the second team will play with the 6 other teams, as the game between the first and the second team has already been counted. Similarly, the third team will play with 5 other teams, and so on. So, the total number of games will be $7 + 6 + 5 + 4 + 3 + 2 + 1 = 28$. So, in the first round there will be 28 games.
    b. Postulate 2.1 states that through any two points, there is exactly one line. Plot 8 points and draw lines joining any two points.
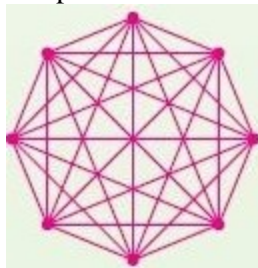
    c. If there are 8 teams in the tournament, the number of games in the first round is $(8 - 1) + (8 - 2) + \ldots + 1$.
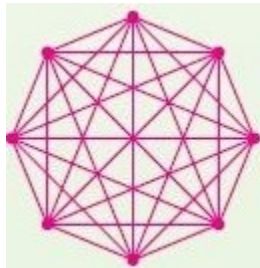
Therefore, if there are *n* teams in the tournament, the number of games in the first round is $(n-1) + (n-2) + \ldots + 1$.
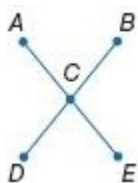
ANSWER:

a. 28 games

b.

Postulate 2.1

c. If there are *n* teams in the tournament, the number of games in the first round is $(n-1) + (n-2) + \ldots + 1$.

15. **CCSS ARGUMENTS** In the figure, $\overline{AE} \cong \overline{DB}$ and *C* is the midpoint of $\overline{AE}$ and $\overline{DB}$. Write a paragraph proof to show that $AC = CB$.

SOLUTION:

You are given a midpoint and a pair of congruent segments, *AE* and *DB*. Use your knowledge of midpoints and congruent segments to obtain information about *AC* and *CB*, the segments that you are trying to prove congruent.

Since *C* is the midpoint of $\overline{AE}$ and $\overline{DB}$, $CA = CE = \dfrac{1}{2}AE$ and $CD = CB = \dfrac{1}{2}DB$ by the definition of midpoint. We are given $\overline{AE} \cong \overline{DB}$ so $AE = DB$ by the definition of congruent segments.

By the multiplication property, $\dfrac{1}{2}AE = \dfrac{1}{2}DB$. So, by substitution, $AC = CB$.

ANSWER:

Since *C* is the midpoint of $\overline{AE}$ and $\overline{DB}$, $CA = CE = \dfrac{1}{2}AE$ and $CD = CB = \dfrac{1}{2}D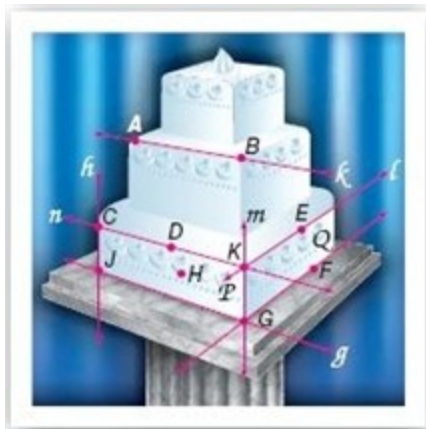B$ by the definition of midpoint. We are given $\overline{AE} \cong \overline{DB}$ so $AE = DB$ by the definition of congruent segments.

By the multiplication property, $\dfrac{1}{2}AE = \dfrac{1}{2}DB$. So, by substitution,

$AC = CB$.

**CAKES Explain how the picture illustrates that each statement is true. Then state the postulate that can be used to show each
statement is true.**

16. Lines n and ℓ intersect at point K.

SOLUTION:

Identify lines n and ℓ and locate the point at the intersection.
The top edges of the bottom layer form intersecting lines. Lines n and ℓ of this cake intersect only once at point K.
Postulate 2.6 states that if two lines intersect, then their intersection is exactly one point.

ANSWER:

The top edges of the bottom layer form intersecting lines. Lines n and ℓ of this cake intersect only once at point K.
Postulate 2.6; if two lines intersect, then their intersection is exactly one point.

17. Planes P and Q intersect in line m.

SOLUTION:

Identify planes P and Q and locate line m.
The edges of the sides of the bottom layer of the cake intersect. Plane P and Q of this cake intersect only once in
line m. Postulate 2.7 states that if two planes intersect, then their intersection is a line.

ANSWER:

The edges of the sides of the bottom layer of the cake intersect. Plane P and Q of this cake intersect only once in
line m. Postulate 2.7; if two planes intersect, then their intersection is a line.

18. Points D, K, and H determine a plane.

SOLUTION:

Locate points D, K, and H.
The bottom left part of the cake is a side. This side contains the points D, K, and H and forms a plane. Postulate 2.2
states that through any three noncollinear points, there is exactly one plane.

ANSWER:

The bottom left part of the cake is a side. This side contains the points D, K, and H and forms a plane. Postulate 2.2;
through any three noncollinear points, there is exactly one plane.

2-5 Postulates and Paragraph Proofs

19. Point *D* is also on the line *n* through points *C* and *K*.

   SOLUTION:

   Identify line *n*. and locate points *D*, *C* and *K*.
   The top edge of the bottom layer of the cake is a straight line *n*. Points *C*, *D*, and *K* lie along this edge, so they lie along line *n*. Postulate 2.3 states that a line contains at least two points.

   ANSWER:

   The top edge of the bottom layer of the cake is a straight line *n*. Points *C*, *D*, and *K* lie along this edge, so they lie along line *n*. Postulate 2.3; a line contains at least two points.

20. Points *D* and *H* are collinear.

   SOLUTION:

   Identify points *D* and *H*.
   Only one line can be drawn between the points *D* and *H*.
   Postulate 2.1 states that through any two points, there is exactly one line.

   ANSWER:

   Only one line can be drawn between the points *D* and *H*.
   Postulate 2.1; through any two points, there is exactly one line.

21. Points *E, F,* and *G* are coplanar.

   SOLUTION:

   Locate points *E, F,* and *G*.
   The bottom right part of the cake is a side. The side contains points *K, E, F,* and *G* and forms a plane. Postulate 2.2 states that through any three noncollinear points, there is exactly one plane.

   ANSWER:

   The bottom right part of the cake is a side. The side contains points *K, E, F,* and *G* and forms a plane. Postulate 2.2; through any three noncollinear points, there is exactly one plane.

22. $\overleftrightarrow{EF}$ lies in plane *Q*.

   SOLUTION:

   Identify plane *Q* and locate line *EF*.
   The bottom part of the cake is a side. Connecting the points *E* and *F* forms a line, which is contained on this side. Postulate 2.5 states that if two points lie in a plane, then the entire line containing those points lies in that plane.

   ANSWER:

   The bottom part of the cake is a side. Connecting the points *E* and *F* forms a line, which is contained on this side. Postulate 2.5; if two points lie in a plane, then the entire line containing those points lies in that plane.

23. Lines *h* and *g* intersect

   SOLUTION:

   Locate lines *h* and *g*.
   The top edges of the bottom layer form intersecting lines. Lines *h* and *g* of this cake intersect only once at point *J*. Postulate 2.6 states that if two lines intersect, then their intersection is exactly one point.

   ANSWER:

   The top edges of the bottom layer form intersecting lines. Lines *h* and *g* of this cake intersect only once at point *J*. Postulate 2.6; if two lines intersect, then their intersection is exactly one point.
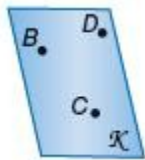
Determine whether each statement is *always, sometimes,* or *never* true. Explain.

24. There is exactly one plane that contains noncollinear points *A, B,* and *C*.

SOLUTION:

Postulate 2.2 states that through any three noncollinear points, there is exactly one plane. Therefore, the statement is *always* true.
For example, plane *K* contains three noncollinear points.

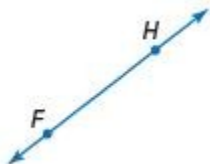
ANSWER:

Always; Postulate 2.2 states that through any three non-collinear points, there is exactly one plane.

25. There are at least three lines through points *J* and *K*.

SOLUTION:

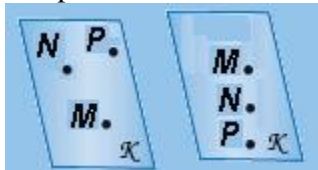Postulate 2.1 states through any two points, there is exactly one line. Therefore, the statement is *never* true.

ANSWER:

Never; Postulate 2.1 states through any two points, there is exactly one line.

26. If points *M, N,* and *P* lie in plane *X,* then they are collinear.

SOLUTION:

The points do not have to be collinear to lie in a plane. Therefore, the statement is *sometimes* true.

ANSWER:
Sometimes; the points do not have to be collinear to lie in a plane.

27. Points *X* and *Y* are in plane *Z*. Any point collinear with *X* and *Y* is in plane *Z*.

    SOLUTION:
    Postulate 2.5 states if two points lie in a plane, then the entire line containing those points lies in that plane. Therefore, the statement is *always* true. In the figure below, points *VWXY* are all on line *n* which is in plane *Z*. Any other point on the line *n* will also be on plane *Z*.
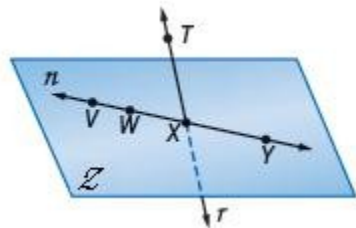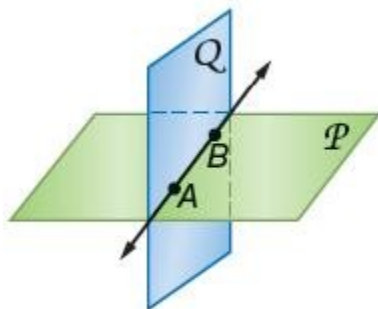

    ANSWER:
    Always; Postulate 2.5 states if two points lie in a plane, then the entire line containing those points lies in that plane.

28. The intersection of two planes can be a point.

    SOLUTION:
    Postulate 2.7 states if two planes intersect, then their intersection is a line. Therefore, the statement is *never* true.
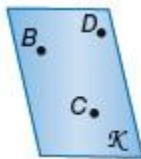

    ANSWER:
    Never; Postulate 2.7 states if two planes intersect, then their intersection is a line.

29. Points *A, B,* and *C* determine a plane.

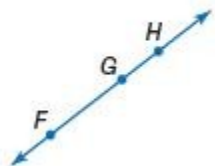   *SOLUTION:*

    The points must be non-collinear to determine a plane by postulate 2.2. Therefore, the statement is *sometimes* true.

    Three non-collinear points determine a plane.   Three collinear points determine a line.

    ANSWER:

    Sometimes; the points must be non-collinear.

30. **PROOF** Point *Y* is the midpoint of $\overline{XZ}$. *Z* is the midpoint of $\overline{YW}$. Prove that $\overline{XY} \cong \overline{ZW}$.

    SOLUTION:

    You are given midpoints for two segments, $\overline{XZ}$ and $\overline{YW}$. Use your knowledge of midpoints and congruent segments to obtain information about $\overline{XZ}$ and $\overline{YW}$, the segments that you are trying to prove congruent.

    Given: Point *Y* is the midpoint of $\overline{XZ}$.
    Z is the midpoint of $\overline{YW}$.
    Prove: $\overline{XY} \cong \overline{ZW}$.
    Proof: We are given that *Y* is the midpoint of $\overline{XZ}$ and *Z* is the midpoint of $\overline{YW}$. By the definition of midpoint, $\overline{XY} \cong \overline{YZ}$ and $\overline{YZ} \cong \overline{ZW}$. . Using the definition of congruent segments, *XY = YZ* and *YZ = ZW*.
    XY = ZW by the Transitive Property of Equality. Thus, $\overline{XY} \cong \overline{ZW}$ by the definition of congruent segments.

    ANSWER:

    Given: Point *Y* is the midpoint of $\overline{XZ}$.
    Z is the midpoint of $\overline{YW}$.
    Prove: $\overline{XY} \cong \overline{ZW}$
    Proof: We are given that *Y* is the midpoint of $\overline{XZ}$ and *Z* is the midpoint of $\overline{YW}$. By the definition of midpoint, $\overline{XY} \cong \overline{YZ}$ and $\overline{YZ} \cong \overline{ZW}$. Using the definition of congruent segments, *XY = YZ* and *YZ = ZW*. *XY = ZW* by the Transitive Property of Equality. Thus, $\overline{XY} \cong \overline{ZW}$ by the definition of congruent segments.

31. **PROOF** Point L is the midpoint of $\overline{JK}$. $\overline{JK}$ intersects $\overline{MK}$ at K. If $\overline{MK} \cong \overline{JL}$, prove that $\overline{LK} \cong \overline{MK}$.

    SOLUTION:

    You are given a midpoint and a pair of intersecting segments, $\overline{JK}$ and $\overline{MK}$. Use your knowledge of midpoints and congruent segments to obtain information about $\overline{LK}$ and $\overline{MK}$, the segments that you are trying to prove congruent.

    Given: L is the midpoint of $\overline{JK}$. $\overline{JK}$ intersects $\overline{MK}$ at K. $\overline{MK} \cong \overline{JL}$
    Prove: $\overline{LK} \cong \overline{MK}$
    Proof: We are given that L is the midpoint of $\overline{JK}$ and $\overline{MK} \cong \overline{JL}$. By the Midpoint Theorem, $\overline{JL} \cong \overline{LK}$. By the Transitive Property of Equality, $\overline{LK} \cong \overline{MK}$.

    ANSWER:

    Given: L is the midpoint of $\overline{JK}$. $\overline{JK}$ intersects $\overline{MK}$ at K. $\overline{MK} \cong \overline{JL}$
    Prove: $\overline{LK} \cong \overline{MK}$
    Proof: We are given that L is the midpoint of $\overline{JK}$ and $\overline{MK} \cong \overline{JL}$. By the Midpoint Theorem, $\overline{JL} \cong \overline{LK}$. By the Transitive Property of Equality, $\overline{LK} \cong \overline{MK}$.

32. **CCSS ARGUMENTS** Last weekend, Emilio and his friends spent Saturday afternoon at the park. There were several people there with bikes and skateboards. There were a total of 11 bikes and skateboards that had a total of 36 wheels. Use a paragraph proof to show how many bikes and how many skateboards there were.

    SOLUTION:

    You are given a the total of bikes and skateboards and the total number of wheels. Use your knowledge of algebra and equations to obtain information about the number of bikes and skateboards.

    From the given information, there are a total of 11 bikes and skateboards, so if b represents bikes and s represents skateboards, $b + s = 11$. The equation can also be written $s = 11 - b$. There are a total of 36 wheels, so $2b + 4s = 36$, since each bike has two wheels and each skateboard has four wheels. Substitute the equation $s = 11 - b$ into the equation $2b + 4s = 36$ to eliminate one variable, resulting in $2b + 4(11 - b) = 36$. Simplify the equation to $2b + 44 - 4b = 36$ and solve to get $b = 4$. If there are 4 bikes, there are $11 - 4$, or 7 skateboards. Therefore, there are 4 bikes and 7 skateboards.

    ANSWER:

    Sample answer: From the given information, there are a total of 11 bikes and skateboards, so if b represents bikes and s represents skateboards, $b + s = 11$. The equation can also be written $s = 11 - b$. There are a total of 36 wheels, so $2b + 4s = 36$, since each bike has two wheels and each skateboard has four wheels. Substitute the equation $s = 11 - b$ into the equation $2b + 4s = 36$ to eliminate one variable, resulting in $2b + 4(11 - b) = 36$. Simplify the equation to $2b + 44 - 4b = 36$ and solve to get $b = 4$. If there are 4 bikes, there are $11 - 4$, or 7 skateboards. Therefore, there are 4 bikes and 7 skateboards.

33. **DRIVING** Keisha is traveling from point A to point B. Two possible routes are shown on the map. Assume that the speed limit on Southside Boulevard is 55 miles per hour and the speed limit on I−295 is 70 miles per hour.

a. Which of the two routes covers the shortest distance? Explain your reasoning.

b. If the distance from point A to point B along Southside Boulevard is 10.5 miles and the distance along I-295 is 11.6 miles, which route is faster, assuming that Keisha drives the speed limit?
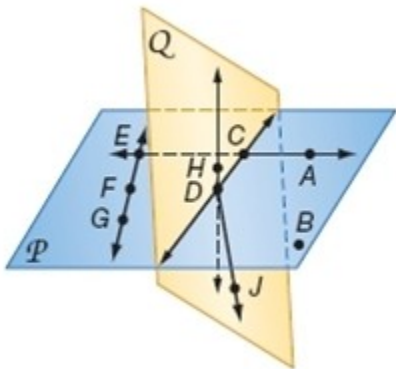
SOLUTION:

a. Since there is a line between any two points, and Southside Blvd is the line between point A and point B, it is the shortest route between the two.
between point A and point B, it is the shortest route between the two.

b. The speed limit on Southside Boulevard is 55 miles per hour and the speed limit on I−295 is 70 miles per hour. The distance from point A to point B along Southside Boulevard is 10.5 miles and the distance along I-295 is 11.6 miles.

So, it takes $\dfrac{10.5}{55} \approx 0.191 \approx 12 \text{ minutes}$ along the Southside Boulevard and it takes $\dfrac{11.6}{70} \approx 0.166 \approx 10 \text{ minutes}$ along the I−295 if Keisha drives the speed limit. So, the route I−295 is faster.

ANSWER:

a. Southside Blvd.; sample answer: Since there is a line between any two points, and Southside Blvd is the line between point A and point B, it is the shortest route between the two.
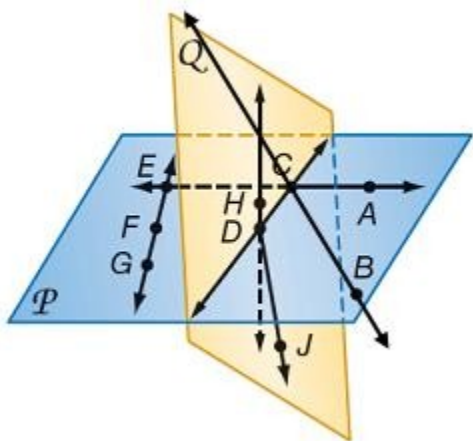
b. I−295

In the figure, $\overrightarrow{CD}$ and $\overleftrightarrow{CE}$ lie in plane P and $\overleftrightarrow{DH}$ and $\overleftrightarrow{DJ}$ lie in plane Q. State the postulate that can be used to show each statement is true.

34. Points C and B are collinear.

    SOLUTION:

    Identify C and B in the figure. If points C and B are collinear, then a line can be drawn through the two points. Postulate 2.1 states that through any two points, there is exactly one line.

    ANSWER:

    Postulate 2.1; through any two points, there is exactly one line.

35. $\overleftrightarrow{EG}$ contains points $E, F,$ and G.

    SOLUTION:

    Identify $\overleftrightarrow{EG}$, locate the points on the line. Postulate 2.3 states that a line contains at least two points. Points $E, F,$ and G are on $\overleftrightarrow{EG}$.

    ANSWER:

    Postulate 2.3; a line contains at least two points.
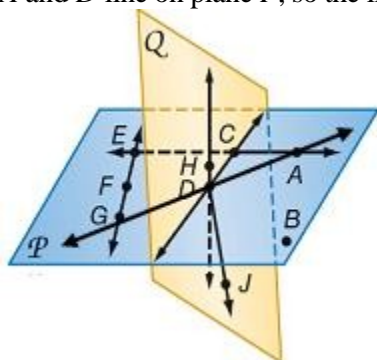
36. $\overleftrightarrow{DA}$ lies in plane *P*.

   SOLUTION:

   Identify plane *P* and locate $\overrightarrow{DA}$.
   Postulate 2.5 states that if two points lie in a plane, then the entire line containing those points lies in that plane. Both
   A and *D* line on plane *P*, so the line through them, $\overrightarrow{DA}$, is also on plane *P*.
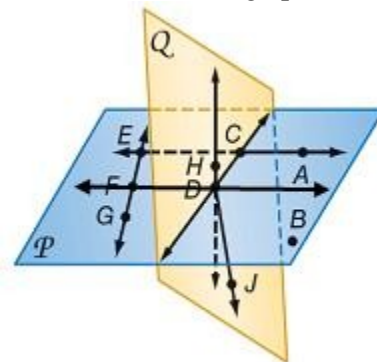

   ANSWER:

   Postulate 2.5; if two points lie in a plane, then the entire line containing those points lies in that plane.

37. Points *D* and *F* are collinear.

   SOLUTION:

   Locate points *D* and *F* . Postulate 2.1 states that through any two points, there is exactly one line. Therefore, you
   can draw a line through points *D* and *F*.

   ANSWER:

   Postulate 2.1; through any two points, there is exactly one line.

38. Points *C, D,* and *B* are coplanar.

   SOLUTION:

   Locate points *C, D,* and *B.* Identify the plan(s) they are on.
   Postulate 2.2 states that through any three noncollinear points, there is exactly one plane.

   ANSWER:

   Postulate 2.2; through any three noncollinear points, there is exactly one plane.

39. Plane *Q* contains the points *C, H, D,* and *J.*

    SOLUTION:

    Identify plane *Q* and locate the points on it.
    Postulate 2.4 states that a plane contains at least three noncollinear points.
    Plane *Q* contains the points *C, H, D,* and *J.*

    ANSWER:

    Postulate 2.4; a plane contains at least three noncollinear points.

40. $\overleftrightarrow{AC}$ and $\overleftrightarrow{FG}$ intersect at point *E*

    SOLUTION:

    Locate $\overleftrightarrow{AC}$ and $\overleftrightarrow{FG}$.
    Postulate 2.6 states that if two lines intersect, then their intersection is exactly one point.
    $\overleftrightarrow{AC}$ and $\overleftrightarrow{FG}$ are both on plane *P* and intersect at point *E.*

    ANSWER:

    Postulate 2.6; if two lines intersect, then their intersection is exactly one point.

41. Plane *P* and plane *Q* intersect at $\overleftrightarrow{CD}$

    SOLUTION:

    Identify plane *P* and plane *Q* and locate $\overleftrightarrow{CD}$.

    Postulate 2.7 states that if two planes intersect, then their intersection is a line. Thus $\overleftrightarrow{CD}$ is the line of intersection of plane *P* and plane *Q.*

    ANSWER:

    Postulate 2.7; if two planes intersect, then their intersection is a line.

42. **CCSS ARGUMENTS** Roofs are designed based on the materials used to ensure that water does not leak into the buildings they cover. Some roofs are constructed from waterproof material, and others are constructed for watershed, or gravity removal of water. The pitch of a roof is the rise over the run, which is generally measured in rise per foot of run. Use the statements below to write a paragraph proof justifying the following statement: The pitch of the roof in Den's design is not steep enough.

• Waterproof roofs should have a minimum slope of $\frac{1}{4}$ inch per foot.

• Watershed roofs should have a minimum slope of 4 inches per foot.
• Den is designing a house with a watershed roof.
• The pitch in Den's design is 2 inches per foot.

SOLUTION:

Den is designing a watershed roof, so the minimum pitch for a waterproof roof are irrelevant to the question. We need to compare the pitch of Den's watershed roof with the minimum pitch for watershed roofs.

Sample answer: Since Den is designing a watershed roof, the pitch of the roof should be a minimum of 4 inches per foot. The pitch of the roof in Den's design is 2 inches per foot, which is less than 4 inches per foot. Therefore, the pitch of the roof in Den's design is not steep enough.

ANSWER:

Sample answer: Since Den is designing a watershed roof, the pitch of the roof should be a minimum of 4 inches per foot. The pitch of the roof in Den's design is 2 inches per foot, which is less than 4 inches per foot. Therefore, the pitch of the roof in Den's design is not steep enough.

43. **NETWORKS** Diego is setting up a network of multiple computers so that each computer is connected to every other. The diagram illustrates this network if Diego has 5 computers.
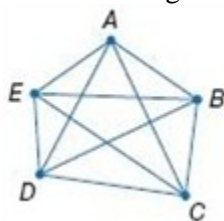

a. Draw diagrams of the networks if Diego has 2, 3, 4, or 6 computers.
b. Create a table with the number of computers and the number of connections for the diagrams you drew.
c. If there are *n* computers in the network, write an expression for the number of computers to which each of the computers is connected.
d. If there are *n* computers in the network, write an expression for the number of connections there are.

SOLUTION:

a. Set up groups (networks) of 2, 3, 4, and 6 points. Label them A-F. For each group, connect all of the points to each other.
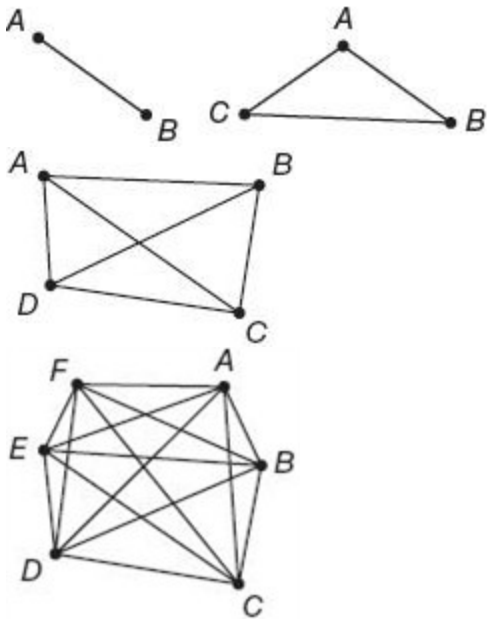
b. Count the number of edges in each group and tabulate it.

Number of Computers	Number of Connections
2	1
3	3
4	6
5	10
6	15

c. Each computer is connected to all the other computers. The computer can not be connected to itself. So if there are n computers, each computer is connected to all of the others in the network, or $n - 1$ computers.
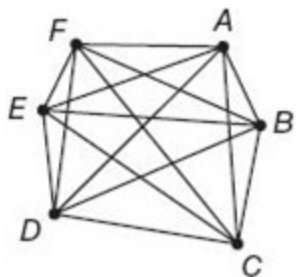
d. The first computer will be connected to the other $n - 1$ computers and the second one with the other $n - 2$ as the connection between the first and the second has already been counted. Similarly, the third computer will be connected to the other $n - 3$ computers and so on. So, the total number of connections will be $\dfrac{n(n-1)}{2}$.

ANSWER:

a.
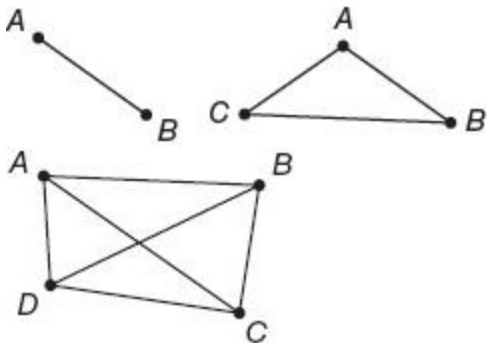
b.

Number of Computers	Number of Connections
2	1
3	3
4	6
5	10
6	15

c. $(n - 1)$;

If there are n computers, each computer is connected to all other computers.

d. $\dfrac{n(n-1)}{2}$

44. **CCSS SENSE-MAKING** The photo on page 133 is of the rotunda in the capital building in St. Paul, Minnesota. A rotunda is a round building, usually covered by a dome. Use Postulate 2.1 to help you answer Exercises **a–c**.

   a. If you were standing in the middle of the rotunda, which arched exit is the closest to you?
   b. What information did you use to formulate your answer?
   c. What term describes the shortest distance from the center of a circle to a point on the circle?

   SOLUTION:

   a. The base of a dome is a circle. All points from the center to points on the circle are equidistant. Thus, all of the exits are the same distance from the center.
   b. The distance from the center of a circle to any point on the circle is equal, and through any two points, there is exactly one line (Postulate 2.1). That means that there is a line between the center and each of the exits, and they are all the same length.
   c. The radius is the shortest distance from the center of a circle to a point on the circle.

   ANSWER:

   a. All of the exits are the same distance from the center.
   b. Sample answer: The distance from the center of a circle to any point on the circle is equal, and through any two points, there is exactly one line. That means that there is a line between the center and each of the exits, and they are all the same length.
   c. radius

45. **ERROR ANALYSIS** Omari and Lisa were working on a paragraph proof to prove that if $\overline{AB}$ is congruent to $\overline{BD}$ and A, B, and D are collinear, then B is the midpoint of $\overline{AD}$. Each student started his or her proof in a different way. Is either of them correct? Explain your reasoning.


SOLUTION:

The proof should begin with the given, which is that $\overline{AB}$ is congruent to $\overline{BD}$ and A, B, and D are collinear. Therefore, Lisa began the proof correctly. Omari starts his proof with what is to be proved and adds details from the proof. So, Lisa is correct.

ANSWER:

Lisa is correct. Sample answer: The proof should begin with the given, which is that $\overline{AB}$ is congruent to $\overline{BD}$ and A, B, and D are collinear. Therefore, Lisa began the proof correctly.

46. **OPEN ENDED** Draw a figure that satisfies five of the seven postulates you have learned. Explain which postulates you chose and how your figure satisfies each postulate.

SOLUTION:
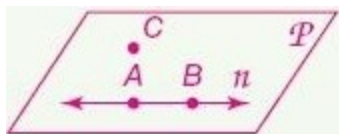
Sample answer: It satisfies Postulates 2.1 and 2.3 because points A and B are on line n It satisfies 2.2 and 2.4 because 3 points lie in the plane. It satisfies Postulate 2.5 because points A and B lie in plane P, so line n also lies in plane P.

ANSWER:
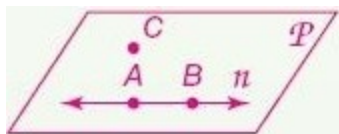
Sample answer: It satisfies Postulates 2.1 and 2.3 because points A and B are on line n. It satisfies 2.2 and 2.4 because 3 points lie in the plane. It satisfies Postulate 2.5 because points A and B lie in plane P, so line n also lies in plane P.

47. **CHALLENGE** Use the following true statement and the definitions and postulates you have learned to answer each question.
    Two planes are perpendicular if and only if one plane contains a line perpendicular to the second plane.
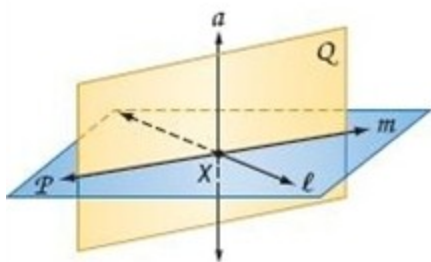
a. Through a given point, there passes one and only one plane perpendicular to a given line. If plane Q is perpendicular to line P at point X and line a lies in plane P, what must also be true?

b. Through a given point, there passes one and only one line perpendicular to a given plane. If plane Q is perpendicular to plane P at point X and line a lies in plane Q, what must also be true?

SOLUTION:

a. Two planes are perpendicular if and only if one plane contains a line perpendicular to the second plane. Here, plane Q is perpendicular to line ℓ at point X and line ℓ lies in plane P, so plane Q is perpendicular to P.

b. Through a given point, there passes one and only one line perpendicular to a given plane. If plane Q is perpendicular to plane P at point X and line a lies in plane Q, then line a is perpendicular to plane P.

ANSWER:

a. Plane Q is perpendicular to plane P.
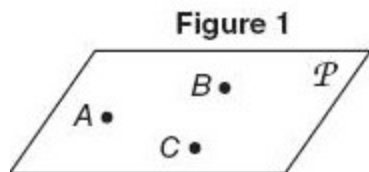
b. Line a is perpendicular to plane P.

REASONING Determine if each statement is *sometimes*, *always*, or *never* true. Explain your reasoning or provide a counterexample.
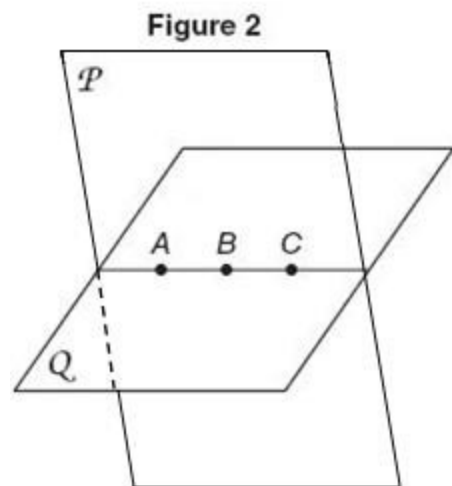
48. Through any three points, there is exactly one plane.

SOLUTION:

If the points were non-collinear, there would be exactly one plane by Postulate 2.2 shown by Figure 1.

Figure 1

If the points were collinear, there would be infinitely many planes. Figure 2 shows what two planes through collinear points would look like. More planes would rotate around the three points. Therefore, the statement is *sometimes* true.

Figure 2

ANSWER:

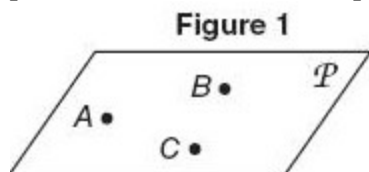Sometimes; if the points were noncollinear, there would be exactly one plane by Postulate 2.2 shown by Figure 1. If the points were collinear, there would be infinitely many planes. Figure 2 shows what two planes through collinear points would look like. More planes would rotate around the three points.
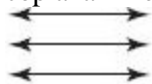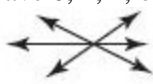
Figure 1

Figure 2
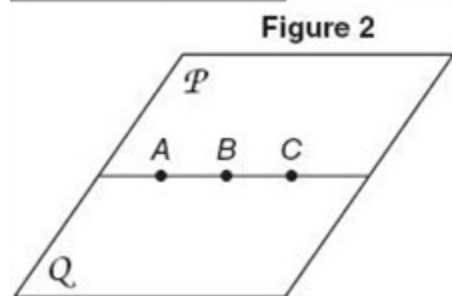
49. Three coplanar lines have two points of intersection.

   SOLUTION:

   Three coplanar lines, may or may not cross. If they cross, there can be 1, 2 or 3 points of intersection. Thus three coplanar lines may have 0, 1, 2, or 3 points of intersection, as shown in the figures below.
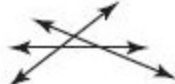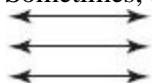

   ANSWER:

   Sometimes; three coplanar lines may have 0, 1, 2, or 3 points of intersection, as shown in the figures below.
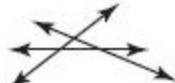
50. **WRITING IN MATH** How does writing a proof require logical thinking?

   SOLUTION:

   Think about what you need to get started? What are all of the aspects of proofs? What is the goal of the proof?

   Sample answer: When writing a proof, you start with something that you know is true (the given), and then use logic to come up with a series of steps that connect the given information to what you are trying to prove.

   ANSWER:

   Sample answer: When writing a proof, you start with something that you know is true (the given), and then use logic to come up with a series of steps that connect the given information to what you are trying to prove.

51. **ALGEBRA** Which is one of the solutions of the equation $3x^2 - 5x + 1 = 0$?

    A $\dfrac{5 + \sqrt{13}}{6}$

    B $\dfrac{-5 - \sqrt{13}}{6}$

    C $\dfrac{5}{6} - \sqrt{13}$

    D $-\dfrac{5}{6} + \sqrt{13}$

    SOLUTION:
    Use the Quadratic Formula to find the roots of the equation.

    $$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

    Here, $a = 3$, $b = -5$ and $c = 1$.

    $$x = \frac{-(-5) \pm \sqrt{(-5)^2 - 4(3)(1)}}{2(3)}$$

    $$= \frac{5 \pm \sqrt{13}}{6}$$

    $$= \frac{5 + \sqrt{13}}{6}, \frac{5 - \sqrt{13}}{6}$$

    Therefore, the correct choice is A.

    ANSWER:
    A

52. **GRIDDED RESPONSE** Steve has 20 marbles in a bag, all the same size and shape. There are 8 red, 2 blue, and 10 yellow marbles in the bag. He will select a marble from the bag at random. What is the probability that the marble Steve selects will be yellow?

    SOLUTION:
    The probability is the ratio of the number of favorable outcomes to the total number of outcomes.
    Here, there are 20 marbles, of which 10 are yellow. So, the probability of selecting a yellow marble is:

    $$P(\text{yellow}) = \frac{10}{20}$$

    $$= \frac{1}{2} \text{ or } 0.5$$

    ANSWER:

    $\dfrac{1}{2}$ or 0.5

53. Which statement *cannot* be true?
    F Three noncollinear points determine a plane.
    G Two lines intersect in exactly one point.
    H At least two lines can contain the same two points.
    J A midpoint divides a segment into two congruent segments.

    SOLUTION:

    The statement in option F is true by Postulate 2.2. By Postulate 2.6, the statement in option G is true. By the Midpoint Theorem, option J is true. By Postulate 2.1, through any two points there is exactly one line. Therefore, the statement in option H cannot be true and so the correct choice is H.

    ANSWER:
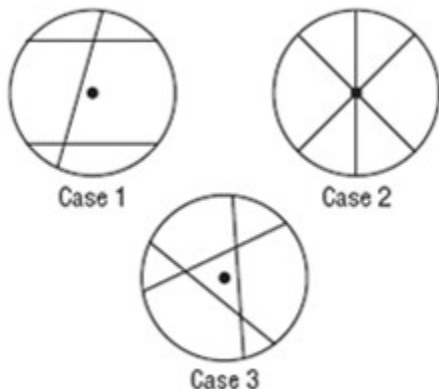
    H

54. **SAT/ACT** What is the greatest number of regions that can be formed if 3 distinct lines intersect a circle?
    A 3
    B 4
    C 5
    D 6
    E 7

    SOLUTION:

    Three lines can intersect a circle in 3 different ways as shown.

    The maximum number of regions is in case 3 and it is 7. Therefore, the correct choice is E.

    ANSWER:

    E

2-5 Postulates and Paragraph Proofs

Determine whether a valid conclusion can be reached from the two true statements using the Law of Detachment or the Law of Syllogism. If a valid conclusion is possible, state it and the law that is used. If a valid conclusion does not follow, write *no conclusion*.

55. (**1**) If two angles are vertical, then they do not form a linear pair.
(**2**) If two angles form a linear pair, then they are not congruent.

SOLUTION:
The Law of Detachment states "If $p \rightarrow q$ is a true statement and p is true, then q is true".
The Law of Syllogism states "If $p \rightarrow q$ and $q \rightarrow r$, then $p \rightarrow r$ is a true statement".

From (1) then let p = "two angles are vertical" and q =" they do not form a linear pair."
From (2) let q = "two angles form a linear pair" and r = "they are not congruent".
The Law of Detachment can not be used because statement (2) does not give us a true statement for p.
The Law of Syllogism can not be used since the q in (1) is not the same as q in (2).
Thus, there is no valid conclusion can be made from the statements.

ANSWER:
No conclusion

56. (**1**) If an angle is acute, then its measure is less than 90.
(**2**) $\angle EFG$ is acute.

SOLUTION:
The Law of Detachment states "If $p \rightarrow q$ is a true statement and p is true, then q is true".
The Law of Syllogism states "If $p \rightarrow q$ and $q \rightarrow r$, then $p \rightarrow r$ is a true statement".

From (1) then let p = "an angle is acute" and q =" its measure is less than 90".
From (2) let p = "$\angle EFG$ is acute".

The Law of Syllogism is not applicable, because statement 2 does not have a part conclusion r.
By the Law of Detachment, the statement "if an angle is acute, then its measure is less than 90" is a true statement and $\angle EFG$ is acute. So, $m\angle EFG$ is less than 90.

ANSWER:
$m\angle EFG$ is less than 90; detachment.

Write each statement in if-then form.
57. Happy people rarely correct their faults.

SOLUTION:
To write these statements in if-then form, identify the hypothesis and conclusion. The word if is not part of the hypothesis. The word then is not part of the conclusion.
If people are happy, then they rarely correct their faults.

ANSWER:
If people are happy, then they rarely correct their faults.

58. A champion is afraid of losing.

    SOLUTION:

    To write these statements in if-then form, identify the hypothesis and conclusion. The word if is not part of the hypothesis. The word then is not part of the conclusion.
    If a person is a champion, then that person is afraid of losing.

    ANSWER:

    If a person is a champion, then that person is afraid of losing.

    Use the following statements to write a compound statement for each conjunction. Then find its truth value. Explain your reasoning.

    p : M **is on** $\overline{AB}$**.**

    q**:** $AM + MB = AB$

    r**:** M **is the midpoint of** $\overline{AB}$**.**

59. $p \wedge q$

    SOLUTION:

    Find the conjunction $p \wedge q$. A conjunction is true only when both statements that form it are true.

    Here, M is on $\overline{AB}$ which is true. $AM + MB = AB$ is true. So $p \wedge q$ is true.

    ANSWER:

    True; M is on $\overline{AB}$ and $AM + MB = AB$, so $p \wedge q$ is true.

60. $\sim p \vee \sim r$

    SOLUTION:

    Negate both p and r, finding the opposite truth values. Then find the disjunction $\sim p \vee \sim r$. A disjunction is true if at least one of the statements is true.

    Here, M is on $\overline{AB}$, so ~p is false. But, M is not the midpoint of $\overline{AB}$. So ~r is true. Therefore, $\sim p \vee \sim r$ is true.

    ANSWER:

    True; M is on $\overline{AB}$, so ~p is false, but M is not the midpoint of $\overline{AB}$, so ~r is true. Therefore, $\sim p \vee \sim r$ is true.

61. **GARDENING** A landscape designer is putting black plastic edging around a rectangular flower garden that has length 5.7 meters and width 3.8 meters. The edging is sold in 5-meter lengths. Find the perimeter of the garden and determine how much edging the designer should buy.
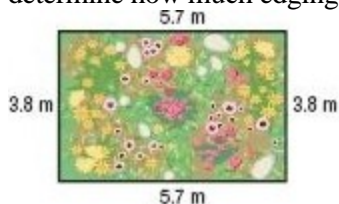
SOLUTION:

Add the lengths of the sides to find the perimeter of the garden.
5.7 + 3.8 + 5.7 + 3.8 = 19m.
Therefore, the designer has to buy 20m of edging.

ANSWER:

19 m; 20 m of edging

62. **HEIGHT** Taylor is 5 feet 8 inches tall. How many inches tall is Taylor?

SOLUTION:

One foot is equivalent to 12 inches. So, 5 feet is equivalent to 5(12) = 60 inches. Therefore, Taylor is 60 + 8 = 68 inches tall.

ANSWER:

68 in

ALGEBRA Solve each equation.

63. $4x - 3 = 19$

SOLUTION:

$$4x - 3 = 19 \quad \text{Original equation}$$
$$4x - 3 + 3 = 19 + 3 \quad \text{Add 3 to each side}$$
$$4x = 22 \quad \text{Simplify.}$$
$$\frac{4x}{4} = \frac{22}{4} \quad \text{Divide each side by 4.}$$
$$x = \frac{22}{4} \quad \text{Simplify.}$$
$$x = 5.5$$

ANSWER:

5.5

64. $\frac{1}{3}x + 6 = 14$

SOLUTION:

$$\frac{1}{3}x + 6 = 14 \qquad \text{Original equation}$$

$$\frac{1}{3}x + 6 - 6 = 14 - 6 \qquad \text{Subtract 6 from each side}$$

$$\frac{1}{3}x = 8 \qquad \text{Simplify.}$$

$$3\left(\frac{1}{3}x\right) = 3(8) \qquad \text{Multiply each side by 3.}$$

$$x = 24 \qquad \text{Simplify.}$$

ANSWER:

24

65. $5\left(x^2 + 2\right) = 30$

SOLUTION:

$$5\left(x^2 + 2\right) = 30 \qquad \text{Original equation}$$

$$\frac{5\left(x^2 + 2\right)}{5} = \frac{30}{5} \qquad \text{Divide each side by 5.}$$

$$x^2 + 2 = 6 \qquad \text{Simplify.}$$

$$x^2 + 2 - 2 = 6 - 2 \qquad \text{Subtract 2 from each side.}$$

$$x^2 = 4 \qquad \text{Simplify.}$$

$$x = \pm\sqrt{4} \qquad \text{Square root of each side.}$$

$$x = \pm 2 \qquad \text{Simplify.}$$

ANSWER:

2, −2