2-6 Algebraic Proof

State the property that justifies each statement.

1. If $m\angle 1 = m\angle 2$ and $m\angle 2 = m\angle 3$, then $m\angle 1 = m\angle 3$.

SOLUTION:

There are two parts to the hypotheses. "If $m\angle 1 = m\angle 2$ and $m\angle 2 = m\angle 3$, then $m\angle 1 = m\angle 3$. "The end of the first part of the hypotheses " $m\angle 2$" is the same as the start of the first part of the second part of the hypothesis. The Transitive Property justifies this statement.

ANSWER:

Trans. Prop.

2. $XY = XY$

SOLUTION:

In the statement, each side is the same "$XY = XY$". The Reflexive Property justifies this statement.

ANSWER:

Refl. Prop.

3. If $5 = x$, then $x = 5$.

SOLUTION:

In the statements, the order is reversed" If $5 = x$, then $x = 5$. ". Thus the Symmetric Property describes the statement.

ANSWER:

Sym. Prop.

4. If $2x + 5 = 11$, then $2x = 6$.

SOLUTION:

$2x + 5 = 11$	Original equation
$2x + 5 - 5 = 11 - 5$	Subtract 5 from each side.
$2x = 6$	Simplify.

5 was subtracted from both sides to change $2x + 5 = 11$ to $2x = 6$. Thus, the Subtraction Property of Equality is the property used in the statement.

ANSWER:

Subt. Prop.

5. Complete the following proof.

Given: $\frac{y+2}{3}=3$

Prove: $y = 7$

Proof:

Statements	Reasons
a. ?	a. Given
b. $3\left(\frac{y+2}{3}\right)=3(3)$	b. ?
c. ?	c. ?
d. $y=7$	d. Subtraction Property

SOLUTION:

Work backwards to find **a**. From the 1st row to the 2nd, we are multiplying by 3, so take out the multiplication.
The 2nd row is identified as multiplication by the 3s outside the parentheses.
The 3rd row is found by distributing the 3s, using the substitution property.
The 4th row is the subtraction of 2.

Statements	Reasons
$\frac{y+2}{3}=3$	Given
$3\left(\frac{y+2}{3}\right)=3(3)$	Multiplicative Property of Equality
$y+2=9$	Substitution
$y=7$	Subtraction Property

ANSWER:

a. $\frac{y+2}{3}=3$

b. Multiplicative Property of Equality

c. $y + 2 = 9$**;** Substitution

PROOF Write a two-column proof to verify each conjecture.

6. If $-4(x - 3) + 5x = 24$, then $x = 12$.

SOLUTION:

You need to walk through the proof step by step. Look over what you are given and what you need to prove. Here, you are given two congruent segments, a value for one of the segments, and an expression for the other segment. One you prove the values are equal, you will need to find the variable in the expression. Use the properties that you have learned about congruent segments and equivalent expressions in algebra to walk through the proof.

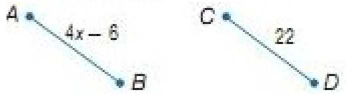
Given: $-4(x - 3) + 5x = 24$
Prove: $x = 12$
Proof:
Statements (Reasons)
1. $-4(x - 3) + 5x = 24$
(Given)
2. $-4x + 12 + 5x = 24$
(Dist. Prop.)
3. $x + 12 = 24$ (Subs.)
4. $x = 12$ (Subt. Prop.)

ANSWER:

Given: $-4(x - 3) + 5x = 24$
Prove: $x = 12$
Proof:
Statements (Reasons)
1. $-4(x - 3) + 5x = 24$
(Given)
2. $-4x + 12 + 5x = 24$
(Dist. Prop.)
3. $x + 12 = 24$ (Subs.)
4. $x = 12$ (Subt. Prop.)

2-6 Algebraic Proof

7. If $\overline{AB} \cong \overline{CD}$, then $x = 7$.



SOLUTION:

You need to walk through the proof step by step. Look over what you are given and what you need to prove.Here, you are given two congruent segments, a value for one of the segments, and an expression for the other segment. One you prove the lengths of the segment are equal, you will need to find the variable in the expression. Use the properties that you have learned about congruent segments and equivalent expressions in algebra to walk through the proof.

Given: $\overline{AB} \cong \overline{CD}$
Prove: $x = 7$
Proof:
Statements (Reasons)

1. $\overline{AB} \cong \overline{CD}$ (Given)
2. $AB = CD$ (Definition of congruent segments)
3. $4x – 6 = 22$ (Substitution Property)
4. $4x = 28$ (Addition Property)
5. $x = 7$ (Division Property)

ANSWER:

Given: $\overline{AB} \cong \overline{CD}$
Prove: $x = 7$
Proof:
Statements (Reasons)

1. $\overline{AB} \cong \overline{CD}$ (Given)
2. $AB = CD$ (Def. of congruent segments)
3. $4x – 6 = 22$ (Subs. Prop.)
4. $4x = 28$ (Add. Prop.)
5. $x = 7$ (Div. Prop.)

8. **CCSS ARGUMENTS** Mai-Lin measures her heart rate whenever she exercises and tries to make sure that she is staying in her target heart rate zone. The American Heart Association suggests a target heart rate of $T = 0.75(220 - a)$, where T is a person's target heart rate and a is his or her age.

a. Prove that given a person's target heart rate, you can calculate his or her age using the formula $a = 220 - \frac{T}{0.75}$.

b. If Mai-Lin's target heart rate is 153, then how old is she? What property justifies your calculation?

SOLUTION:

a. You need to walk through the proof step by step. Look over what you are given and what you need to prove. Here, you are given the formula for suggested target heart rate. Use the properties that you have learned about equivalent expressions in algebra to walk through the proof.
Proof:
Statements (Reasons)
1. $T = 0.75(220 - a)$ (Given)
2. $\frac{T}{0.75} = 220 - a$ (Division Property)
3. $\frac{T}{0.75} - 220 = -a$ (Subtraction Property)
4. $-\frac{T}{0.75} + 220 = a$ (Multiplication Property)
5. $a = -\frac{T}{0.75} + 220$ (Symmetric Property)
6. $a = 220 - \frac{T}{0.75}$ (Commutative. Property)

b. Substitute 153 for T and solve for a.

$$a = 220 - \frac{153}{0.75}$$
$$= 220 - 204$$
$$= 16$$

We substituted $T = 153$ to find how old is Mai-Lin. So, the property used is the Substitution Property of Equality.

ANSWER:

a. Proof:
Statements (Reasons)
1. $T = 0.75(220 - a)$ (Given)
2. $\frac{T}{0.75} = 220 - a$ (Div. Prop.)
3. $\frac{T}{0.75} - 220 = -a$ (Subt. Prop.)
4. $-\frac{T}{0.75} + 220 = a$ (Mult. Prop.)
5. $a = -\frac{T}{0.75} + 220$ (Symm. Prop.)
6. $a = 220 - \frac{T}{0.75}$ (Comm. Prop.)

b. 16 years; Sample answer: Substitution

2-6 Algebraic Proof

State the property that justifies each statement.

9. If $a + 10 = 20$, then $a = 10$

SOLUTION:

$$\begin{aligned} a+10&=20 && \text{Original equation} \\ a+10-10&=20-10 && \text{Subtract 10 from each side.} \\ a&=10 && \text{Simplify.} \end{aligned}$$

10 is subtracted from both sides of the equation to change $a + 10 = 20$ to $a = 10$. Thus, the Subtraction Property of Equality is the property used in the statement.

ANSWER:

Subt. Prop.

10. If $\frac{x}{3} = -15$, then $x = -45$

SOLUTION:

$$\begin{aligned} \frac{x}{3}&=-15 && \text{Original equation} \\ 3\left(\frac{x}{3}\right)&=3(-15) && \text{Multiply each side by 3.} \\ x&=-45 && \text{Simplify.} \end{aligned}$$

Multiply both side of the equation by 3 to simplify $\frac{x}{3} = -15$ to $x = -45$. The Multiplication Property of Equality is the property used in the statement.

ANSWER:

Mult. Prop.

11. If $4x - 5 = x + 12$, then $4x = x + 17$.

SOLUTION:

$$\begin{aligned} 4x-5&=x+12 && \text{Original equation} \\ 4x-5+5&=x+12+5 && \text{Add 5 to each side.} \\ 4x&=x-17 && \text{Simplify.} \end{aligned}$$

Add 5 to each side to simplify $4x - 5 = x + 12$ to $4x = x + 17$. The Addition Property of Equality is the property used in the statement.

ANSWER:

Add. Prop.

12. If $\frac{1}{5}BC = \frac{1}{5}DE$, then $BC = DE$

SOLUTION:

$$\begin{aligned} \tfrac{1}{5}BC &= \tfrac{1}{5}DE && \text{Original equation} \\ 5\left(\tfrac{1}{5}BC\right) &= 5\left(\tfrac{1}{5}DE\right) && \text{Multiply each side by 5.} \\ BC &= DE && \text{Simplify.} \end{aligned}$$

Multiply each side by 5 to simplify $\frac{1}{5}BC = \frac{1}{5}DE$ to $BC = DE$. The Multiplication or Division Property of Equality is the property used in this statement.

ANSWER:

Mult. or Div. Prop.

State the property that justifies each statement.

13. If $5(x + 7) = –3$, then $5x + 35 = –3$.

SOLUTION:

$$\begin{aligned} 5(x+7) &= -3 && \text{Original equation} \\ 5(x) + 5(7) &= -3 && \text{Distributive Property} \\ 5x + 35 &= -3 && \text{Simplify.} \end{aligned}$$

The Distributive Property is used to simplify $5(x + 7) = –3$ to $5x + 35 = –3$. The Distributive Property is the property used in the statement.

ANSWER:

Dist. Prop.

14. If $m\angle 1 = 25$ and $m\angle 2 = 25$, then $m\angle 1 = m\angle 2$.

SOLUTION:

We are given that $m\angle 1 = 25$ and $m\angle 2 = 25$. Since both angles measure 25, $m\angle 1 = m\angle 2$. The Substitution Property of Equality is use in this statement.

ANSWER:

Subs.

15. If $AB = BC$ and $BC = CD$, then $AB = CD$.

SOLUTION:

In the two part hypothesis, $AB = BC$ and $BC = CD$, the end of first part and start of second part are the same BC. The conclusion $AB = CD$, has the first part of first the hypothesis and the second part of the second hypothesis. Thus, the Transitive Property is the property used in the statement.

ANSWER:

Trans. Prop.

16. If $3\left(x-\frac{2}{3}\right)=4$, then $3x - 2 = 4$.

SOLUTION:

$$3\left(x-\frac{2}{3}\right)=4 \quad \text{Original equation}$$
$$3(x)+3\left(-\frac{2}{3}\right)=4 \quad \text{Distributive Property}$$
$$3x-2=4 \quad \text{Simplify.}$$

Multiply both sides by 3 to simplify $3\left(x-\frac{2}{3}\right)=4$ to $3x - 2 = 4$.
The Distributive Property is the property used in the statement.

ANSWER:

Dist. Prop.

CCSS ARGUMENTS Complete each proof.

17. **Given:** $\frac{8-3x}{4}=32$

Prove: $x = -40$

Proof:

Statements	Reasons
a. $\frac{8-3x}{4}=32$	**a.** Given
b. $4\left(\frac{8-3x}{4}\right)=4(32)$	**b.** ___?___
c. $8-3x=128$	**c.** ___?___
d. ___?___	**d.** Subtraction Property
e. $x=-40$	**e.** ___?___

SOLUTION:

The 2nd row is identified as multiplication by the 4s outside the parentheses.
The 3rd row is found by distributing the 4s, using the Substitution Property.
The 4th row is the subtraction of 8.

$$8-3x=128$$
$$8-8-3x=-8 \quad \text{Subtract 8 from each side.}$$
$$-3x=120 \quad \text{Simplify.}$$

The 5th row is found by dividing each side by –3.

$$-3x=120$$
$$\frac{-3x}{-3}=\frac{120}{-3} \quad \text{Subtract 8 from each side.}$$
$$x=-40 \quad \text{Divide each side by } -3.$$

Statements	Reasons
a. $\frac{8-3x}{4}=32$	a. Given
b. $4\left(\frac{8-3x}{4}\right)=4(32)$	b. Multiplication Property
c. $8-3x=128$	c. Substitution
d. $-3x=120$	d. Subtraction Property
e. $x=-40$	e. Division Property

ANSWER:

b. Multiplication Property of Equality
c. Substitution
d. $-3x = 120$
e. Division Property of Equality

18. **Given:** $\frac{1}{5}x+3=2x-24$

Prove: $x = 15$
Proof:

Statements	Reasons
a. ?	a. Given
b. ?	b. Multiplication Property
c. $x+15=10x-120$	c. ?
d. ?	d. Subtraction Property
e. $135=9x$	e. ?
f. ?	f. Division Property
g. ?	g. Symmetric Property

SOLUTION:

The 1st row is is the given $\frac{1}{5}x+3=2x-24$.

The 2nd row is identified as multiplication by the 5s outside the parentheses.
The 3rd row is found by distributing the 5s, using the substitution property.

The 4th row is the subtraction of x.

2-6 Algebraic Proof

$$\begin{aligned} x+15 &= 10x-120 \\ x-x+15 &= 10x-x-120 && \text{Subtract } x \text{ from each side.} \\ 15 &= x-120 && \text{Simplify.} \end{aligned}$$

The 5th row is the addition of 120.

$$\begin{aligned} 15 &= 9x-120 \\ 15+120 &= 9x-120+120 && \text{Add 120 to each side.} \\ 135 &= 9x && \text{Simplify.} \end{aligned}$$

The 6th row is the division of each side by 9.

$$\begin{aligned} 135 &= 9x \\ \frac{135}{9} &= \frac{9x}{9} && \text{Divide each side by 9.} \\ 15 &= x && \text{Simplify.} \end{aligned}$$

The 7th row is switching *x* from the right side of the equation to the left.

Statements	Reasons
a. $\frac{1}{5}x+3=2x-24$	a. Given
b. $5\left(\frac{1}{5}x+3\right)=5(2x-24)$	b. Multiplication Property
c. $x+15=10x-120$	c. Substitution
d. $15=9x-120$	d. Subtraction Property
e. $135=9x$	e. Addition Property
f. $15=x$	f. Division Property
g. $x=15$	g. Symmetric Property

ANSWER:

a. $\frac{1}{5}x+3=2x-24$

b. $5\left(\frac{1}{5}x+3\right)=5(2x-24)$

c. Substitution

d. $15 = 9x - 120$

e. Addition Property of Equality

f. $15 = x$

g. $x = 15$

PROOF Write a two-column proof to verify each conjecture.

19. If $-\frac{1}{3}n = 12$, then $n = -36$

SOLUTION:

You need to walk through the proof step by step. Look over what you are given and what you need to prove. Here, you are given an equation. Use the properties that you have learned about equivalent equations in algebra to walk through the proof.

Given: $-\frac{1}{3}n = 12$

Prove: $n = -36$

Proof:

Statements (Reasons)

1. $-\frac{1}{3}n = 12$ (Given)
2. $-3\left(-\frac{1}{3}n\right) = -3(12)$ (Multiplication. Property)
3. $n = -36$ (Substitution)

ANSWER:

Given: $-\frac{1}{3}n = 12$

Prove: $n = -36$

Proof:

Statements (Reasons)

1. $-\frac{1}{3}n = 12$ (Given)
2. $-3\left(-\frac{1}{3}n\right) = -3(12)$ (Mult. Prop.)
3. $n = -36$ (Substitution)

20. If $-3r+\frac{1}{2}=4$, then $r=-\frac{7}{6}$

SOLUTION:

You need to walk through the proof step by step. Look over what you are given and what you need to prove. Here, you are given an equation. Use the properties that you have learned about equivalent equations in algebra to walk through the proof.

Given: $-3r+\frac{1}{2}=4$

Prove: $r=-\frac{7}{6}$

Proof:

Statements (Reasons)

1. $-3r+\frac{1}{2}=4$ (Given)
2. $2\left(-3r+\frac{1}{2}\right)=2(4)$ (Multiplication Property)
3. $-6r + 1 = 8$ (Distributive Property and Substitution)
4. $-6r = 7$ (Subtraction Property)
5. $r=-\frac{7}{6}$ (Division Property.)

ANSWER:

Given: $-3r+\frac{1}{2}=4$

Prove: $r=-\frac{7}{6}$

Proof:

Statements (Reasons)

1. $-3r+\frac{1}{2}=4$ (Given)
2. $2\left(-3r+\frac{1}{2}\right)=2(4)$ (Mult. Prop.)
3. $-6r + 1 = 8$ (Dist. Prop. and Substitution)
4. $-6r = 7$ (Subt. Prop.)
5. $r=-\frac{7}{6}$ (Div. Prop.)

21. **SCIENCE** Acceleration a in feet per second squared, distance traveled d in feet, velocity v in feet per second, and time t in seconds are related in the formula $d=vt+\frac{1}{2}at^2$.

a. Prove that if the values for distance, velocity, and time are known, then the acceleration of an object can be calculated using the formula $a=\frac{2d-2vt}{t^2}$.

b. If an object travels 2850 feet in 30 seconds with an initial velocity of 50 feet per second, what is the acceleration of the object? What property justifies your calculation?

SOLUTION:

You need to walk through the proof step by step. Look over what you are given and what you need to prove. Here, you are given the formula for distance traveled. Use the properties that you have learned about equivalent equations in algebra to walk through the proof.

a.

Given: $d = vt + \frac{1}{2}at^2$

Prove: $\frac{2d - 2vt}{t^2} = a$

Proof:

Statements (Reasons)

1. $d = vt + \frac{1}{2}at^2$ (Given)
2. $d - vt = vt - vt + \frac{1}{2}at^2$ (Subtraction Property)
3. $d - vt = \frac{1}{2}at^2$ (Substitution)
4. $2(d - vt) = 2\left(\frac{1}{2}at^2\right)$(Multiplication Property)
5. $2(d - vt) = at^2$(Substitution.)
6. $2d - 2vt = at^2$ (Distributive Property)
7. $\frac{2d - 2vt}{t^2} = \frac{at^2}{t^2}$(Division Property)
8. $\frac{2d - 2vt}{t^2} = a$ (Substitution.)
9. $a = \frac{2d - 2vt}{t^2}$(Symmetric. Property)

b. Substitute d = 2850, t = 30, and v = 50 in the formula.

$$a = \frac{2(2850) - 2(50)(30)}{(30)^2}$$

$$= 3$$

Therefore, the acceleration is 3 ft/ sec^2. We substituted d, t, and v to find the acceleration. So, the property used is the Substitution Property of Equality.

ANSWER:

a.

Given: $d = vt + \frac{1}{2}at^2$

Prove: $\frac{2d - 2vt}{t^2} = a$

Proof:

Statements (Reasons)

2-6 Algebraic Proof

1. $d = vt + \frac{1}{2}at^2$ (Given)
2. $d - vt = vt - vt + \frac{1}{2}at^2$ (Subt. Prop.)
3. $d - vt = \frac{1}{2}at^2$ (Subs.)
4. $2(d - vt) = 2\left(\frac{1}{2}at^2\right)$ (Mult. Prop.)
5. $2(d - vt) = at^2$ (Subs.)
6. $2d - 2vt = at^2$ (Dist. Prop.)
7. $\frac{2d - 2vt}{t^2} = \frac{at^2}{t^2}$ (Div. Prop.)
8. $\frac{2d - 2vt}{t^2} = a$ (Subs.)
9. $a = \frac{2d - 2vt}{t^2}$ (Sym. Prop.)

b. 3 ft/sec^2;

22. **CCSS ARGUMENTS** The Ideal Gas Law is given by the formula $PV = nRT$, where P = pressure in atmospheres, V = volume in liters, n = the amount of gas in moles, R is a constant value, and T = temperature in degrees Kelvin.

a. Prove that if the pressure, volume, and amount of the gas are known, then the formula $T = \frac{PV}{nR}$ gives the temperature of the gas.

b. If you have 1 mole of oxygen with a volume of 25 liters at a pressure of 1 atmosphere, what is the temperature of the gas? The value of R is 0.0821. What property justifies your calculation?

SOLUTION:

a. You need to walk through the proof step by step. Look over what you are given and what you need to prove. Here, you are given the formula for the Ideal Gas Law. Use the properties that you have learned equivalent equations in algebra to walk through the proof.

Proof:

Statements (Reasons)

1. $PV = nRT$ (Given)
2. $\frac{PV}{nR} = \frac{nRT}{nR}$ (Division Property)
3. $\frac{PV}{nR} = T$ (Substitution)

b. Substitute $n = 1$, $V = 25$, $P = 1$ and $R = 0.0821$ in the formula.

$$T = \frac{(1)(25)}{(1)(0.0821)}$$
$$\approx 305$$

Therefore, the temperature is about 305 degrees Kelvin. We substituted n, V, P, and R to find the temperature. So, the property used is the Substitution Property of Equality.

ANSWER:

a. Proof:

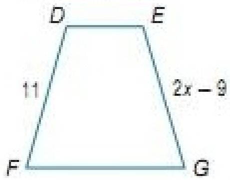
Statements (Reasons)

1. $PV = nRT$ (Given)
2. $\frac{PV}{nR} = \frac{nRT}{nR}$ (Div. Prop.)
3. $\frac{PV}{nR} = T$ (Substitution)

b. 305 degrees Kelvin; subs.

PROOF Write a two-column proof.

23. If $\overline{DF} \cong \overline{EG}$, then x = 10.



SOLUTION:

You need to walk through the proof step by step. Look over what you are given and what you need to prove. Here, you are given two congruent segments of a trapezoid. Use the properties that you have learned about congruent segments and equivalent expressions in algebra to walk through the proof.

Given: $\overline{DF} \cong \overline{EG}$

Prove: x = 10

Proof:

Statements (Reasons)

1. $\overline{DF} \cong \overline{EG}$ (Given)
2. $DF = EG$ (Definition of congruent segments)
3. 11 = 2x – 9 (Substitution)
4. 20 = 2x (Addition Property)
5. 10 = x (Division Property)
6. x = 10 (Symmetric Property)

ANSWER:

Given: $\overline{DF} \cong \overline{EG}$

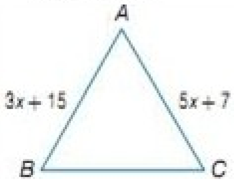
Prove: x = 10

Proof:

Statements (Reasons)

1. $\overline{DF} \cong \overline{EG}$ (Given)
2. $DF = EG$ (Def. of $\cong$ segs)
3. 11 = 2x – 9 (Subs.)
4. 20 = 2x (Add. Prop.)
5. 10 = x (Div. Prop.)
6. x = 10 (Symm. Prop.)

24. If $\overline{AB} \cong \overline{AC}$, then $x = 4$.



SOLUTION:

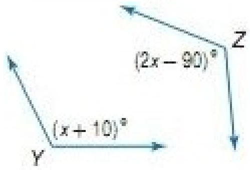
You need to walk through the proof step by step. Look over what you are given and what you need to prove. Here, you are given two congruent segments of a triangle. Use the properties that you have learned about congruent segments and equivalent expressions in algebra to walk through the proof.

Given: $\overline{AB} \cong \overline{AC}$
Prove: $x = 4$
Proof:
Statements (Reasons)
1. $\overline{AB} \cong \overline{AC}$ (Given)
2. $AB = AC$ (Definition of congruent segments)
3. $3x + 15 = 5x + 7$ (Substitution)
4. $8 = 2x$ (Subtraction)
5. $4 = x$ (Division Property)
6. $x = 4$ (Symmetric. Property)

ANSWER:

Given: $\overline{AB} \cong \overline{AC}$
Prove: $x = 4$
Proof:
Statements (Reasons)
1. $\overline{AB} \cong \overline{AC}$ (Given)
2. $AB = AC$ (Def. of $\cong$ segs)
3. $3x + 15 = 5x + 7$ (Subs.)
4. $8 = 2x$ (Subt.)
5. $4 = x$ (Div. Prop.)
6. $x = 4$ (Symm. Prop.)

25. If $\angle Y \cong \angle Z$, then $x = 100$.



SOLUTION:

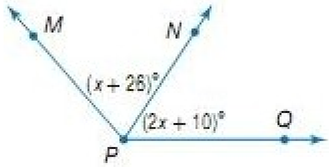
You need to walk through the proof step by step. Look over what you are given and what you need to prove. Here, you are given two congruent angles. Use the properties that you have learned about congruent angles and equivalent expressions in algebra to walk through the proof.

Given: $\angle Y \cong \angle Z$
Prove: $x = 100$
Proof:
Statements (Reasons)
1. $\angle Y \cong \angle Z$ (Given)
2. $m\angle Y = m\angle Z$ (Definition of congruent angles)
3. $x + 10 = 2x - 90$ (Substitution)
4. $10 = x - 90$ (Subtraction Property)
5. $100 = x$ (Addition Property)
6. $x = 100$ (Symmetric Property)

ANSWER:

Given: $\angle Y \cong \angle Z$
Prove: $x = 100$
Proof:
Statements (Reasons)
1. $\angle Y \cong \angle Z$ (Given)
2. $m\angle Y = m\angle Z$ (Def. of $\cong$ $\angle$s)
3. $x + 10 = 2x - 90$ (Subs.)
4. $10 = x - 90$ (Subt. Prop.)
5. $100 = x$ (Add. Prop.)
6. $x = 100$ (Sym. Prop.)

26. If $\angle MPN \cong \angle QPN$, then $x = 16$.



SOLUTION:

You need to walk through the proof step by step. Look over what you are given and what you need to prove. Here, you are given two congruent angles. Use the properties that you have learned about congruent angles and equivalent expressions in algebra to walk through the proof.

Given: $\angle MPN \cong \angle QPN$
Prove: $x = 16$
Proof:
Statements (Reasons)
1. $\angle MPN \cong \angle QPN$ (Given)
2. $m\angle MPN = m\angle QPN$ (Definition of congruent angles)
3. $x + 26 = 2x + 10$ (Substitution.)
4. $16 = x$ (Subtraction Property)
5. $x = 16$ (Symmetric Property)

ANSWER:

Given: $\angle MPN \cong \angle QPN$
Prove: $x = 16$
Proof:
Statements (Reasons)
1. $\angle MPN \cong \angle QPN$ (Given)
2. $m\angle MPN = m\angle QPN$ (Def. of $\cong \angle s$)
3. $x + 26 = 2x + 10$ (Subs.)
4. $16 = x$ (Subt. Prop.)
5. $x = 16$ (Symm. Prop.)

27. **ELECTRICITY** The voltage V of a circuit can be calculated using the formula $V = \frac{P}{I}$, where P is the power and I is the current of the circuit.
a. Write a proof to show that when the power is constant, the voltage is halved when the current is doubled.
b. Write a proof to show that when the current is constant, the voltage is doubled when the power is doubled.

SOLUTION:

You need to walk through the proof step by step. Look over what you are given and what you need to prove. Here, you are given the formula for the voltage of a circuit. Use the properties that you have learned about equivalent equations in algebra to walk through the proof.

a. Given: $V = \frac{P}{I}$
Prove: $\frac{V}{2} = \frac{P}{2I}$
Proof:
Statements (Reasons)

1. $V = \frac{P}{I}$ (Given)

2. $\frac{1}{2} \cdot V = \frac{1}{2} \cdot \frac{P}{I}$ (Multiplication Property)

3. $\frac{V}{2} = \frac{P}{2I}$ (Multiplication Property)

b. Given: $V = \frac{P}{I}$

Prove: $2V = \frac{2P}{I}$

Proof:
Statements (Reasons)

1. $V = \frac{P}{I}$ (Given)

2. $2 \cdot V = 2 \cdot \frac{P}{I}$ (Multiplication Property)

3. $2V = \frac{2P}{I}$ (Multiplication Property.)

ANSWER:

a. Given: $V = \frac{P}{I}$

Prove: $\frac{V}{2} = \frac{P}{2I}$

Proof:
Statements (Reasons)

1. $V = \frac{P}{I}$ (Given)

2. $\frac{1}{2} \cdot V = \frac{1}{2} \cdot \frac{P}{I}$ (Mult. Prop.)

3. $\frac{V}{2} = \frac{P}{2I}$ (Mult. Prop.)

b. Given: $V = \frac{P}{I}$

Prove: $2V = \frac{2P}{I}$

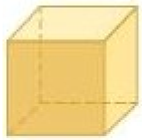
Proof:
Statements (Reasons)

1. $V = \frac{P}{I}$ (Given)

2. $2 \cdot V = 2 \cdot \frac{P}{I}$ (Mult. Prop.)

3. $2V = \frac{2P}{I}$ (Mult. Prop.)

28. **MULTIPLE REPRESENTATIONS** Consider a cube with a side length of s.

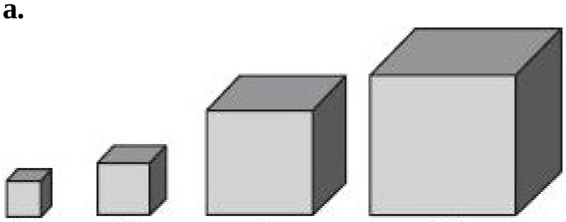


a. CONCRETE Sketch or build a model of cubes with side lengths of 2, 4, 8, and 16 units.
b. TABULAR Find the volume of each cube. Organize your results into a table like the one shown.

Side Length (s)	Volume (V)
2	
4	
8	
16	

c. VERBAL Use your table to make a conjecture about the change in volume when the side length of a cube is doubled. Express your conjecture in words.
d. ANALYTICAL Write your conjecture as an algebraic equation.
e. LOGICAL Write a proof of your conjecture. Be sure to write the Given and Prove statements at the beginning of your proof.

SOLUTION:
a.



b. The volume of a cube of side s is given by the formula $V = s^3$. Use the formula to find the volumes of the above cubes.

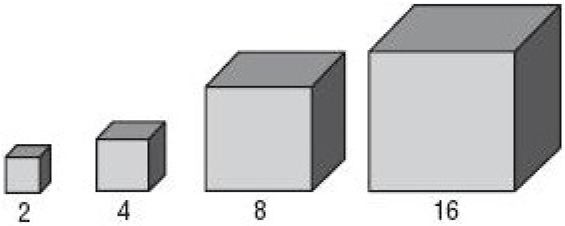
Side Length (s)	Volume (V)
2	8
4	64
8	512
16	4096

c. When the side length of a cube doubles, the volume is 8 times greater.
d. $8V = (2s)^3$
e. You need to walk through the proof step by step. Look over what you are given and what you need to prove. Here, you are given a cube and with volume V and the volume formula. Use the properties that you have learned about equivalent equations in algebra to walk through the proof.
Given: a cube with side length s and volume V
Prove: $8V = (2s)^3$
Proof:
Statements (Reasons)
1. side length = s (Given)
2. volume = V (Given)
3. $V = s^3$ (Definition of volume of a cube)
4. $V = s \cdot s \cdot s$ (Definition of exponent)
5. $(2)(2)(2)(V) = (2)(s)(2)(s)(2)(s)$ (Multiplication Property)

6. $8V = (2s)(2s)(2s)$ (Multiplication Property)
7. $8V = (2s)^3$ (Definition of exponent)

ANSWER:

a.



b.

Side Length (s)	Volume (V)
2	8
4	64
8	512
16	4096

c. Sample answer: When the side length of a cube doubles, the volume is 8 times greater.

d. $8V = (2s)^3$

e. Given: a cube with side length s and volume V

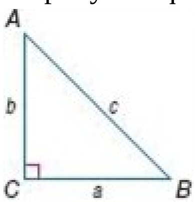
Prove: $8V = (2s)^3$

Proof:

Statements (Reasons)

1. side length = s (Given)
2. volume = V (Given)
3. $V = s^3$ (Def. of volume of a cube)
4. $V = (s)(s)(s)$ (Def. of exponent)
5. $(2)(2)(2)(V) = (2)(s)(2)(s)(2)(s)$ (Mult. Prop.)
6. $8V = (2s)(2s)(2s)$ (Mult. Prop.)
7. $8V = (2s)^3$ (Def. of exponent)

29. **PYTHAGOREAN THEOREM** The Pythagorean Theorem states that in a right triangle *ABC*, the sum of the squares of the measures of the lengths of the legs, a and b, equals the square of the measure of the hypotenuse c, or $a^2 + b^2 = c^2$. Write a two-column proof to verify that $a = \sqrt{c^2 - b^2}$. Use the Square Root Property of Equality, which states that if $a^2 = b^2$, then $a = \pm\sqrt{b^2}$.



SOLUTION:

You need to walk through the proof step by step. Look over what you are given and what you need to prove. Here, you are given and equation. Use the properties that you have learned about equivalent equations in algebra to walk through the proof.

Given: $c^2 = a^2 + b^2$

Prove: $a = \sqrt{c^2 - b^2}$

Proof:

Statements (Reasons)

1. $a^2 + b^2 = c^2$ (Given)
2. $a^2 + b^2 - b^2 = c^2 - b^2$ (Subtraction Property)
3. $a^2 = c^2 - b^2$ (Substitution)
4. $a = \pm\sqrt{c^2 - b^2}$ (Square Root Property)
5. $a = \sqrt{c^2 - b^2}$ (Length cannot be negative.)

ANSWER:

Given: $c^2 = a^2 + b^2$

Prove: $a = \sqrt{c^2 - b^2}$

Proof:

Statements (Reasons)

1. $a^2 + b^2 = c^2$ (Given)
2. $a^2 + b^2 - b^2 = c^2 - b^2$ (Subt. Prop.)
3. $a^2 = c^2 - b^2$ (Substitution)
4. $a = \pm\sqrt{c^2 - b^2}$ (Sq. Root Prop.)
5. $a = \sqrt{c^2 - b^2}$ (Length cannot be negative.)

An *equivalence relation* is any relationship that satisfies the Reflexive, Symmetric, and Transitive Properties. For real numbers, equality is one type of equivalence relation. Determine whether each relation is an equivalence relation. Explain your reasoning.

30. "has the same birthday as", for the set of all human beings

SOLUTION:

Reflexive: $a = a$
Symmetric: If $a = b$, then $b = a$
Transitive: If $a = b$ and $b = c$, then $a = c$

The relation "has the same birthday as" is an equivalence relation because it satisfies all three properties. You can have the same birthday as yourself (reflexive); if you have the same birthday as your friend, then your friend has the same birthday as you (symmetric); if you have the same birthday as Bob and Bob has the same birthday as Bill, then you have the same birthday as Bill (transitive).

ANSWER:

The relation "has the same birthday as" is an equivalence relation because it satisfies all three properties. Sample answer: You can have the same birthday as yourself (reflexive); if you have the same birthday as your friend, then your friend has the same birthday as you (symmetric); if you have the same birthday as Bob and Bob has the same birthday as Bill, then you have the same birthday as Bill (transitive).

31. "is taller than", for the set of all human beings

SOLUTION:

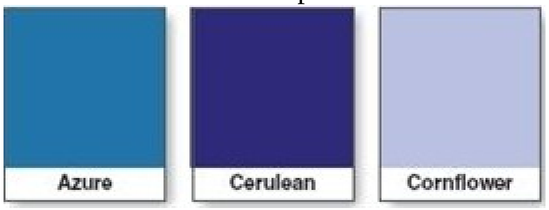
Reflexive: $a = a$
Symmetric: If $a = b$, then $b = a$
Transitive: If $a = b$ and $b = c$, then $a = c$

The relation "is taller than" is not an equivalence relation because it fails the Reflexive and Symmetric properties. You cannot be taller than yourself (reflexive); if you are taller than your friend, then it does not imply that your friend is taller than you (symmetric).

ANSWER:

The relation "is taller than" is not an equivalence relation because it fails the Reflexive and Symmetric properties. You cannot be taller than yourself (reflexive); if you are taller than your friend, then it does not imply that your friend is taller than you (symmetric).

32. "is bluer than" for all the paint colors with blue in them



SOLUTION:

Reflexive: $a = a$
Symmetric: If $a = b$, then $b = a$
Transitive: If $a = b$ and $b = c$, then $a = c$

The relation "is bluer than" is not an equivalence relation because it fails the Reflexive property. A color cannot be bluer than itself.

ANSWER:

The relation "is bluer than" is not an equivalence relation because it fails the Reflexive property. A color cannot be bluer than itself.

33. $\neq$, for the set of real numbers

SOLUTION:

Reflexive: $a = a$
Symmetric: If $a = b$, then $b = a$
Transitive: If $a = b$ and $b = c$, then $a = c$

The relation "$\neq$" is not an equivalence relation because it fails the Reflexive Property, since $a \neq a$ is not true.

ANSWER:

The relation "$\neq$" is not an equivalence relation because it fails the Reflexive Property, since $a \neq a$ is not true.

34. $\geq$, for the set of real numbers

SOLUTION:

Reflexive: $a = a$
Symmetric: If $a = b$, then $b = a$
Transitive: If $a = b$ and $b = c$, then $a = c$

The relation "$\geq$" is not an equivalence relation because it fails the Symmetric Property, since $2 \geq 3$ does not imply $3 \geq 2$.

ANSWER:

The relation "$\geq$" is not an equivalence relation because it fails the Symmetric Property, since $2 \geq 3$ does not imply $3 \geq 2$.

35. $\approx$, for the set of real numbers

SOLUTION:

Reflexive: $a = a$
Symmetric: If $a = b$, then $b = a$
Transitive: If $a = b$ and $b = c$, then $a = c$

The relation "≈" is not an equivalence relation because it fails the Reflexive Property, since $a \approx a$ is not true.

ANSWER:

The relation "≈" is not an equivalence relation because it fails the Reflexive Property, since $a \approx a$ is not true.

36. **OPEN ENDED** Give one real-world *example* and one real-world *non-example* of the Symmetric, Transitive, and Substitution properties.

SOLUTION:

Reflexive: $a = a$
Symmetric: If $a = b$, then $b = a$
Transitive: If $a = b$ and $b = c$, then $a = c$

Symmetric example: Sarah is Stacy's sister, and Stacy is Sarah's sister.
Symmetric Non-example: If Miko is shorter than Sebastian, then Sebastian is shorter than Miko.

Substitution example: When a player leaves the court in a basketball game to rest, the coach substitutes another player.
Substitution Non-example: A person with a blood type of O negative can give blood to someone of any blood type, but can only receive blood type O negative.

Transitive example: If Jorge is younger than Tomas and Tomas is younger that Gabby, then Jorge is younger than Gabby.
Transitive Non-example: School A defeated school B, and school B defeated school C, then school A will defeat school C.

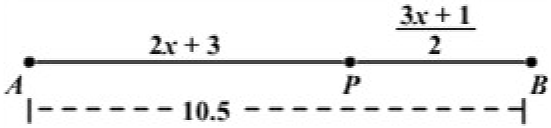
ANSWER:

Sample answer: Symmetric example: Sarah is Stacy's sister, and Stacy is Sarah's sister. Symmetric Non-example: If Miko is shorter than Sebastian, then Sebastian is shorter than Miko. Substitution example: When a player leaves the court in a basketball game to rest, the coach substitutes another player. Substitution Non-example: A person with a blood type of O negative can give blood to someone of any blood type, but can only receive blood type O negative. Transitive example: If Jorge is younger than Tomas and Tomas is younger that Gabby, then Jorge is younger than Gabby. Transitive Non-example: School A defeated school B, and school B defeated school C, then school A will defeat school C.

37. **CCSS SENSE-MAKING** Point *P* is located on $\overline{AB}$. The length of $\overline{AP}$ is $2x + 3$, and the length of $\overline{PB}$ is $\frac{3x+1}{2}$. Segment *AB* is 10.5 units long. Draw a diagram of this situation, and prove that point *P* is located two thirds of the way between point *A* and point *B*.

SOLUTION:

You need to walk through the proof step by step. Look over what you are given and what you need to prove. Here, you are given three segments, a value for one of the segments, and an expression for the other two segments. Once you prove the values are equal, you will need to find the variable in the expression. Use the properties that you have learned about congruent segments and equivalent expressions in algebra to walk through the proof.



Given: $AP = 2x + 3$, $PB = \frac{3x+1}{2}$, and $AB = 10.5$

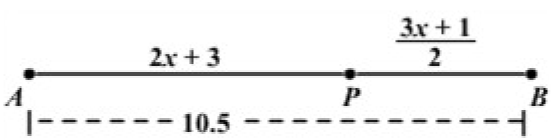
Prove: $\frac{AP}{AB} = \frac{2}{3}$

Proof:

Statements (Reasons)

1. $AP = 2x + 3$, $PB = \frac{3x+1}{2}$, $AB = 10.5$ (Given)
2. $AP + PB = AB$ (Definition of a segment)
3. $2x+3+\frac{3x+1}{2}=10.5$ (Subtraction)
4. $2\left(2x+3+\frac{3x+1}{2}\right)=2(10.5)$ (Multiplication Property)
5. $2\left(2x+3+\frac{3x+1}{2}\right)=21$ (Subs.)
6. $2(2x)+2(3)+2\left(\frac{3x+1}{2}\right)=21$ (Distributive Property)
7. $4x + 6 + 3x + 1 = 21$ (Multiply)
8. $7x + 7 = 21$ (Add)
9. $7x + 7 - 7 = 21 - 7$ (Subtraction Property)
10. $7x = 14$ (Substitution)
11. $x = 2$ (Division Property)
12. $AP = 2(2) + 3$ (Substitution)
13. $AP = 4 + 3$ (Multiply)
14. $AP = 7$ (Addition)
15. $\frac{AP}{AB}=\frac{7}{10.5}$ (Substitution.)
16. $\frac{AP}{AB}=0.\bar{6}$ (Divide)
17. $\frac{2}{3}=0.\bar{6}$ (Known)
18. $\frac{AP}{AB}=\frac{2}{3}$ (Transitive Property)

ANSWER:



Given: $AP = 2x + 3$

$PB = \frac{3x+1}{2}$

$AB = 10.5$

Prove: $\frac{AP}{AB} = \frac{2}{3}$

Proof:

Statements (Reasons)

1. $AP = 2x + 3$, $PB = \frac{3x+1}{2}$, $AB = 10.5$ (Given)
2. $AP + PB = AB$ (Def. of a segment)
3. $2x + 3 + \frac{3x+1}{2} = 10.5$ (Subs.)
4. $2\left(2x + 3 + \frac{3x+1}{2}\right) = 2 \cdot 10.5$ (Mult. Prop.)
5. $2\left(2x + 3 + \frac{3x+1}{2}\right) = 21$ (Subs.)
6. $2(2x) + 2(3) + 2\left(\frac{3x+1}{2}\right) = 21$ (Dist. Prop.)
7. $4x + 6 + 3x + 1 = 21$ (Multiply)
8. $7x + 7 = 21$ (Add)
9. $7x + 7 - 7 = 21 - 7$ (Subt. Prop.)
10. $7x = 14$ (Subs.)
11. $x = 2$ (Div. Prop.)
12. $AP = 2(2) + 3$ (Subs.)
13. $AP = 4 + 3$ (Multiply)
14. $AP = 7$ (Add)
15. $\frac{AP}{AB} = \frac{7}{10.5}$ (Subs.)
16. $\frac{AP}{AB} = 0.\overline{6}$ (Divide)
17. $\frac{2}{3} = 0.\overline{6}$ (Known)
18. $\frac{AP}{AB} = \frac{2}{3}$ (Trans. Prop.)

REASONING Classify each statement below as *sometimes, always,* or *never* true. Explain your reasoning.

38. If a and b are real numbers and $a + b = 0$, then $a = -b$.

SOLUTION:

If $a + b = 0$, then $a + b - b = 0 - b$ by the Subtraction. Property. Simplify and $a = -b$. Therefore, this statement is *always* true.

ANSWER:

Always; Sample answer: If $a + b = 0$, then $a + b - b = 0 - b$ (Subt. Prop.) and $a = -b$ (Simplify). Therefore, this statement is always true.

39. If a and b are real numbers and $a^2 = b$, then $a = \sqrt{b}$.

SOLUTION:

The statement "If a and b are real numbers and $a^2 = b$, then $a = \sqrt{b}$." is sometimes true.
If $a^2 = 1$ and $a = 1$, then $b = \sqrt{1}$ or 1. The statement is false if $a = -1$ and $b = 1$, since $-1 \neq \sqrt{1}$. If $b = 1$, then $\sqrt{b} = 1$ since the square root of a number is nonnegative. Therefore, the statement is sometimes true.

ANSWER:

Sometimes; sample answer: If $a^2 = 1$ and $a = 1$, then $b = \sqrt{1}$ or 1. The statement is also true if $a = -1$ and $b = 1$. If $b = 1$, then $\sqrt{b} = 1$ since the square root of a number is nonnegative. Therefore, the statement is sometimes true.

40. **CHALLENGE** Ayana makes a conjecture that the sum of two odd integers is an even integer.
a. List information that supports this conjecture. Then explain why the information you listed does not prove that this conjecture is true.
b. Two odd integers can be represented by the expressions $2n - 1$ and $2m - 1$, where n and m are both integers. Give information that supports this statement.
c. If a number is even, then it is a multiple of what number? Explain in words how you could use the expressions in part **a** and your answer to part **b** to prove Ayana's conjecture.
d. Write an algebraic proof that the sum of two odd integers is an even integer.

SOLUTION:

a. 3 + 3 = 6, 5 + 7 =12, 7 + 9 = 16. The information listed does not represent every odd integer but only a sample of odd numbers. The sample does not show the conjecture is true for every odd number.
b. For example, the odd number 3 is represented as 3 = 2(2) – 1. The number 5 as 5 =2(3) – 1 and 7 as 7 = 2(4) – 1
c. Even numbers are multiples of 2. Add to odd numbers to get an even number.

$$2n - 1 + 2m - 1 = 2n + 2m - 2$$
$$= 2(n + m + 2)$$

Fir any integers n and m, the sum is a multiple of 2.
d. You need to walk through the proof step by step. Look over what you are given and what you need to prove. Here, you are given expressions for two odd numbers. Use the properties that you have learned about equivalent equations in algebra to walk through the proof.

Let two odd integers be represented by $2n - 1$ and $2m - 1$. The sum $(2n - 1) + (2m - 1)$ is equal to $2n + 2m - 2$. Each term has a two as a factor so by factoring out a 2 we get $2(n + m - 1)$. Since this expression is a multiple of 2 it is an even number. Hence, the sum of two odd integers is an even integer.

ANSWER:

a. Sample answer: 3 + 3 = 6, 5 + 7 =12, 7 + 9 = 16. The information listed does not represent every odd integer but only a sample of odd numbers. The sample does not show the conjecture is true for every odd number.
b. Sample answer: 3 = 2(2) – 1, 5 =2(3) – 1, 7 = 2(4) – 1
c. 2; Sample answer: I would add the expressions in part b and show that the sum is a multiple of 2.
d. Let two odd integers be represented by $2n - 1$ and $2m - 1$. The sum $(2n - 1) + (2m - 1)$ is equal to $2n + 2m - 2$. Each term has a two as a factor so by factoring out a 2 we get $2(n + m - 1)$. Since this expression is a multiple of 2 it is an even number. Hence, the sum of two odd integers is an even integer.

41. **WRITING IN MATH** Why is it useful to have different formats that can be used when writing a proof?

SOLUTION:

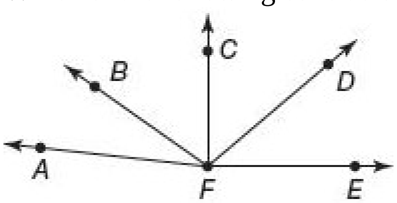
Think about proofs, the different types of proofs, the process of completing proofs and the differences in how this process is done among the different types of proofs. What are the pros and cons of the different types?

Sample answer: Depending on the purpose of the proof, one format may be preferable to another. For example, when writing an informal proof, you could use a paragraph proof to quickly convey your reasoning. When writing a more formal proof, a two-column proof may be preferable so that the justifications for each step are organized and easy to follow.

ANSWER:

Sample answer: Depending on the purpose of the proof, one format may be preferable to another. For example, when writing an informal proof, you could use a paragraph proof to quickly convey your reasoning. When writing a more formal proof, a two-column proof may be preferable so that the justifications for each step are organized and easy to follow.

42. In the diagram, $m\angle CFE = 90$ and $\angle AFB \cong \angle CFD$.
Which of the following conclusions does not have to be true?



A $m\angle BFD = m\angle BFD$

B $\overline{BF}$ bisects $\angle$AFD.

C $m\angle CFD = m\angle AFB$

D $\angle CFE$ is a right angle.

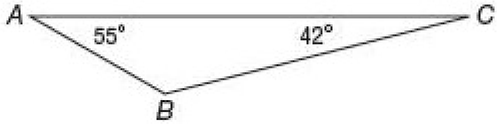
SOLUTION:

Given that $m\angle CFE = 90$ and $\angle AFB \cong \angle CFD$. So, $\angle CFE$ is a right angle and $m\angle AFB = m\angle CFD$ by definition.
The statement in option A is true by reflexive property.
The statement in option C is true by the definition of congruent angles.
The statement for option D is true by the definition of right angles.
The statement for option B does not have to be true. There is not enough information to determine if $\overline{BF}$ is an angle bisector.
So the correct choice is B.

ANSWER:

B

43. **SHORT RESPONSE** Find the measure of $\angle B$ when $m\angle A = 55$ and $m\angle C = 42$.



SOLUTION:

The sum of the measures of the three angles of a triangle is 180°. Here, $m\angle A = 55$ and $m\angle C = 42$.

$$\begin{aligned} m\angle B &= 180 - (55 + 42) \\ &= 180 - 97 \\ &= 83 \end{aligned}$$

ANSWER:

83^o

44. **ALGEBRA** Kendra's walk-a-thon supporters have pledged $30 plus $7.50 for each mile she walks. Rebecca's supporters have pledged $45 plus $3.75 for each mile she walks. After how many miles will Kendra and Rebecca have raised the same amount of money?

F 10
G 8
H 5
J 4

SOLUTION:

Let Kendra and Rebecca has to walk *x* miles to make the same amount of money. Then, the amount of money that Kendra makes is 30 + 7.5*x* and that Rebecca makes is 45 + 3.75*x* which are equal. Then the equation is $30 + 7.5x = 45 + 3.75x$.

$$\begin{aligned} 30 + 7.5x &= 45 + 3.75x && \text{Original equation} \\ 30 + 7.5x - 3.75x &= 45 + 3.75x - 3.75x && -3.75x \text{ from each side.} \\ 30 + 3.75x &= 45 && \text{Simplify} \\ 30 - 30 + 3.75x &= 45 - 30 && -30 \text{ from each side.} \\ 3.75x &= 15 && \text{Simplify.} \\ \frac{3.75x}{3.75} &= \frac{15}{3.75} && \div \text{ each side by } 3.75 \\ x &= 4 && \text{Simplify.} \end{aligned}$$

Therefore, the correct choice is J.

ANSWER:

J

45. **SAT/ACT** When 17 is added to $4m$, the result is $15z$. Which of the following equations represents the statement above?

A $17 + 15z = 4m$
B $(4m)(15z) = 17$
C $4m - 15z = 17$
D $17(4m) = 15z$
E $4m + 17 = 15z$

SOLUTION:

Add 17 to $4m$.
$4m + 17$
Equate it to $15z$.
$4m + 17 = 15z$
Therefore, the correct choice is E.

ANSWER:

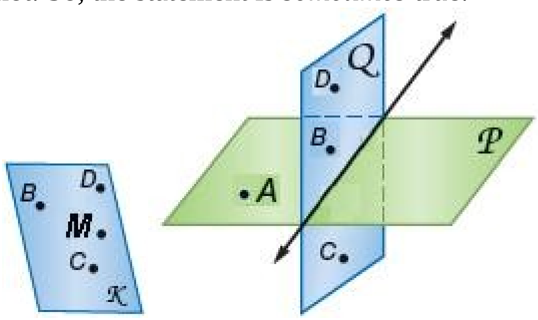
E

Determine whether the following statements are always, sometimes, or never true. Explain.

46. Four points will lie in one plane.

SOLUTION:

By Postulate 2.4, a plane contains at least 3 non-collinear points. The fourth point can be coplanar with the first 3, or not. So, the statement is *sometimes* true.



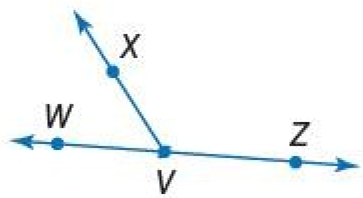
ANSWER:

Sometimes; Since a plane must consist of at least 3 points, the fourth point could lie in the same plane or in a different one.

47. Two obtuse angles will be supplementary.

SOLUTION:

The sum of the measure of two supplementary angles is 180, so two obtuse angles which will have a sum more than 180 can never be supplementary. Therefore, the statement is *never* true.



In the figure $\angle XVZ$ is obtuse, so $\angle XVW$ must be acute.

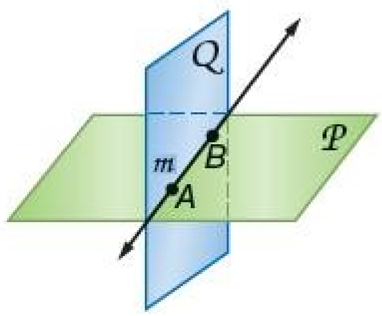
ANSWER:

Never; the sum of the measure of two supplementary angles is 180, so two obtuse angles can never be supplementary.

48. Planes *P* and *Q* intersect in line *m*. Line *m* lies in both plane *P* and plane *Q*.

SOLUTION:

Since the line is the intersection of two planes, the line lies in both of the planes. Therefore, the statement is *always* true.



ANSWER:

Always; since the line is the intersection of two planes, the line lies in both of the planes.

49. **ADVERTISING** An ad for Speedy Delivery Service says *When it has to be there fast, it has to be Speedy*. Catalina needs to send a package fast. Does it follow that she should use Speedy? Explain.

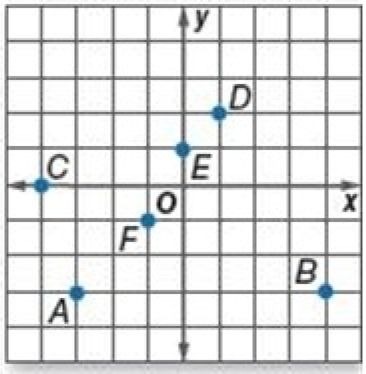
SOLUTION:

If $p \rightarrow q$ is a true statement and p is true, then q. Here, the statement "when it has to be there fast, it has to be Speedy" is a true statement and Catalina needs to send a package fast. So, by the Law of Detachment, she should use Speedy.

ANSWER:

yes; by the Law of Detachment

Write the ordered pair for each point shown.



50. *A*

SOLUTION:

A is 3 units left and 3 units down from the origin. so the *x*-coordinate is –3 and *y*-coordinate is –3. The point is (–3, –3).

ANSWER:

(–3, –3)

51. *B*

SOLUTION:

B is 4 units right and 3 units down from the origin. So the *x*-coordinate is 4 and *y*-coordinate is –3. The point is (4, –3)

ANSWER:

(4, –3)

52. *C*

SOLUTION:

C is 4 units left and 0 units down from the origin. So the *x*-coordinate is –4 and *y*-coordinate is 0. The point is(–4, 0)

ANSWER:

(–4, 0)

53. *D*

SOLUTION:

D is 1 units right and 2 units up from the origin. So the *x*-coordinate is 1 and *y*-coordinate is 2. The point is(1, 2)

ANSWER:

(1, 2)

54. *E*

SOLUTION:

E is 0 units left or right and 1 units up from the origin. So the *x*-coordinate is 0 and *y*-coordinate is 1. The point is (0, 1)

ANSWER:

(0, 1)

55. *F*

SOLUTION:

B is –1 units left and –1units down from the origin. So the *x*-coordinate is –1 and *y*-coordinate is **–1**. The point is(–1, –1)

ANSWER:

(–1, –1)

Find the measurement of each segment. Assume that each figure is not drawn to scale.

56. $\overline{ST}$

SOLUTION:

Since the point *S* lies between *R* and *T*, *RT* = *RS* + *ST*.
We have *RT* = 4.0 cm and *RS* = 1.2 cm.
So, *ST* = *RT* – *RS* = 4.0 – 1.2 = 2.8 cm.

ANSWER:

2.8 cm

57. $\overline{WX}$

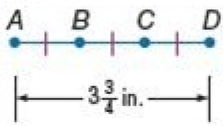
SOLUTION:

Since the point *X* is the midpoint of $\overline{WY}$, *WX* = *XY* and *WY* = *WX* + *XY*. Let *WX* = *XY* = *x*. Then, 4.8 = *x* + *x* = 2*x*.
Divide each side by 2.
x = 2.4
So, *WX* = *XY* = 2.4 cm.

ANSWER:

2.4 cm

58. $\overline{BC}$



SOLUTION:

Here, $\overline{AB} \cong \overline{BC} \cong \overline{CD} \Rightarrow AB = BC = CD$ and $AB + BC + CD = 3\frac{3}{4}$ inches.

Let $AB = BC = CD = x$. Then,

$$x + x + x = 3\frac{3}{4}.$$
$$3x = \frac{15}{4}$$

Divide each side by 3.

$$x = \frac{5}{4}$$
$$= 1\frac{1}{4}$$

Therefore, $BC = 1\frac{1}{4}$ inches.

ANSWER:

$1\frac{1}{4}$ in.