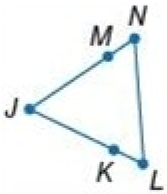
2-7 Proving Segment Relationships

1. **CCSS ARGUMENTS** Copy and complete the proof.

Given: $\overline{LK} \cong \overline{NM}, \overline{KJ} \cong \overline{MJ}$

Prove: $\overline{LJ} \cong \overline{NJ}$

Proof:



Statements	Reasons
a. $\overline{LK} \cong \overline{NM}, \overline{KJ} \cong \overline{MJ}$	a. ___?___
b. ___?___	b. Def. of congruent segments
c. $LK + KJ = NM + MJ$	c. ___?___
d. ___?___	d. Segment Addition Postulate
e. $LJ = NJ$	e. ___?___
f. $\overline{LJ} \cong \overline{NJ}$	f. ___?___

SOLUTION:

The 1st row is the information given.

The 2nd row the definition of congruent segment, changes congruent symbols changed to equal signs and the remove the bars above the segment to indicate the lengths of the segments.

The 3rd row is found adding the two congruent segments.

The 4th row is uses the Segment Addition Postulate to rewrite the segments.

The 5th row is substitution to replace the segments with equivalent segments.

The 6th row is replacing the = with congruent symbols and changing the segments lengths to segment using the definition of congruent segments. .

Statements	Reasons
a. $\overline{LK} \cong \overline{NM}, \overline{KJ} \cong \overline{MJ}$	a. Given
b. $LK = NM, KJ = MJ$	b. Def. of congruent segments
c. $LK + KJ = NM + MJ$	c. Addition Property of Equality
d. $LJ = LK + KJ; NJ = NM + MJ$	d. Segment Addition Postulate
e. $LJ = NJ$	e. Substitution
f. $\overline{LJ} \cong \overline{NJ}$	f. Definition of congruent segments.

ANSWER:

Statements	Reasons
a. $\overline{LK} \cong \overline{NM}, \overline{KJ} \cong \overline{MJ}$	a. ___?___ Given
b. ___?___	b. Def. of congruent segments
c. $LK + KJ = NM + MJ$	c. ___?___ Add. Prop.
d. ___?___	d. Segment Addition Postulate
e. $LJ = NJ$	e. ___?___ Subs.
f. $\overline{LJ} \cong \overline{NJ}$	f. ___?___ Def. $\cong$ segs.

b. $LK = NM,\ KJ = MJ$
d. $LJ = LK + KJ$; $NJ = NM + MJ$

2. PROOF Prove the following.

Given: $\overline{WX} \cong \overline{YZ}$

Prove: $\overline{WY} \cong \overline{XZ}$

SOLUTION:

You need to walk through the proof step by step. Look over what you are given and what you need to prove. Here, you are given two congruent segments. You need to find a way to relate the smaller segment with the larger segments. Use the properties that you have learned about congruent segments and equivalent expressions in algebra to walk through the proof.

Given: $\overline{WX} \cong \overline{YZ}$

Prove: $\overline{WY} \cong \overline{XZ}$

Proof:

Statements (Reasons)

1. $\overline{WX} \cong \overline{YZ}$ (Given)
2. $WX = YZ$ (Definition of congruent segments)
3. $XY = XY$ (Reflexive Property)
4. $WX + XY = XY + YZ$ (Addition Property)
5. $WY = WX + XY$; $XZ = XY + YZ$ (Segment Addition Postulate)
6. $WY = XZ$ (Substitution.)
7. $\overline{WX} \cong \overline{YZ}$ (Definition of congruent segments)

ANSWER:

Given: $\overline{WX} \cong \overline{YZ}$

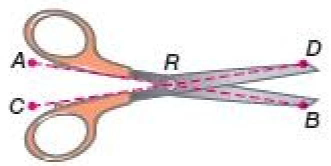
Prove: $\overline{WY} \cong \overline{XZ}$

Proof:

Statements (Reasons)

1. $\overline{WX} \cong \overline{YZ}$ (Given)
2. $WX = YZ$ (Def. $\cong$ segs.)
3. $XY = XY$ (Refl. Prop.)
4. $WX + XY = XY + YZ$ (Add.Property)
5. $WY = WX + XY$; $XZ = XY + YZ$
(Seg. Add. Post.)
6. $WY = XZ$ (Subs.)
7. $\overline{WY} \cong \overline{XZ}$ (Def. $\cong$ segs.)

3. **SCISSORS** Refer to the diagram shown. $\overline{AP}$ is congruent to $\overline{CP}$. $\overline{DP}$ is congruent to $\overline{BP}$. Prove that *AP* + *DP* = *CP* + *BP*.



SOLUTION:

You need to walk through the proof step by step. Look over what you are given and what you need to prove. Here, you are given two pairs of congruent segments. Use the properties that you have learned about congruent segments and equivalent expressions in algebra to walk through the proof.

Given: $\overline{AR} \cong \overline{CP}$; $\overline{DP} \cong \overline{BP}$

Prove: *AR* + *DP* = *CP* + *BP*

Proof:

Statements (Reasons)

1. $\overline{AR} \cong \overline{CP}$; $\overline{DP} \cong \overline{BP}$ (Given)
2. *AR* = *CP*, = *DP* = *BP* (Definition of congruent segments)
3. *AR* + *DP* = *CP* + *DP* (Addition Property)
4. *AR* + *DP* = *CP* + *BP* (Substitution.)

ANSWER:

Given: $\overline{AR} \cong \overline{CP}$; $\overline{DP} \cong \overline{BP}$

Prove: *AR* + *DP* = *CP* + *BP*

Proof:

Statements (Reasons)

1. $\overline{AR} \cong \overline{CP}$; $\overline{DP} \cong \overline{BP}$(Given)
2. *AR* = *CP*, *DP* = *BP* (Def. of $\cong$ segs)
3. *AR* + *DP* = *CP* + *DP* (Add.Prop.)
4. *AR* + *DP* = *CP* + *BP* (Subs.)

4. **CCSS ARGUMENTS** Copy and complete the proof.

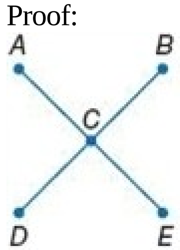
Given: *C* is the midpoint of $\overline{AE}$.

C is the midpoint of $\overline{BD}$.

$\overline{AE} \cong \overline{BD}$

Prove: $\overline{AC} \cong \overline{CD}$

Proof:



Statements	Reasons
a. ___?___	a. Given
b. $AC = CE, BC = CD$	b. ___?___
c. $AE = BD$	c. ___?___
d ___?___	d. Segment Addition Postulate
e. $AC + CE = BC + CD$	e. ___?___
f. $AC + AC = CD + CD$	f. ___?___
g. ___?___	g. Substitution
h. ___?___	h. Division Property
i. $\overline{AC} \cong \overline{CD}$	i. ___?___

SOLUTION:

The 1st contains the given information about the midpoints of segments and segment congruence.
The 2nd row is uses the midpoint of each segment to write equivalence using the definition of midpoints.
The 3rd row changes the segment congruence to distance equivalence using the definition of congruent segments.
The 4th row rewrites two equal segments each with two parts.
The 5th row is replaces or substitutes the two equal segments with their parts.
The 6th row is replaces or substitutes segments with congruent segments.
The 7th row is simplifying or replacing segments by combining them.
The 8th row is found by dividing by 2.
The 9th row is changing segment length to segment congruence using definition of congruent segments.

Statements	Reasons
a. C is the midpoint of $\overline{AE}$. C is the midpoint of $\overline{BD}$. $\overline{AE} \cong \overline{BD}$	a. Given
b. $AC = CE, BC = CD$	b. Definition of midpoint
c. $AE = BD$	c. Definition of congruent segments.
d. $AE = AC + CE, BD = BC + CD$	d. Segment Addition Postulate
e. $AC + CE = BC + CD$	e. Substitution
f. $AC + AC = CD + CD$	f. Substitution
g. $2AC = 2CD$	g. Substitution
h. $AC = CD$	h. Division Property
i. $\overline{AC} \cong \overline{CD}$	i. Definition of congruent segments.

ANSWER:

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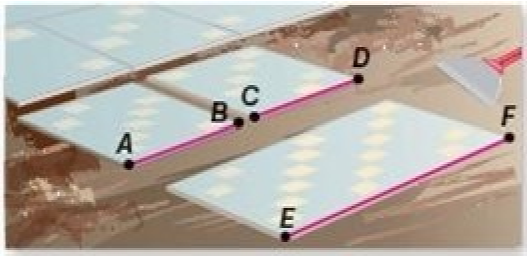
Statements	Reasons
a. ___?___	**a.** Given
b. $AC = CE, BC = CD$	**b.** ___?___ **Def of midpoint**
c. $AE = BD$	**c.** ___?___ **Def ≅ segs.**
d ___?___	**d.** Segment Addition Postulate
e. $AC + CE = BC + CD$	**e.** ___?___ **Subs.**
f. $AC + AC = CD + CD$	**f.** ___?___ **Subs.**
g. ___?___ **$2AC = 2CD$**	**g.** Substitution
h. ___?___ **$AC = CD$**	**h.** Division Property
i. $\overline{AC} \cong \overline{CD}$	**i.** ___?___ **Def ≅ segs.**

a. *C* is the midpoint of $\overline{AE}$. *C* is the midpoint of $\overline{BD}$. $\overline{AE} \cong \overline{BD}$
d. $AE = AC + CE$, $BD = BC + CD$

5. **TILING** A tile setter cuts a piece of tile to a desired length. He then uses this tile as a pattern to cut a second tile congruent to the first. He uses the first two tiles to cut a third tile whose length is the sum of the measures of the first two tiles. Prove that the measure of the third tile is twice the measure of the first tile.



SOLUTION:

You need to walk through the proof step by step. Look over what you are given and what you need to prove. Here, you are given two congruent tiles and another tile which is equal in length to the sum of the lengths of the other tiles. You need to find a way to relate the large tile with the first small tile. Use the properties that you have learned about congruent segments and equivalent expressions in algebra to walk through the proof.

Given: $\overline{AB} \cong \overline{CD}$, $AB + CD = EF$
Prove: $2AB = EF$
<u>Statements (Reasons)</u>
1. $\overline{AB} \cong \overline{CD}$, $AB + CD = EF$ (Given)
2. $AB = CD$ (Definition of congruent segment)
3. $AB + AB = EF$ (Substitution)
4. $2AB = EF$ (Substitution Property)

ANSWER:

Given: $\overline{AB} \cong \overline{CD}$, $AB + CD = EF$
Prove: $2AB = EF$
<u>Statements (Reasons)</u>
1. $\overline{AB} \cong \overline{CD}$, $AB + CD = EF$ (Given)
2. $AB = CD$ (Def. of $\cong$ segs.)
3. $AB + AB = EF$ (Subs.)
4. $2AB = EF$ (Subs. Prop.)

CCSS ARGUMENTS Prove each theorem.

6. Symmetric Property of Congruence (Theorem 2.2)

SOLUTION:

You need to walk through the proof step by step. Look over what you are given and what you need to prove. Here, you are given two congruent congruent segment. You need to find a way to relate the congruent segments to itself in different order. Use the properties that you have learned about congruent segments and equivalent expressions in algebra to walk through the proof.

Given: $\overline{AB} \cong \overline{CD}$
Prove: $\overline{CD} \cong \overline{AB}$
Proof:
Statements (Reasons)
1. $\overline{AB} \cong \overline{CD}$ (Given)
2. $AB = CD$ (Definition of congruent segments)
3. $CD = AB$ (Symmetric. Properties)
4. $\overline{CD} \cong \overline{AB}$ (Definition of congruent segments)

ANSWER:

Given: $\overline{AB} \cong \overline{CD}$
Prove: $\overline{CD} \cong \overline{AB}$
Proof:
Statements (Reasons)
1. $\overline{AB} \cong \overline{CD}$ (Given)
2. $AB = CD$ (Def. of $\cong$ segs.)
3. $CD = AB$ (Symm. Prop.)
4. $\overline{CD} \cong \overline{AB}$ (Def. of $\cong$ segs.)

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7. Reflexive Property of Congruence (Theorem 2.2)

SOLUTION:

You need to walk through the proof step by step. Look over what you are given and what you need to prove. Here, you are segment. You need to find a way to relate the the segment to itself. Use the properties that you have learned about congruent segments and equivalent expressions in algebra to walk through the proof.

Given: $\overline{AB}$
Prove: $\overline{AB} \cong \overline{AB}$
Proof:
Statements (Reasons)
1. $\overline{AB}$ (Given)
2. $AB = AB$ (Reflexive. Property)
3. $\overline{AB} \cong \overline{AB}$ (Definition of congruent segments.)

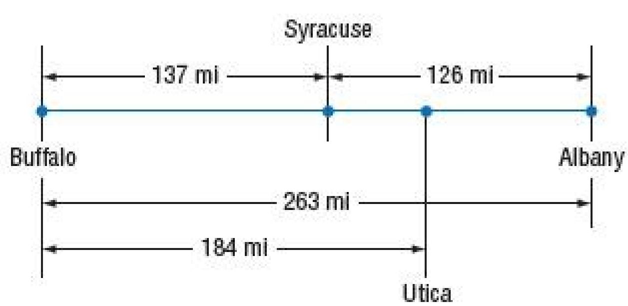
ANSWER:

Given: $\overline{AB}$
Prove: $\overline{AB} \cong \overline{AB}$
Proof:
Statements (Reasons)
1. $\overline{AB}$ (Given)
2. $AB = AB$ (Refl. Prop.)
3. $\overline{AB} \cong \overline{AB}$ (Def. of $\cong$ segs.)

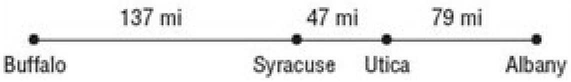
8. **TRAVEL** Four cities in New York are connected by Interstate 90: Buffalo, Utica, Albany, and Syracuse. Buffalo is the farthest west.
• Albany is 126 miles from Syracuse and 263 miles from Buffalo.
• Buffalo is 137 miles from Syracuse and 184 miles from Utica.
a. Draw a diagram to represent the locations of the cities in relation to each other and the distances between each city. Assume that Interstate 90 is straight.
b. Write a paragraph proof to support your conclusion.

SOLUTION:

a. Use the information given to determine the order of the cities. Since Buffalo is furthest west, start with the information given about Buffalo. Albany is 263 miles from Buffalo. Buffalo is 137 miles from Syracuse. So Syracuse must be between Buffalo and Albany. Next, Buffalo is 184 miles from Utica. So, Utica must be between Syracuse and Albany.



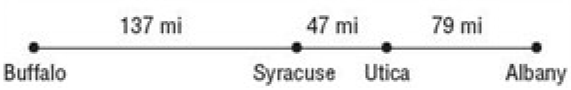
Next, find the distances between each consecutive pair of cities. The distance from Syracuse to Utica is 184 - 137 or 47 miles. The distance from Utica to Albany is 263 - 184 or 79 miles.



b. Given: Buffalo, Utica, Albany, and Syracuse are collinear.
Buffalo is the farthest west.
Albany is 126 miles from Syracuse.
Albany is 263 miles from Buffalo.
Buffalo is 137 miles from Syracuse.
Buffalo is 184 miles from Utica.
Prove: The cities from west to east are Buffalo, Syracuse, Utica, and Albany.
It is 137 miles from Buffalo to Syracuse.
It is 47 miles from Syracuse to Utica.
It is 79 miles from Utica to Albany.
Proof:
We are given that all of the points are collinear. Since Syracuse is 137 miles from Buffalo and Albany is 263 miles from Buffalo, Syracuse is between Buffalo and Albany. Since Utica is 184 miles from Buffalo, and Syracuse is 137 miles from Buffalo, Syracuse is between Utica and Buffalo. Since Albany is 253 miles from Buffalo, and Utica is 184 miles from Buffalo, Utica is between Albany and Buffalo. Therefore, from east to west, the cities are Buffalo, Syracuse, Utica, and Albany.
Syracuse is 137 miles from Buffalo and Utica is 184 miles from Buffalo, so, using the Segment Addition Postulate, Syracuse is 184 − 137, or 47 miles from Utica. The distance from Buffalo to Albany is 263 miles and the distance from Buffalo to Utica is 184 miles, so, using the Segment Addition Postulate, the distance from Utica to Albany is 263 − 184, or 79 miles.

ANSWER:

a.



b. Given: Buffalo, Utica, Albany, and Syracuse are collinear.
Buffalo is the farthest west.
Albany is 126 miles from Syracuse.
Albany is 263 miles from Buffalo.
Buffalo is 137 miles from Syracuse.
Buffalo is 184 miles from Utica.
Prove: The cities from west to east are Buffalo, Syracuse, Utica, and Albany.
It is 137 miles from Buffalo to Syracuse.
It is 47 miles from Syracuse to Utica.
It is 79 miles from Utica to Albany.
Proof:
We are given that all of the points are collinear. Since Syracuse is 137 miles from Buffalo and Albany is 263 miles from Buffalo, Syracuse is between Buffalo and Albany. Since Utica is 184 miles from Buffalo, and Syracuse is 137 miles from Buffalo, Syracuse is between Utica and Buffalo. Since Albany is 253 miles from Buffalo, and Utica is 184 miles from Buffalo, Utica is between Albany and Buffalo. Therefore, from east to west, the cities are Buffalo, Syracuse, Utica, and Albany.
Syracuse is 137 miles from Buffalo and Utica is 184 miles from Buffalo, so, using the Segment Addition Postulate, Syracuse is 184 − 137, or 47 miles from Utica. The distance from Buffalo to Albany is 263 miles and the distance from Buffalo to Utica is 184 miles, so, using the Segment Addition Postulate, the distance from Utica to Albany is 263 − 184, or 79 miles.

PROOF Prove the following.

9. If $\overline{SC} \cong \overline{HR}$ and $\overline{HR} \cong \overline{AB}$, then $\overline{SC} \cong \overline{AB}$.

SOLUTION:

You need to walk through the proof step by step. Look over what you are given and what you need to prove. Here, you are given two congruent segment. Use the properties that you have learned about congruent segments and equivalent expressions in algebra to walk through the proof.

Given: $\overline{SC} \cong \overline{HR}$ and $\overline{HR} \cong \overline{AB}$

Prove: $\overline{SC} \cong \overline{AB}$

Proof:

Statements (Reasons)

1. $\overline{SC} \cong \overline{HR}$ and $\overline{HR} \cong \overline{AB}$ (Given)
2. $SC = HR$ and $HR = AB$ (Definition of congruent segments)
3. $SC = AB$ (Transitive Property)
4. $\overline{SC} \cong \overline{AB}$ (Definition of congruent segments)

ANSWER:

Given: $\overline{SC} \cong \overline{HR}$ and $\overline{HR} \cong \overline{AB}$

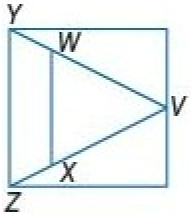
Prove: $\overline{SC} \cong \overline{AB}$

Proof:

Statements (Reasons)

1. $\overline{SC} \cong \overline{HR}$ and $\overline{HR} \cong \overline{AB}$ (Given)
2. $SC = HR$ and $HR = AB$ (Def. of $\cong$ segs.)
3. $SC = AB$ (Trans. Prop.)
4. $\overline{SC} \cong \overline{AB}$ (Def. of segs.)

10. If $\overline{VZ} \cong \overline{VY}$ and $\overline{WY} \cong \overline{XZ}$, then $\overline{VW} \cong \overline{VX}$.



SOLUTION:

You need to walk through the proof step by step. Look over what you are given and what you need to prove. Here, you are given two congruent segments. Use the properties that you have learned about congruent segments and equivalent expressions in algebra to walk through the proof.

Given: $\overline{VZ} \cong \overline{VY}$ and $\overline{WY} \cong \overline{XZ}$

Prove: $\overline{VW} \cong \overline{VX}$.

Proof:

Statements (Reasons)

1. $\overline{VZ} \cong \overline{VY}$ *and* $\overline{WY} \cong \overline{XZ}$ (Given)
2. $VZ = VY$ and $WY = XZ$ (Definition of congruent segments)
3. $VZ = VX + XZ$ and $VY = VW + WY$ (Seg.ment Addition Postulate)
4. $VX + XZ = VW + WY$ (Substitution)
5. $VX + WY = VW + WY$ (Substitution)
6. $VX = VW$ (Subtraction Property)
7. $VW = VX$ (Symmetric Property)
8. $\overline{VW} \cong \overline{VX}$. (Definition of congruent segments.)

ANSWER:

Given: $\overline{VZ} \cong \overline{VY}$ and $\overline{WY} \cong \overline{XZ}$

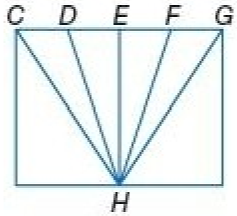
Prove: $\overline{VW} \cong \overline{VX}$.

Proof:

Statements (Reasons)

1. $\overline{VZ} \cong \overline{VY}$ and $\overline{WY} \cong \overline{XZ}$ (Given)
2. $VZ = VY$ and $WY = XZ$ (Def. of $\cong$ segs.)
3. $VZ = VX + XZ$ and $VY = VW + WY$ (Seg. Add. Postulate)
4. $VX + XZ = VW + WY$ (Subs.)
5. $VX + WY = VW + WY$ (Subs.)
6. $VX = VW$ (Subtraction Property of Equality)
7. $VW = VX$ (Symm. Prop.)
8. $\overline{VW} \cong \overline{VX}$. (Def. of $\cong$ segs.)

11. If E is the midpoint of $\overline{DF}$ and $\overline{CD} \cong \overline{FG}$, then $\overline{CE} \cong \overline{EG}$.



SOLUTION:

You need to walk through the proof step by step. Look over what you are given and what you need to prove. Here, you are given a congruent segments and a midpoint of a segment. Use the properties that you have learned about congruent segments and equivalent expressions in algebra to walk through the proof.

Given: E is the midpoint of $\overline{DF}$ and $\overline{CD} \cong \overline{FG}$.

Prove: $\overline{CE} \cong \overline{EG}$

Proof:

Statements (Reasons)

1. E is the midpoint of $\overline{DF}$ and $\overline{CD} \cong \overline{FG}$. (Given)
2. $DE = EF$ (Definition of midpoint)
3. $CD = FG$ (Definition of congruent segments)
4. $CD + DE = EF + FG$ (Addition Property)
5. $CE = CD + DE$ and $EG = EF + FG$ (Segment Addition Postulate)
6. $CE = EG$ (Substitution)
7. $\overline{CE} \cong \overline{EG}$ (Definition of congruent segments)

ANSWER:

Given: E is the midpoint of $\overline{DF}$ and $\overline{CD} \cong \overline{FG}$.

Prove: $\overline{CE} \cong \overline{EG}$

Proof:

Statements (Reasons)

1. E is the midpoint of $\overline{DF}$ and $\overline{CD} \cong \overline{FG}$. (Given)
2. $DE = EF$ (Def. of midpoint)
3. $CD = FG$ (Def. of $\cong$ segs.)
4. $CD + DE = EF + FG$ (Add. Prop.)
5. $CE = CD + DE$ and $EG = EF + FG$ (Seg. Add. Post.)
6. $CE = EG$ (Subs.)
7. $\overline{CE} \cong \overline{EG}$ (Def. of $\cong$ segs.)

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12. If B is the midpoint of $\overline{AC}$, D is the midpoint of $\overline{CE}$, and $\overline{AB} \cong \overline{DE}$, then $AE = 4AB$.

SOLUTION:

You need to walk through the proof step by step. Look over what you are given and what you need to prove. Here, you are given congruent segments, two midpoints of segments. Use the properties that you have learned about congruent segments and equivalent expressions in algebra to walk through the proof.

Given: B is the midpoint of $\overline{AC}$, D is the midpoint of $\overline{CE}$, and $\overline{AB} \cong \overline{DE}$.
Prove: $AE = 4AB$
Proof:
Statements (Reasons)

1. B is the midpoint of $\overline{AC}$, D is the midpoint of $\overline{CE}$, and $\overline{AB} \cong \overline{DE}$. (Given)
2. $AB = BC$ and $CD = DE$ (Definition of midpoint)
3. $AB = DE$ (Definition of congruent segments)
4. $AC = AB + BC$ and $CE = CD + DE$ (Segment Addition Postulate)
5. $AE = AC + CE$ (Segment Addition Postulate)
6. $AE = AB + BC + CD + DE$ (Substitution)
7. $AE = AB + AB + AB + AB$ (Substitution)
8. $AE = 4AB$ (Substitution)

ANSWER:

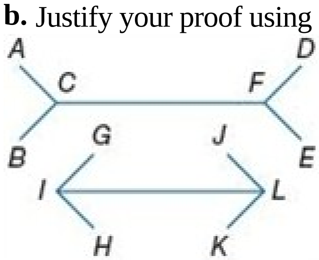
Given: B is the midpoint of $\overline{AC}$, D is the midpoint of $\overline{CE}$, and $\overline{AB} \cong \overline{DE}$.
Prove: $AE = 4AB$
Proof:
Statements (Reasons)

1. B is the midpoint of $\overline{AC}$ D is the midpoint of $\overline{CE}$, and $\overline{AB} \cong \overline{DE}$. (Given)
2. $AB = BC$ and $CD = DE$ (Def. of midpoint)
3. $AB = DE$ (Def. of $\cong$ segs.)
4. $AC = AB + BC$ and $CE = CD + DE$ (Seg. Add. Post.)
5. $AE = AC + CE$ (Seg. Add. Post.)
6. $AE = AB + BC + CD + DE$ (Subs.)
7. $AE = AB + AB + AB + AB$ (Subs.)
8. $AE = 4AB$ (Subs.)

13. **OPTICAL ILLUSION** $\overline{AC} \cong \overline{GI}$, $\overline{FE} \cong \overline{LK}$, and $AC + CF + FE = GI + IL + LK$.

a. Prove that $\overline{CF} \cong \overline{IL}$

b. Justify your proof using measurement. Explain your method.



SOLUTION:

a. You need to walk through the proof step by step. Look over what you are given and what you need to prove. Here, you are given two congruent segments, equal distances for sum of 3 sides. Use the properties that you have learned about congruent segments and equivalent expressions in algebra to walk through the proof.

Given: $\overline{AC} \cong \overline{GI}, \overline{FE} \cong \overline{LK},$ $AC + CF + FE = GI + IL + LK$

Prove: $\overline{CF} \cong \overline{IL}$

Proof:
Statements (Reasons)
1. $\overline{AC} \cong \overline{GI}, \overline{FE} \cong \overline{LK},$ $AC + CF + FE = GI + IL + LK$
(Given)
2. $AC + CF + FE = AC + IL + LK$ (Substitution)
3. $AC - AC + CF + FE = AC - AC + IL + LK$ (Subtraction Property)
4. $CF + FE = IL + LK$ (Substitution Property)
5. $CF + FE = IL + FE$ (Substitution)
6. $CF + FE – FE = IL + FE – FE$ (Subtraction Property)
7. $CF = IL$ (Substitution Property)
8. $\overline{CF} \cong \overline{IL}$ (Definition of congruent segments)

b. Sample answer: When using the student edition, the measures of $\overline{CF}$ and $\overline{IL}$ are 1.5 inches long, so the two segments are congruent.

ANSWER:

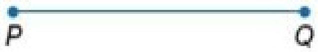
Given: $\overline{AC} \cong \overline{GI}, \overline{FE} \cong \overline{LK},$ $AC + CF + FE = GI + IL + LK$

Prove: $\overline{CF} \cong \overline{IL}$

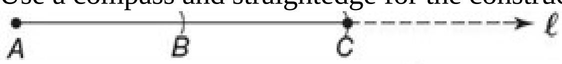
Proof:
Statements (Reasons)
1. $\overline{AC} \cong \overline{GI}, \overline{FE} \cong \overline{LK},$ $AC + CF + FE = GI + IL + LK$
(Given)
2. $AC + CF + FE = AC + IL + LK$ (Subs.)
3. $AC - AC + CF + FE = AC - AC + IL + LK$ (Subt. Prop.)
4. $CF + FE = IL + LK$ (Subs. Prop.)
5. $CF + FE = IL + FE$ (Subs.)
6. $CF + FE – FE = IL + FE – FE$ (Subt. Prop.)
7. $CF = IL$ (Subs. Prop.)
8. $\overline{CF} \cong \overline{IL}$ (Def. of $\cong$ segs.)

b. Sample answer: When using the student edition, the measures of $\overline{CF}$ and $\overline{IL}$ are 1.5 inches long, so the two segments are congruent.

14. **CONSTRUCTION** Construct a segment that is twice as long as $\overline{PQ}$.
Explain how the Segment Addition Postulate can be used to justify your construction.



SOLUTION:
Use a compass and straightedge for the construction.



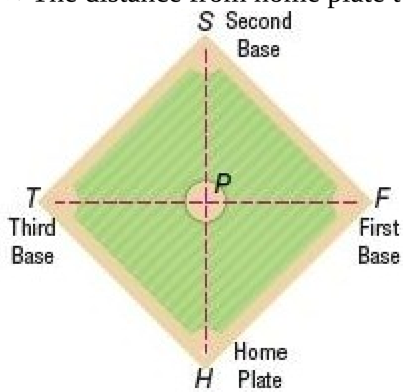
I placed an initial point *A* on a line ℓ and constructed a point *B* on the line so that *AB* is equal to *PQ*.

Using point *B* as an initial point, I marked point *C* on the line so that *BC* is also equal to *PQ*. The length of the whole segment *AC* is *AB* + *BC* according to the Additional Postulate and *AB* = *BC* = *PQ*.
Using substitution *AC* = *PQ* + *PQ*, or *AC* = 2*PQ*, so $\overline{AC}$ is twice as long as $\overline{PQ}$.

ANSWER:

Sample answer: I placed an initial point *A* on a line ℓ and constructed a point *B* on the line so that *AB* is equal to *PQ*.
Using point *B* as an initial point, I marked point *C* on the line so that *BC* is also equal to *PQ*. The length of the whole segment *AC* is *AB* + *BC* according to the Additional Postulate and *AB* = *BC* = *PQ*. Using substitution *AC* = *PQ* + *PQ*, or *AC* = 2*PQ*, so $\overline{AC}$ is twice as long as $\overline{PQ}$.

15. **BASEBALL** Use the diagram of a baseball diamond shown..
a. On a baseball field, $\overline{SH} \cong \overline{TF}$. *P* is the midpoint of $\overline{SH}$ and $\overline{TF}$. Using a two-column proof, prove that $\overline{SP} \cong \overline{TP}$.
b. The distance from home plate to second base is 127.3 feet. What is the distance from first base to second base?



SOLUTION:
a. You need to walk through the proof step by step. Look over what you are given and what you need to prove. Here, you are given two congruent segments and the midpoint of two segments. Use the properties that you have learned about congruent segments and equivalent expressions in algebra to walk through the proof.
Given: $\overline{SH} \cong \overline{TF}$; *P* is the midpoint of $\overline{SH}$ *and* $\overline{TF}$.
Prove: $\overline{SP} \cong \overline{TP}$
Proof:
<u>Statements (Reasons)</u>

1. $\overline{SH} \cong \overline{TF}$; *P* is the midpoint of $\overline{SH}$; *P* is the midpoint of $\overline{TF}$ (Given)
2. *SH* = *TF* (Definition of congruent segments)
3. *SP* = *PH*, *TP* = *PF* (Definition. of Midpoint)
4. *SH* = *SP* + *PH*, *TF* = *TP* + *PF* (Segment Addition Postulate)
5. *SP* + *PH* = *TP* + *PF* (Substitution)
6. *SP* + *SP* = *TP* + *TP* (Substitution)
7. 2*SP* = 2*TP* (Substitution)
8. *SP* = *TP* (Division Property)
9. $\overline{SP} \cong \overline{TP}$ (Definition of congruent segments)

b. The home plate, first base and second base form an isosceles triangle with right angle at *F*.
Let *HF* = *x*, then *SF* = *x*.
Use the Pythagorean Theorem.

$SH^2 = HF^2 + SF^2$	Pythagorean Theorem.
$127.3^2 = x^2 + x^2$	Substitution.
$16,205.29 = 2x^2$	Simplify.
$8102.645 = x^2$	Divide each side by 2.
$90 \approx x$	Take the square root of each side.

The distance from first base to second base is about 90 ft.

ANSWER:

a. Given: $\overline{SH} \cong \overline{TF}$; *P* is the midpoint of $\overline{SH}$ *and* $\overline{TF}$.

Prove: $\overline{SP} \cong \overline{TP}$
Proof:
Statements (Reasons)

1. $\overline{SH} \cong \overline{TF}$; *P* is the midpoint of $\overline{SH}$; P is the midpoint of $\overline{TF}$ (Given)
2. *SH* = *TF* (Def. of ≅ Segs.)
3. *SP* = *PH*, *TP* = *PF* (Def. of Midpoint)
4. *SH* = *SP* + *PH*, *TF* = *TP* + *PF* (Seg. Add. Post.)
5. *SP* + *PH* = *TP* + *PF* (Subs.)
6. *SP* + *SP* = *TP* + *TP* (Subs.)
7. 2*SP* = 2*TP* (Subs.)
8. *SP* = *TP* (Div. Prop.)
9. $\overline{SP} \cong \overline{TP}$ (Def. of ≅ segs.)

b. 90 ft

16. **MULTIPLE REPRESENTATIONS** *A* is the midpoint of $\overline{PQ}$, *B* is the midpoint of $\overline{PA}$, and *C* is the midpoint of $\overline{PB}$.

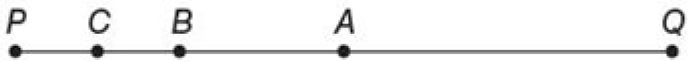
a. GEOMETRIC Make a sketch to represent this situation.
b. ALGEBRAIC Make a conjecture as to the algebraic relationship between *PC* and *PQ*.
c. GEOMETRIC Copy segment $\overline{PQ}$ from your sketch. Then construct points *B* and *C* on $\overline{PQ}$. Explain how you can use your construction to support your conjecture.
d. CONCRETE Use a ruler to draw a segment congruent to $\overline{PQ}$ from your sketch and to draw points *B* and *C* on

$\overline{PQ}$. Use your drawing to support your conjecture.

e. LOGICAL Prove your conjecture.

SOLUTION:

a.



b. We know that PA=AQ and, PC = CB, and PB=BA. Also, by segment addition PC+CB+BA+AQ = PQ.
Then PC+CB+BA+AQ = PQ.
PC+PC+PB+PA = PQ (Substitute PC for CB, PB for BA, PA for AQ)
PC+PC+(PC+CB)+(PC+CB+BA) = PQ (Substitute PC + CB for PB, PC+CB+BA for PA)
PC+PC+PC+PC+PC+PC+(PC+CB) = PQ (Substitute PC for CB and PC + CB for BA)
PC+PC+PC+PC+PC+PC+PC+PC = PQ (Substitute PC for CB)
Thus, $8\,PC = PQ$.

c.

I can measure $\overline{PC}$ and mark off segments of that length along $\overline{PQ}$, and count how many segments were formed.

d.

$8\,PC = PQ$

e. You need to walk through the proof step by step. Look over what you are given and what you need to prove. Here, you are given midpoints of three segments,Use the properties that you have learned about congruent segments and equivalent expressions in algebra to walk through the proof.

Given: A is the midpoint of $\overline{PQ}$, B is the midpoint of $\overline{PA}$, and C is the midpoint of $\overline{PB}$.
Prove: $8PC = PQ$
<u>Statements (Reasons)</u>
1. A is the midpoint of $\overline{PQ}$, B is the midpoint of $\overline{PA}$, and C is the midpoint of $\overline{PB}$. (Given)
2. $PA = AQ$, $PB = BA$, $PC = CB$ (Definition of Midpoint)
3. $PC + CB = PB$ (Segment Addition Postulate)
4. $PC + PC = PB$ (Substitution)
5. $2PC = PB$ (Substitution.)
6. $PB + BA = PA$ (Segment Addition Postulate)
7. $PB + PB = PA$ (Substitution.)
8. $2PB = PA$ (Addition Property)
9. $2\,(2PC) = PA$ (Substitution.)
10. $4\,PC = PA$ (Substitution.)
11. $PA + AQ = PQ$ (Segment Addition Postulate)
12. $PA + PA = PQ$ (Substitution.)
13. $2PA = PQ$ (Substitution.)
14. $2(4PC) = PQ$ (Substitution.)
15. $8PC = PQ$ (Substitution.)

ANSWER:

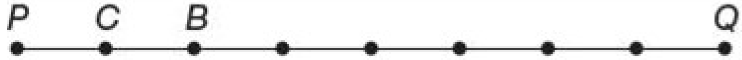
a.

b. $8\,PC = PQ$

c.

I can measure $\overline{PC}$ and mark off segments of that length along $\overline{PQ}$, and count how many segments were formed.

d.

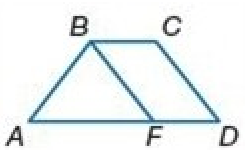


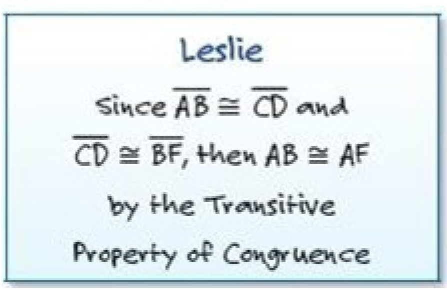
8 $PC = PQ$

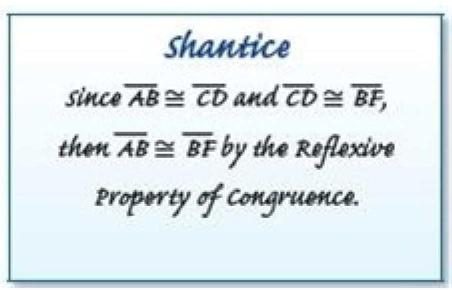
e. Given: A is the midpoint of $\overline{PQ}$, B is the midpoint of $\overline{PA}$, and C is the midpoint of $\overline{PB}$.
Prove: $8PC = PQ$
Statements (Reasons)

1. A is the midpoint of $\overline{PQ}$, B is the midpoint of $\overline{PA}$, and C is the midpoint of $\overline{PB}$. (Given)
2. $PA = AQ$, $PB = BA$, $PC = CB$ (Def. of Midpoint)
3. $PC + CB = PB$ (Seg. Add. Post.)
4. $PC + PC = PB$ (Subs.)
5. $2PC = PB$ (Subs.)
6. $PB + BA = PA$ (Seg. Add. Post.)
7. $PB + PB = PA$ (Subs.)
8. $2PB = PA$ (Add. Prop.)
9. $2\ (2PC) = PA$ (Subs.)
10. $4\ PC = PA$ (Subs.)
11. $PA + AQ = PQ$ (Seg. Add. Post.)
12. $PA + PA = PQ$ (Subs.)
13. $2PA = PQ$ (Subs.)
14. $2(4PC) = PQ$ (Subs.)
15. $8PC = PQ$ (Subs.)

17. **CCSS CRITIQUE** In the diagram, $\overline{AB} \cong \overline{CD}$ and $\overline{CD} \cong \overline{BF}$.
Examine the conclusions made by Leslie and Shantice. Is either of them correct?







SOLUTION:

Neither is correct. Since $\overline{AB} \cong \overline{CD}$ and $\overline{CD} \cong \overline{BF}$, then $\overline{AB} \cong \overline{BF}$ by the Transitive Property of Congruence. Leslie indicated the correct property, but applied it incorrectly. Stantice stated the Transitive Property correct, but indicated that the property was the Reflexive Property.

ANSWER:

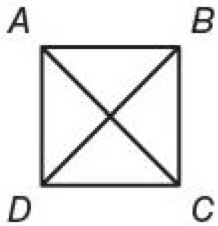
Neither; Since $\overline{AB} \cong \overline{CD}$ and $\overline{CD} \cong \overline{BF}$, then $\overline{AB} \cong \overline{BF}$ by the Transitive Property of Congruence

18. **CHALLENGE** $ABCD$ is a square. Prove that $\overline{AC} \cong \overline{BD}$.

SOLUTION:

You need to walk through the proof step by step. Look over what you are given and what you need to prove. Here, you are given a square.Use the properties that you have learned about congruent segments, squares, and equivalent expressions in algebra to walk through the proof.

Given: *ABCD* is a square.



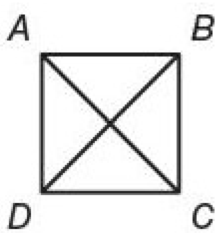
Prove: $\overline{AC} \cong \overline{BD}$

Statements (Reasons)

1. *ABCD* is a square. (Given)
2. $AB = BC = CD = DA$ (Definition of a square)
3. $(AC)^2 = (AB)^2 + (BC)^2, (BD)^2 = (AB)^2 + (AD)^2$ (Pythagorean Theorem)
4. $(BD)^2 = (AB)^2 + (BC)^2$ (Substitution)
5. $(AC)^2 = (BD)^2$ (Transitive Property)
6. $AC = \pm\sqrt{(BD)^2}$ (Square Root Property)
7. $AC = \sqrt{(BD)^2}$ (By definition, length must be positive.)
8. $AC = BD$ (Definition of Square Root)
9. $\overline{AC} \cong \overline{BD}$ (Definition. of congruent segments)

ANSWER:

Given: *ABCD* is a square.



Prove: $\overline{AC} \cong \overline{BD}$

Statements (Reasons)

1. *ABCD* is a square. (Given)
2. $AB = BC = CD = DA$ (Def. of a square)
3. $(AC)^2 = (AB)^2 + (BC)^2, (BD)^2 = (AB)^2 + (AD)^2$
(Pythagorean Theorem)
4. $(BD)^2 = (AB)^2 + (BC)^2$ (Subt.)
5. $(AC)^2 = (BD)^2$ (Trans. Prop.)
6. $AC = \pm\sqrt{(BD)^2}$ (Sq. Root Prop.)

7. $AC = \sqrt{(BD)^2}$ (By definition, length must be positive.)
8. $AC = BD$ (Def. of Sq. Root)
9. $\overline{AC} \cong \overline{BD}$ (Def. of segs.)

19. **WRITING IN MATH** Does there exist an Addition Property of Congruence? Explain.

SOLUTION:

There does not exist an Addition Property of Congruence. Congruence refers to segments. Segments cannot be added, only the measures of segments.

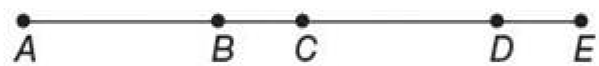
ANSWER:

No; congruence refers to segments. Segments cannot be added, only the measures of segments.

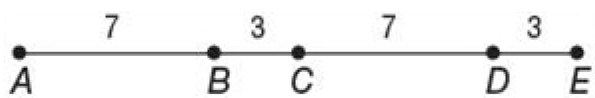
20. **REASONING** Classify the following statement as *true* or *false*. If false, provide a counterexample.
If A, B, C, D, and E are collinear with B between A and C, C between B and D, and D between C and E, and AC = BD = CE, then AB = BC = DE.

SOLUTION:

The statement is false.

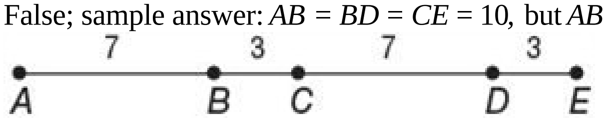


Let $AC = BD = CE = 10$, then $AB = BC = DE = 10$. However, $AB = 7$, $BC = 3$, $CD = 7$ and $DE = 3$.



ANSWER:

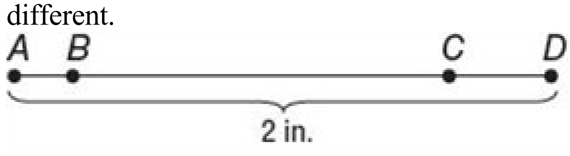
False; sample answer: $AB = BD = CE = 10$, but $AB = 7$, $BC = 3$, $CD = 7$ and $DE = 3$.



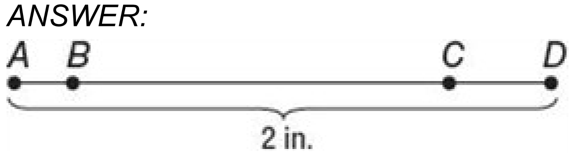
21. **OPEN ENDED** Draw a representation of the Segment Addition Postulate in which the segment is two inches long, contains four collinear points, and contains no congruent segments.

SOLUTION:

Draw a segment two inches long. label points *A*, *B*, *C*, and *D* such that the distance between any of the points is different.



ANSWER:



22. **WRITING IN MATH** Compare and contrast paragraph proofs and two-column proofs.

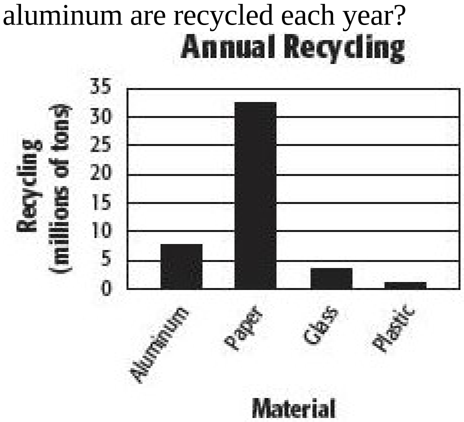
SOLUTION:

Paragraph proofs and two-column proofs both use deductive reasoning presented in a logical order along with the postulates, theorems, and definitions used to support the steps of the proofs.
Paragraph proofs are written as a paragraph with the reasons for each step incorporated into the sentences.
Two-column proofs are numbered and itemized. Each step of the proof is provided on a separate line with the support for that step in the column beside the step.

ANSWER:

Paragraph proofs and two-column proofs both use deductive reasoning presented in a logical order along with the postulates, theorems, and definitions used to support the steps of the proofs. Paragraph proofs are written as a paragraph with the reasons for each step incorporated into the sentences. Two-column proofs are numbered and itemized. Each step of the proof is provided on a separate line with the support for that step in the column beside the step.

23. **ALGEBRA** The chart below shows annual recycling by material in the United States. About how many pounds of aluminum are recycled each year?



A 7.5
B 15,000
C 7,500,000
D 15,000,000,000

SOLUTION:

From the figure we can see that 7.5 million tons of aluminum is recycled each year.
Convert 7.5 million to pounds.
1 ton = 2000 lb.
$7{,}500{,}000 \times 2000 = 15{,}000{,}000{,}000$
The correct choice is D.

ANSWER:

D

24. **ALGEBRA** Which expression is equivalent to $\frac{12x^{-4}}{4x^{-8}}$?

F $\frac{1}{3x^4}$
G $3x^4$
H $8x^2$
J $\frac{x^4}{3}$

SOLUTION:

$$\frac{12x^{-4}}{4x^{-8}} = \frac{12x^8}{4x^4} = 3x^{8-4} = 3x^4$$

The correct choice is G.

ANSWER:
G

25. **SHORT RESPONSE** The measures of two complementary angles are in the ratio 4:1. What is the measure of the smaller angle?

SOLUTION:
If two angles are complementary then their sum is 90°.
Let *x* be the measure of the smaller angle. Then the measure of the larger angle is 4*x*.

$$x + 4x = 90$$
$$5x = 90$$
$$\frac{5x}{5} = \frac{90}{5}$$
$$x = 18$$

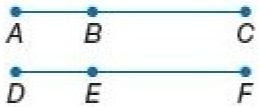
ANSWER:
18

26. **SAT/ACT** Julie can word process 40 words per minute. How many minutes will it take Julie to word process 200 words?
A 0.5
B 2
C 5
D 10
E 12

SOLUTION:
Divide 200 by 40.
200 ÷ 40 = 5
The correct choice is C.

ANSWER:
C

27. **PROOF** Write a two-column proof.
Given: $AC = DF$
$AB = DE$
Prove: $BC = EF$



SOLUTION:

You need to walk through the proof step by step. Look over what you are given and what you need to prove. Here, you are given the measures of two segments are equal. Use the properties that you have learned about congruent segments and equivalent expressions in algebra to walk through the proof.

Given: $AC = DF$, $AB = DE$
Prove: $BC = EF$
Proof:
Statements (Reasons)
1. $AC = DF$, $AB = DE$ (Given)
2. $AC = AB + BC$; $DF = DE + EF$ (Segment Addition Postulate)
3. $AB + BC = DE + EF$ (Substitution)
4. $BC = EF$ (Subtraction Property)

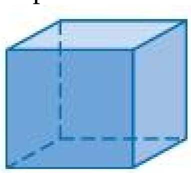
ANSWER:

Given: $AC = DF$, $AB = DE$
Prove: $BC = EF$
Proof:
Statements (Reasons)
1. $AC = DF$, $AB = DE$ (Given)
2. $AC = AB + BC$; $DF = DE + EF$
(Seg. Add. Post.)
3. $AB + BC = DE + EF$ (Subs.)
4. $BC = EF$ (Subt. Prop.)

28. **MODELS** Brian is using six squares of cardboard to form a rectangular prism. What geometric figure do the pieces of cardboard represent, and how many lines will be formed by their intersections?

SOLUTION:

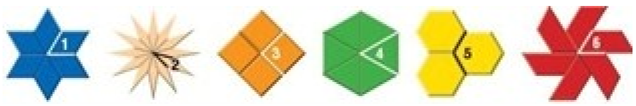
The pieces of cardboard represent planes. There are 12 edges in a rectangular prism. So, 12 lines will be formed.



ANSWER:

Planes; 12

29. **PATTERN BLOCKS** Pattern blocks can be arranged to fit in a circular pattern without leaving spaces. Remember that the measurement around a full circle is 360°. Determine the degree measure of the numbered angles shown below.



SOLUTION:

Count the number of parts in to which the circle is divided and divided 360° by the number of parts.

$360° \div 6 = 60°$
$360° \div 12 = 30°$
$360° \div 4 = 90°$
$360° \div 6 = 60°$
$360° \div 3 = 120°$
$360° \div 6 = 60°$

ANSWER:

60, 30, 90, 60, 120, 60

Simplify.

30. $\sqrt{48}$

SOLUTION:

$$\begin{aligned}\sqrt{48} &= \sqrt{16 \cdot 3}\\ &= \sqrt{4 \cdot 4 \cdot 3}\\ &= 4\sqrt{3}\end{aligned}$$

ANSWER:

$4\sqrt{3}$

31. $\sqrt{162}$

SOLUTION:

$$\begin{aligned}\sqrt{162} &= \sqrt{81 \cdot 2}\\ &= \sqrt{9 \cdot 9 \cdot 2}\\ &= 9\sqrt{2}\end{aligned}$$

ANSWER:

$9\sqrt{2}$

32. $\sqrt{25a^6b^4}$

SOLUTION:

$$\sqrt{25a^6b^4} = \sqrt{5 \cdot 5 \cdot a^3 \cdot a^3 \cdot b^2 \cdot b^2}$$
$$= 5\left|a^3\right|b^2$$

ANSWER:

$5|a^3|b^2$

33. $\sqrt{45xy^8}$

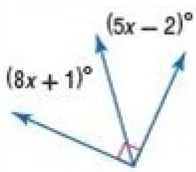
SOLUTION:

$$\sqrt{45xy^8} = \sqrt{9 \cdot 5 \cdot x \cdot y^8}$$
$$= \sqrt{3 \cdot 3 \cdot 5 \cdot x \cdot y^4 \cdot y^4}$$
$$= 3y^4\sqrt{5x}$$

ANSWER:

$3y^4\sqrt{5x}$

ALGEBRA Find *x*.

34. 

SOLUTION:

The sum of the two angles is 90°.
$(8x + 1)° + (5x – 2)° = 90°$
$(13x – 1) = 90$
$13x = 91$
Divide both sides by 13.
$x = 7$

ANSWER:

7

35. 

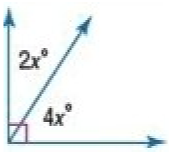
SOLUTION:

The two angles form a linear pair. So, the sum of their angles is 180°.

$14x + 8x + 4 = 180$
$22x + 4 = 180$
$22x = 176$
$x = 8$

ANSWER:

8

36. 

SOLUTION:

$2x + 4x = 90$
$6x = 90$
$x = 15$

ANSWER:

15