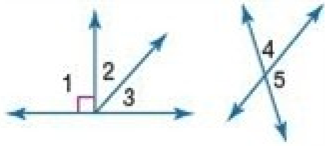
2-8 Proving Angle Relationships

Find the measure of each numbered angle, and name the theorems that justify your work.



1. $m\angle 2 = 26$

SOLUTION:

The angles $\angle 2$ and $\angle 3$ are complementary, or adjacent angles that form a right angle.
So, $m\angle 2 + m\angle 3 = 90$.

Substitute.
$26 + m\angle 3 = 90$
$m\angle 3 = 90 - 26$
$= 64$

Here, the Complement Theorem has been used.

ANSWER:
$m\angle 1 = 90, m\angle 3 = 64$; Comp. Thm.

2. $m\angle 2 = x, m\angle 3 = x - 16$

SOLUTION:

The angles $\angle 2$ and $\angle 3$ are complementary, or adjacent angles that form a right angle.
So, $m\angle 2 + m\angle 3 = 90$.

Substitute.

$$x + x - 16 = 90$$
$$2x - 16 = 90$$
$$2x = 106$$
$$x = 53$$

So, $m\angle 2 = 53$ and
$m\angle 3 = 53 - 16$ or 37.

Here, the Complement Theorem has been used.

ANSWER:
$m\angle 2 = 53, m\angle 3 = 37$; Comp. Thm.

3. $m\angle 4 = 2x, m\angle 5 = x + 9$

SOLUTION:

The angles $\angle 4$ and $\angle 5$ are supplementary or form a linear pair.
So, $m\angle 4 + m\angle 5 = 180$.

Substitute.

$$
\begin{aligned}
2x + x + 9 &= 180 \\
3x + 9 &= 180 \\
3x + 9 - 9 &= 180 - 9 \\
3x &= 171 \\
x &= 57
\end{aligned}
$$

Substitute $x = 57$ in $m\angle 4 = 2x$ and $m\angle 5 = x + 9$.

$$
\begin{aligned}
m\angle 4 &= 2(57) \\
&= 114 \\
m\angle 5 &= 57 + 9 \\
&= 66
\end{aligned}
$$

Here, the Supplement Theorem has been used.

ANSWER:

$m\angle 4 = 114, m\angle 5 = 66$; Suppl. Thm.

4. $m\angle 4 = 3(x - 1), m\angle 5 = x + 7$

SOLUTION:

The angles $\angle 4$ and $\angle 5$ are supplementary or form a linear pair.
So, $m\angle 4 + m\angle 5 = 180$.
Substitute.

$$
\begin{aligned}
3(x - 1) + x + 7 &= 180 \\
3x - 3 + x + 7 &= 180 \\
4x + 4 &= 180 \\
4x &= 176 \\
x &= 44
\end{aligned}
$$

Substitute $x = 44$ in $m\angle 4 = 3(x - 1)$ and $m\angle 5 = x + 7$.

$$
\begin{aligned}
m\angle 4 &= 3(44 - 1) \\
&= 3(43) \\
&= 129 \\
m\angle 5 &= 44 + 7 \\
&= 51
\end{aligned}
$$

Here, the Supplement Theorem has been used.

ANSWER:

$m\angle 4 = 129, m\angle 5 = 51$; Suppl. Thm.

5. **PARKING** Refer to the diagram of the parking lot. Given that $\angle 2 \cong \angle 6$, prove that $\angle 4 \cong \angle 8$.

SOLUTION:

You need to walk through the proof step by step. Look over what you are given and what you need to prove. Here, you are given two congruent angles. Use the properties that you have learned about congruent angles and equivalent expressions in algebra to walk through the proof.

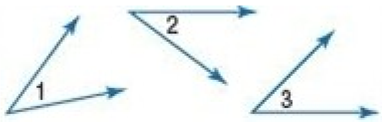
Given: $\angle 2 \cong \angle 6$
Prove: $\angle 4 \cong \angle 8$
Proof:
Statements (Reasons)
1. $\angle 2 \cong \angle 6$ (Given)
2. $\angle 2 + m\angle 4 = 180$
   $m\angle 6 + m\angle 8 = 180$ (Supplement Theorem)
3. $\angle 2 + m\angle 8 = 180$ (Substitution)
4. $m\angle 2 - m\angle 2 + m\angle 4 = 180 - m\angle 2$
   $m\angle 2 - m\angle 2 + m\angle 8 = 180 - m\angle 2$ (Subtraction Property)
5. $m\angle 4 = 180 - m\angle 2$
   $m\angle 8 = 180 - m\angle 2$ (Subtraction Property)
6. $m\angle 4 = m\angle 8$ (Substitution)
7. $\angle 4 \cong \angle 8$ (Definition of congruent angles)

ANSWER:

Given: $\angle 2 \cong \angle 6$
Prove: $\angle 4 \cong \angle 8$
Proof:
Statements (Reasons)
1. $\angle 2 \cong \angle 6$ (Given)
2. $\angle 2 + m\angle 4 = 180, m\angle 6 + m\angle 8 = 180$ (Suppl. Thm.)
3. $\angle 2 + m\angle 8 = 180$ (Subs.)
4. $m\angle 2 - m\angle 2 + m\angle 4 = 180 - m\angle 2, m\angle 2 - m\angle 2 + m\angle 8 = 180 - m\angle 2$ (Subt. Prop.)
5. $m\angle 4 = 180 - m\angle 2, m\angle 8 = 180 - m\angle 2$ (Subt. Prop.)
6. $m\angle 4 = m\angle 8$ (Subs.)
7. $\angle 4 \cong \angle 8$ (Def. $\cong \angle s$)

6. **PROOF** Copy and complete the proof of one case of Theorem 2.6.
Given: $\angle 1$ and $\angle 3$ are complementary.
$\angle 2$ and $\angle 3$ are complementary.

2-8 Proving Angle Relationships



Prove: $\angle 1 \cong \angle 2$

Statements	Reasons
a. $\angle 1$ and $\angle 3$ are complementary. $\angle 2$ and $\angle 3$ are complementary.	**a.** ___?___
b. $m\angle 1 + m\angle 3 = 90$; $m\angle 2 + m\angle 3 = 90$	**b.** ___?___
c. $m\angle 1 + m\angle 3 = m\angle 2 + m\angle 3$	**c.** ___?___
d. ___?___	**d.** Reflexive Property
e. $m\angle 1 = m\angle 2$	**e.** ___?___
f. $\angle 1 \cong \angle 2$	**f.** ___?___

SOLUTION:

The 1st row contains the given information about complementary angles..
The 2nd row is uses the definition of complementary angles.
The 3rd row use substitution to write the two statements as one.
The 4th row look ahead at the segment for Row 4 to see what is changes, so you can identify the statement for reflexive.
The 5th row is subtraction is used to remove $\angle 3$ from each side.
The 6th row is replaces or substitutes angles with congruent angles.

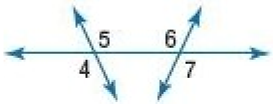
Statements	Reasons
a. $\angle 1$ and $\angle 3$ are complementary. $\angle 2$ and $\angle 3$ are complementary.	**a.** Given
b. $m\angle 1 + m\angle 3 = 90$; $m\angle 2 + m\angle 3 = 90$	**b.** Definition of complementary angles
c. $m\angle 1 + m\angle 3 = m\angle 2 + m\angle 3$	**c.** Substitution
d. $m\angle 3 = m\angle 3$	**d.** Reflexive Property
e. $m\angle 1 = m\angle 2$	**e.** Subtraction Property
f. $\angle 1 \cong \angle 2$	**f.** Definition of congruent angles

ANSWER:

2-8 Proving Angle Relationships

Statements	Reasons
a. $\angle 1$ and $\angle 3$ are complementary. $\angle 2$ and $\angle 3$ are complementary.	**a.** ___?___ Given
b. $m\angle 1 + m\angle 3 = 90$; $m\angle 2 + m\angle 3 = 90$	**b.** ___?___ Def. of comp. $\angle$s
c. $m\angle 1 + m\angle 3 = m\angle 2 + m\angle 3$	**c.** ___?___ Subs.
d. ___?___ $m\angle 3 = m\angle 3$	**d.** Reflexive Property
e. $m\angle 1 = m\angle 2$	**e.** ___?___ Subt. Prop.
f. $\angle 1 \cong \angle 2$	**f.** ___?___ Def $\cong$ $\angle$s

7. **PROOF** Write a two-column proof.
Given: $\angle 4 \cong \angle 7$
Prove: $\angle 5 \cong \angle 6$



SOLUTION:

You need to walk through the proof step by step. Look over what you are given and what you need to prove. Here, you are given two congruent angles. Use the properties that you have learned about congruent angles and equivalent expressions in algebra to walk through the proof.

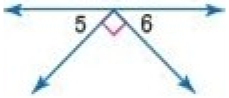
Given: $\angle 4 \cong \angle 7$
Prove: $\angle 5 \cong \angle 6$
Proof:
Statements(Reasons)
1. $\angle 4 \cong \angle 7$ (Given)
2. $\angle 4 \cong \angle 5$ and $\angle 6 \cong \angle 7$ (Vertical angles Theorem)
3. $\angle 7 \cong \angle 5$ (Substitution)
4. $\angle 5 \cong \angle 6$ (Substitution)

ANSWER:

Given: $\angle 4 \cong \angle 7$
Prove: $\angle 5 \cong \angle 6$
Proof:
Statements (Reasons)
1. $\angle 4 \cong \angle 7$ (Given)
2. $\angle 4 \cong \angle 5$ and $\angle 6 \cong \angle 7$ (Vert. $\angle s$ Thm.)
3. $\angle 7 \cong \angle 5$ (Subs.)
4. $\angle 5 \cong \angle 6$ (Subs.)

Find the measure of each numbered angle, and name the theorems used that justify your work.

8. $m\angle 5 = m\angle 6$



SOLUTION:

In the figure, $m\angle 5 + 90 + m\angle 6 = 180$.
Given that $m\angle 5 = m\angle 6$.

So, $m\angle 5 + 90 + m\angle 5 = 180$.

$$2m\angle 5 + 90 = 180$$
$$2m\angle 5 = 90$$
$$m\angle 5 = 45$$

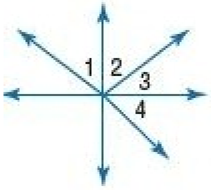
We know that $m\angle 5 = m\angle 6$.

So, $m\angle 6 = 45$.
Here, the Congruent Supplements Theorem has been used.

ANSWER:

$m\angle 5 = m\angle 6 = 45$ ($\cong$ Supp. Thm.)

9. $\angle 2$ and $\angle 3$ are complementary.
$\angle 1 \cong \angle 4$ and $m\angle 2 = 28$



SOLUTION:

Since $\angle 2$ and $\angle 3$ are complementary, $m\angle 2 + m\angle 3 = 90$.

Substitute $m\angle 2 = 28$.

$$28 + m\angle 3 = 90$$
$$m\angle 3 = 62$$

In the figure, $m\angle 1 + m\angle 2 + m\angle 3 + m\angle 4 = 180$.

Substitute.

$$m\angle 1 + 90 + m\angle 4 = 180$$
$$m\angle 1 + m\angle 4 = 90$$
$$m\angle 1 + m\angle 1 = 90 \ (\angle 1 \cong \angle 4)$$
$$2m\angle 1 = 90$$
$$m\angle 1 = 45$$

So, $m\angle 1 = 45$ and $m\angle 4 = 45$.
Here, the Congruent Complements and Congruent Supplements Theorems have been used.

ANSWER:

$m\angle 3 = 62, m\angle 1 = m\angle 4 = 45$ ($\cong$ Comp.
and Supp. Thm.)

10. $\angle 2$ and $\angle 4$, and $\angle 4$ and $\angle 5$ are supplementary. $m\angle 4 = 105$

SOLUTION:

Since $\angle 2$ and $\angle 4$ are supplementary angles, $m\angle 2 + m\angle 4 = 180$. Since $\angle 4$ and $\angle 5$ are supplementary angles, $m\angle 4 + m\angle 5 = 180$.

Therefore, $m\angle 2 = m\angle 5$.

Substitute $m\angle 4 = 105$.

$$m\angle 2 + 105 = 180$$
$$m\angle 2 = 75$$

So, $m\angle 5 = 75$.

We know that $m\angle 2 + m\angle 3 = 180$.

Substitute.

$$75 + m\angle 3 = 180$$
$$m\angle 3 = 105$$

Here, the Congruent Supplements Theorem has been used.

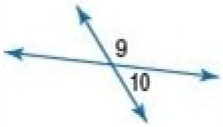
ANSWER:

$m\angle 2 = 75, m\angle 3 = 105, m\angle 5 = 75$ ($\cong$ Supp. Thm.)

Find the measure of each numbered angle and name the theorems used that justify your work.

11. $m\angle 9 = 3x + 12$

$m\angle 10 = x - 24$



SOLUTION:

The angles $\angle 9$ and $\angle 10$ are supplementary or form a linear pair.
So, $m\angle 9 + m\angle 10 = 180$.

Substitute.

$$3x + 12 + x - 24 = 180$$
$$4x - 12 = 180$$
$$4x = 192$$
$$x = 48$$

Substitute $x = 48$ in $m\angle 9 = 3x + 12$ and $m\angle 10 = x - 24$.

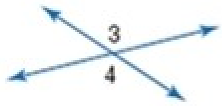
$$m\angle 9 = 3(48) + 12$$
$$= 144 + 12$$
$$= 156$$
$$m\angle 10 = 48 - 24$$
$$= 24$$

Here, the Supplement Theorem has been used.

ANSWER:

$m\angle 9 = 156, m\angle 10 = 24$ (Supp. Thm.)

12. $m\angle 3 = 2x + 23$
$m\angle 4 = 5x - 112$



SOLUTION:

Since $\angle 3$ and $\angle 4$ are vertical angles, they are congruent.
$m\angle 3 = m\angle 4$

Substitute.

$$\begin{aligned} 2x + 23 &= 5x - 112 \\ 2x + 23 - 5x &= 5x - 112 - 5x \\ 23 - 3x &= -112 \\ 23 - 3x - 23 &= -112 - 23 \\ -3x &= -135 \\ \frac{-3x}{-33} &= \frac{-135}{-3} \\ x &= 45 \end{aligned}$$

Substitute $x = 45$ in $m\angle 3 = 2x + 23$.

$$\begin{aligned} m\angle 3 &= 2(45) + 23 \\ &= 113 \end{aligned}$$

Substitute $x = 45$ in $m\angle 4 = 5x - 112$.

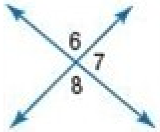
$$\begin{aligned} m\angle 4 &= 5(45) - 112 \\ &= 113 \end{aligned}$$

Here, the Vertical Angles Theorem has been used.

ANSWER:

$m\angle 3 = 113, m\angle 4 = 113$ (Vert. $\angle s$ Thm.)

13. $m\angle 6 = 2x - 21$
$m\angle 7 = 3x - 34$



SOLUTION:

In the figure, $m\angle 6 + m\angle 7 = 180$ and $m\angle 7 + m\angle 8 = 180$.
By congruent supplementary theorem, $m\angle 6 = m\angle 8$.

Since $\angle 6$ and $\angle 8$ are vertical angles, they are congruent.
$m\angle 6 = m\angle 8$
$m\angle 6 + m\angle 7 = 180$

Substitute.

$$\begin{aligned} 2x - 21 + 3x - 34 &= 180 \\ 5x - 55 &= 180 \\ 5x &= 235 \\ x &= 47 \end{aligned}$$

Substitute $x = 47$ in $m\angle 6 = 2x - 21$.

$$\begin{aligned} m\angle 6 &= 2(47) - 21 \\ &= 94 - 21 \\ &= 73 \end{aligned}$$

So, $m\angle 8 = 73$.

We know that $m\angle 6 + m\angle 7 = 180$.

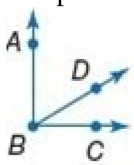
$$\begin{aligned} 73 + m\angle 7 &= 180 \\ m\angle 7 &= 103 \end{aligned}$$

Here, Congruent Supplements Theorem and the Vertical Angles Theorem have been used.

ANSWER:
$m\angle 6 = 73, m\angle 7 = 107, m\angle 8 = 73$ ($\cong$ Supp. Thm. and Vert. $\angle s$ Thm.)

PROOF Write a two-column proof.

14. Given: $\angle ABC$ is a right angle.
Prove: $\angle ABD$ and $\angle CBD$ are complementary.



SOLUTION:

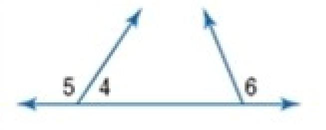
You need to walk through the proof step by step. Look over what you are given and what you need to prove. Here, you are given a right angle. Use the properties that you have learned about right angles, angle addition, complementary angles, congruent angles and equivalent expressions in algebra to walk through the proof.

Given: $\angle ABC$ is a right angle.
Prove: $\angle ABD$ and $\angle CBD$ are complementary.
Proof:
Statements (Reasons)
1. $\angle ABC$ is a right angle. (Given)
2. $m\angle ABC = 90$ (Definition of a right angle)
3. $m\angle ABC = m\angle ABD + m\angle CBD$ (Angle Addition Postulate)
4. $m\angle ABD + m\angle CBD = 90$ (Substitution)
5. $\angle ABD$ and $\angle CBD$ are complementary. (Definition of complimentary angles)

ANSWER:

Proof:
Statements (Reasons)
1. $\angle ABC$ is a right angle. (Given)
2. $m\angle ABC = 90$ (Def. of rt. angle)
3. $m\angle ABC = m\angle ABD + m\angle CBD$ ($\angle$ Add. Post.)
4. $m\angle ABD + m\angle CBD = 90$ (Subs.)
6. $\angle ABD$ and $\angle CBD$ are complementary. (Def. of compl. $\angle s$)

15. Given:. $\angle 5 \cong \angle 6$
Prove: $\angle 4$ and $\angle 6$ are supplementary.



SOLUTION:

You need to walk through the proof step by step. Look over what you are given and what you need to prove. Here, you are given two congruent angles Use the properties that you have learned about congruent angles and equivalent expressions in algebra to walk through the proof.

Given:. $\angle 5 \cong \angle 6$

Prove: $\angle 4$ and $\angle 6$ are supplementary.

Proof:

Statements (Reasons)

1. $\angle 4 \cong \angle 6$. (Given)
2. $m\angle 4 = m\angle 6$ (Definition of Congruent Angles)
3. $\angle 4 + \angle 5 = 180$ (Supplement Theorem)
4. $\angle 6 + \angle 5 = 180$ (Substitution)
5. $\angle 5$ and $\angle 6$ are supplementary. (Definition of supplementary angles)

ANSWER:

Proof:

Statements (Reasons)

1. $\angle 5 \cong \angle 6$ (Given)
2. $m\angle 5 = m\angle 6$ (Def. of $\cong \angle s$)
3. $\angle 4$ and $\angle 5$ are supplementary. (Def. of linear pairs)
4. $m\angle 4 + m\angle 5 = 180$ (Def. of supp. $\angle s$)
5. $m\angle 4 + m\angle 6 = 180$ (Subs.)
6. $\angle 4$ and $\angle 6$ are supplementary. (Def. of supp. $\angle s$)

Write a proof for each theorem.

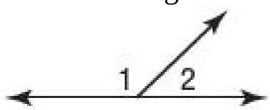
16. Supplement Theorem

SOLUTION:

You need to walk through the proof step by step. Look over what you are given and what you need to prove. Here, you are given two angles form a linear pair. Use the properties that you have learned about linear pairs, supplementary angles and equivalent expressions in algebra to walk through the proof.

Given: Two angles form a linear pair.

Prove: The angles are supplementary.



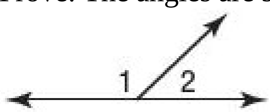
Paragraph Proof:

When two angles form a linear pair, the resulting angle is a straight angle whose measure is 180. By definition, two angles are supplementary if the sum of their measures is 180. By the Angle Addition Postulate, $m\angle 1 + m\angle 2 = 180$. Thus, if two angles form a linear pair, then the angles are supplementary.

ANSWER:

Given: Two angles form a linear pair.

Prove: The angles are supplementary.



Paragraph Proof:

When two angles form a linear pair, the resulting angle is a straight angle whose measure is 180. By definition, two angles are supplementary if the sum of their measures is 180. By the Angle Addition Postulate, $m\angle 1 + m\angle 2 = 180$. Thus, if two angles form a linear pair, then the angles are supplementary.

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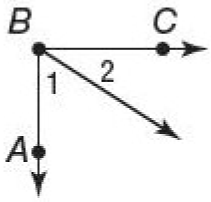
17. Complement Theorem

SOLUTION:

You need to walk through the proof step by step. Look over what you are given and what you need to prove. Here, you are given a right angle. Use the properties that you have learned about congruent angles, right angles, complementary angles, and equivalent expressions in algebra to walk through the proof.

Given: $\angle ABC$ is a right angle.

Prove: $\angle 1$ and $\angle 2$ are complementary angles.



Proof:

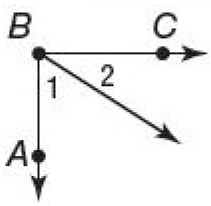
Statements (Reasons)

1. $\angle ABC$ is a right angle. (Given)
2. $m\angle ABC = 90$ (Definition of a right angle)
3. $m\angle ABC = m\angle 1 + m\angle 2$ (Angle Addition Postulate)
4. $m\angle 1 + m\angle 2 = 90$ (Substitution)
5. $\angle 1$ and $\angle 2$ are complementary. (Definition of complimentary angles)

ANSWER:

Given: $\angle ABC$ is a right angle..

Prove: $\angle 1$ and $\angle 2$ are complementary angles.



Proof:

Statements (Reasons)

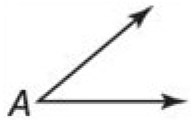
1. $\angle ABC$ is a right angle. (Given)
2. $m\angle ABC = 90$ (Def. of rt. $\angle s$)
3. $m\angle ABC = m\angle 1 + m\angle 2$ ($\angle$ Add. Post.)
4. $m\angle 1 + m\angle 2 = 90$ (Subst.)
5. $\angle 1$ and $\angle 2$ are complementary angles. (Def. of comp. $\angle s$)

18. Reflexive Property of Angle Congruence

SOLUTION:

You need to walk through the proof step by step. Look over what you are given and what you need to prove. Here, you are given an angle. Use the properties that you have learned about congruent angles and equivalent expressions in algebra to walk through the proof.

Given: $\angle A$



Prove: $\angle A \cong \angle A$

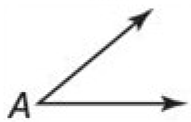
Proof:

Statements (Reasons)

1. $\angle A$ is an angle. (Given)
2. $m\angle A \cong m\angle A$ (Reflection Property)
3. $\angle A \cong \angle A$ (Definition of congruent angles)

ANSWER:

Given: $\angle A$



Prove: $\angle A \cong \angle A$

Proof:

Statements (Reasons)

1. $\angle A$ is an angle. (Given)
2. $m\angle A \cong m\angle A$ (Refl. Prop.)
3. $\angle A \cong \angle A$ (Def. of $\cong$ $\angle s$)

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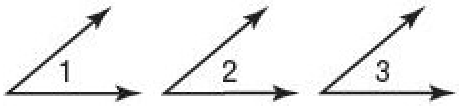
19. Transitive Property of Angle Congruence

SOLUTION:

You need to walk through the proof step by step. Look over what you are given and what you need to prove. Here, you are given two congruent angles. Use the properties that you have learned about congruent angles, Transitive property, and equivalent expressions in algebra to walk through the proof.

Given: $\angle 1 \cong \angle 2$, $\angle 2 \cong \angle 3$

Prove: $\angle 1 \cong \angle 3$



Proof:

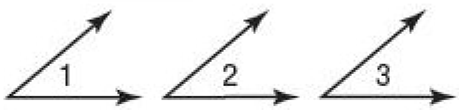
Statements (Reasons)

1. $\angle 1 \cong \angle 2$, $\angle 2 \cong \angle 3$ (Given)
2. $m\angle 1 = m\angle 2$, $m\angle 2 = m\angle 3$ Definition of congruent angles)
3. $m\angle 1 = m\angle 3$ (Transitive Property)
4. $\angle 1 \cong \angle 3$ (Definition of congruent angles)

ANSWER:

Given: $\angle 1 \cong \angle 2$, $\angle 2 \cong \angle 3$

Prove: $\angle 1 \cong \angle 3$



Proof:

Statements (Reasons)

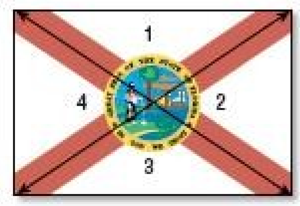
1. $\angle 1 \cong \angle 2$, $\angle 2 \cong \angle 3$ (Given)
2. $m\angle 1 = m\angle 2$, $m\angle 2 = m\angle 3$ (Def. of $\cong$ $\angle$s)
3. $m\angle 1 = m\angle 3$ (Trans. Prop.)
4. $\angle 1 \cong \angle 3$ (Def. of $\cong$ $\angle$s)

20. **FLAGS** Refer to the Florida state flag at the right. Prove that the sum of the four angle measures is 360.

SOLUTION:

You need to walk through the proof step by step. Look over what you are given and what you need to prove. Here, you are given the Florida state flag with diagonals. Use the properties that you have learned about congruent angles and equivalent expressions in algebra to walk through the proof.

Given:



Prove: $m\angle 1 + m\angle 2 + m\angle 3 + m\angle 4 = 360$
Proof:
Statements (Reasons)
1.

(Given)

2. $m\angle 1 + m\angle 2 = 180$ and $m\angle 3 + m\angle 4 = 180$ (Supplementary Theorem)
3. $m\angle 1 + m\angle 2 + m\angle 3 = 180 + m\angle 3$ (Addition Property)
4. $m\angle 1 + m\angle 2 + m\angle 3 + m\angle 4 = 180 + m\angle 3 + m\angle 4$ (Addition Property)
5. $m\angle 1 + m\angle 2 + m\angle 3 + m\angle 4 = 180 + 180$ (Substitution)
6. $m\angle 1 + m\angle 2 + m\angle 3 + m\angle 4 = 360$ (Addition Property)

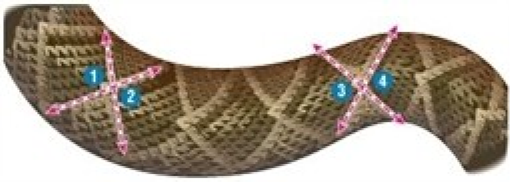
ANSWER:
Given:

Prove: $m\angle 1 + m\angle 2 + m\angle 3 + m\angle 4 = 360$
Proof:
Statements (Reasons)
1.

(Given)

2. $m\angle 1 + m\angle 2 = 180$ and $m\angle 3 + m\angle 4 = 180$ (Suppl. Thm.)
3. $m\angle 1 + m\angle 2 + m\angle 3 = 180 + m\angle 3$ (Add. Prop.)
4. $m\angle 1 + m\angle 2 + m\angle 3 + m\angle 4 = 180 + m\angle 3 + m\angle 4$
(Add. Prop.)
5. $m\angle 1 + m\angle 2 + m\angle 3 + m\angle 4 = 180 + 180$ (Subs.)
6. $m\angle 1 + m\angle 2 + m\angle 3 + m\angle 4 = 360$ (Add. Prop.)

21. **CCSS ARGUMENTS** The diamondback rattlesnake is a pit viper with a diamond pattern on its back. An enlargement of the skin of the snake at the left is shown below. If $\angle 1 \cong \angle 4$, prove that $\angle 2 \cong \angle 3$.



SOLUTION:

You need to walk through the proof step by step. Look over what you are given and what you need to prove. Here, you are given two congruent angles. Use the properties that you have learned about congruent angles, vertical angles, and equivalent expressions in algebra to walk through the proof.

Given: $\angle 1 \cong \angle 4$

Prove: $\angle 2 \cong \angle 3$

Proof:

<u>Statements (Reasons)</u>

1. $\angle 1 \cong \angle 4$ (Given)
2. $\angle 1 \cong \angle 2$ and $\angle 3 \cong \angle 4$ (Vertical angles are congruent)
3. $\angle 1 \cong \angle 3$ (Transitive Property)
4. $\angle 2 \cong \angle 3$ (Substitution)

ANSWER:

Given: $\angle 1 \cong \angle 4$

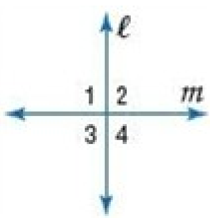
Prove: $\angle 2 \cong \angle 3$

Proof:

<u>Statements (Reasons)</u>

1. $\angle 1 \cong \angle 4$ (Given)
2. $\angle 1 \cong \angle 2$, $\angle 3 \cong \angle 4$ (Vert. $\angle$s are $\cong$.)
3. $\angle 1 \cong \angle 3$ (Trans. Prop.)
4. $\angle 2 \cong \angle 3$ (Subs.)

PROOF Use the figure to write a proof of each theorem.



22. Theorem 2.9

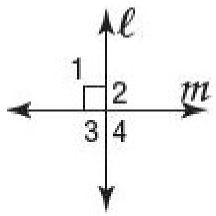
SOLUTION:

You need to walk through the proof step by step. Look over what you are given and what you need to prove. Here, you are given two perpendicular lines. Use the properties that you have learned about congruent angles, right angles, perpendicular lines, and equivalent expressions in algebra to walk through the proof.

Given: $\ell \perp m$

Prove: $\angle 2, \angle 3, \angle 4$ are right angles.

2-8 Proving Angle Relationships



Proof:

Statements (Reasons)

1. $\ell \perp m$ (Given)
2. $\angle 1$ is a right angle. (Definition of perpendicular)
3. $m\angle 1 = 90$ (Definition of right angles)
4. $\angle 1 \cong \angle 4$ (Vertical angles are congruent)
5. $m\angle 1 = m\angle 4$ (Definition of vertical angles)
6. $\angle 1 \cong \angle 4$ (Substitution)
7. $\angle 1$ and $\angle 2$ form a linear pair.

$\angle 3$ and $\angle 4$ form a linear pair. (Definition of linear pairs)

8. $m\angle 1 + m\angle 2 = 180$, $m\angle 3 + m\angle 4 = 180$ (Linear pairs are supplementary)
9. $90 + m\angle 2 = 180$, $90 + m\angle 3 = 180$ (Substitution)
10. $m\angle 2 = 90, m\angle 3 = 90$ (Subtraction Property)
11. $\angle 2, \angle 3, \angle 4$ are right angles. (Definition. of right angles (Steps 6, 10))

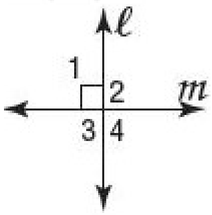
ANSWER:

2-8 Proving Angle Relationships

Given:

$\ell \perp m$

Prove: $\angle 2, \angle 3, \angle 4$ are rt. $\angle$s



Proof:

Statements (Reasons)

1. $\ell \perp m$ (Given)
2. $\angle 1$ is a right angle. (Def. of $\perp$)
3. $m\angle 1 = 90$ (Def. of rt. $\angle s$)
4. $\angle 1 \cong \angle 4$ (Vert. $\angle s \cong$)
5. $m\angle 1 = m\angle 4$ (Def. of vertical $\angle s$)
6. $m\angle 4 = 90$ (Subs.)
7. $\angle 1$ and $\angle 2$ form a linear pair.

$\angle 3$ and $\angle 4$ form a linear pair. (Def. of linear pairs)

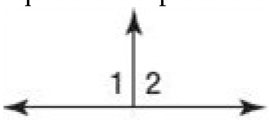
8. $m\angle 1 + m\angle 2 = 180$, $m\angle 3 + m\angle 4 = 180$ (Linear pairs are supplementary.)
9. $90 + m\angle 2 = 180$, $90 + m\angle 3 = 180$ (Subs.)
10. $m\angle 2 = 90, m\angle 3 = 90$ (Subt. Prop.)
11. $\angle 2, \angle 3, \angle 4$ are rt. $\angle$s . (Def. of rt. $\angle s$ (Steps 6, 10))

2-8 Proving Angle Relationships

23. Theorem 2.10

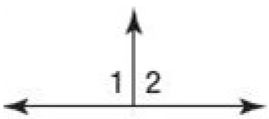
SOLUTION:

You need to walk through the proof step by step. Look over what you are given and what you need to prove. Here, you are given two right angles. Use the properties that you have learned about congruent angles, right angles, and equivalent expressions in algebra to walk through the proof.



Given: $\angle 1$ and $\angle 2$ are right angles.
Prove: $\angle 1 \cong \angle 2$
Proof:
Statements (Reasons)
1. $\angle 1$ and $\angle 2$ are rt. $\angle$s. (Given)
2. $m\angle 1 = 90, m\angle 2 = 90$ (Definition of right angles)
3. $m\angle 1 = m\angle 2$ (Substitution)
4. $\angle 1 \cong \angle 2$ (Definition of congruent angles)

ANSWER:



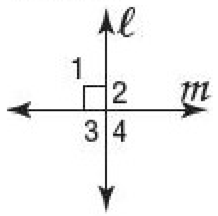
Given: $\angle 1$ and $\angle 2$ are rt. $\angle$s.
Prove: $\angle 1 \cong \angle 2$
Proof:
Statements (Reasons)
1. $\angle 1$ and $\angle 2$ are rt. $\angle$s. (Given)
2. $m\angle 1 = 90, m\angle 2 = 90$ (Def. of rt. $\angle$s)
3. $m\angle 1 = m\angle 2$ (Subs.)
4. $\angle 1 \cong \angle 2$ (Def. of $\cong$ $\angle$s)

24. Theorem 2.11

SOLUTION:

You need to walk through the proof step by step. Look over what you are given and what you need to prove. Here, you are given two perpendicular lines. Use the properties that you have learned about congruent angles, right angles, perpendicular lines, and equivalent expressions in algebra to walk through the proof.

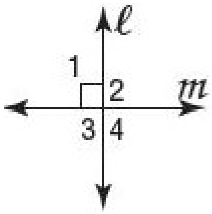
Given:
$\ell \perp m$
Prove: $\angle 1 \cong \angle 2$



Proof:
Statements (Reasons)
1. $\ell \perp m$ (Given)
2. $\angle 1$ and $\angle 2$ right angles. (Perpendicular lines intersect to form 4 right angles.)
3. $\angle 1 \cong \angle 2$ (All right angles are congruent .)

ANSWER:

Given:
$\ell \perp m$
Prove: $\angle 1 \cong \angle 2$

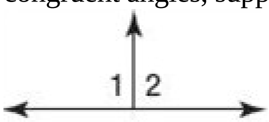


Proof:
Statements (Reasons)
1. $\ell \perp m$ (Given)
2. $\angle 1$ and $\angle 2$ rt. $\angle s$. ($\perp$ lines intersect to form 4 rt. $\angle s$.)
3. $\angle 1 \cong \angle 2$ (All rt. $\angle s \cong$.)

25. Theorem 2.12

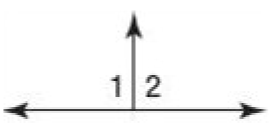
SOLUTION:

You need to walk through the proof step by step. Look over what you are given and what you need to prove. Here, you are given two congruent angles, and two supplementary angles Use the properties that you have learned about congruent angles, supplmentary angles, right angles, and equivalent expressions in algebra to walk through the proof.



Given: $\angle 1 \cong \angle 2$, $\angle 1$ and $\angle 2$ are supplementary.
Prove: $\angle 1$ and $\angle 2$ are right angles.
Proof:
Statements (Reasons)
1. $\angle 1 \cong \angle 2$, $\angle 1$ and $\angle 2$ are supplementary. (Given)
2. $m\angle 1 + m\angle 2 = 180$ (Definition of angles)
3. $m\angle 1 = m\angle 2$ (Definition of congruent angles)
4. $m\angle 1 + m\angle 1 = 180$ (Substitution)
5. $2(m\angle 1) = 180$ (Substitution)
6. $m\angle 1 = 180$ (Division. Property)
7. $m\angle 2 = 180$ (Substitution (Steps 3, 6))
8. $\angle 1$ and $\angle 2$ are right angles (Definition of right angles)

ANSWER:



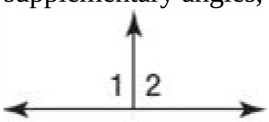
Given: $\angle 1 \cong \angle 2$, $\angle 1$ and $\angle 2$ are supplementary.
Prove: $\angle 1$ and $\angle 2$ are rt. $\angle s$.
Proof:
Statements (Reasons)
1. $\angle 1 \cong \angle 2$, $\angle 1$ and $\angle 2$ are rt. $\angle s$. (Given)
2. $m\angle 1 + m\angle 2 = 180$ (Def. of $\angle s$)
3. $m\angle 1 = m\angle 2$ (Def. of $\cong$ $\angle s$)
4. $m\angle 1 + m\angle 1 = 180$ (Subs.)
5. $2(m\angle 1) = 180$ (Subs.)
6. $m\angle 1 = 180$ (Div. Prop.)
7. $m\angle 2 = 180$ (Subs.(Steps 3, 6))
8. $\angle 1$ and $\angle 2$ are rt. $\angle s$ (Def. of rt. $\angle s$)

2-8 Proving Angle Relationships

26. Theorem 2.13

SOLUTION:

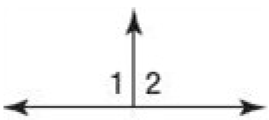
You need to walk through the proof step by step. Look over what you are given and what you need to prove. Here, you are given two congruent angles. Use the properties that you have learned about congruent angles, linear pair, supplementary angles, right angles, and equivalent expressions in algebra to walk through the proof.



Given: $\angle 1 \cong \angle 2$
Prove: $\angle 1$ and $\angle 2$ are right angles.

Proof:
Statements (Reasons)
1. $\angle 1 \cong \angle 2$ (Given)
2. $\angle 1$ and $\angle 2$ form a linear pair. (Definition of linear pair)
3. $\angle 1$ and $\angle 2$ are supplementary. (Linear pairs are supplementary.)
4. $\angle 1$ and $\angle 2$ are right angles. (If angles are congruent and supplementary, they are right angles.)

ANSWER:



Given: $\angle 1 \cong \angle 2$
Prove: $\angle 1$ and $\angle 2$ are rt. $\angle$s.
Proof:
Statements (Reasons)
1. $\angle 1 \cong \angle 2$ (Given)
2. $\angle 1$ and $\angle 2$ form a linear pair. (Def. of linear pair)
3. $\angle 1$ and $\angle 2$ are supplementary. (Linear pairs are supplementary.)
4. $\angle 1$ and $\angle 2$ are rt. $\angle$s. (If $\angle$s $\cong$ and suppl., they are rt. $\angle$s .)

27. **CCSS ARGUMENTS** To mark a specific tempo, the weight on the pendulum of a metronome is adjusted so that it swings at a specific rate. Suppose $\angle ABC$ in the photo is a right angle. If $m\angle 1 = 45$, write a paragraph proof to show that $\overrightarrow{BR}$ bisects $\angle ABC$.

SOLUTION:

You need to walk through the proof step by step. Look over what you are given and what you need to prove. Here, you are given the measures of two angles. Use the properties that you have learned about congruent angles, right angles, angle bisectors, and equivalent expressions in algebra to walk through the proof.

Since the path of the pendulum forms a right angle, $\angle ABC$ is a right angle, or measures 90. $\overrightarrow{BR}$ divides $\angle ABC$ into $\angle ABR$ and $\angle CBR$. By the Angle Addition Postulate, $m\angle ABR + m\angle CBR = m\angle ABC$, and using substitution, $m\angle ABR + m\angle CBR = 90$. Substituting again, $m\angle 1 + m\angle 2 = 90$. We are given that $m\angle 1$ is 45, so, substituting, $45 + m\angle 2 = 90$. Using the Subtraction Property, $45 - 45 + m\angle 2 = 90 - 45$, or $m\angle 2 = 45$. Since $\angle 1$ and $\angle 2$ are congruent, $\overrightarrow{BR}$ is the bisector of $\angle ABC$ by the definition of angle bisector.

ANSWER:

Since the path of the pendulum forms a right angle, $\angle ABC$ is a right angle, or measures 90. $\overrightarrow{BR}$ divides $\angle ABC$ into $\angle ABR$ and $\angle CBR$. By the Angle Addition Postulate, $m\angle ABR + m\angle CBR = m\angle ABC$ and, using substitution, $m\angle ABR + m\angle CBR = 90$. Substituting again, $m\angle 1 + m\angle 2 = 90$. We are given that $m\angle 1$ is 45, so, substituting, $45 + m\angle 2 = 90$. Using the Subtraction Property, $45 - 45 + m\angle 2 = 90 - 45$. Since $\angle 1$ and $\angle 2$ are congruent, $\overrightarrow{BR}$ is the bisector of $\angle ABC$ by the definition of angle bisector.

28. **PROOF** Write a proof of Theorem 2.8.

SOLUTION:

You need to walk through the proof step by step. Look over what you are given and what you need to prove. Here, you are given two vertical angles. Use the properties that you have learned about congruent angles, vertical angles, supplementary angles, linear pairs, and equivalent expressions in algebra to walk through the proof.

Given: $\angle 1$ and $\angle 3$ are vertical angles.
Prove: $\angle 1 \cong \angle 3$
Proof: Since $\angle 1$ and $\angle 3$ are vertical angles so they are formed by intersecting lines. Then we know that $\angle 1$ and $\angle 2$ are a linear pair and $\angle 2$ and $\angle 3$ are also a linear pair. By Theorem 2.3, $\angle 1$ and $\angle 2$ are supplementary angles and $\angle 2$ and $\angle 3$ are supplementary angles. Then by the Transitive Property of Congruence, $\angle 1 \cong \angle 3$.

ANSWER:

Given: $\angle 1$ and $\angle 3$ are vertical angles.
Prove: $\angle 1 \cong \angle 3$.
Proof: Since $\angle 1$ and $\angle 3$ are vertical angles so they are formed by intersecting lines. Then we know that $\angle 1$ and $\angle 2$ are a linear pair and $\angle 2$ and $\angle 3$ are also a linear pair. By Theorem 2.3, $\angle 1$ and $\angle 2$ are supplementary angles and $\angle 2$ and $\angle 3$ are supplementary angles. Then by the Transitive Property of Congruence, $\angle 1 \cong \angle 3$.

29. **GEOGRAPHY** Utah, Colorado, Arizona, and New Mexico all share a common point on their borders called Four Corners. This is the only place where four states meet in a single point. If $\angle 2$ is a right angle, prove that lines ℓ and *m* are perpendicular.

SOLUTION:

You need to walk through the proof step by step. Look over what you are given and what you need to prove. Here, you are given a right angle. Use the properties that you have learned about congruent angles, right angles, vertical angles, and equivalent expressions in algebra to walk through the proof.
Given: $\angle 2$ is a right angle.

Prove:
$\ell \perp m$
Proof:
Statements (Reasons)
1. $\angle 2$ is a right angle. (Given)
2. $m\angle 2 = 90$ (Definition of a right angles)
3. $m\angle 2 \cong m\angle 3$ (Vertical angles are congruent.)
4. $m\angle 3 = 90$ (Substitution)
5. $m\angle 1 + m\angle 2 = 180$ (Supplementary Theorem)
6. $m\angle 1 + 90 = 180$ (Substitution)
7. $m\angle 1 + 90 - 90 = 180 - 90$ (Subtraction Property)
8. $m\angle 1 = 90$ (Subtraction. Property)
9. $\angle 1 \cong \angle 4$ (Vertical angles are congruent.)
10. $\angle 4 \cong \angle 1$ (Symmetric Property)
11. $m\angle 4 = m\angle 1$ (Definition of congruent angles)
12. $m\angle 4 \cong 90$ (Substitution)
13. $\ell \perp m$ (Perpendicular lines intersect to form four right angles.)

ANSWER:

Given: $\angle 2$ is a right angle.

Prove:
$\ell \perp m$
Proof:
Statements (Reasons)
1. $\angle 2$ is a right angle. (Given)
2. $m\angle 2 = 90$ (Def. of a rt. $\angle$)
3. $m\angle 2 \cong m\angle 3$ (Vert. $\angle$s are $\cong$.)
4. $m\angle 3 = 90$ (Subs.)
5. $m\angle 1 + m\angle 2 = 180$ (Supp. Th.)
6. $m\angle 1 + 90 = 180$ (Subs.)
7. $m\angle 1 + 90 - 90 = 180 - 90$ (Subt. Prop.)
8. $m\angle 1 = 90$ (Subt. Prop.)
9. $\angle 1 \cong \angle 4$ (Vert. $\angle$s are $\cong$.)
10. $\angle 4 \cong \angle 1$ (Symm. Prop.)
11. $m\angle 4 = m\angle 1$ (Def. of $\cong$ $\angle$s)
12. $m\angle 4 \cong 90$ (Subs.)
13. $\ell \perp m$ (Perpendicular lines intersect to form four right angles.)

30. **MULTIPLE REPRESENTATIONS** In this problem, you will explore angle relationships.

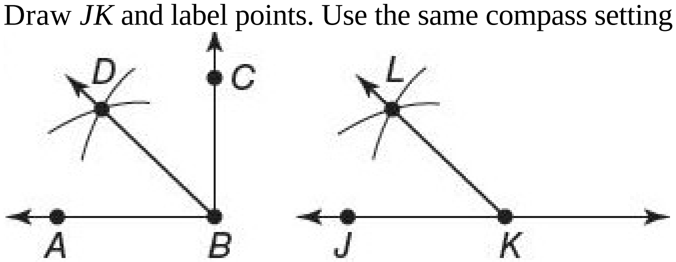
a. GEOMETRIC Draw a right angle *ABC*. Place point *D* in the interior of this angle and draw $\overrightarrow{BD}$. Draw $\overrightarrow{KL}$ and construct $\angle JKL$ congruent to $\angle ABD$.

b. VERBAL Make a conjecture as to the relationship between $\angle JKL$ and $\angle DBC$.

c. LOGICAL Prove your conjecture.

SOLUTION:

a. Use a compass, protractor, and straightedge for the construction. First draw *AB* with the straightedge. Use the protractor to draw the 90 angle from *AB*. Label all the points. Set the compass to a distance shorter then *AC*. Draw arcs with the same compass setting from point points *A* and *C*. Label the point of intersection *D*. Connect *B* and *D*. Draw *JK* and label points. Use the same compass setting to draw point *L*. Connect *K* and *L*.



b. $\angle DBC$ and $\angle ABD$ are complementary . Since $\angle JKL$ was constructed to be congruent to $\angle ABD$, then $\angle DBC$ and $\angle JKL$ are complementary.

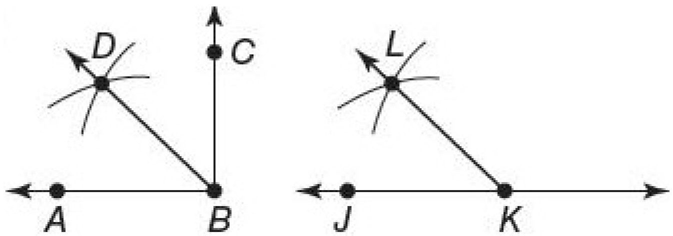
c. You need to walk through the proof step by step. Look over what you are given and what you need to prove. Here, you are given two complementary angles. Use the properties that you have learned about congruent angles, complementary angles, and equivalent expressions in algebra to walk through the proof.

Given: $\angle ABD$ and $\angle DBC$ are complementary.
$\angle ABD \cong \angle JKL$
Prove: $\angle DBC$ and $\angle JKL$ are complementary.
Proof:
Statements (Reasons)
1. $\angle ABD$ and $\angle DBC$ are complementary $\angle ABD \cong \angle JKL$. (Given)
2. $m\angle DBC + m\angle ABD = 90$ (Definition of complementary angles)
3. $m\angle ABD = m\angle JKL$ (Definition of congruent angles)

4. $m\angle DBC + m\angle JKL = 90$ (Substitution)
5. $\angle DBC$ and $\angle JKL$ are complementary. (Definition of complementary angles)

ANSWER:

a.

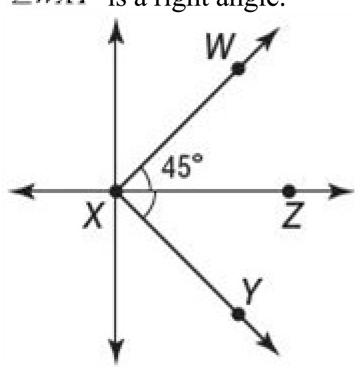


b. Sample answer: $\angle DBC$ and $\angle JKL$ are complementary.
c. Given: $\angle ABD$ and $\angle DBC$ are complementary.
$\angle ABD \cong \angle JKL$
Prove: $\angle DBC$ and $\angle JKL$ are complementary.
Proof:
Statements (Reasons)
1. $\angle ABD$ and $\angle DBC$ are complementary $\angle ABD \cong \angle JKL$.(Given)
2. $m\angle DBC + m\angle ABD = 90$ (Def. of comp. $\angle$s)
3. $m\angle ABD = m\angle JKL$ (Def. of $\cong$ $\angle$s)
4. $m\angle DBC + m\angle JKL = 90$ (Subs.)
5. $\angle DBC$ and $\angle JKL$ are complementary (Def. of comp. $\angle$s)

31. **OPEN ENDED** Draw an angle *WXZ* such that $m\angle WXZ = 45$. Construct $\angle YXZ$ congruent to $\angle WXZ$. Make a conjecture as to the measure of $\angle WXY$, and then prove your conjecture.

SOLUTION:

You need to walk through the proof step by step. Look over what you are given and what you need to prove. Here, you are given an angle bisector and the measure of an angle.Use the properties that you have learned about congruent angles, angle bisectors, right angles, and equivalent expressions in algebra to walk through the proof.
Given: $\overline{XZ}$ bisects $\angle WXY$, and $m\angle WXZ = 45$.
Prove: $\angle WXY$ is a right angle.

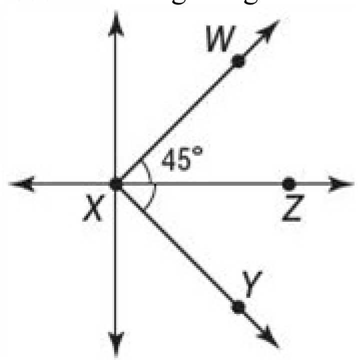


Proof:
Statements (Reasons)
1. $\overline{XZ}$ bisects $\angle WXY$, and $m\angle WXZ = 45$ (Given)
2. $\angle WXZ \cong \angle ZXY$ (Definition of angle bisector)
3. $m\angle WXZ = m\angle ZXY$ (Definition of congruent angles)
4. $m\angle ZXY = 45$ (Substitution)
5. $m\angle WXY = m\angle WXZ + m\angle ZXY$ (Angle Addition Postulate)

6. $m\angle WXY = 45 + 45$ (Substitution)
7. $m\angle WXY = 90$ (Substitution)
8. $\angle WXY$ is a right angle. (Definition of right angle)

ANSWER:

Given: $\overline{XZ}$ bisects $\angle WXY$, and $m\angle WXZ = 45$.
Prove: $\angle WXY$ is a right angle.



Proof:
Statements (Reasons)
1. $\overline{XZ}$ bisects $\angle WXY$, and $m\angle WXZ = 45$. (Given)
2. $\angle WXZ \cong \angle ZXY$ (Def. of $\angle$ bisector)
3. $m\angle WXZ = m\angle ZXY$ (Def. of $\cong$ $\angle$s)
4. $m\angle ZXY = 45$ (Subs.)
5. $m\angle WXY = m\angle WXZ + m\angle ZXY$ ($\angle$ Add. Post.)
6. $m\angle WXY = 45 + 45$ (Subs.)
7. $m\angle WXY = 90$ (Subs.)
8. $\angle WXY$ is a right angle. (Def. of rt. $\angle$)

32. **WRITING IN MATH** Write the steps that you would use to complete the proof below.
Given: $\overline{BC} \cong \overline{CD}, AB = \frac{1}{2}BD$
Prove: $\overline{AB} \cong \overline{CD}$

SOLUTION:

First, show that $BC = CD$ by the definition of congruent segments. Then show $BC + CD = BD$ by the Segment Addition Postulate. . Then use substitution to show that $CD + CD = BD$ and $2CD = BD$. Divide to show that $CD = \frac{1}{2}BD$, so $AB = CD$. That means that $\overline{AB} \cong \overline{CD}$ by the definition of congruent segments.

ANSWER:

Sample answer: First, show that $BC = CD$ and $BC + CD = BD$. Then use substitution to show that $CD + CD = BD$ and $2CD = BD$. Divide to show that $CD = \frac{1}{2}BD$, so $AB = CD$. That means that $\overline{AB} \cong \overline{CD}$.

33. **CHALLENGE** In this lesson, one case of the Congruent Supplements Theorem was proven. In Exercise 6, you proved the same case for the Congruent Complements Theorem. Explain why there is another case for each of these theorems. Then write a proof of this second case for each theorem.

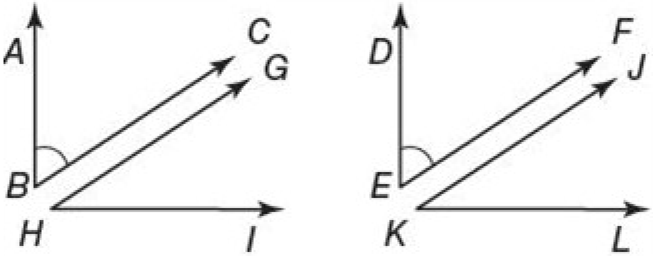
SOLUTION:

2-8 Proving Angle Relationships

Each of these theorems uses the words “or to congruent angles” indicating that this case of the theorem must also be proven true. The other proofs only addressed the “to the same angle” case of the theorem.

Given: $\angle ABC \cong \angle DEF$, $\angle GHI$ is complementary to $\angle ABC, \angle JKL$ is complementary to $\angle DEF$.

Prove: $\angle GHI \cong \angle JKL$



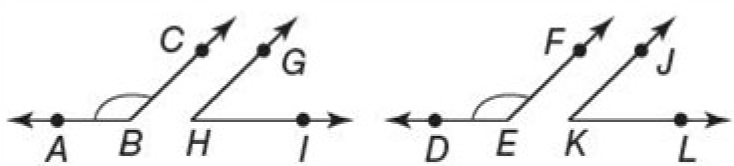
Proof:

Statements (Reasons)

1. $\angle ABC \cong \angle DEF$, $\angle GHI$ is complementary to $\angle ABC, \angle JKL$ is complementary to $\angle DEF$. (Given)
2. $m\angle ABC + m\angle GHI = 90, m\angle DEF + m\angle JKL = 90$ (Definition of complementary angles)
3. $m\angle ABC + m\angle JKL = 90$ (Substitution)
4. $90 = m\angle ABC + m\angle JKL$ (Symmetric. Property)
5. $m\angle ABC + m\angle GHI = m\angle ABC + m\angle JKL$ (Transitive Property)
6. $m\angle ABC + m\angle GHI - m\angle ABC = m\angle ABC + m\angle JKL - m\angle ABC$ (Subtraction Property)
7. $m\angle GHI = m\angle JKL$ (Simplify)
8. $\angle GHI \cong \angle JKL$ (Definition of congruent angles)

Given: $\angle ABC \cong \angle DEF$, $\angle GHI$ is supplementary to $\angle ABC, \angle JKL$ is supplementary to $\angle DEF$.

Prove: $\angle GHI \cong \angle JKL$



Proof:

Statements (Reasons)

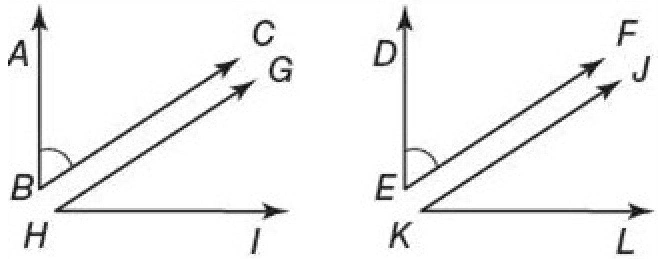
1. $\angle ABC \cong \angle DEF$, $\angle GHI$ is supplementary to $\angle ABC, \angle JKL$ is supplementary to $\angle DEF$. (Given)
2. $m\angle ABC + m\angle GHI = 180, m\angle DEF + m\angle JKL = 180$ (Definition of supplementary angles)
3. $m\angle ABC + m\angle JKL = 180$ (Substitution)
4. $180 = m\angle ABC + m\angle JKL$ (Symmetric Property)
5. $m\angle ABC + m\angle GHI = m\angle ABC + m\angle JKL$ (Transitive Property)
6. $m\angle ABC + m\angle GHI - m\angle ABC = m\angle ABC + m\angle JKL - m\angle ABC$ (Subtraction Property)
7. $m\angle GHI = m\angle JKL$ (Simplify.)
8. $\angle GHI \cong \angle JKL$ (Definition of congruent angles)

ANSWER:

Each of these theorems uses the words “or to congruent angles” indicating that this case of the theorem must also be proven true. The other proofs only addressed the “to the same angle” case of the theorem.

Given: $\angle ABC \cong \angle DEF$, $\angle GHI$ is complementary to $\angle ABC, \angle JKL$ is complementary to $\angle DEF$.

Prove: $\angle GHI \cong \angle JKL$

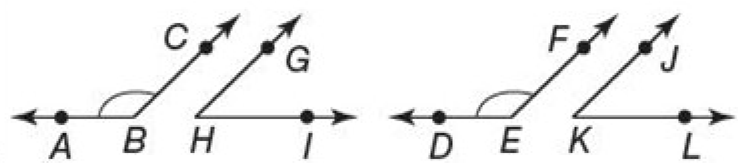


Proof:

Statements (Reasons)

1. $\angle ABC \cong \angle DEF$, $\angle GHI$ is complementary to $\angle ABC, \angle JKL$ is complementary to $\angle DEF$. (Given)
2. $m\angle ABC + m\angle GHI = 90, m\angle DEF + m\angle JKL = 90$ (Def. of compl. $\angle s$)
3. $m\angle ABC + m\angle JKL = 90$ (Subs.)
4. $90 = m\angle ABC + m\angle JKL$ (Symm. Prop.)
5. $m\angle ABC + m\angle GHI = m\angle ABC + m\angle JKL$ (Trans. Prop.)
6. $m\angle ABC + m\angle GHI - m\angle ABC = m\angle ABC + m\angle JKL - m\angle ABC$ (Subt. Prop.)
7. $m\angle GHI = m\angle JKL$ (Simplify.)
8. $\angle GHI \cong \angle JKL$ (Def. of $\cong$ $\angle s$)

Given: $\angle ABC \cong \angle DEF$, $\angle GHI$ is complementary to $\angle ABC, \angle JKL$ is complementary to $\angle DEF$.



Prove: $\angle GHI \cong \angle JKL$

Proof:

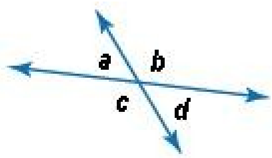
Statements (Reasons)

1. $\angle ABC \cong \angle DEF$, $\angle GHI$ is complementary to $\angle ABC, \angle JKL$ is complementary to $\angle DEF$. (Given)
2. $m\angle ABC + m\angle GHI = 180, m\angle DEF + m\angle JKL = 180$ (Def. of suppl. $\angle s$)
3. $m\angle ABC + m\angle JKL = 180$ (Subs.)
4. $180 = m\angle ABC + m\angle JKL$ (Symm. Prop.)
5. $m\angle ABC + m\angle GHI = m\angle ABC + m\angle JKL$ (Trans. Prop.)
6. $m\angle ABC + m\angle GHI - m\angle ABC = m\angle ABC + m\angle JKL - m\angle ABC$ (Subt. Prop.)
7. $m\angle GHI = m\angle JKL$ (Simplify.)
8. $\angle GHI \cong \angle JKL$ (Def. of $\cong$ $\angle s$)

34. **REASONING** Determine whether the following statement is sometimes, always, or never true. Explain your reasoning.
If one of the angles formed by two intersecting lines is acute, then the other three angles formed are also acute.

SOLUTION:

If one of the angles formed by two intersecting lines is acute, the other three angles formed are never all acute. Adjacent angles formed by two interesting lines form a linear pair. If one angle in this linear pair is acute, then its measure is less than 90. The supplement of any angle will be greater than 90 because subtracting a number less than 90 from 180 must always result in a measure greater than 90.



In the example above, if a is acute, the b will be obtuse. c will be obtuse because is is an vertical angle with b. b is acute.

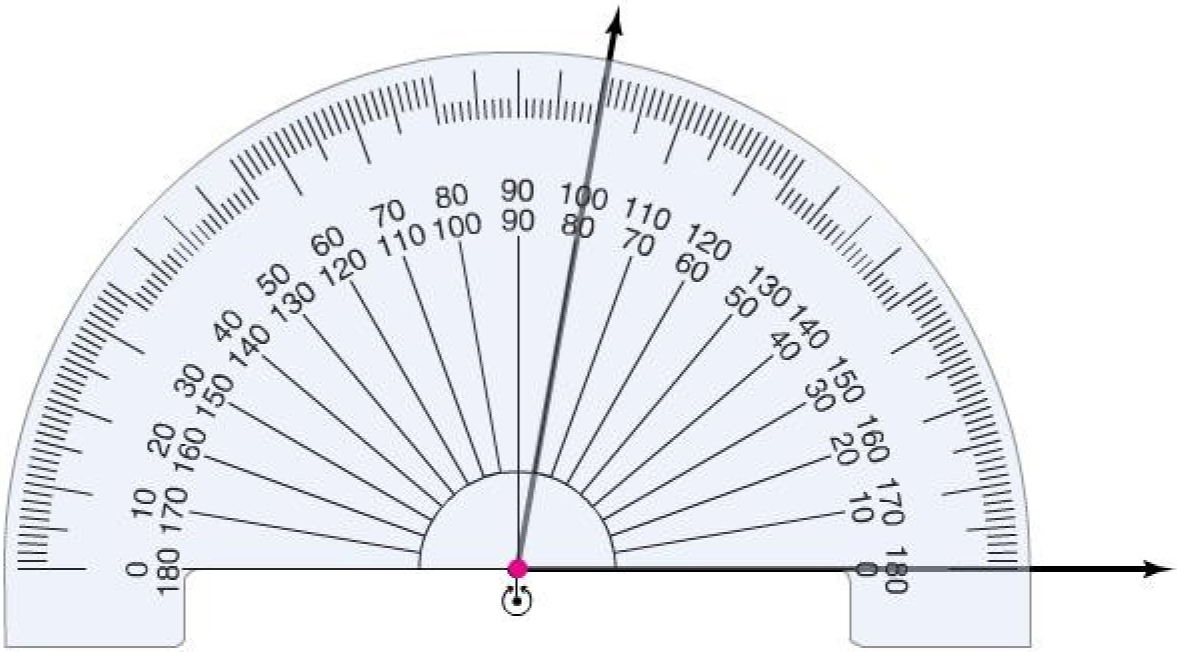
ANSWER:

Never; adjacent angles formed by two interesting lines form a linear pair. If one angle in this linear pair is acute, then its measure is less than 90. The supplement of any angle will be greater than 90 because subtracting a number less than 90 from 180 must always result in a measure greater than 90.

35. **WRITING IN MATH** Explain how you can use your protractor to quickly find the measure of the supplement of an angle.

SOLUTION:

Since protractors have the scale for both acute and obtuse angles along the top, the supplement is the measure of the given angle on the other scale. In the example below. The acute measure for the angle is 80° and the obtuse is 100°.



ANSWER:

Sample answer: Since protractors have the scale for both acute and obtuse angles along the top, the supplement is the measure of the given angle on the other scale.

36. **GRIDDED RESPONSE** What is the mode of this set of data?
4, 3, –2, 1 , 4, 0, 1, 4

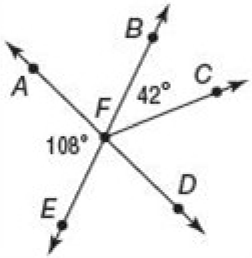
SOLUTION:

The mode is the value or values that occur most frequently in a data set. Here, the mode is 4.

ANSWER:

4

37. Find the measure of $\angle CFD$.



A 66°
B 72°
C 108°
D 138°

SOLUTION:

In the figure, $\angle BFD$ and $\angle AFD$ are vertical angles.
So, $m\angle BFD = m\angle AFD$.
Here, $m\angle BFD = m\angle BFC + m\angle CFD$.
Substitute.

$$m\angle BFC + m\angle CFD = m\angle AFE$$
$$42 + m\angle CFD = 108$$
$$m\angle CFD = 66$$

So, the correct option is A.

ANSWER:

A

38. **ALGEBRA** Simplify.
$4(3x-2)(2x+4)+3x^2+5x-6$

F $9x^2+3x-14$
G $9x^2+13x-14$
H $27x^2+37x-38$
J $27x^2+27x-26$

SOLUTION:

$4(3x-2)(2x+4)+3x^2+5x-6$	Original expression
$=4\left(6x^2+12x-4x-8\right)+3x^2+5x-6$	Multiply.
$=4\left(6x^2+8x-8\right)+3x^2+5x-6$	Simplify.
$=24x^2+32x-32+3x^2+5x-6$	Distributive Property
$=27x^2+37x-38$	Simplify.

The correct choice is H.

ANSWER:

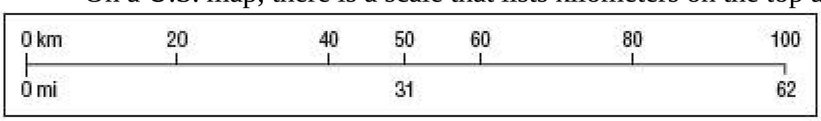
H

39. **SAT/ACT** On a coordinate grid where each unit represents 1 mile, Isabel's house is located at (3, 0) and a mall is located at (0, 4). What is the distance between Isabel's house and the mall?
A 3 miles
B 5 miles
C 12 miles
D 13 miles
E 25 miles

SOLUTION:
Use the Distance Formula.
$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$
Substitute.
$$\begin{aligned} d &= \sqrt{(0-3)^2 + (4-0)^2} \\ &= \sqrt{(-3)^2 + (4)^2} \\ &= \sqrt{9+16} \\ &= 5 \end{aligned}$$
The distance between Isabel's house and the mall is 5 miles.
So, the correct option is B.

ANSWER:
B

40. **MAPS** On a U.S. map, there is a scale that lists kilometers on the top and miles on the bottom.



Suppose $\overline{AB}$ and $\overline{CD}$ are segments on this map. If *AB* = 100 kilometers and *CD* = 62 miles, is $\overline{AB} \cong \overline{CD}$? Explain.

SOLUTION:
If *AB* = 100 kilometers and *CD* = 62 miles, then $\overline{AB} \cong \overline{CD}$. According to the scale, 100 km = 62 miles, so *AB* = *CD*. By the definition of congruence, $\overline{AB} \cong \overline{CD}$.

ANSWER:
Yes; according to the scale, 100 km = 62 miles, so *AB* = *CD*. By the definition of congruence, $\overline{AB} \cong \overline{CD}$.

State the property that justifies each statement.

41. If $y + 7 = 5$, then $y = -2$.

SOLUTION:

To change the equation $y + 7 = 5$ to $y = -2$, 7 must be subtracted from each side. Use the Subtraction Property of Equality.

$$\begin{aligned} y + 7 &= 5 \\ y + 7 - 7 &= 5 - 7 \\ y &= -2 \end{aligned}$$

ANSWER:

Subt. Prop.

42. If $MN = PQ$, then $PQ = MN$.

SOLUTION:

Use the Symmetric Property of Equality to change the equation $MN = PQ$, to $PQ = MN$.

ANSWER:

Symm. Prop

43. If $a - b = x$ and $b = 3$, then $a - 3 = x$.

SOLUTION:

Since $a - b = x$ and $b = 3$, substitute 3 for b. Then $a - 3 = x$. This utilizes Substitution.

ANSWER:

Subs.

44. If $x(y + z) = 4$, then $xy + xz = 4$

SOLUTION:

Use the Distributive Property to change the equation $x(y + z) = 4$ to $xy + xz = 4$.

ANSWER:

Dist. Prop.

Determine the truth value of the following statement for each set of conditions.
If you have a fever, then you are sick.

45. You do not have a fever, and you are sick.

SOLUTION:

The conditional statement "You do not have a fever, and you are sick." is true. When this hypothesis is true and the the conclusion is also true, the conditional is true.

ANSWER:

True

46. You have a fever, and you are not sick.

SOLUTION:

The conditional statement "You have a fever, and you are not sick." is false. When the conclusion is false, the conditional is false is regardless of the hypothesis So, the conditional statement is false.

ANSWER:

false

47. You do not have a fever, and you are not sick.

SOLUTION:

The conditional statement "You do not have a fever, and you are not sick." is true. When this hypothesis is false, the conclusion is also false, So, the conditional statement is true.

ANSWER:

true

48. You have a fever, and you are sick.

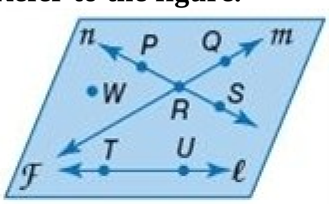
SOLUTION:

The conditional statement "I have a fever, and you are sick." is true. When this hypothesis is true, the conclusion is also true. So, the conditional statement is true.

ANSWER:

true

Refer to the figure.



49. Name a line that contains point *P*.

SOLUTION:

Locate point *P* in the figure. Identify the line that contains point *P*. Point *P* is on line *n*.

ANSWER:

line *n*

50. Name the intersection of lines *n* and *m*.

SOLUTION:

Locate lines *n* and *m*. Identify the point of intersection. Point *R* is the intersection of lines *n* and *m*.

ANSWER:

point *R*

51. Name a point not contained in lines ℓ, m, or n.

SOLUTION:

Locate lines ℓ, m, or n. Identify a point not on the three lines. Point W is not on lines ℓ, m, or n.

ANSWER:

point W

52. What is another name for line n?

SOLUTION:

There are two points P and R marked on the line n. So, the line n can also be called as $\overleftrightarrow{PR}$.

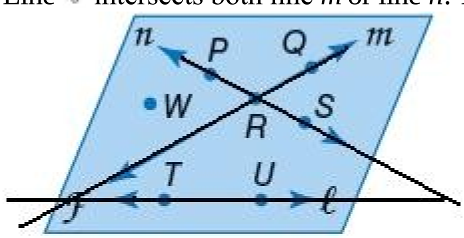
ANSWER:

Sample answer: $\overleftrightarrow{PR}$.

53. Does line ℓ intersect line m or line n? Explain.

SOLUTION:

Line ℓ intersects both line m or line n. If all three lines are extended they intersects.



ANSWER:

Yes; it intersects both m and n when all three lines are extended.