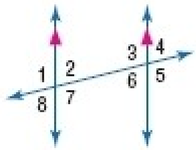
3-2 Angles and Parallel Lines

In the figure, $m\angle 1 = 94$. Find the measure of each angle. Tell which postulate(s) or theorem(s) you used.



1. $\angle 3$

SOLUTION:

In the figure, angles 1 and 3 are corresponding angles. Use the Corresponding Angles Postulate.
If two parallel lines are cut by a transversal, then each pair of corresponding angles are congruent.

$\angle 3 \cong \angle 1$ Corresponding Angles Postulate

$m\angle 3 = m\angle 1$ Definition of congreunt angles

$m\angle 3 = 94$ Substitution.

ANSWER:

94; Corresponding Angle Postulate

2. $\angle 5$

SOLUTION:

In the figure, angles 1 and 5 are alternate exterior angles.

Use the Alternate Exterior Angles Theorem.
If two parallel lines are cut by a transversal, then each pair of alternate interior angles is congruent.

$\angle 5 \cong \angle 1$ Alternate Exterior Angles Theorem

$m\angle 5 = m\angle 1$ Definition of congruent angles

$m\angle 5 = 94$ Substitution.

ANSWER:

94; Alt. Ext. $\angle$s Thm.

3. $\angle 4$

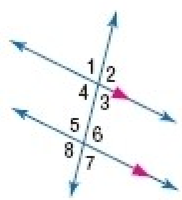
SOLUTION:
In the figure, angles 1 and 3 are corresponding angles. Use the Corresponding Angles Postulate:
If two parallel lines are cut by a transversal, then each pair of corresponding angles is congruent.

$$\begin{aligned} \angle 3 &\cong \angle 1 && \text{Corresponding Angles Postulate} \\ m\angle 3 &= m\angle 1 && \text{Definition of congruent angles} \\ m\angle 3 &= 94 && \text{Substitution.} \end{aligned}$$

$$\begin{aligned} \angle 3 + \angle 4 &\cong 180° && \text{Def. of supplementary angles} \\ m\angle 3 + m\angle 4 &= 180 && \text{Def. of congruent angles} \\ 94 + m\angle 4 &= 180 && \text{Substitution.} \\ 94 - 94 + m\angle 4 &= 180 - 94 && \text{Subtract 94 from each side.} \\ m\angle 4 &= 86 && \text{Simplify.} \end{aligned}$$

ANSWER:
86; Corresponding Angle Postulate and Supplement Angle Thm.

In the figure, $m\angle 4 = 101$. Find the measure of each angle. Tell which postulate(s) or theorem(s) you used.



4. $\angle 6$

SOLUTION:
In the figure, angles 4 and 6 are alternate interior angles.
Use the Alternate interior Angles Theorem.

If two parallel lines are cut by a transversal, then each pair of alternate interior angles is congruent.

$$\begin{aligned} \angle 4 &\cong \angle 6 && \text{Alternate Interior Angles Theorem} \\ m\angle 4 &= m\angle 6 && \text{Definition of congruent angles} \\ 101 &= m\angle 6 && \text{Substitution.} \end{aligned}$$

ANSWER:
101; Alt. Int. $\angle$s Thm.

5. $\angle 7$

SOLUTION:

In the figure, angles 4 and 5 are consecutive interior angles.

$$\begin{aligned}
\angle 4 + \angle 5 &\cong 180° && \text{Def. of supplementary angles} \\
m\angle 4 + m\angle 5 &= 180 && \text{Def. of congruent angles} \\
101 + m\angle 5 &= 180 && \text{Substitution.} \\
101 + m\angle 5 &= 180 - 101 && \text{Subtract101 from each side.} \\
m\angle 5 &= 79 && \text{Simplify.}
\end{aligned}$$

In the figure, angles 5 and 7 are vertical angles.

$$\begin{aligned}
\angle 5 &\cong \angle 7 && \text{Vertical Angles} \\
m\angle 5 &= m\angle 7 && \text{Definition of congruent angles} \\
79 &= m\angle 7 && \text{Substitution.}
\end{aligned}$$

ANSWER:

79; Vertical Angle Thm. Cons. Int. $\angle$s Thm.

6. $\angle 5$

SOLUTION:

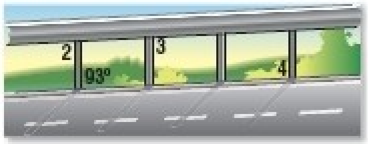
In the figure, angles 4 and 5 are consecutive interior angles.

$$\begin{aligned}
\angle 4 + \angle 5 &\cong 180° && \text{Consecutive Interior Angles Theorem} \\
m\angle 4 + m\angle 5 &= 180 && \text{Definition of congruent angles} \\
101 + m\angle 5 &= 180 && \text{Substitution.} \\
101 - 101 + m\angle 5 &= 180 - 101 && \text{Subtract101from each side.} \\
m\angle 5 &= 79 && \text{Simplify.}
\end{aligned}$$

ANSWER:

79; Cons. Int. $\angle$s Thm.

7. **ROADS** In the diagram, the guard rail is parallel to the surface of the roadway and the vertical supports are parallel to each other. Find the measures of angles 2, 3, and 4.



SOLUTION:

Use the Alternate Interior Angles Theorem, Definition of Supplementary Angles and Corresponding Angles Postulate to find $m\angle 4$.

$\angle 2 \cong 93°$ Alternate Interior Angles Theorem

$m\angle 2 = 93$ Definition of congruent angles

$\angle 3 + 93° \cong 180°$ Definition of Supplementary Angles

$m\angle 3 + 93 = 180$ Definition of congruent angles

$m\angle 3 + 93 - 93 = 180 - 93$ Subtract 93 from each side.

$m\angle 3 = 87$ Simplify.

$\angle 4 \cong 87°$ Corresponding Angles Postulate

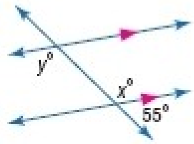
$m\angle 4 = 87$ Definition of congruent angles

So, $m\angle 4 = 87$.

ANSWER:

$m\angle 2 = 93$, $m\angle 3 = 87$, $m\angle 4 = 87$

Find the value of the variable(s) in each figure. Explain your reasoning.

8. 

SOLUTION:

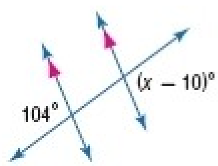
Use the definition of supplementary angles to find $m\angle x$. Then use the Alternate Interior Angles Theorem to find $m\angle y$.

$$m\angle x + 55 = 180 \quad \text{Def. of supplementary angles}$$
$$m\angle x + 55 - 55 = 180 - 55 \quad \text{Subtract 55 from each side.}$$
$$m\angle x = 125 \quad \text{Simplify.}$$

$$\angle x \cong \angle y \quad \text{Alternate Interior Angles Theorem}$$
$$m\angle x = m\angle y \quad \text{Definition of congruent angles}$$
$$125 = m\angle y \quad \text{Substitution.}$$

ANSWER:

x = 125 by the Supplement Thm.;
y = 125 by the Alt. Int. $\angle$s Thm.

9. 

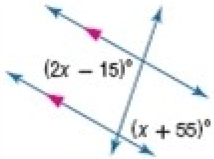
SOLUTION:

Use the Alternate Exterior Angles Theorem to find x.

$$104 = x - 10 \quad \text{Alternate Exterior Angles Theorem}$$
$$104 + 10 = x - 10 + 10 \quad \text{Add 10 to each side.}$$
$$114 = x \quad \text{Simplify.}$$

ANSWER:

x = 114 by the Alt. Ext. $\angle$s Thm.

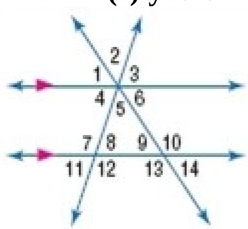
10. 

SOLUTION:
Use the Alternate Interior Angles Theorem to find *x*.

$$
\begin{aligned}
x+55 &= 2x-15 && \text{Alternate Interior Angles Theorem}\\
x-2x+55 &= 2x-2x-15 && \text{Subtract 2x from each side.}\\
-x+55 &= -15 && \text{Simplify.}\\
-x+55-55 &= -15-55 && \text{Subtract 55 from each side.}\\
-x &= -70 && \text{Simplify.}\\
-1(-x) &= -1(-70) && \text{Multiply each side by } -1.\\
x &= 70 && \text{Simplify.}
\end{aligned}
$$

ANSWER:
$x = 70$ by the Alt. Int. ∠s Thm.

In the figure, $m\angle 11 = 62$ and $m\angle 14 = 38$. Find the measure of each angle. Tell which postulate(s) or theorem(s) you used.



11. ∠4

SOLUTION:
In the figure, angles 4 and 11 are corresponding angles.

$$
\begin{aligned}
\angle 4 &\cong \angle 11 && \text{Corresponding Angles Postulate}\\
m\angle 4 &= m\angle 11 && \text{Definition of congruent angles}\\
m\angle 4 &= 62 && \text{Substitution.}
\end{aligned}
$$

ANSWER:
62; Corr. ∠s Post.

12. $\angle 3$

SOLUTION:

In the figure, angles 4 and 11 are corresponding angles and angles 3 and 4 are vertical angles.

$\angle 4 \cong \angle 11$ Corresponding Angles Postulate

$m\angle 4 = m\angle 11$ Definition of congruent angles

$m\angle 4 = 62$ Substitution.

$\angle 3 \cong \angle 4$ Vertical Angles

$m\angle 3 = m\angle 4$ Definition of congruent angles

$m\angle 3 = 62$ Substitution.

ANSWER:

62; Corresponding $\angle$s Post. and Vertical $\angle$ Thm. or Alt. Ext. Thm.

13. $\angle 12$

SOLUTION:

In the figure, angles 12 and 11 are supplementary angles.

$\angle 11 + \angle 12 \cong 180°$ Definition of supplimentary angles

$m\angle 11 + m\angle 12 = 180$ Definition of congruent angles

$62 + m\angle 12 = 180$ Substitution.

$62 - 62 + m\angle 12 = 180 - 62$ Subtract 62 from each side.

$m\angle 12 = 118$ Simplify.

ANSWER:

118; Def. Supp. $\angle$s

14. $\angle 8$

SOLUTION:

In the figure, angles 8 and 11 are vertical angles.

$\angle 8 \cong \angle 11$ Vertical Angles

$m\angle 8 = m\angle 11$ Definition of congruent angles

$m\angle 8 = 62$ Substitution.

ANSWER:

62; Vertical Angle Thm.

15. $\angle 6$

SOLUTION:

In the figure, angles 14 and 6 are corresponding angles.

$$\begin{aligned} \angle 6 &\cong \angle 14 && \text{Corresponding Angles Postulate} \\ m\angle 6 &= m\angle 14 && \text{Definition of congruent angles} \\ m\angle 6 &= 38 && \text{Substitution.} \end{aligned}$$

ANSWER:

38; Corr. $\angle$s Post.

16. $\angle 2$

SOLUTION:

The angles $\angle 1$ and $\angle 14$ are alternate exterior angles and so are congruent. and angles $\angle 3$ and $\angle 11$ are alternate exterior angles and so are congruent. By Supplementary Theorem, $m\angle 1 + m\angle 2 + m\angle 3 = 180$.

$$\begin{aligned} \angle 1 &\cong \angle 14 && \text{Alternate Exterior Angles Theorem} \\ m\angle 1 &= m\angle 14 && \text{Definition of congruent angles} \\ m\angle 1 &= 38 && \text{Substitution.} \end{aligned}$$

$$\begin{aligned} \angle 3 &\cong \angle 11 && \text{Alternate Exterior Angles Theorem} \\ m\angle 3 &= m\angle 11 && \text{Definition of congruent angles} \\ m\angle 3 &= 62 && \text{Substitution.} \end{aligned}$$

$$\begin{aligned} \angle 1 + \angle 2 + \angle 3 &\cong 180^\circ && \text{Def. of supplementary angles} \\ m\angle 1 + m\angle 2 + m\angle 3 &= 180 && \text{Def. of congruent angles} \\ 38 + m\angle 2 + 62 &= 180 && \text{Substitution.} \\ 100 + m\angle 2 &= 180 && \text{Simplify.} \\ 100 - 100 + m\angle 2 &= 180 - 100 && \text{Subtract 100 from each side.} \\ m\angle 2 &= 80 && \text{Simplify.} \end{aligned}$$

ANSWER:

80; Alt. Ext. $\angle$s Post. and Supp. $\angle$ Thm.

17. $\angle 10$

SOLUTION:

In the figure, angles 14 and 10 are supplementary angles.

$$\begin{aligned} \angle 14 + \angle 10 &\cong 180° && \text{Def. of supplementary angles} \\ m\angle 14 + m\angle 10 &= 180 && \text{Def. of congruent angles} \\ 38 + m\angle 10 &= 180 && \text{Substitution.} \\ 38 - 38 + m\angle 10 &= 180 - 38 && \text{Subtract 38 from each side.} \\ m\angle 10 &= 142 && \text{Simplify.} \end{aligned}$$

ANSWER:

142; Supplement Angles Thm.

18. $\angle 5$

SOLUTION:

Use definition of supplementary angles, Corresponding Angles Postulate and the Alternate Interior Angles Theorem .

$$\begin{aligned} \angle 11 + \angle 7 &\cong 180° && \text{Definition of supplementary angles} \\ m\angle 11 + m\angle 7 &= 180 && \text{Def. of congruent angles} \\ 62 + m\angle 7 &= 180 && \text{Substitution.} \\ 62 - 62 + m\angle 7 &= 180 - 62 && \text{Subtract 62 from each side.} \\ m\angle 7 &= 118 && \text{Simplify.} \end{aligned}$$

$$\begin{aligned} \angle 6 &\cong \angle 14 && \text{Corresponding Angles Postulate} \\ m\angle 6 &= m\angle 14 && \text{Definition of congruent angles} \\ m\angle 6 &= 38 && \text{Substitution.} \end{aligned}$$

$$\begin{aligned} \angle 7 &\cong \angle 5 + \angle 6 && \text{Alternate Interior Angles Theorem} \\ m\angle 7 &= m\angle 5 + m\angle 6 && \text{Definition of congruent angles} \\ 118 &= m\angle 5 + 38 && \text{Substitution.} \\ 118 - 38 &= m\angle 5 + 38 - 38 && \text{Subtract 38 from each side.} \\ 80 &= m\angle 5 && \text{Substitution.} \end{aligned}$$

ANSWER:

80; Vertical Angles Thm.

19. $\angle 1$

SOLUTION:

In the figure, angles 1 and 14 are alternate exterior angles.

$\angle 1 \cong \angle 14$ Alternate Exterior Angles Theorem

$m\angle 1 = m\angle 14$ Definition of congruent angles

$m\angle 1 = 38$ Substitution.

ANSWER:

38; Alt. Ext. $\angle$s Thm.

CCSS MODELING A solar dish collects energy by directing radiation from the Sun to a receiver located at the focal point of the dish. Assume that the radiation rays are parallel. Determine the relationship between each pair of angles and explain your reasoning.

Refer to Page 183.

20. $\angle 1$ and $\angle 2$

SOLUTION:

If the radiation rays form parallel lines, then $\angle 1$ and $\angle 2$ are consecutive interior angles. So, according to the Consecutive Interior Angles Theorem, $\angle 1$ and $\angle 2$ are supplementary.

ANSWER:

supplementary; Consecutive Interior Angles

21. $\angle 1$ and $\angle 3$

SOLUTION:

If the radiation rays form parallel lines, then $\angle 1$ and $\angle 3$ are corresponding angles. So, according to the Corresponding Angles Postulate, $\angle 1$ and $\angle 3$ are congruent.

ANSWER:

congruent; Corresponding Angles

22. $\angle 4$ and $\angle 5$

SOLUTION:

If the radiation rays form parallel lines, then $\angle 4$ and $\angle 5$ are alternate exterior angles. So, according to the Alternate Exterior Angles Theorem, $\angle 4$ and $\angle 5$ are congruent.

ANSWER:

congruent; Alternate Exterior Angles

23. $\angle 3$ and $\angle 4$

SOLUTION:

If the radiation rays form parallel lines, then $\angle 3$ and $\angle 5$ are a linear pair of angles.. So, according to the definition of linear pairs, $\angle 3$ and $\angle 5$ are supplementary. $\angle 4$ and $\angle 5$ are alternate exterior angles. So, by the Alternate Exterior Angles Theorem, $\angle 4$ is congruent to $\angle 5$. Then,

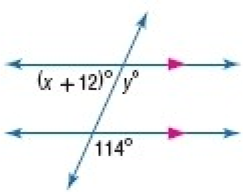
$$
\begin{aligned}
m\angle 3 + m\angle 5 &= 180 && \text{Definition of supplementary angles} \\
m\angle 4 &= m\angle 5 && \text{Definition of congruent angles} \\
m\angle 3 + m\angle 4 &= 180 && \text{Substitution}
\end{aligned}
$$

Therefore, $\angle 3$ is supplementary to $\angle 4$ by the definition of supplementary angles.

ANSWER:

supplementary; since $\angle 3$ and $\angle 5$ are a linear pair, they are supplementary. $\angle 4$ and $\angle 5$ are congruent because they are alternate exterior angles, so $\angle 3$ is supplementary to $\angle 4$.

Find the value of the variable(s) in each figure. Explain your reasoning.

24. 

SOLUTION:

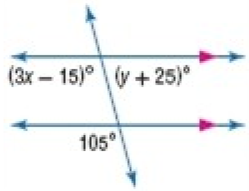
Use Corresponding Angles Postulate and definition of supplementary angles to find *x*.

$$m\angle y = 114 \quad \text{Corresponding Angles Postulate}$$

$$
\begin{aligned}
m\angle y + (x + 12) &= 180 && \text{Definition of supplementary angles} \\
114 + x + 12 &= 180 && \text{Substitution.} \\
126 + x &= 180 && \text{Simplify.} \\
126 - 126 + x &= 180 - 126 && \text{Subtract 126 from each side.} \\
x &= 54 && \text{Simplify.}
\end{aligned}
$$

ANSWER:

$y = 114$ by the Corresponding Angles Postulate; $x = 54$ by the Supplement Theorem

25. 

SOLUTION:

Use the Corresponding Angles Postulate and Supplement Theorem to find *x* and *y*.

$$
\begin{aligned}
(3x-15)^\circ &\cong 105^\circ && \text{Corresponding Angles Postulate}\\
3x-15 &= 105 && \text{Definition of congruent angles}\\
3x-15+15 &= 105+15 && \text{Substitution.}\\
3x &= 120 && \text{Simplify.}\\
\frac{3x}{3} &= \frac{120}{3} && \text{Divide each side by 3.}\\
x &= 40 && \text{Substitution.}
\end{aligned}
$$

$$
\begin{aligned}
(3x-15)^\circ+(y+25)^\circ &\cong 180^\circ && \text{Consecutive Interior Angles Theorem}\\
3x-15+y+25 &= 180 && \text{Definition of congruent angles}\\
3(40)-15+y+25 &= 180 && \text{Substitution.}\\
120-15+y+25 &= 180 && \text{Simplify.}\\
130+y &= 180 && \text{Simplify.}\\
130-130+y &= 180-130 && \text{Subtract 130 from each side.}\\
y &= 50 && \text{Simplify.}
\end{aligned}
$$

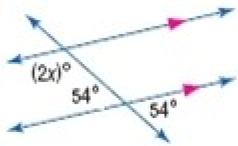
ANSWER:

x = 40 by the Corresponding Angles Postulate; y = 50 by the Supplement Theorem

26. 

SOLUTION:

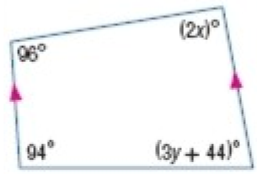
Use the Vertical Angle Theorem and Consecutive Interior Angles Theorem to find *x*.



$(2x)° + 54° \cong 180°$	Consecutive Interior Angles Theorem
$2x + 54 = 180$	Definition of congruent angles
$2x + 54 = 180 - 54$	Subtract 54 from each side.
$2x = 126$	Simplify.
$\frac{2x}{2} = \frac{126}{2}$	Divide each side by 2.
$x = 63$	Simplify.

ANSWER:

$x = 63$ by the Vertical Angle Theorem and the Consecutive Interior Angles Theorem

27. 

SOLUTION:

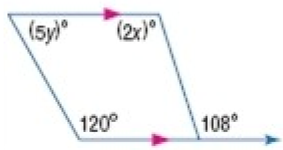
Use the Consecutive Interior Angles Theorem to find x and y.

$(2x)° + 96° \cong 180°$	Consecutive Interior Angles Theorem
$2x + 96 = 180$	Definition of congruent angles
$2x + 96 - 96 = 180 - 96$	Subtract 96 from each side.
$2x = 84$	Simplify.
$\frac{2x}{2} = \frac{84}{2}$	Divide each side by 2.
$x = 42$	Simplify.

$94° + (3y + 44)° \cong 180°$	Consecutive Interior Angles Theorem
$94 + 3y + 44 = 180$	Definition of congruent angles
$3y + 138 = 180$	Simplify.
$3y + 138 - 138 = 180 - 138$	Subtract 138 from each side.
$3y = 42$	Simplify.
$\frac{3y}{3} = \frac{42}{3}$	Divide each side by 3.
$y = 14$	Simplify.

ANSWER:

x = 42 by the Consecutive Interior Angles Theorem; y = 14 by the Consecutive Interior Angles Theorem

28. 

SOLUTION:

Use the Alternate Interior Angles Theorem and Consecutive Interior Angles Theorem to find x and y.

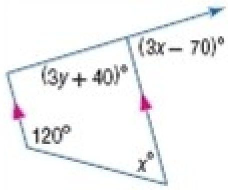
$$\begin{aligned}
(2x)^\circ &\cong 108^\circ && \text{Alternate Interior Angles Theorem}\\
2x &= 108 && \text{Definition of congruent angles}\\
\frac{2x}{2} &= \frac{108}{2} && \text{Divide each side by 2.}\\
x &= 54 && \text{Simplify.}
\end{aligned}$$

$$\begin{aligned}
(5y)^\circ + 120^\circ &\cong 180^\circ && \text{Consecutive Interior Angles Theorem}\\
5y + 120 &= 180 && \text{Definition of congruent angles}\\
5y + 120 - 120 &= 180 - 120 && \text{Subtract 120 from each side.}\\
5y &= 60 && \text{Simplify.}\\
\frac{5y}{5} &= \frac{60}{5} && \text{Divide each side by 5.}\\
y &= 12 && \text{Simplify.}
\end{aligned}$$

ANSWER:

x = 54 by the Alternate Interior Angles Theorem; y = 12 by the Consecutive Interior Angles Theorem

3-2 Angles and Parallel Lines

29. 

SOLUTION:

Use the Consecutive Interior Angles Theorem and definition of supplementary angles to find x and y.

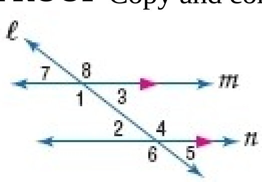
$$\begin{aligned} x^\circ + 120^\circ &\cong 180^\circ && \text{Consecutive Interior Angles Theorem} \\ x + 120 &= 180 && \text{Definition of congruent angles} \\ x + 120 - 120 &= 180 - 120 && \text{Subtract 120 from each side.} \\ x &= 60 && \text{Simplify.} \end{aligned}$$

$$\begin{aligned} (3x - 70)^\circ + (3y + 40)^\circ &\cong 180^\circ && \text{Def. of supplementary angles} \\ 3x - 70 + 3y + 40 &= 180 && \text{Definition of congruent angles} \\ 3(60) - 70 + 3y + 40 &= 180 && \text{Substitution.} \\ 180 - 70 + 3y + 40 &= 180 && \text{Multiply.} \\ 150 + 3y &= 180 && \text{Subtraction.} \\ 150 - 150 + 3y &= 180 - 150 && \text{Subtract 150 from each side.} \\ 3y &= 30 && \text{Simplify.} \\ \frac{3y}{3} &= \frac{30}{3} && \text{Divide each side by 3.} \\ y &= 10 && \text{Simplify.} \end{aligned}$$

ANSWER:

x = 60 by the Consecutive Interior Angles Theorem; y = 10 by the Supplement Theorem

30. **PROOF** Copy and complete the proof of Theorem 3.2.



Given: $m \parallel n$; ℓ is a transversal.
Prove: $\angle 1$ and $\angle 2$ are supplementary;
$\angle 3$ and $\angle 4$ are supplementary.
Proof:

Statements	Reasons
a. ?	**a.** Given
b. $\angle 1$ and $\angle 3$ form a linear pair; $\angle 2$ and $\angle 4$ form a linear pair.	**b.** ?
c. ?	**c.** If two angles form a linear pair, then they are supplementary.
d. $\angle 1 \cong \angle 4$, $\angle 2 \cong \angle 3$	**d.** ?
e. $m\angle 1 = m\angle 4$, $m\angle 2 = m\angle 3$	**e.** Definition of Congruence
f. ?	**f.** ?

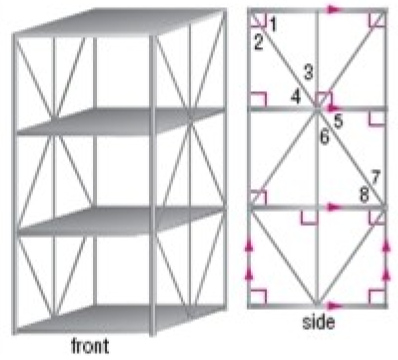
SOLUTION:

Statements	Reasons
a. $m \parallel n$; ℓ is a transversal.	**a.** Given
b. $\angle 1$ and $\angle 3$ form a linear pair; $\angle 2$ and $\angle 4$ form a linear pair.	**b.** Def. of linear pair
c. $\angle 1$ and $\angle 3$ are supplementary. $\angle 2$ and $\angle 4$ are supplementary.	**c.** If two angles form a linear pair, then they are supplementary.
d. $\angle 1 \cong \angle 4$, $\angle 2 \cong \angle 3$	**d.** Alt. Int. $\angle$s Theorem
e. $m\angle 1 = m\angle 4$, $m\angle 2 = m\angle 3$	**e.** Definition of Congruence
f. $\angle 1$ and $\angle 2$ are supp. $\angle 3$ and $\angle 4$ are supp.	**f.** Substitution

ANSWER:

Statements	Reasons
a. $m \parallel n$; ℓ is a transversal.	**a.** Given
b. $\angle 1$ and $\angle 3$ form a linear pair; $\angle 2$ and $\angle 4$ form a linear pair.	**b.** Def. of linear pair
c. $\angle 1$ and $\angle 3$ are supplementary. $\angle 2$ and $\angle 4$ are supplementary	**c.** If two angles form a linear pair, then they are supplementary.
d. $\angle 1 \cong \angle 4$, $\angle 2 \cong \angle 3$	**d.** Alt. Int. $\angle$s Theorem
e. $m\angle 1 = m\angle 4$, $m\angle 2 = m\angle 3$	**e.** Definition of Congruence
f. $\angle 1$ and $\angle 2$ are supp. $\angle 3$ and $\angle 4$ are supp.	**f.** Substitution

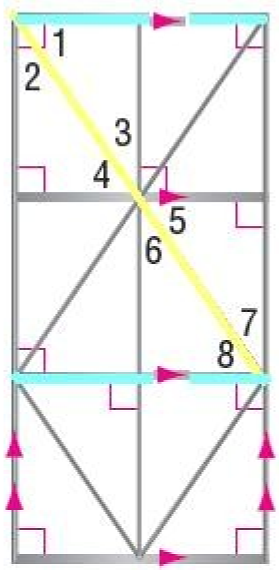
STORAGE When industrial shelving needs to be accessible from either side, additional support is provided on the side by transverse members. Determine the relationship between each pair of angles and explain your reasoning.



31. ∠1 and ∠8

SOLUTION:

∠1 and ∠8 are Alternate interior angles. Therefore ∠1 and ∠8 are congruent.



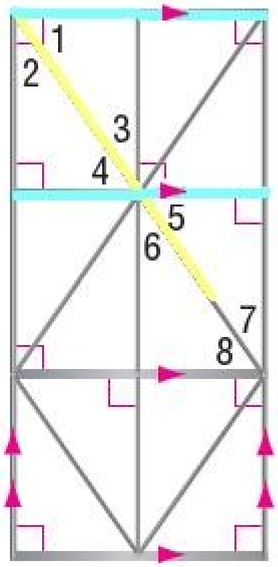
ANSWER:

Congruent; Alternate interior angles

32. ∠1 and ∠5

SOLUTION:

∠1 and ∠5 are Corresponding angles. Therefore, they are congruent.



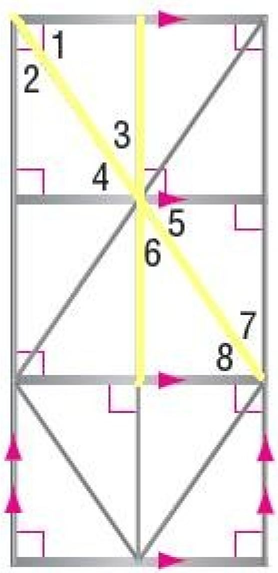
ANSWER:

Congruent; Corresponding angles

33. ∠3 and ∠6

SOLUTION:

∠3 and ∠6 are Vertical angles. Therefore Vertical angles are congruent.



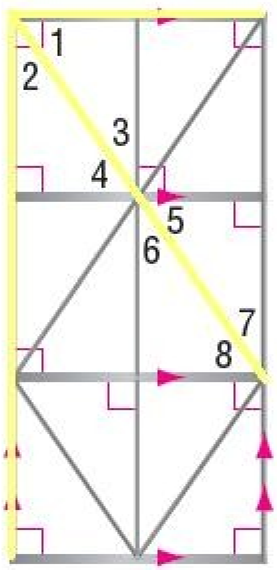
ANSWER:

Congruent; Vertical angles are congruent

34. $\angle 1$ and $\angle 2$

SOLUTION:

All vertical and horizontal lines are perpendicular at their point of intersection.By definition of perpendicular, they form right angles. $\angle 1$ and $\angle 2$ are adjacent angles. By the Angle Addition Postulate, $m\angle 1 + m\angle 2 = 90$. Since the sum of the two angles is 90, $\angle 1$ and $\angle 2$ are complementary angles.



ANSWER:

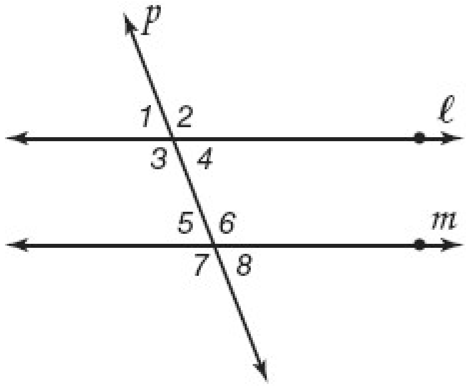
Complementary; because the vertical and horizontal lines are perpendicular, they form right angles.

35. **CCSS ARGUMENTS** Write a two-column proof of the Alternate Exterior Angles Theorem.

SOLUTION:

Given: $\ell \parallel m$

Prove: $\angle 1 \cong \angle 8$
$\angle 2 \cong \angle 7$



Proof:

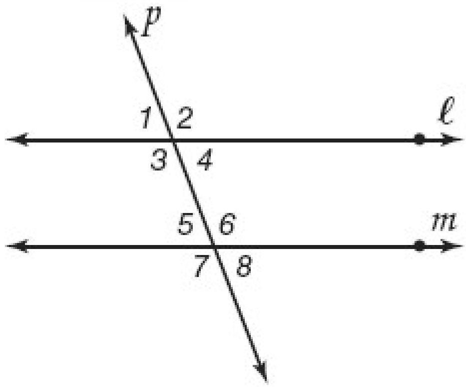
Statements (Reasons)

1. $\ell \parallel m$ (Given)
2. $\angle 1 \cong \angle 5, \angle 2 \cong \angle 6$ (Corr. $\angle$s Post.)
3. $\angle 8 \cong \angle 5, \angle 7 \cong \angle 6$ (Vertical $\angle$s Thm.)
4. $\angle 8 \cong \angle 1, \angle 7 \cong \angle 2$ (Trans. Prop.)

ANSWER:

Given: $\ell \parallel m$

Prove: $\angle 1 \cong \angle 8$
$\angle 2 \cong \angle 7$



Proof:

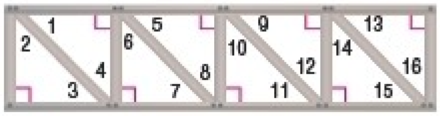
Statements (Reasons)

1. $\ell \parallel m$ (Given)
2. $\angle 1 \cong \angle 5$, $\angle 2 \cong \angle 6$ (Corr. $\angle$s Post.)
3. $\angle 5 \cong \angle 8$, $\angle 6 \cong \angle 7$ (Vertical $\angle$s Thm.)
4. $\angle 1 \cong \angle 8$, $\angle 2 \cong \angle 7$ (Trans. Prop.)

36. **BRIDGES** Refer to the diagram of the double decker Michigan Avenue Bridge in Chicago, Illinois. The two levels of the bridge, and its diagonal braces, are parallel.

a. How are the measures of the odd-numbered angles related? Explain.

b. How are the measures of the even-numbered angles related? Explain.
c. How are any pair of angles in which one is odd and the other is even related?
d. What geometric term(s) can be used to relate the two roadways contained by the bridge?



SOLUTION:

a. The top and bottom levels of the bridge are parallel, so the lines formed by the edge of each level are parallel, and by using the diagonal braces as transversals and the Alternate Interior Angles Theorem $\angle 1 \cong \angle 3$, $\angle 5 \cong \angle 7$, $\angle 9 \cong \angle 11$, and $\angle 13 \cong \angle 15$.
The diagonal braces are parallel, so by using the vertical braces as transversals and the Alternate Interior Angles Theorem $\angle 4 \cong \angle 6$, $\angle 8 \cong \angle 10$, and $\angle 12 \cong \angle 14$.
Since the vertical braces are perpendicular to the levels of the bridge, $\angle 3$ and $\angle 4$, $\angle 5$ and $\angle 6$, $\angle 7$ and $\angle 8$, $\angle 9$ and $\angle 10$, $\angle 11$ and $\angle 12$, and $\angle 13$ and $\angle 14$ are pairs of complementary angles.
By the Congruent Complements Theorem, $\angle 3 \cong \angle 5$, $\angle 7 \cong \angle 9$, and $\angle 11 \cong \angle 13$. So, $\angle 1 \cong \angle 3 \cong \angle 5 \cong \angle 7 \cong \angle 9 \cong \angle 11 \cong \angle 13 \cong \angle 13$ by the Transitive Property of Congruence. So, all of the odd numbered angles are alternate interior angles related by the diagonal transversals or are complements of even numbered alternate interior angles related by the vertical transversals. Therefore, they are all congruent.

b. All the vertical braces are parallel since all vertical lines are parallel. Using the diagonal braces as transversals to the vertical braces and the Alternate Interior Angles Theorem, $\angle 2 \cong \angle 4$, $\angle 6 \cong \angle 8$, $\angle 10 \cong \angle 12$, and $\angle 14 \cong \angle 16$.
Using the vertical braces as transversals between the diagonal braces and the Alternate Interior Angles Theorem, $\angle 4 \cong \angle 6$, $\angle 8 \cong \angle 10$, and $\angle 12 \cong \angle 14$. So, $\angle 2 \cong \angle 4 \cong \angle 6 \cong \angle 8 \cong \angle 10 \cong \angle 12 \cong \angle 14 \cong \angle 16$ by the Transitive Property of Congruence.
All of the even numbered angles are alternate interior angles related by either the diagonal transversals or the vertical transversals. Therefore, they are all congruent.

c. Complementary; since the vertical supports and the horizontal supports are perpendicular, angle pairs like angle 1 and angle 2 must be complementary. Since all of the odd numbered angles are congruent and all of the even numbered angles are congruent, any pair of angles that has one odd and one even number will be complementary.

d. Since the two levels (or surfaces) of the bridge are parallel, the geometric term that best represents the two roadways contained by the bridge is parallel planes.

ANSWER:

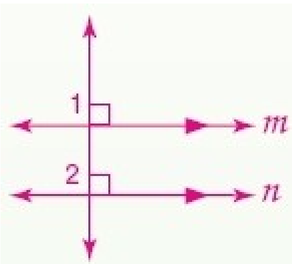
a. Congruent; all of the odd numbered angles are alternate interior angles related by the diagonal transversals or are complements of even numbered alternate interior angles related by the vertical transversals, so they are all congruent.
b. Congruent; all of the even numbered angles are alternate interior angles related by either the diagonal transversals or the vertical transversals, so they are all congruent.
c. Complementary; since the vertical supports and the horizontal supports are perpendicular, angle pairs like $\angle 1$ and $\angle 2$ must be complementary. Since all of the odd numbered angles are congruent and all of the even numbered angles are congruent, any pair of angles that has one odd and one even number will be complementary.
d. parallel planes

37. **PROOF** In a plane, prove that if a line is perpendicular to one of two parallel lines, then it is perpendicular to the other.

SOLUTION:

Given: $m \parallel n, t \perp m$

Prove: $t \perp n$



Proof:

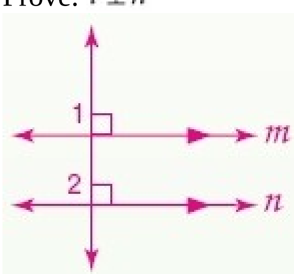
Statements (Reasons)

1. $m \parallel n, t \perp m$ (Given)
2. Angle 1 is a right angle. (Def. of $\perp$)
3. $m\angle 1 = 90$ (Def. of rt. $\angle s$)
4. $\angle 1 \cong \angle 2$ (Corr. $\angle s$ Post.)
5. $m\angle 1 = m\angle 2$ (Def. of $\cong \angle s$)
6. $m\angle 2 = 90$ (Subs.)
7. $\angle 2$ is a right angle. (Def. of rt. $\angle s$)
8. $t \perp n$ (Def. of $\perp$ lines)

ANSWER:

Given: $m \parallel n, t \perp m$

Prove: $t \perp n$

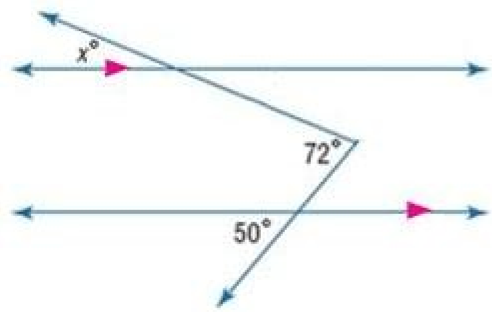


Proof:

Statements (Reasons)

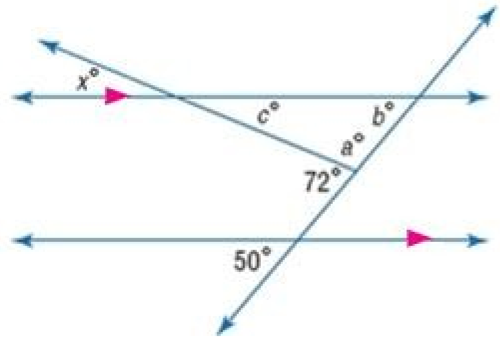
1. $m \parallel n$, $t \perp m$ (Given)
2. $\angle 1$ is a right angle. (Def. of $\perp$)
3. $m\angle 1 = 90$ (Def. of rt. $\angle$s)
4. $\angle 1 \cong \angle 2$ (Corr. $\angle$s Post.)
5. $m\angle 1 = m\angle 2$ (Def. of $\cong \angle$s)
6. $m\angle 2 = 90$ (Subs.)
7. $\angle 2$ is a right angle. (Def. of rt. $\angle$s)
8. $t \perp n$ (Def. of $\perp$ lines)

CCSS TOOLS Find *x*. (*Hint*: Draw an auxiliary line.)

38. 

SOLUTION:

Draw an auxiliary line to construct a triangle. Then label the angles $a°$, $b°$, and $c°$. By finding the measures for angles a and b, we can use the Triangle Angle Sum theorem to find angle c. Angles c and x are vertical angles.



Use the definition of supplementary angles to find a.

$72° + a° \cong 180°$	Def. of supplementary angles
$72 + a = 180$	Def. of congruent angles
$72 - 72 + a = 180 - 72$	Subtract 72 from each side.
$a = 108$	Simplify.

Find angle b.

$b° \cong 50$	Corresponding Angles Theorem
$b = 50$	Definition of congruent angles

Find angle c.

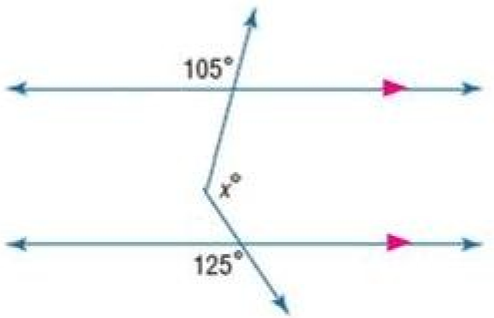
$a° + b° + c° \cong 180°$	Triangle Angle Sum Theorem
$a + b + c = 180$	Definition of congruent angles
$108 + 50 + c = 180$	Substitution.
$158 + c = 180$	Simplify.
$158 - 158 + c = 180 - 158$	Subtract 158 from each side.
$c = 22$	Simplify.

Find angle x.

$c° \cong x°$	Vertical Angles
$c = x$	Def. of congruent angles
$22 = x$	Substitution.

So, $x = 22$.

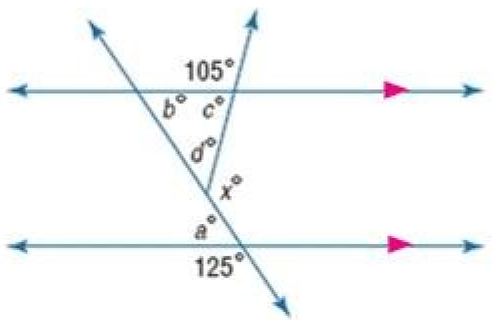
ANSWER:
22



39.

SOLUTION:

Draw an auxiliary line to construct a triangle.By creating a triangle, we can sue the Triangle Angle Sum Theorem and definition of supplementary angles to find *x*. Label the angles.



First find angle *a*.

$$\begin{aligned} a° + 125° &\cong 180° && \text{Def. of supplementary angles} \\ a + 125 &= 180 && \text{Def. of congruent angles} \\ a + 125 - 125 &= 180 - 125 && \text{Subtract 125 from each side.} \\ a &= 55 && \text{Simplify.} \end{aligned}$$

Find angle *b*.

$$\begin{aligned} a° &\cong b° && \text{Alternate Interior Angles Theorem} \\ a &= b && \text{Definition of congruent angles} \\ 55 &= b && \text{Substitution.} \end{aligned}$$

Find angle *c*.

$$\begin{aligned} c° + 105° &\cong 180° && \text{Def. of supplementary angles} \\ c + 105 &= 180 && \text{Def. of congruent angles} \\ c + 105 - 105 &= 180 - 105 && \text{Subtract 105 from each side.} \\ c &= 75 && \text{Simplify.} \end{aligned}$$

Find angle *d*.

$$\begin{aligned}
b^\circ + c^\circ + d^\circ &\cong 180^\circ && \text{Triangle Angle Sum Theorem}\\
b + c + d &= 180 && \text{Def. of congruent angles}\\
55 + 75 + d &= 180 && \text{Substitution.}\\
130 + d &= 180 && \text{Simplify.}\\
130 - 130 + d &= 180 - 130 && \text{Subtract 130 from each side.}\\
d &= 50 && \text{Simplify.}
\end{aligned}$$

Find angle *x*

$$\begin{aligned}
x^\circ + d^\circ &\cong 180^\circ && \text{Def. of supplementary angles}\\
x + d &= 180 && \text{Def. of congruent angles}\\
x + 50 &= 180 && \text{Substitution.}\\
x + 50 - 50 &= 180 - 50 && \text{Subtract 50 from each side.}\\
x &= 130 && \text{Simplify.}
\end{aligned}$$

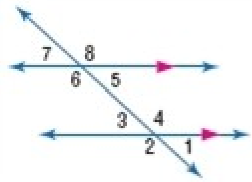
.

So $x = 130°$.

ANSWER:

130

40. **PROBABILITY** Suppose you were to pick any two angles in the figure below.
a. How many possible angle pairings are there? Explain.
b. Describe the possible relationships between the measures of the angles in each pair. Explain.
c. Describe the likelihood of randomly selecting a pair of congruent angles. Explain your reasoning.



SOLUTION:

a. Sample answer: There are 28 possible angle pairings. The first angle can be paired with seven others, then the second angle can be paired with six others since it has already been paired with the first angle. The number of pairings is the sum of the number of angles each subsequent angle can be paired with, 7 + 6 + 5 + 4 + 3 + 2 + 1 or 28 pairings.
b. Sample answer: Because the two lines being transversed are parallel, there are only two possible relationships between the pairs of angles. Each pair of angles chosen will be either congruent or supplementary.
Congruent pairs: $\angle 1$ and $\angle 3$, $\angle 1$ and $\angle 5$, $\angle 1$ and $\angle 7$, $\angle 3$ and $\angle 5$, $\angle 3$ and $\angle 7$, $\angle 5$ and $\angle 7$, $\angle 2$ and $\angle 4$, $\angle 2$ and $\angle 6$, $\angle 2$ and $\angle 8$, $\angle 4$ and $\angle 6$, $\angle 4$ and $\angle 8$, $\angle 6$ and $\angle 8$
Supplementary pairs: $\angle 1$ and $\angle 2$, $\angle 1$ and $\angle 4$, $\angle 1$ and $\angle 6$, $\angle 1$ and $\angle 8$, $\angle 2$ and $\angle 3$, $\angle 2$ and $\angle 5$, $\angle 2$ and $\angle 7$, $\angle 3$ and $\angle 4$, $\angle 3$ and 6, $\angle 3$ and $\angle 8$, $\angle 4$ and $\angle 5$, $\angle 4$ and $\angle 7$, $\angle 5$ and $\angle 6$, $\angle 5$ and $\angle 8$, $\angle 6$ and $\angle 7$, $\angle 7$ and $\angle 8$
c. Sample answer: Twelve of the 28 angle pairs are congruent. So, the likelihood of selecting a pair of congruent angles is $\frac{12}{28}$ or $\frac{3}{7}$.

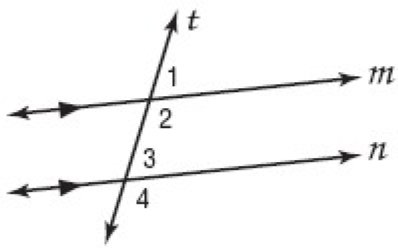
ANSWER:

a. Sample answer: There are 28 possible angle pairings. The first angle can be paired with seven others, then the second angle can be paired with six others since it has already been paired with the first angle. The number of pairings is the sum of the number of angles each subsequent angle can be paired with, 7 + 6 + 5 + 4 + 3 + 2 + 1 or 28 pairings.
b. Sample answer: There are two possible relationships between the pairs of angles. Two angles chosen will be either congruent or supplementary.
c. Sample answer: Twelve of the 28 angle pairs are congruent. So, the likelihood of selecting a pair of congruent angles is $\frac{12}{28}$ or $\frac{3}{7}$.

41. **MULTIPLE REPRESENTATIONS** In this problem, you will investigate the relationship between same-side exterior angles.
a. GEOMETRY Draw five pairs of parallel lines, *m* and *n*, *a* and *b*, *r* and *s*, *j* and *k*, and *x* and *y*, cut by a transversal *t*, and measure the four angles on one side of *t*.
b. TABULAR Record your data in a table.
c. VERBAL Make a conjecture about the relationship between the pair of angles formed on the exterior of parallel lines and on the same side of the transversal.
d. LOGICAL What type of reasoning did you use to form your conjecture? Explain.
e. PROOF Write a proof of your conjecture.

SOLUTION:

a. Sample answer for *m* and *n*:



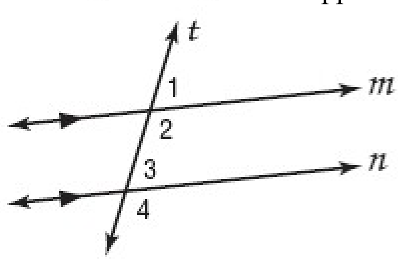
b. Sample answer:

$m\angle 1$	$m\angle 2$	$m\angle 3$	$m\angle 4$
60	120	60	120
45	135	45	135
70	110	70	110
90	90	90	90
25	155	25	155

c. Sample answer: In the diagram, $\angle 1$ and $\angle 4$ are a pair of exterior angles on the same side of the transversal. The sum of $m\angle 1$ and $m\angle 4$ for each row is 60 + 120 = 180, 45 + 135 = 180, 70 + 110 = 180, 90 + 90 = 180, and 25 + 155 = 180. A pair of angles whose sum is 180 are supplementary angles. Therefore, angles on the exterior of a pair of parallel lines located on the same side of the transversal are supplementary.

d. Inductive; a pattern was used to make a conjecture.

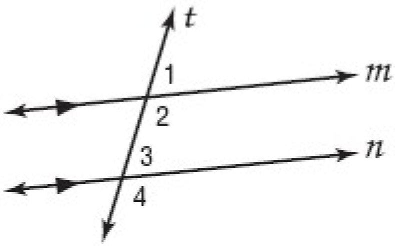
e. Given: parallel lines *m* and *n* cut by transversal *t*.
Prove: $\angle 1$ and $\angle 4$ are supplementary.



Proof:
1. Lines *m* and *n* are parallel and cut by transversal *t*. (Given)
2. $m\angle 1 + m\angle 2 = 180$ (Suppl. Thm.)
3. $\angle 2 \cong \angle 4$ (Corr. angles are $\cong$.)
4. $m\angle 2 = m\angle 4$ (Def. of congruence.)
5. $m\angle 1 + m\angle 4 = 180$ (Subs.)
6. Angle 1 and angle 4 are supplementary. (Def. of supplementary angles.)

ANSWER:

a. Sample answer for *m* and *n*:



b. Sample answer:

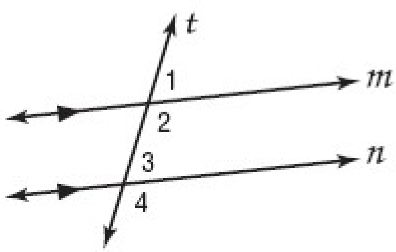
$m\angle 1$	$m\angle 2$	$m\angle 3$	$m\angle 4$
60	120	60	120
45	135	45	135
70	110	70	110
90	90	90	90
25	155	25	155
30	150	30	150

c. Sample answer: Angles on the exterior of a pair of parallel lines located on the same side of the transversal are supplementary.

d. Inductive; a pattern was used to make a conjecture.

e. Given: parallel lines *m* and *n* cut by transversal *t*

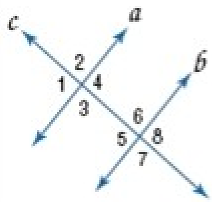
Prove: $\angle 1$ and $\angle 4$ are supplementary.



Proof:

1. Lines *m* and *n* are parallel and cut by transversal *t*. (Given)
2. $m\angle 1 + m\angle 2 = 180$ (Suppl. Thm.)
3. $\angle 2 \cong \angle 4$ (Corr. $\angle$s are $\cong$.)
4. $m\angle 2 = m\angle 4$ (Def. of congruence.)
5. $m\angle 1 + m\angle 4 = 180$ (Subs.)
6. $\angle 1$ and $\angle 4$ are supplementary. (Def. of supplementary $\angle$s.)

42. **WRITING IN MATH** If line a is parallel to line b and $\angle 1 \cong \angle 2$, describe the relationship between lines b and c. Explain your reasoning.



SOLUTION:

Lines b and c are perpendicular. Since $\angle 1$ and $\angle 2$ form a linear pair, $m\angle 1 + m\angle 2 = 180$. $\angle 1 \cong \angle 2$, so $m\angle 1 = m\angle 2$. . Substituting, $m\angle 1 + m\angle 1 = 180$, so $m\angle 1 = 90$ and $m\angle 2 = 90$. So, lines a and c are perpendicular. By Theorem 3.4, since transversal c is perpendicular to line a and lines a and b are parallel, then line c is perpendicular to line b.

ANSWER:

Lines b and c are perpendicular. Since $\angle 1$ and $\angle 2$ form a linear pair, $m\angle 1 + m\angle 2 = 180$. $\angle 1 \cong \angle 2$, so $m\angle 1 = m\angle 2$. Substituting, $m\angle 1 + m\angle 1 = 180$, so $m\angle 1 = 90$ and $m\angle 2 = 90$. So, lines a and c are perpendicular. By Theorem 3.4, since transversal c is perpendicular to line a and lines a and b are parallel, then line c is perpendicular to line b.

43. **WRITING IN MATH** Compare and contrast the Alternate Interior Angles Theorem and the Consecutive Interior Angles Theorem.

SOLUTION:

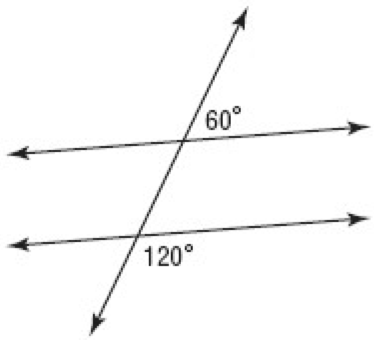
In both theorems, a pair of angles is formed when two parallel lines are cut by a transversal. However, in the Alternate Interior Angles Theorem, each pair of alternate interior angles that is formed are congruent, whereas in the Consecutive Interior Angles Theorem, each pair of angles formed is supplementary.

ANSWER:

In both theorems, a pair of angles is formed when two parallel lines are cut by a transversal. However, in the Alternate Interior Angles Theorem, each pair of alternate interior angles that is formed are congruent, whereas in the Consecutive Interior Angles Theorem, each pair of angles formed is supplementary.

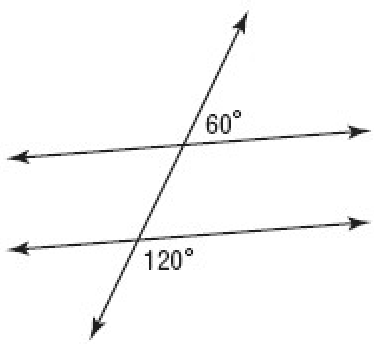
44. **OPEN ENDED** Draw a pair of parallel lines cut by a transversal and measure the two exterior angles on the same side of the transversal. Include the measures on your drawing. Based on the pattern you have seen for naming other pairs of angles, what do you think the name of the pair you measured would be?

SOLUTION:



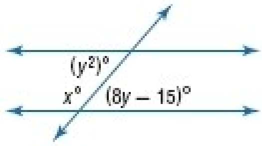
Consecutive Exterior Angles or Same-Side Exterior Angles

ANSWER:



Consecutive Exterior Angles or Same-Side Exterior Angles

45. **CHALLENGE** Find x and y.



SOLUTION:

To find x and y, we will write two equation and solve the system. In the figure, we are given a pair of consecutive interior angles [xº and y^{2}º], alternate interior angles ([y^{2}º and ($8y$ – 15)º]and supplementary angles [xº and ($8y$ – 15) º]. Use the supplementary angles and consecutive interior angles since they are both equation to 180.

Supplementary angles equation: $x+8y-15=180$

Consecutive Interior angles equation: $x+y^2=180$

Name the equation.

$$\begin{aligned} x+8y-15&=180 && \text{Equation 1} \\ \underline{(-)\ x+y^2}&\underline{=180} && \text{Equation 2} \\ y^2+8y-15&=0 && \text{Subtract equations.} \\ (y-3)(y-5)&=0 && \text{Factor} \\ y&=3 \text{ or } 5 && \text{Simplify.} \end{aligned}$$

Substitute $y=3$ in Equation 2.

$$\begin{aligned} x+y^2&=180 && \text{Equation 2} \\ x+(3)^2&=180 \\ x&=171 \end{aligned}$$

Substitute $y=5$ in Equation 2.

$$\begin{aligned} x+y^2&=180 && \text{Equation 2.} \\ x+(5)^2&=180 \\ x&=155 \end{aligned}$$

Thus, $x=171$ or $x=155$.

ANSWER:

x = 171 or x = 155;
y = 3 or y = 5

46. **REASONING** Determine the minimum number of angle measures you would have to know to find the measures of all the angles formed by two parallel lines cut by a transversal. Explain.

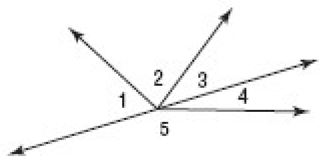
SOLUTION:

One;
Sample answer: Once the measure of one angle is known, the rest of the angles are either congruent or supplementary to the given angle.

ANSWER:

One; sample answer: Once the measure of one angle is known, the rest of the angles are either congruent or supplementary to the given angle.

47. Suppose $\angle 4$ and $\angle 5$ form a linear pair. If $m\angle 1 = 2x$, $m\angle 2 = 3x - 20$, and $m\angle 3 = x - 4$, what is $m\angle 3$?



A 26°
B 28°
C 30°
D 32°

SOLUTION:

Use the definition of supplementary angles to find *x*.

$\angle 1 + \angle 2 + \angle 3 \cong 180°$	Definition of supplementary angles
$m\angle 1 + m\angle 2 + m\angle 3 = 180$	Definition of congruent angles
$2x + 3x - 20 + x - 4 = 180$	Substitution.
$6x - 24 = 180$	Simplify.
$6x - 24 + 24 = 180 + 24$	Add 24 to each side.
$6x = 204$	Simplify.
$\frac{6x}{6} = \frac{204}{6}$	Divide each side by 6.
$x = 34$	Simplify.

To find $m\angle 3$, substitute $x = 34$ in $m\angle 3 = x - 4$.

$$\begin{aligned} m\angle 3 &= x - 4 \\ &= 34 - 4 \\ &= 30 \end{aligned}$$

So, the correct option is C.

ANSWER:

C

48. **SAT/ACT** A farmer raises chickens and pigs. If his animals have a total of 120 heads and a total of 300 feet, how many chickens does the farmer have?
F 60
G 70
H 80
J 90

SOLUTION:
Chickens and pigs have one head each and also they have 2 feet and 4 feet respectively. Let *x* be the number of pigs and *y* be the number chickens.

$$x + y = 120 \quad (1)$$
$$4x + 2y = 300 \quad (2)$$

Solve the first equation for *x*.

$$\begin{aligned} x + y &= 120 && \text{Equation 1} \\ x + y - y &= 120 - y && \text{Subtract } y \text{ from each side} \\ x &= 120 - y && \text{Simplify.} \end{aligned}$$

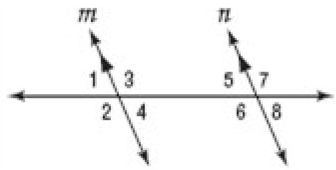
Substitute 120 – *y* for *x* in equation (2).

$$\begin{aligned} 4x + 2y &= 300 && \text{Equation 2} \\ 4(120 - y) + 2y &= 300 && \text{Substitution.} \\ 480 - 4y + 2y &= 300 && \text{Simplify.} \\ 480 - 2y &= 300 && \text{Simplify.} \\ 480 - 480 - 2y &= 300 - 480 && \text{Subtract 480 from each side.} \\ -2y &= -180 && \text{Simplify.} \\ \frac{-2y}{-2} &= \frac{-180}{-2} && \text{Divide each side by } -2. \\ y &= 90 && \text{Simplify.} \end{aligned}$$

Therefore, the farmer has 90 chickens. So, the correct option is J.

ANSWER:
J

49. **SHORT RESPONSE** If $m \parallel n,$ then which of the following statements must be true?



I. $\angle 3$ and $\angle 6$ are Alternate Interior Angles.
II. $\angle 4$ and $\angle 6$ are Consecutive Interior Angles.
III. $\angle 1$ and $\angle 7$ are Alternate Exterior Angles.

SOLUTION:

In the figure, angles 3 and 6 are alternate interior angles, angles 4 and 6 consecutive interior angles, and angles 1 and 7 are consecutive exterior angles. So, the statements I and II are true.

ANSWER:

I and II

50. **ALGEBRA** If –2 + *x* = –6, then –17 – *x* =
A –13
B –4
C 13
D 21

SOLUTION:

Solve for *x*.

$$\begin{aligned} -2+x &= -6 && \text{Original equation} \\ -2+x+2 &= -6+2 && \text{Add 2 to each side.} \\ x &= -4 && \text{Simplify.} \end{aligned}$$

Substitute $x = -4$ in $-17 - x.$

$$\begin{aligned} -17-x &= -17-(-4) && \text{Substitution.} \\ &= -17+4 && \text{Simplify.} \\ &= -13 && \text{Simplify.} \end{aligned}$$

The correct choice is A.

ANSWER:

A

51. **AVIATION** Airplanes are assigned an altitude level based on the direction they are flying. If one airplane is flying northwest at 34,000 feet and another airplane is flying east at 25,000 feet, describe the type of lines formed by the paths of the airplanes. Explain your reasoning.

SOLUTION:

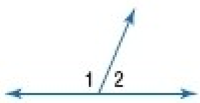
Since the planes are flying at different altitude levels, they are flying in different planes. The lines formed by the path of the planes will not intersect since they are flying in different directions. Lines that are not coplanar and do not intersect are skew lines.
Therefore, the lines formed by the paths of the airplanes are skew lines.

ANSWER:

Skew lines; the planes are flying in different directions and at different altitudes.

Use the given information to find the measure of each numbered angle.

52. If $\angle 1$ and $\angle 2$ form a linear pair and $m\angle 2 = 67$.



SOLUTION:

Since the angles 1 and 2 are linear pairs, they are supplementary.

$m\angle 1 + m\angle 2 = 180$

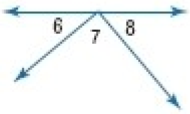
Substitute.

$$m\angle 1 + 67 = 180$$
$$m\angle 1 = 180 - 67$$
$$m\angle 1 = 113$$

ANSWER:

$m\angle 1 = 113$

53. $\angle 6$ and $\angle 8$ are complementary; $m\angle 8 = 47$.



SOLUTION:

Angles 6 and 8 are complementary.

$\angle 6 + \angle 8 \cong 90°$	Complementary Angles
$m\angle 6 + m\angle 8 = 90$	Def. of congruent angles
$m\angle 6 + 47 = 90$	Substitution.
$m\angle 6 + 47 - 47 = 90 - 47$	Subtract 47 from each side.
$m\angle 6 = 43$	Simplify.

In the figure, angles 6, 7 and 8 form a linear pair

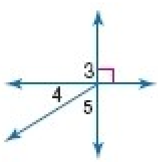
$\angle 6 + \angle 7 + \angle 8 \cong 180°$	Def. of linear pair
$m\angle 6 + m\angle 7 + m\angle 8 = 180$	Def. of congruent angles
$43 + m\angle 7 + 47 = 180$	Substitution.
$m\angle 7 + 90 = 180$	Simplify.
$m\angle 7 + 90 - 90 = 180 - 90$	Subtract 90 from each side.
$m\angle 7 = 90$	Simplify.

.

ANSWER:

$m\angle 6 = 43$, $m\angle 7 = 90$

54. $m\angle 4 = 32$



SOLUTION:

We know that vertical angles are congruent.

So, $m\angle 4 + m\angle 5 = 90$.

Substitute.

$32 + m\angle 5 = 90$

$m\angle 5 = 58$

In the figure, $m\angle 3 + 90 = 180$.

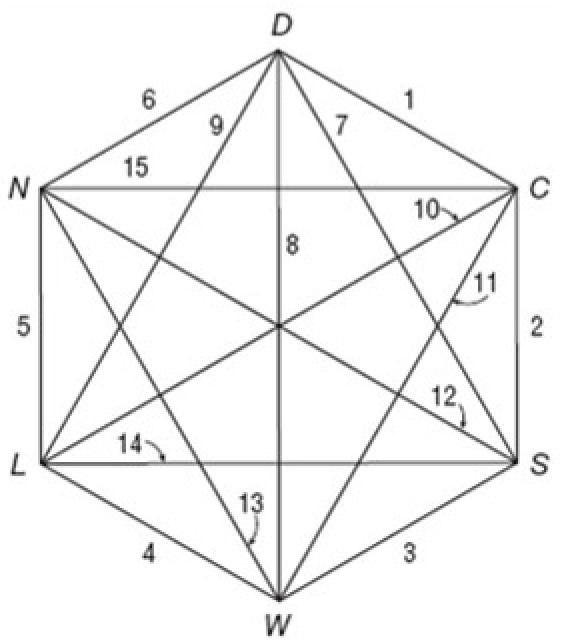
So, $m\angle 3 = 90$.

ANSWER:

$m\angle 3 = 90$, $m\angle 5 = 58$

55. **TRAINS** A train company wants to provide routes to New York City, Dallas, Chicago, Los Angeles, San Francisco, and Washington, D.C. An engineer draws lines between each pair of cities on a map. No three of the cities are collinear. How many lines did the engineer draw?

SOLUTION:



The engineer drew 15 lines.

ANSWER:
15

Simplify each expression.

56. $\frac{6-5}{4-2}$

SOLUTION:

$\frac{6-5}{4-2}=\frac{1}{2}$

ANSWER:

$\frac{1}{2}$

57. $\frac{-5-2}{4-7}$

SOLUTION:

$$\begin{aligned}\frac{-5-2}{4-7} &= \frac{-7}{-3} \\ &= \frac{7}{3}\end{aligned}$$

ANSWER:

$\frac{7}{3}$

58. $\frac{-11-4}{12-(-9)}$

SOLUTION:

$$\begin{aligned}\frac{-11-4}{12-(-9)} &= \frac{-15}{21} \\ &= -\frac{5}{7}\end{aligned}$$

ANSWER:

$-\frac{5}{7}$

59. $\frac{16-12}{15-11}$

SOLUTION:

$$\begin{aligned}\frac{16-12}{15-11} &= \frac{4}{4} \\ &= 1\end{aligned}$$

ANSWER:

1

60. $\frac{10-22}{8-17}$

SOLUTION:

$$\begin{aligned}\frac{10-22}{8-17} &= \frac{-12}{-9} \\ &= \frac{4}{3}\end{aligned}$$

ANSWER:

$\frac{4}{3}$

61. $\dfrac{8-17}{12-(-3)}$

SOLUTION:

$$\begin{aligned}\frac{8-17}{12-(-3)} &= -\frac{9}{15} \\ &= -\frac{3}{5}\end{aligned}$$

ANSWER:

$-\dfrac{3}{5}$