## 3-3 Slopes of Lines

Find the slope of each line.
1.


## SOLUTION:

The coordinates of the point $J$ is $(-2,3)$ and that of $K$ is $(3,-2)$. Substitute the values in the slope formula with $\left(x_{1}\right.$, $\left.y_{1}\right)=(-2,3)$ and $\left(x_{2}, y_{2}\right)=(3,-2)$.
$m=\frac{y_{2}-y_{1}}{x_{2}-x_{1}}$
$=\frac{-2-3}{3-(-2)}$
$=\frac{-5}{5}$
$=-1$
Therefore, the slope of the line is -1 .
ANSWER:
-1
2.

| $\square$ |  | ${ }^{1}$ |  |
| :---: | :---: | :---: | :---: |
|  |  |  |  |
|  |  |  |  |
|  | 0 |  |  |
|  |  |  |  |
|  |  |  |  |

## SOLUTION:

Here, $\overrightarrow{T U}$ is a vertical line. So, the slope is undefined.
ANSWER:
undefined
3.


## SOLUTION:

The coordinates of the point $A$ is $\left(1, \frac{3}{2}\right)$ and that of $B$ is $\left(-\frac{3}{2},-\frac{3}{2}\right)$. Substitute the values in the slope formula with

$$
\begin{aligned}
& \left(x_{1}, y_{1}\right)=\left(1, \frac{3}{2}\right) \text { and }\left(x_{2}, y_{2}\right)=\left(-\frac{3}{2},-\frac{3}{2}\right) . \\
& m=\frac{y_{2}-y_{1}}{x_{2}-x_{1}} \\
& \\
& =\frac{\left(-\frac{3}{2}-\frac{3}{2}\right)}{\left(-\frac{3}{2}-1\right)} \\
& \\
& =\frac{-3}{\left(-\frac{5}{2}\right)} \\
&
\end{aligned}
$$

Therefore, the slope of the line is $\frac{6}{5}$.
ANSWER:
$\frac{6}{5}$
4. BOTANY Kudzu is a fast-growing vine found in the southeastern United States. An initial measurement of the length of a kudzu vine was 0.5 meter. Seven days later the plant was 4 meters long.
a. Graph the line that models the length of the plant over time.
b. What is the slope of your graph? What does it represent?
c. Assuming that the growth rate of the plant continues, how long will the plant be after 15 days?


## SOLUTION:

## 3-3 Slopes of Lines

a. Let $x$ represent the day and $y$ the height. The plant starts at 0.5 m . So one ordered pair is $(0,0.5)$. After 7 days the plant grows to 4 m so a second ordered pair is $(7,4)$.

Plot the points $(0,0.5)$ and $(7,4)$ and join them by a straight line on a coordinate plane.

b. Substitute the values in the slope formula with $\left(x_{1}, y_{1}\right)=(0,0.5)$ and $\left(\boldsymbol{x}_{2}, \boldsymbol{y}_{2}\right)=(7,4)$.

$$
\begin{aligned}
m & =\frac{y_{2}-y_{1}}{x_{2}-x_{1}} \\
& =\frac{4-0.5}{7-0} \\
& =\frac{3.5}{7} \\
& =\frac{1}{2}
\end{aligned}
$$

The slope of the line is $\frac{1}{2}$.
The slope of the line represents the rate of growth of the plant. That is, the plant grows 0.5 m per day.
c. Substitute $m=\frac{1}{2}, x_{1}=7, y_{1}=4$, and $x_{2}=15$ in the slope formula.

$$
\begin{aligned}
m & =\frac{y_{2}-y_{1}}{x_{2}-x_{1}} & & \text { Slope Form ula } \\
\frac{1}{2} & =\frac{y_{2}-4}{15-7} & & \text { Substitution. } \\
\frac{1}{2} & =\frac{y_{2}-4}{8} & & \text { Simplify. } \\
8\left(\frac{1}{2}\right) & =8\left(\frac{y_{2}-4}{8}\right) & & \times \text { each side by } 8 . \\
4 & =y_{2}-4 & & \text { Simplify. } \\
4+4 & =y_{2}-4+4 & & +4 \text { to each side. } \\
8 & =y_{2} & & \text { Simplify. }
\end{aligned}
$$

Therefore, if the trend continues, the plant will be 8 m long in 15 days.
ANSWER:
a.

b. $\frac{1}{2}$; The plant grows 0.5 m a day.
c. 8 m

Determine whether $\overrightarrow{W X}$ and $\overrightarrow{Y Z}$ areparallel, perpendicular, or neither. Graph each line to verify your answer.
5. $W(2,4), X(4,5), Y(4,1), Z(8,-7)$

## SOLUTION:

Substitute the coordinates of the points in slope formula to find the slopes of the lines.
Slope $\overleftrightarrow{W X}$

$$
\begin{aligned}
m_{1} & =\frac{y_{2}-y_{1}}{x_{2}-x_{1}} \\
& =\frac{5-4}{4-2} \\
& =\frac{1}{2}
\end{aligned}
$$

$$
\begin{aligned}
& \text { Slope } \begin{aligned}
m_{2} & =\frac{\overleftrightarrow{Y} y_{2}-y_{1}}{x_{2}-x_{1}} \\
& =\frac{-7-1}{8-4} \\
& =\frac{-8}{4} \\
& =-2
\end{aligned}
\end{aligned}
$$

The product of the slopes of the lines is -1 . Therefore, the lines are perpendicular.
Graph the lines on a coordinate plane to verify the answer.


ANSWER:
perpendicular

6. $W(1,3), X(-2,-5), Y(-6,-2), Z(8,3)$

## SOLUTION:

Substitute the coordinates of the points in slope formula to find the slopes of the lines.

$$
\begin{aligned}
& \text { Slope } \begin{aligned}
m_{1} & =\frac{y_{2}-y_{1}}{x_{2}-x_{1}} \\
= & \frac{-5-3}{-2-1} \\
= & \frac{-8}{-3} \\
= & \frac{8}{3}
\end{aligned}
\end{aligned}
$$

$$
\begin{aligned}
& \text { Slope } \begin{aligned}
m_{2} & =\frac{y_{2}-y_{1}}{x_{2}-x_{1}} \\
& =\frac{3-(-2)}{8-(-6)} \\
& =\frac{5}{14}
\end{aligned}
\end{aligned}
$$

The two lines neither have equal slopes nor is their product -1 . Therefore, the lines are neither parallel nor perpendicular.

Graph the lines on a coordinate plane to verify the answer.


ANSWER:
neither

7. $W(-7,6), X(-6,9), Y(6,3), Z(3,-6)$

## SOLUTION:

Substitute the coordinates of the points in slope formula to find the slopes of the lines.
Slope $\overleftrightarrow{W X}$

$$
\begin{aligned}
m_{1} & =\frac{y_{2}-y_{1}}{x_{2}-x_{1}} \\
& =\frac{9-6}{-6-(-7)} \\
& =\frac{3}{1} \\
& =3
\end{aligned}
$$

Slope $\overleftrightarrow{Y Z}$
$m_{2}=\frac{y_{2}-y_{1}}{x_{2}-x_{1}}$
$=\frac{-6-3}{3-6}$
$=\frac{-9}{-3}$
$=3$
The two lines have equal slopes, 3 . Therefore, the lines are parallel.
Graph the lines on a coordinate plane to verify the answer.


ANSWER:
parallel

8. $W(1,-3), X(0,2), Y(-2,0), Z(8,2)$

## SOLUTION:

Substitute the coordinates of the points in slope formula to find the slopes of the lines.
Slope $\overleftrightarrow{W X}$

$$
\begin{aligned}
m_{1} & =\frac{y_{2}-y_{1}}{x_{2}-x_{1}} \\
& =\frac{2-(-3)}{0-1} \\
& =\frac{5}{-1} \\
& =-5
\end{aligned}
$$

Slope $\overleftrightarrow{Y Z}$

$$
\begin{aligned}
m_{2} & =\frac{y_{2}-y_{1}}{x_{2}-x_{1}} \\
& =\frac{2-0}{8-(-2)} \\
& =\frac{2}{10} \\
& =\frac{1}{5}
\end{aligned}
$$

The product of the slopes of the lines is -1 . Therefore, the lines are perpendicular.
Graph the lines on a coordinate plane to verify the answer.

## 3-3 Slopes of Lines



ANSWER:
perpendicular


## 3-3 Slopes of Lines

## Graph the line that satisfies each condition.

9. passes through $A(3,-4)$, parallel to $\overparen{B C}$ with $B(2,4)$ and $C(5,6)$

## SOLUTION:

Find the slope of the line $\overleftrightarrow{B C}$ with $\left(x_{1}, y_{1}\right)=(2,4)$ and $\left(x_{2}, y_{2}\right)=(5,6)$.

$$
\begin{aligned}
m & =\frac{y_{2}-y_{1}}{x_{2}-x_{1}} \\
& =\frac{6-4}{5-2} \\
& =\frac{2}{3}
\end{aligned}
$$

The required line is parallel to $\overleftrightarrow{B C}$. So, the slope of the required line is also $\frac{2}{3}$.
Start at $A(3,-4)$. Move two units up and three units right to reach the point $(6,-2)$. Join the two points and extend.


ANSWER:


## 3-3 Slopes of Lines

10. slope $=3$, passes through $A(-1,4)$

## SOLUTION:

Start at $A(-1,4)$. Move three units up and one unit right to reach the point $(0,7)$. Join the two points and extend.


ANSWER:


## 3-3 Slopes of Lines

11. passes through $P(7,3)$, perpendicular to $\overleftarrow{L M}$ with $L(-2,-3)$ and $M(-1,5)$

## SOLUTION:

Find the slope of the line $\overleftrightarrow{L M}$. with and $\left(x_{1}, y_{1}\right)=(-2,-3)$ and $\left(x_{2}, y_{2}\right)=(-1,5)$.

$$
\begin{aligned}
m & =\frac{y_{2}-y_{1}}{x_{2}-x_{1}} \\
& =\frac{5-(-3)}{-1-(-2)} \\
& =\frac{8}{1} \\
& =8
\end{aligned}
$$

The required line is perpendicular to $\overrightarrow{L M}$. So, the slope of the required line is $\frac{-1}{8}$.

Start at $P(7,3)$. Move one unit up and eight units left to reach the point $(-1,4)$. Join the two points and extend.


ANSWER:


## 3-3 Slopes of Lines

Find the slope of each line.
12.


## SOLUTION:

The coordinates of the point $C$ is $(-2,-1)$ and that of $D$ is $(5,5)$. Substitute the values in the slope formula.

$$
\begin{aligned}
m & =\frac{y_{2}-y_{1}}{x_{2}-x_{1}} \\
& =\frac{5-(-1)}{5-(-2)} \\
& =\frac{6}{7}
\end{aligned}
$$

Therefore, the slope of the line is $\frac{6}{7}$.
ANSWER:
$\frac{6}{7}$
13.


## SOLUTION:

The coordinates of the point $A$ is $(-3,1)$ and that of $B$ is $(4,-2)$. Substitute the values in the slope formula.

$$
\begin{aligned}
m & =\frac{y_{2}-y_{1}}{x_{2}-x_{1}} \\
& =\frac{-2-1}{4-(-3)} \\
& =\frac{-3}{7} \\
& =-\frac{3}{7}
\end{aligned}
$$

Therefore, the slope of the line is $-\frac{3}{7}$.
ANSWER:
$-\frac{3}{7}$
14.


## SOLUTION:

Here, $\stackrel{\rightharpoonup F}{ }$ is a horizontal line. So, the slope is zero.
ANSWER:
0

## 3-3 Slopes of Lines

15. 



## SOLUTION:

The coordinates of the point $X$ is $(2,4)$ and that of $Y$ is $(1,-4)$. Substitute the values in the slope formula.

$$
\begin{aligned}
m & =\frac{y_{2}-y_{1}}{x_{2}-x_{1}} \\
& =\frac{-4-4}{1-2} \\
& =\frac{-8}{-1} \\
& =8
\end{aligned}
$$

Therefore, the slope of the line is 8 .
ANSWER:
8
16.

|  |  |  |  | $4 y$ |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  |  |  |  |  |  |
|  | $\cdots$ |  |  |  |  |  |  |
|  | N |  |  |  |  |  |  |
|  |  |  | 0 |  |  |  | $\bar{x}$ |
|  |  |  |  |  |  |  |  |
|  |  |  |  |  | N | N |  |
|  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |
|  |  |  |  | 1 |  |  | - |

## SOLUTION:

The coordinates of the point $M$ is $(-4,1)$ and that of $N$ is $(1,-3)$. Substitute the values in the slope formula.

$$
\begin{aligned}
m & =\frac{y_{2}-y_{1}}{x_{2}-x_{1}} \\
& =\frac{-3-1}{1-(-4)} \\
& =\frac{-4}{5} \\
& =-\frac{4}{5}
\end{aligned}
$$

Therefore, the slope of the line is $-\frac{4}{5}$.
ANSWER:
$-\frac{4}{5}$
17.


## SOLUTION:

Here, $\overleftrightarrow{R S}$ is a vertical line. So, the slope is undefined.
ANSWER:
undefined
Determine the slope of the line that contains the given points.
18. $C(3,1), D(-2,1)$

SOLUTION:
Substitute the coordinates of the points in the slope formula.

$$
\begin{aligned}
m & =\frac{y_{2}-y_{1}}{x_{2}-x_{1}} \\
& =\frac{1-1}{-2-3} \\
& =\frac{0}{-5} \\
& =0
\end{aligned}
$$

Therefore, the slope of the line is 0 .
ANSWER:
0
19. $E(5,-1), F(2,-4)$

## SOLUTION:

Substitute the coordinates of the points in the slope formula.

$$
\begin{aligned}
m & =\frac{y_{2}-y_{1}}{x_{2}-x_{1}} \\
& =\frac{-4-(-1)}{2-5} \\
& =\frac{-3}{-3} \\
& =1
\end{aligned}
$$

Therefore, the slope of the line is 1 .

## ANSWER:

1

## 3-3 Slopes of Lines

20. $G(-4,3), H(-4,7)$

## SOLUTION:

Substitute the coordinates of the points in the slope formula.

$$
\begin{aligned}
m & =\frac{y_{2}-y_{1}}{x_{2}-x_{1}} \\
& =\frac{7-3}{-4-(-4)} \\
& =\frac{4}{0}
\end{aligned}
$$

Division of any number by zero is undefined.
Therefore, the slope of the line is undefined.

## ANSWER:

undefined
21. $J(7,-3), K(-8,-3)$

## SOLUTION:

Substitute the coordinates of the points in the slope formula.

$$
\begin{aligned}
m & =\frac{y_{2}-y_{1}}{x_{2}-x_{1}} \\
& =\frac{-3-(-3)}{-8-7} \\
& =\frac{0}{-15} \\
& =0
\end{aligned}
$$

Therefore, the slope of the line is 0 .
ANSWER:
0

## 3-3 Slopes of Lines

22. $L(8,-3), M(-4,-12)$

## SOLUTION:

Substitute the coordinates of the points in the slope formula.

$$
\begin{aligned}
m & =\frac{y_{2}-y_{1}}{x_{2}-x_{1}} \\
& =\frac{-12-(-3)}{-4-8} \\
& =\frac{-9}{-12} \\
& =\frac{3}{4}
\end{aligned}
$$

Therefore, the slope of the line is $\frac{3}{4}$.
ANSWER:
$\frac{3}{4}$
23. $P(-3,-5), Q(-3,-1)$

## SOLUTION:

Substitute the coordinates of the points in the slope formula.

$$
\begin{aligned}
m & =\frac{y_{2}-y_{1}}{x_{2}-x_{1}} \\
& =\frac{-1-(-5)}{-3-(-3)} \\
& =\frac{-4}{0}
\end{aligned}
$$

Division of any number by zero is undefined.
Therefore, the slope of the line is undefined.
ANSWER:
undefined
24. $R(2,-6), S(-6,5)$

## SOLUTION:

Substitute the coordinates of the points in the slope formula.

$$
\begin{aligned}
m & =\frac{y_{2}-y_{1}}{x_{2}-x_{1}} \\
& =\frac{5-(-6)}{-6-2} \\
& =\frac{11}{-8} \\
& =-\frac{11}{8}
\end{aligned}
$$

Therefore, the slope of the line is $-\frac{11}{8}$.
ANSWER:
$-\frac{11}{8}$
25. $T(-6,-11), V(-12,-10)$

## SOLUTION:

Substitute the coordinates of the points in the slope formula.

$$
\begin{aligned}
m & =\frac{y_{2}-y_{1}}{x_{2}-x_{1}} \\
& =\frac{-10-(-11)}{-12-(-6)} \\
& =\frac{1}{-6} \\
& =-\frac{1}{6}
\end{aligned}
$$

Therefore, the slope of the line is $-\frac{1}{6}$.
ANSWER:
$-\frac{1}{6}$
26. CCSS MODELING In 2004, 8 million Americans over the age of 7 participated in mountain biking, and in 2006, 8.5 million participated.
a. Create a graph to show the number of participants in mountain biking based on the change in participation from 2004 to 2006.
b. Based on the data, what is the growth per year of the sport?
c. If participation continues at the same rate, what will be the participation in 2013 to the nearest 10,000 ?

## SOLUTION:

a. Plot the points $(2004,8)$ and $(2006,8.5)$, join them and extend the line.

## 3-3 Slopes of Lines


b. Substitute the coordinates of any two points on the line in the slope formula. Consider the points $(2004,8)$ and $(2012,10)$ since neither has a decimal and will make the slope calculation easier.
Let $\left(x_{1}, y_{1}\right)=(2004,8)$ and $\left(x_{2}, y_{2}\right)=(2012,10)$. Find $m$.

$$
\begin{aligned}
m & =\frac{y_{2}-y_{1}}{x_{2}-x_{1}} \\
& =\frac{10-8}{2012-2004} \\
& =\frac{2}{8} \\
& =\frac{1}{4}
\end{aligned}
$$

The rate of growth is $\frac{1}{4}$ or 0.25 . That is, 250,000 people per year.
c. Substitute $m=\frac{1}{4}, x_{1}=2004, y_{1}=8$, and $x_{2}=2013$ in the slope formula.

$$
\begin{aligned}
m & =\frac{y_{2}-y_{1}}{x_{2}-x_{1}} & & \text { Slope Formula } \\
\frac{1}{4} & =\frac{y_{2}-8}{2013-2004} & & \text { Substitution. } \\
\left(\frac{1}{4}\right) & =\frac{y_{2}-8}{9} & & \text { Subtraction. } \\
9\left(\frac{1}{4}\right) & =9\left(\frac{y_{2}-8}{9}\right) & & \text { xeach side by } 9 . \\
\frac{9}{4} & =y_{2}-8 & & \text { Simplify. } \\
\frac{9}{4}+8 & =y_{2}-8+8 & & +8 \text { to each side. } \\
\frac{9}{4}+8 & =y_{2} & & \text { Addition. } \\
10.25 & =y_{2} & & \text { Simplify. }
\end{aligned}
$$

Therefore, if the trend continues the participation in 2013 will be about 10.25 million.
ANSWER:
a.

b. 250,000 people per year
c. $10,250,000$
27. FINANCIAL LITERACY Suppose an MP3 player cost $\$ 499$ in 2003 and $\$ 249.99$ in 2009.
a. Graph a trend line to predict the price of the MP3 player for 2003 through 2009.
b. Based on the data, how much does the price drop per year?
c. If the trend continues, what will be the cost of an MP3 player in 2013?

## SOLUTION:

a. Plot the points $(2003,499)$ and $(2009,249.99)$, join them and extend the line.

b. Substitute the coordinates of any two points on the line in the slope formula. Consider the points $(2003,499)$ and (2009, 249.99)
$m=\frac{y_{2}-y_{1}}{x_{2}-x_{1}}$
$=\frac{249.99-499}{2009-2003}$
$=\frac{-249.01}{6}$
$\approx-41.50$
The price drops $\$ 41.50$ per year.
c. Substitute $m=-41.50, x_{1}=2003, y_{1}=499$, and $x_{2}=2013$ in the slope formula.

$$
\begin{aligned}
m & =\frac{y_{2}-y_{1}}{x_{2}-x_{1}} \\
-41.50 & =\frac{y_{2}-499}{2013-2003} \\
(-41.50) 10 & =y_{2}-499 \\
-415.00+499 & =y_{2} \\
84 & =y_{2}
\end{aligned}
$$

Therefore, if the trend continues an MP3 Player will cost \$84 in 2013.
ANSWER:
a.

b. $\$ 41.50$
c. $\$ 84$

Determine whether $\overleftrightarrow{A B}$ and $\overrightarrow{C D}$ areparallel, perpendicular, or neither. Graph each line to verify your answer.
28. $A(1,5), B(4,4), C(9,-10), D(-6,-5)$

## SOLUTION:

Substitute the coordinates of the points in slope formula to find the slopes of the lines.
Find slope of $\overleftrightarrow{A B}$ with $\left(x_{1}, y_{1}\right)=(1,5)$ and $\left(x_{2}, y_{2}\right)=(4,4)$.

$$
\begin{aligned}
m_{1} & =\frac{y_{2}-y_{1}}{x_{2}-x_{1}} \\
& =\frac{4-5}{4-1} \\
& =\frac{-1}{3} \\
& =-\frac{1}{3}
\end{aligned}
$$

Find slope of $\stackrel{\leftrightarrow}{C D}$ with $\left(x_{1}, y_{1}\right)=(9,-10)$ and $\left(x_{2}, y_{2}\right)=(-6,-5)$.

$$
\begin{aligned}
m_{2} & =\frac{y_{2}-y_{1}}{x_{2}-x_{1}} \\
& =\frac{-5-(-10)}{-6-9} \\
& =\frac{5}{-15} \\
& =-\frac{1}{3}
\end{aligned}
$$

The two lines have equal slopes, $-\frac{1}{3}$. Therefore, the lines are parallel.
Graph the lines on a coordinate plane to verify the answer.


ANSWER:
parallel

29. $A(-6,-9), B(8,19), C(0,-4), D(2,0)$

## SOLUTION:

Substitute the coordinates of the points in slope formula to find the slopes of the lines.
Find slope of $\overleftrightarrow{A B}$ with $\left(x_{1}, y_{1}\right)=(-6,-9)$ and $\left(x_{2}, y_{2}\right)=(8,19)$.

$$
\begin{aligned}
m_{1} & =\frac{y_{2}-y_{1}}{x_{2}-x_{1}} \\
& =\frac{19-(-9)}{8-(-6)} \\
& =\frac{28}{14} \\
& =2
\end{aligned}
$$

Find slope of $\stackrel{\leftrightarrow D}{ }$ with $\left(x_{1}, y_{1}\right)=(0,-4)$ and $\left(x_{2}, y_{2}\right)=(2,0)$.

$$
\begin{aligned}
m_{2} & =\frac{y_{2}-y_{1}}{x_{2}-x_{1}} \\
& =\frac{0-(-4)}{2-0} \\
& =\frac{4}{2} \\
& =2
\end{aligned}
$$

The two lines have equal slopes, 2 . Therefore, the lines are parallel.
Graph the lines on a coordinate plane to verify the answer.


ANSWER:
parallel

30. $A(4,2), B(-3,1), C(6,0), D(-10,8)$

## SOLUTION:

Substitute the coordinates of the points in slope formula to find the slopes of the lines.

## 3-3 Slopes of Lines

Find slope of $\overleftrightarrow{A B}$ with $\left(x_{1}, y_{1}\right)=(4,2)$ and $\left(x_{2}, y_{2}\right)=(-3,1)$.

$$
\begin{aligned}
m_{1} & =\frac{y_{2}-y_{1}}{x_{2}-x_{1}} \\
& =\frac{1-2}{-3-4} \\
& =\frac{-1}{-7} \\
& =\frac{1}{7}
\end{aligned}
$$

Find slope of $\overleftrightarrow{C D}$ with $\left(x_{1}, y_{1}\right)=(6,0)$ and $\left(x_{2}, y_{2}\right)=(-10,8)$.

$$
\begin{aligned}
m_{2} & =\frac{y_{2}-y_{1}}{x_{2}-x_{1}} \\
& =\frac{8-0}{-10-6} \\
& =\frac{8}{-16} \\
& =-\frac{1}{2}
\end{aligned}
$$

The two lines neither have equal slopes nor is their product -1 . Therefore, the lines are neither parallel nor perpendicular.

Graph the lines on a coordinate plane to verify the answer.


## ANSWER:

neither

31. $A(8,-2), B(4,-1), C(3,11), D(-2,-9)$

## SOLUTION:

Substitute the coordinates of the points in slope formula to find the slopes of the lines.
Find slope of $\overleftrightarrow{A B}$ with $\left(x_{1}, y_{1}\right)=(8,-2)$ and $\left(x_{2}, y_{2}\right)=(4,-1)$.

$$
\begin{aligned}
m_{1} & =\frac{y_{2}-y_{1}}{x_{2}-x_{1}} \\
& =\frac{-1-(-2)}{4-8} \\
& =\frac{1}{-4} \\
& =-\frac{1}{4}
\end{aligned}
$$

Find slope of $\stackrel{C D}{ }$ with $\left(x_{1}, y_{1}\right)=C(3,11)$ and $\left(x_{2}, y_{2}\right)=D(-2,-9)$.

$$
\begin{aligned}
m_{2} & =\frac{y_{2}-y_{1}}{x_{2}-x_{1}} \\
& =\frac{-9-11}{-2-3} \\
& =\frac{-20}{-5} \\
& =4
\end{aligned}
$$

The product of the slopes of the lines is -1 . Therefore, the lines are perpendicular.
Graph the lines on a coordinate plane to verify the answer.


ANSWER:
perpendicular

32. $A(8,4), B(4,3), C(4,-9), D(2,-1)$

## SOLUTION:

Substitute the coordinates of the points in slope formula to find the slopes of the lines.
Find slope of $\overleftrightarrow{A B}$ with $\left(x_{1}, y_{1}\right)=(8,4)$ and $\left(x_{2}, y_{2}\right)=(4,3)$.

$$
\begin{aligned}
m_{1} & =\frac{y_{2}-y_{1}}{x_{2}-x_{1}} \\
& =\frac{3-4}{4-8} \\
& =\frac{-1}{-4} \\
& =\frac{1}{4}
\end{aligned}
$$

Find slope of $\overleftrightarrow{C D}$ with $\left(x_{1}, y_{1}\right)=(4,-9)$ and $\left(x_{2}, y_{2}\right)=(2,-1)$.

$$
\begin{aligned}
m_{2} & =\frac{y_{2}-y_{1}}{x_{2}-x_{1}} \\
& =\frac{-1-(-9)}{2-4} \\
& =\frac{8}{-2} \\
& =-4
\end{aligned}
$$

The product of the slopes of the lines is -1 . Therefore, the lines are perpendicular.
Graph the lines on a coordinate plane to verify the answer.


ANSWER:
perpendicular

33. $A(4,-2), B(-2,-8), C(4,6), D(8,5)$

## SOLUTION:

Substitute the coordinates of the points in slope formula to find the slopes of the lines.
Find slope of $\overleftrightarrow{A B}$ with $\left(x_{1}, y_{1}\right)=(4,-2)$ and $\left(x_{2}, y_{2}\right)=(-2,-8)$.

$$
\begin{aligned}
m_{1} & =\frac{y_{2}-y_{1}}{x_{2}-x_{1}} \\
& =\frac{-8-(-2)}{-2-(4)} \\
& =\frac{-6}{-6} \\
& =-1
\end{aligned}
$$

Find slope of $\stackrel{\leftrightarrow}{C D}$ with $\left(x_{1}, y_{1}\right)=(4,6)$ and $\left(x_{2}, y_{2}\right)=(8,5)$.

$$
\begin{aligned}
m_{2} & =\frac{y_{2}-y_{1}}{x_{2}-x_{1}} \\
& =\frac{5-6}{8-4} \\
& =\frac{-1}{4} \\
& =-\frac{1}{4}
\end{aligned}
$$

The two lines neither have equal slopes nor is their product -1 . Therefore, the lines are neither parallel nor

## 3-3 Slopes of Lines

perpendicular.
Graph the lines on a coordinate plane to verify the answer.


ANSWER:
neither


## 3-3 Slopes of Lines

## Graph the line that satisfies each condition.

34. passes through $A(2,-5)$, parallel to $\stackrel{\rightharpoonup}{B C}$ with $B(1,3)$ and $C(4,5)$

## SOLUTION:

Find the slope of the line $\overleftrightarrow{B C}$ with $\left(x_{1}, y_{1}\right)=(1,3)$ and $\left(x_{2}, y_{2}\right)=(4,5)$.

$$
\begin{aligned}
m & =\frac{y_{2}-y_{1}}{x_{2}-x_{1}} \\
& =\frac{5-3}{4-1} \\
& =\frac{2}{3}
\end{aligned}
$$

The required line is parallel to $B C$. So, the slope of the required line is also 3 .
Start at $A(2,-5)$. Move two units up and three unit right to reach the point $(5,-3)$. Join the two points and extend.


ANSWER:


## 3-3 Slopes of Lines

35. slope $=-2$, passes through $H(-2,-4)$

SOLUTION:
Start at $H(-2,-4)$. Move two units up and one unit left to reach the point $(-3,-2)$. Join the two points and extend.


ANSWER:


## 3-3 Slopes of Lines

36. passes through $K(3,7)$, perpendicular to $\overleftrightarrow{L M}$ with $L(-1,-2)$ and $M(-4,8)$

## SOLUTION:

Find the slope of the line $\overleftrightarrow{L M}$ with $\left(x_{1}, y_{1}\right)=(-1,-2)$ and $\left(x_{2}, y_{2}\right)=(-4,8)$.

$$
\begin{aligned}
m & =\frac{y_{2}-y_{1}}{x_{2}-x_{1}} \\
& =\frac{8-(-2)}{-4-(-1)} \\
& =\frac{10}{-3}
\end{aligned}
$$

The required line is perpendicular to $\overrightarrow{L M}$. So, the slope of the required line is $\frac{3}{10}$.
Start at $K(3,7)$. Move three units up and ten units right to reach the point $(13,10)$. Join the two points and extend.


ANSWER:


## 3-3 Slopes of Lines

37. passes through $X(1,-4)$, parallel to $\overrightarrow{Y Z}$ with $Y(5,2)$ and $Z(-3,-5)$

## SOLUTION:

Find the slope of the line $\overleftrightarrow{Y Z}$ with $\left(x_{1}, y_{1}\right)=(5,2)$ and $\left(x_{2}, y_{2}\right)=(-3,-5)$.

$$
\begin{aligned}
m & =\frac{y_{2}-y_{1}}{x_{2}-x_{1}} \\
& =\frac{-5-2}{-3-5} \\
& =\frac{-7}{-8} \\
& =\frac{7}{8}
\end{aligned}
$$

The required line is parallel to $\stackrel{\rightharpoonup}{Y Z}$. So, the slope of the required line is also $\frac{7}{8}$.
Start at $X(1,-4)$. Move seven units up and eight units right to reach the point $(9,3)$. Join the two points and extend.


ANSWER:


## 3-3 Slopes of Lines

38. slope $=\frac{2}{3}$, passes through $J(-5,4)$

## SOLUTION:

Start at $J(-5,4)$. Move two units up and three units right to reach the point $(-2,6)$. Join the two points and extend.


ANSWER:

39. passes through $D(-5,-6)$, perpendicular to $\stackrel{\rightharpoonup}{F G}$ with $F(-2,-9)$ and $G(1,-5)$

## SOLUTION:

Find the slope of the line $\overleftrightarrow{F G}$ with $\left(x_{1}, y_{1}\right)=(-2,-9)$ and $\left(x_{2}, y_{2}\right)=(1,-5)$.

$$
\begin{aligned}
m & =\frac{y_{2}-y_{1}}{x_{2}-x_{1}} \\
& =\frac{-5-(-9)}{1-(-2)} \\
& =\frac{4}{3}
\end{aligned}
$$

The required line is perpendicular to $\stackrel{\rightharpoonup}{F G}$. So, the slope of the required line is $\frac{-3}{4}$.
Start at $D(-5,-6)$. Move three units down and four units to right to reach the point $(-1,-9)$. Join the two points and extend.


ANSWER:

40. STADIUMS Before it was demolished, the RCA Dome was home to the Indianapolis Colts. The attendance in 2001 was 450,746 , and the attendance in 2005 was 457,373 .
a. What is the approximate rate of change in attendance from 2001 to 2005 ?
b. If this rate of change continues, predict the attendance for 2012.
c. Will the attendance continue to increase indefinitely? Explain.
d. The Colts have now built a new, larger stadium. Do you think their decision was reasonable? Why or why not?

## SOLUTION:

a. Substitute the coordinates of any two points on the line in the slope formula. Consider the points $(2001,450,746)$ and $(2005,457,373)$. Let $\left(x_{1}, y_{1}\right)=(2001,450,746)$ and $\left(x_{2}, y_{2}\right)=(2005,457,373)$.

## 3-3 Slopes of Lines

$$
\begin{aligned}
m & =\frac{y_{2}-y_{1}}{x_{2}-x_{1}} \\
& =\frac{457,373-450,746}{2005-2001} \\
& =\frac{6627}{4} \\
& \approx 1657
\end{aligned}
$$

The rate of growth is about 1657 . That is, the attendance increases about 1657 per year.
b. To find the attendance in 2012, let $m=1657, x_{1}=2001, y_{1}=450746$, and $x_{2}=2012$. Then use the slope formula to find $y_{2}$.

$$
\begin{aligned}
m & =\frac{y_{2}-y_{1}}{x_{2}-x_{1}} & & \text { Slope Formula } \\
1657 & =\frac{y_{2}-450,746}{2012-2001} & & \text { Substitution. } \\
1657 & =\frac{y_{2}-450,746}{11} & & \text { Subtraction. } \\
11(1657) & =11\left(\frac{y_{2}-450,746}{11}\right) & & \text { xeach side by } 11 . \\
11(1657) & =y_{2}-450,746 & & \text { Simplify. } \\
18,227 & =y_{2}-450,746 & & \text { Multiply. } \\
18,227+450,746 & =y_{2}-450,746+450,746 & & +450,746 \text { to each side. } \\
468,973 & =y_{2} & & \text { Addition. }
\end{aligned}
$$

Therefore, if the rate of change continues the attendance will be about 468,973 in 2012.
c. No; the attendance could only continue to increase until the capacity of the stadium was reached.
d. Sample answer: Yes; since their attendance is growing, the new stadium will allow them to accommodate more fans.

ANSWER:
a. 1657
b. 468,973
c. No; the attendance could only continue to increase until the capacity of the stadium was reached.
d. Sample answer: Yes; since their attendance is growing, the new stadium will allow them to accommodate more fans.

Determine which line passing through the given points has a steeper slope.
41. Line $1:(0,5)$ and $(6,1)$

Line 2: $(-4,10)$ and $(8,-5)$

## SOLUTION:

The greater the absolute value of the slope, the steeper the slope of the line becomes.
Substitute the coordinates of the points in slope formula to find the slopes of the two lines and then compare their absolute values to determine which is steeper.

Line 1 Line 2

$$
\begin{aligned}
m_{1} & =\frac{y_{2}-y_{1}}{x_{2}-x_{1}} & m_{2} & =\frac{y_{2}-y_{1}}{x_{2}-x_{1}} \\
& =\frac{1-5}{6-0} & & =\frac{-5-10}{8-(-4)} \\
& =\frac{-4}{6} & & =\frac{-15}{12} \\
& =-\frac{2}{3} & & =-\frac{3}{2}
\end{aligned}
$$

Since $\left|-\frac{3}{2}\right|>\left|-\frac{2}{3}\right|,\left|m_{2}\right|>\left|m_{1}\right|$. Therefore, line 2 is steeper than line 1.
ANSWER:
Line 2
42. Line 1: $(0,-4)$ and $(2,2)$

Line 2: $(0,-4)$ and $(4,5)$

## SOLUTION:

The greater the absolute value of the slope, the steeper the slope of the line becomes.
Substitute the coordinates of the points in slope formula to find the slopes of the two lines and then compare their absolute values to determine which is steeper.

Line 1 Line 2

$$
\begin{aligned}
m_{1} & =\frac{y_{2}-y_{1}}{x_{2}-x_{1}} & m_{2} & =\frac{y_{2}-y_{1}}{x_{2}-x_{1}} \\
& =\frac{2-(-4)}{2-0} & & =\frac{5-(-4)}{4-0} \\
& =\frac{6}{2} & & =\frac{9}{4} \\
& =3 & & =2 \frac{1}{4}
\end{aligned}
$$

Since $|3|>\left|2 \frac{1}{4}\right|,\left|m_{1}\right|>\left|m_{2}\right|$. Therefore, line 1 is steeper than line 2.
ANSWER:
Line 1
43. Line $1:(-9,-4)$ and $(7,0)$

Line 2: $(0,1)$ and $(7,4)$

## SOLUTION:

The greater the absolute value of the slope, the steeper the slope of the line becomes.
Substitute the coordinates of the points in slope formula to find the slopes of the two lines and then compare their absolute values to determine which is steeper.

$$
\begin{aligned}
& \text { Line } 1 \\
& \text { Line } 2 \\
& m_{1}=\frac{y_{2}-y_{1}}{x_{2}-x_{1}} \\
& m_{2}=\frac{y_{2}-y_{1}}{x_{2}-x_{1}} \\
& =\frac{0-(-4)}{7-(-9)}=\frac{4-1}{7-0} \\
& =\frac{4}{16} \quad=\frac{3}{7} \\
& =\frac{1}{4}
\end{aligned}
$$

Since $\left|\frac{3}{7}\right|>\left|\frac{1}{4}\right|,\left|m_{2}\right|>\left|m_{1}\right|$. Therefore, line 2 is steeper than line 1.
ANSWER:
Line 2
44. Line $1:(-6,7)$ and $(9,-3)$

Line 2: $(-9,9)$ and $(3,5)$

## SOLUTION:

The greater the absolute value of the slope, the steeper the slope of the line becomes.
Substitute the coordinates of the points in slope formula to find the slopes of the two lines and then compare their absolute values to determine which is steeper.

$$
\begin{array}{rlrl}
\text { Line 1 } & \text { Line 2 } \\
m_{1}=\frac{y_{2}-y_{1}}{x_{2}-x_{1}} & m_{2} & =\frac{y_{2}-y_{1}}{x_{2}-x_{1}} \\
=\frac{-3-7}{9-(-6)} & =\frac{5-9}{3-(-9)} \\
= & \frac{-10}{15} & & =\frac{-4}{12} \\
=-\frac{2}{3} & & =-\frac{1}{3}
\end{array}
$$

Since $\left|-\frac{2}{3}\right|>\left|-\frac{1}{3}\right|,\left|m_{1}\right|>\left|m_{2}\right|$. Therefore, line 1 is steeper than line 2.
ANSWER:
Line 1
45. CCSS MODELING Michigan provides habitat for two endangered species, the bald eagle and the gray wolf. The graph shows the Michigan population of each species in 1992 and 2006.
a. Which species experienced a greater rate of change in population?
b. Make a line graph showing the growth of both populations.
c. If both species continue to grow at their respective rates, what will the population of each species be in 2012 ?


## SOLUTION:

a. Use the points $(1992,440),(2006,964),(1992,50)$, and $(2006,361)$ to find the rate of change (slope) for each species over the given time periods.

Bald Eagle
Gray Wolf

$$
\begin{aligned}
m_{1} & =\frac{y_{2}-y_{1}}{x_{2}-x_{1}} & m_{2} & =\frac{y_{2}-y_{1}}{x_{2}-x_{1}} \\
& =\frac{964-440}{2006-1992} & & =\frac{361-50}{2006-1992} \\
& =\frac{524}{14} & & =\frac{311}{14} \\
& \approx 37.43 & & \approx 22.21
\end{aligned}
$$

The bald eagle increased at a rate of about 37.43 per year and the gray wolf increased at a rate of about 22.21 per year. So, the bald eagle had a greater rate of change in population.
b. Plot the points $(1992,440)$ and $(2006,964)$, join them and extend to get the line representing the growth of the bald eagle.
Similarly, plot the points $(1992,50)$ and $(2006,361)$, join them and extend to get the line representing the growth of the bald eagle.

c. Substitute $m=37.43, x_{1}=2006, y_{1}=964$, and $x_{2}=2012$ in the slope formula to find the population of bald eagle in 2012.

$$
\begin{aligned}
m & =\frac{y_{2}-y_{1}}{x_{2}-x_{1}} \\
37.43 & =\frac{y_{2}-964}{2012-2006} \\
(37.43) 6 & =y_{2}-964 \\
224.58+964 & =y_{2} \\
1188.58 & =y_{2} \\
1189 & \approx y_{2}
\end{aligned}
$$

Substitute $m=22.21, x_{1}=1992, y_{1}=50$, and $x_{2}=2012$ in the slope formula to find the population of gray wolf in 2012.

$$
m=\frac{y_{2}-y_{1}}{x_{2}-x_{1}}
$$

$$
22.21=\frac{y_{2}-50}{2012-1992}
$$

$$
(22.21) 20=y_{2}-50
$$

$$
444.2+50=y_{2}
$$

$$
494.2=y_{2}
$$

$$
494 \approx y_{2}
$$

Therefore, if the rate of change continues there will be about 1189 bald eagles and 494 gray wolves in 2012.

## ANSWER:

a. the bald eagle
b.

c. 1189 bald eagles; 494 gray wolves

Find the value of $\boldsymbol{x}$ or $\boldsymbol{y}$ that satisfies the given conditions. Then graph the line.
46. The line containing $(4,-1)$ and $(x,-6)$ has a slope of $-\frac{5}{2}$.

## SOLUTION:

Substitute the coordinates of the points and the value of slope in the slope formula.
Let $\left(x_{1}, y_{1}\right)=\left(4,-10,\left(x_{2}, y_{2}\right)=(x,-6)\right.$ and $m=-\frac{5}{2}$.

$$
\begin{aligned}
m & =\frac{y_{2}-y_{1}}{x_{2}-x_{1}} & & \text { Slope Formula } \\
-\frac{5}{2} & =\frac{-6-(-1)}{x-4} & & \text { Substitution. } \\
-\frac{5}{2} & =\frac{-5}{x-4} & & \text { Simplify. } \\
-5(x-4) & =2(-5) & & \text { Cross Multiply. } \\
-5 x+20 & =-10 & & \text { Simplify. } \\
-5 x+20-20 & =-10-20 & & -20 \text { from each side. } \\
-5 x & =-30 & & \text { Simplify. } \\
\frac{-5 x}{-5} & =\frac{-30}{-5} & & \div \text { each side by }-5 . \\
x & =6 & & \text { Simplify. }
\end{aligned}
$$

Plot the points $(4,-1)$ and $(6,-6)$ and join them by a straight line.


ANSWER:
$x=6$;


## 3-3 Slopes of Lines

47. The line containing $(-4,9)$ and $(4,3)$ is parallel to the line containing $(-8,1)$ and $(4, y)$.

## SOLUTION:

Find the slope of the line containing $(-4,9)$ and $(4,3)$ with $\left(x_{1}, y_{1}\right)=(-4,9)$ and $\left(x_{2}, y_{2}\right)=(4,3)$.

$$
\begin{aligned}
m & =\frac{y_{2}-y_{1}}{x_{2}-x_{1}} \\
& =\frac{3-9}{4-(-4)} \\
& =\frac{-6}{8} \\
& =-\frac{3}{4}
\end{aligned}
$$

The two lines are parallel, hence they have the same slope.
Substitute the coordinates of the points and the value of slope in the slope formula. Let $\left(x_{1}, y_{1}\right)=(-8,1),\left(x_{2}, y_{2}\right)=$

$$
\begin{aligned}
(4, y), \text { and } m & =-\frac{3}{4} & & \\
m & =\frac{y_{2}-y_{1}}{x_{2}-x_{1}} & & \text { Slope formula } \\
-\frac{3}{4} & =\frac{y-1}{4-(-8)} & & \text { Substitution. } \\
-\frac{3}{4} & =\frac{y-1}{12} & & \text { Simplify. } \\
-3(12) & =4(y-1) & & \text { Cross Multiply. } \\
-36 & =4 y-4 & & \text { Simplify. } \\
-36+4 & =4 y-4+4 & & +4 \text { to each side. } \\
-32 & =4 y & & \text { Simplify. } \\
-\frac{32}{4} & =\frac{4 y}{4} & & \div \text { each side by } 4 . \\
-8 & =y & & \text { Simplify. }
\end{aligned}
$$

Plot the points $(-8,1)$ and $(4,-8)$ and join them by a straight line.

$y=-8$

48. The line containing $(8,7)$ and $(7,-6)$ is perpendicular to the line containing $(2,4)$ and $(x, 3)$.

## SOLUTION:

Find the slope of the line containing $(8,7)$ and $(7,-6)$ with $\left(x_{1}, y_{1}\right)=(8,7)$ and $\left(x_{2}, y_{2}\right)=(7,-6)$.

$$
\begin{aligned}
m & =\frac{y_{2}-y_{1}}{x_{2}-x_{1}} \\
& =\frac{-6-7}{7-8} \\
& =\frac{-13}{-1} \\
& =13
\end{aligned}
$$

The two lines are perpendicular; hence the product of their slopes is -1 .
So, the slope of the line passing through $(2,4)$ and $(x, 3)$ is $-\frac{1}{13}$.
Substitute the coordinates of the points and the value of slope in the slope formula. Let $\left(x_{1}, y_{1}\right)=(2,4),\left(x_{2}, y_{2}\right)=$ $(x, 3)$ and $m=-\frac{1}{13}$.

$$
\begin{aligned}
m & =\frac{y_{2}-y_{1}}{x_{2}-x_{1}} & & \text { Slope Formula } \\
-\frac{1}{13} & =\frac{3-4}{x-2} & & \text { Substitution. } \\
-\frac{1}{13} & =\frac{-1}{x-2} & & \text { Simplify. } \\
-1(x-2) & =13(-1) & & \text { Cross multiply. } \\
-1 x+2 & =-13 & & \text { Simplify. } \\
-1 x+2-2 & =-13-2 & & -2 \text { from each side. } \\
-1 x & =-15 & & \text { Simplify. } \\
-1(-1 x) & =-1(-15) & & \times \text { each side by }-1 . \\
x & =15 & & \text { Simplify. }
\end{aligned}
$$

## 3-3 Slopes of Lines

Plot the points $(2,4)$ and $(15,3)$ and join them by a straight line.


ANSWER:
$x=15$;

49. The line containing $(1,-3)$ and $(3, y)$ is parallel to the line containing $(5,-6)$ and $(9, y)$.

## SOLUTION:

The two lines are parallel, hence have the same slope.
Substitute the coordinates of the points in the slope formula.

$$
\begin{aligned}
m & =\frac{y_{2}-y_{1}}{x_{2}-x_{1}} & & \text { Slope Formula } \\
\frac{y-(-3)}{3-1} & =\frac{y-(-6)}{9-5} & & \text { Substitution. } \\
\frac{y+3}{2} & =\frac{y+6}{4} & & \text { Simplify. } \\
4(y+3) & =2(y+6) & & \text { Cross multiply. } \\
4 y+12 & =2 y+12 & & \text { Simplify. } \\
4 y-2 y+12 & =2 y-2 y+12 & & -2 \mathrm{y} \text { from each side. } \\
2 y+12 & =12 & & \text { Simplify. } \\
2 y+1 \overline{2}-1 \overline{2} & =1 \overline{2}-1 \overline{2} & & -12 \text { from each side. } \\
2 y & =0 & & \text { Simplify. } \\
\frac{2 y}{2} & =\frac{0}{2} & & \div \text { each side by } 2 . \\
y & =0 & & \text { Simplify. }
\end{aligned}
$$

## 3-3 Slopes of Lines

Plot the points $(5,-6)$ and $(9,0)$ and join them by a straight line.


ANSWER:

$$
y=0 \text {; }
$$


50. SCHOOLS In 2000, Jefferson High School had 1125 students. By 2006, the student body had increased to 1425 students. When Fairview High School was built in 2001, it had 1275 students. How many students did Fairview High School have in 2006 if the student body grew at the same rate as Jefferson High School?

## SOLUTION:

Use the slope formula to find the rate of growth of Jefferson High School. Let $\left(x_{1}, y_{1}\right)=(2000,1125)$ and $\left(x_{2}, y_{2}\right)$

$$
\begin{aligned}
= & (2006,1425) . \\
m & =\frac{y_{2}-y_{1}}{x_{2}-x_{1}} \\
& =\frac{1425-1125}{2006-2000} \\
& =\frac{300}{6} \\
& =50
\end{aligned}
$$

Substitute $m=50, x_{1}=2001, y_{1}=1275$, and $x_{2}=2006$ in the slope formula to find the number of students in Fairview High School in 2006.

$$
\begin{aligned}
m & =\frac{y_{2}-y_{1}}{x_{2}-x_{1}} & & \text { Slope Formula } \\
50 & =\frac{y_{2}-1275}{2006-2001} & & \text { Substitution. } \\
50 & =\frac{y_{2}-1275}{5} & & \text { xeach side by } 5 . \\
5(50) & =5\left(\frac{y_{2}-1275}{5}\right) & & \text { Simplify. } \\
250 & =y_{2}-1275 & & \text { Simplify. } \\
250+1275 & =y_{2}-1275+1275 & & +1275 \text { to each side. } \\
1525 & =y_{2} & & \text { Simplify. }
\end{aligned}
$$

Therefore, in 2006 Fairview High School must have had 1525 students.
ANSWER:
1525 students
51. MUSIC Maggie and Mikayla want to go to the music store near Maggie's house after school. They can walk 3.5 miles per hour and ride their bikes 10 miles per hour.
a. Create a table to show how far Maggie and Mikayla can travel walking and riding their bikes. Include distances for $0,1,2,3$, and 4 hours.
b. Create a graph to show how far Maggie and Mikayla can travel based on time for both walking and riding their bikes. Be sure to label the axes of your graph.
c. What does the slope represent in your graph?
d. Maggie's mom says they can only go if they can make it to the music store and back in less than two hours. If they want to spend at least 30 minutes in the music store and it is four miles away, can they make it? Should they walk or ride their bikes? Explain your reasoning.

## SOLUTION:

a.

| Time <br> (hours) | Distance <br> Walking <br> (miles) | Distance <br> Riding Bikes <br> (miles) |
| :---: | :---: | :---: |
| 0 | 0 | 0 |
| 1 | 3.5 | 10 |
| 2 | 7 | 20 |
| 3 | 10.5 | 30 |
| 4 | 14 | 40 |

b.

c. The slope of the line is the distance covered per hour, which is same as their speed.
d. Sample answer: If they want to spend at least 30 minutes in the music store, then they only have $1 \frac{1}{2}$ hours to travel to the music store and back. It is 4 miles to the store, so the distance to and from is 2(4) or 8 miles. Find the time it takes to walk or ride 8 miles and see if it no more than $1 \frac{1}{2}$ hours.

$$
\begin{aligned}
t & =\frac{d}{r} & t & =\frac{d}{r} \\
t_{\text {walking }} & =\frac{8}{3.5} & t_{\text {bike }} & =\frac{8}{10} \\
d_{\text {walking }} & \approx 2.3 & t_{\text {bike }} & =0.8
\end{aligned}
$$

If they walk, it takes over two hours to go eight miles, so they wouldn't be home in time and they wouldn't get to spend any time in the store. If they ride their bikes, they can travel the the 8 miles in 0.8 hours, or 48 minutes. They can spend 30 minutes in the store and still return home in less than 2 hours from the time they left. So, they can make it if they ride their bikes.

ANSWER:
a.

| Time <br> (hours) | Distance <br> Walking <br> (miles) | Distance <br> Riding Bikes <br> (miles) |
| :---: | :---: | :---: |
| 0 | 0 | 0 |
| 1 | 3.5 | 10 |
| 2 | 7 | 20 |
| 3 | 10.5 | 30 |
| 4 | 14 | 40 |

b.

c. their speed
d. Sample answer: Yes, they can make it if they ride their bikes. If they walk, it takes over two hours to go eight miles, so they wouldn't be home in time and they wouldn't get to spend any time in the store. If they ride their bikes, they can travel there in 24 minutes. If they spend 30 minutes in the store and spend 24 minutes riding home, the total amount of time they will use is $24+30+24=78$ minutes, which is 1 hour and 18 minutes.
52. WRITE A QUESTION A classmate says that all lines have positive or negative slope. Write a question that would challenge his conjecture.

## SOLUTION:

Sample answer: Since vertical lines have an undefined slope, the question "What about vertical lines?" would challenge his conjecture.
Some students might also choose the question "What about horizontal lines?" to challenge his conjecture because horizontal lines have a slope of 0 , which is neither positive or negative.

ANSWER:
Sample answer: What about vertical lines?
53. ERROR ANALYSIS Terrell and Hale calculated the slope of the line passing through the points $Q(3,5)$ and $R(-2$, 2). Is either of them correct? Explain your reasoning.


## SOLUTION:

Hale subtracted the $x$-coordinates in the wrong order and hence got the incorrect answer. Terrell is correct.
ANSWER:
Terrell; Hale subtracted the $x$-coordinates in the wrong order.
54. CCSS REASONING Draw a square $A B C D$ with opposite vertices at $A(2,-4)$ and $C(10,4)$.
a. Find the other two vertices of the square and label them $B$ and $D$.
b. Show that $\overline{A D} \| \overline{B C}$ and $\overline{A B} \| \overline{D C}$.
c. Show that the measure of each angle inside the square is equal to $90^{\circ}$.

## SOLUTION:

a. Sample answer: Plot the points $A$ and $C$ on a coordinate plane. Since $C$ is 8 units up and 8 units to the right of $A$, one missing vertex of the square is 8 units up from $A$ or at $(2,-4+8)=(2,4)$. The other missing vertex is 8 units to the right of $A$ or at $(2+8,-4)=(10,-4)$.
Label the two new points in a clockwise order.


So, the coordinates of the vertices $B$ and $D$ are $(2,4)$ and $(10,-4)$ respectively. (The vertices could also be labeled in a counterclockwise order. This would reverse the coordinates of $B$ and $D$ and reverse the slopes of the sides in the remainder of the problem.)
b. Sample answer: Find the slope of each side to determine if the sides are parallel.

$$
\begin{aligned}
m & =\frac{y_{2}-y_{1}}{x_{2}-x_{1}} & m & =\frac{y_{2}-y_{1}}{x_{2}-x_{1}} \quad m
\end{aligned} \quad \begin{aligned}
& y_{2}-y_{1} \\
& x_{2}-x_{1}
\end{aligned} m_{1}=\frac{y_{2}-y_{1}}{x_{2}-x_{1}}
$$

The slopes of $\overline{A B}$ and $\overline{D C}$ are undefined. Only vertical lines have an undefined slopes, and all vertical lines are parallel to each other. So, $\overline{A B} \| \overline{D C}$. The slopes of $\overline{A D}$ and $\overline{B C}$ are 0 . Lines with equal slopes are parallel. So, $\overline{A D} \| \overline{B C}$.
c. Sample answer: Lines with 0 slope are horizontal lines. Since the slope of $\overline{A B}$ is undefined and the slope of $\overline{B C}$ is zero, the sides are contained by vertical and horizontal lines that intersect at $B$. Horizontal and vertical lines are always perpendicular to each other. Therefore, they form a right angle at $B$, which measures $90^{\circ}$. The same logic applies to the other 3 angles inside the square at $A, C$, and $D$.

## ANSWER:

a. $B(2,4)$ and $D(10,-4)$
b. Sample answer: The slopes of $\overline{A B}$ and $\overline{D C}$ are undefined, so they are parallel to each other. The slopes of $\overline{A D}$ and $\overline{B C}$ are 0 , so they are parallel to each other.
c. Sample answer: Since the slope of $\overline{A B}$ is undefined and the slope of $\overline{B C}$ is zero, the lines are perpendicular to each other. Therefore, they form a right angle, which measures $90^{\circ}$. The same logic applies to all the sides.
55. WRITING IN MATH Describe the slopes of the Sears Tower and the Leaning Tower of Pisa. Refer to Page 195.

## SOLUTION:

The Sears Tower has a vertical or undefined slope and the Leaning Tower of Pisa has a positive slope. ANSWER:
The Sears Tower has a vertical or undefined slope and the Leaning Tower of Pisa has a positive slope.
56. CHALLENGE In this lesson you learned that $m=\frac{y_{2}-y_{1}}{x_{2}-x_{1}}$. Use an algebraic proof to show that the slope can also be calculated using the equation $m=\frac{y_{1}-y_{2}}{x_{1}-x_{2}}$.

## SOLUTION:

Sample answer:
Given: $m=\frac{y_{2}-y_{1}}{x_{2}-x_{1}}$
Prove: $m=\frac{y_{1}-y_{2}}{x_{1}-x_{2}}$
Statements (Reasons)

1. $m=\frac{y_{2}-y_{1}}{x_{2}-x_{1}}$ (Given)
2. $m=\frac{-\left(y_{2}-y_{1}\right)}{-\left(x_{2}-x_{1}\right)}$ (Mult. Prop.)
3. $m=\frac{-y_{2}+y_{1}}{-x_{2}+x_{1}}$ (Dist. Prop.)
4. $m=\frac{y_{1}-y_{2}}{x_{1}-x_{2}}$ (Comm. Prop. of Addition)

ANSWER:
Sample answer:
Given: $m=\frac{y_{2}-y_{1}}{x_{2}-x_{1}}$
Prove: $m=\frac{y_{1}-y_{2}}{x_{1}-x_{2}}$
Statements (Reasons)

1. $m=\frac{y_{2}-y_{1}}{x_{2}-x_{1}}$ (Given)
2. $m=\frac{-\left(y_{2}-y_{1}\right)}{-\left(x_{2}-x_{1}\right)}$ (Mult. Prop.)
3. $m=\frac{-y_{2}+y_{1}}{-x_{2}+x_{1}}$ (Dist. Prop.)
4. $m=\frac{y_{1}-y_{2}}{x_{1}-x_{2}}$ (Comm. Prop. of Addition)
5. WRITING IN MATH Find two additional points that lie along the same line as $X(3,-1)$ and $Y(-1,7)$. Generalize a method you can use to find additional points on the line from any given point.

## SOLUTION:

Sample answer: Use the slope of $\overleftrightarrow{X Y}$ to find additional points.
$m=\frac{y_{2}-y_{1}}{x_{2}-x_{1}}$
$=\frac{-5-(-3)}{5-4}$
$=\frac{-2}{1}$
The slope of $\frac{-2}{1}$ means that from any point on the line another point will be located 2 units down and 1 unit over to the right.

Start at point $X$ and find another point on the line by adding 1 to the $x$-coordinate and -2 to the $y$-coordinate. $(3+1,-1-2)=(4,-3)$

Repeat the procedure to find a second point.
$(4+1,-3-2)=(5,-5)$
So, two additional points that lie on the same line are $(4,-3)$ and $(5,-5)$.

## ANSWER:

Sample answer: $(4,-3)$ and $(5,-5)$ lie along the same line as points $X$ and $Y$. The slope between all of the points is 2. To find additional points, you can take any point on the line and subtract 2 from the $y$-coordinate and add 1 to the $x$-coordinate.
58. What is the slope of a line perpendicular to the line through the points $(-1,6)$ and $(3,-4)$ ?

A $m=-\frac{5}{2}$
B $m=-1$
C $m=-\frac{2}{5}$
D $m=\frac{2}{5}$
SOLUTION:
First, find the slope of the line through $(-1,6)$ and $(3,-4)$.
$m=\frac{y_{2}-y_{1}}{x_{2}-x_{1}}$
$m=\frac{6-(-4)}{-1-3}$
$m=\frac{10}{-4}$ or $-\frac{5}{2}$
The slope of the perpendicular line is the negative reciprocal of $-\frac{5}{2}$.
$-\left(\frac{1}{-\frac{5}{2}}\right)=\frac{2}{5}$
Therefore, the correct choice is D.
ANSWER:
D
59. SHORT RESPONSE A set of 25 cards is randomly placed face down on a table. 15 cards have only the letter A written on the face, and 10 cards have only the letter B. Patrick turned over 1 card. What is the probability of this card having the letter B written on its face?

## SOLUTION:

There are a total of 25 cards of which 10 have the letter B and 15 do not have the letter B. So, the probability of Patrick's card having the letter B on its face is 10:25 or 2:5.

ANSWER:
2:5
60. ALGEBRA Jamie is collecting money to buy an $\$ 81$ gift for her teacher. She has already contributed $\$ 24$. She will collect $\$ 3$ from each contributing student. From how many students must Jamie collect money?
F 3 students
G 9 students
H 12 students
J 19 students

## SOLUTION:

Let $x$ be the number of students that Jamie has to collect the money from, to get $\$ 81$. That is, $3 x+24=81$.
Solve the equation for $x$.
$3 x=57$
$x=19$
Therefore, the correct choice is J .

## ANSWER:

J
61. SAT/ACT The area of a circle is $20 \pi$ square centimeters. What is its circumference?


A $\sqrt{5} \pi \mathrm{~cm}$
B $2 \sqrt{5} \pi \mathrm{~cm}$
C $4 \sqrt{5} \pi \mathrm{~cm}$
D $20 \pi \mathrm{~cm}$

## SOLUTION:

Let $r$ be the radius of the circle. The area of a circle of radius $r$ is $\pi r^{2}$ sq. units.

$$
\begin{aligned}
20 \pi & =\pi r^{2} \\
r & =\sqrt{20}
\end{aligned}
$$

The circumference of a circle of radius $r$ is $2 \pi r$.

$$
\begin{aligned}
2 \pi r & =2 \pi \sqrt{20} \\
& =2 \pi(2 \sqrt{5}) \\
& =4 \sqrt{5} \pi
\end{aligned}
$$

Therefore, the correct choice is C.

## ANSWER:

C

In the figure, $a\|b, c\| d$, and $m \angle 4=57$. Find the measure of each angle.

62. $\angle 5$

## SOLUTION:

The angles 4 and 5 form a linear pair and $m \angle 4=57$. Therefore, $m \angle 5=180-57=123$.
ANSWER:
123
63. $\angle 1$

## SOLUTION:

The angles 2 and 4 are corresponding angles and hence their measures are equal. So, $m \angle 2=57$. The angles 1 and 2 form a linear pair and $m \angle 2=57$. Therefore, $m \angle 1=180-57=123$.

ANSWER:
123
64. $\angle 8$

SOLUTION:
The angles 2 and 4 are corresponding angles and hence they are congruent. So, $m \angle 2=57$. The angles 2 and 8 are vertical angles, so they are equal. Therefore, $m \angle 8=57$.

ANSWER:
57
65. $\angle 10$

SOLUTION:
The angles 2 and 4 are corresponding angles and hence they are congruent. So, $m \angle 2=57$. Again the angles 2 and 10 are corresponding angles. Therefore, $m \angle 10=57$.

ANSWER:
57

Refer to the diagram below.

66. Name all segments parallel to $\overline{T U}$.

SOLUTION:
$\overline{B C}, \overline{E F}, \overline{Q R}$
ANSWER:
$\overline{B C}, \overline{E F}, \overline{Q R}$
67. Name all planes intersecting plane $B C R$.

SOLUTION:
$A B C, A B Q, P Q R, C D S, A P U, D E T$
ANSWER:
$A B C, A B Q, P Q R, C D S, A P U, D E T$
68. Name all segments skew to $\overline{D E}$.

SOLUTION:
$\overline{A P}, \overline{B Q}, \overline{C R}, \overline{F U}, \overline{P U}, \overline{Q R}, \overline{R S}, \overline{T U}$
ANSWER:
$\overline{A P}, \overline{B Q}, \overline{C R}, \overline{F U}, \overline{P U}, \overline{Q R}, \overline{R S}, \overline{T U}$
Determine whether the stated conclusion is valid based on the given information. If not, write invalid. Explain your reasoning.
69. Given: $\angle B$ and $\angle C$ are vertical angles.

Conclusion: $\angle B \cong \angle C$

## SOLUTION:

Any pair of vertical angles is congruent to each other. Since $\angle B$ and $\angle C$ are vertical angles, they are congruent. Therefore, the conclusion is a valid statement.

ANSWER:
valid
70. Given: $\angle W \cong \angle Y$

Conclusion: $\angle W$ and $\angle Y$ are vertical angles.

## SOLUTION:

Congruent angles need not be vertical. So, the conclusion is invalid.
ANSWER:
Invalid; congruent angles do not have to be vertical.
71. CONSTRUCTION There are four buildings on the Mansfield High School Campus, no three of which stand in a straight line. How many sidewalks need to be built so that each building is directly connected to every other building?

## SOLUTION:

Each building should be connected to all the other buildings. The first building will be connected to the other 3 buildings with 3 different paths. Two more sidewalks are needed to connect the second building to buildings 3 and 4 . Finally, one last sidewalk is need to connect building 3 and 4 . So, there will be $3+2+1=6$ pathways in total.

ANSWER:
6
Solve for $\boldsymbol{y}$.
72. $3 x+y=5$

SOLUTION:
Subtract $3 x$ from each side.
$3 x+y-3 x=5-3 x$

$$
y=-3 x+5
$$

ANSWER:
$y=-3 x+5$
73. $4 x+2 y=6$

SOLUTION:
Subtract $4 x$ from each side.

$$
\begin{aligned}
4 x+2 y-4 x & =6-4 x \\
2 y & =-4 x+6
\end{aligned}
$$

Divide each side by 2 .

$$
\begin{aligned}
\frac{2 y}{2} & =\frac{-4 x}{2}+\frac{6}{2} \\
y & =-2 x+3
\end{aligned}
$$

ANSWER:
$y=-2 x+3$

3-3 Slopes of Lines
74. $4 y-3 x=5$

SOLUTION:
Add $3 x$ to each side.

$$
4 y-3 x+3 x=5+3 x
$$

$$
4 y=3 x+5
$$

Divide each side by 4 .

$$
\begin{aligned}
\frac{4 y}{4} & =\frac{3 x}{4}+\frac{5}{4} \\
y & =\frac{3}{4} x+\frac{5}{4}
\end{aligned}
$$

ANSWER:
$y=\frac{3}{4} x+\frac{5}{4}$

