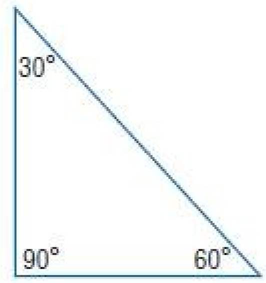
4-1 Classifying Triangles

ARCHITECTURE Classify each triangle as *acute, equiangular, obtuse,* or *right*.

1. **Refer to the figure on page 240.**



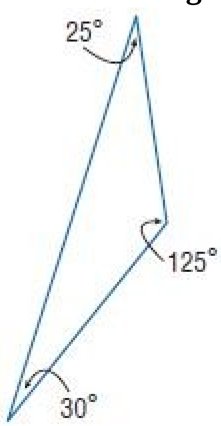
SOLUTION:

One angle of the triangle measures 90, so it is a right angle. Since the triangle has a right angle, it is a right triangle.

ANSWER:

right

2. **Refer to the figure on page 240.**



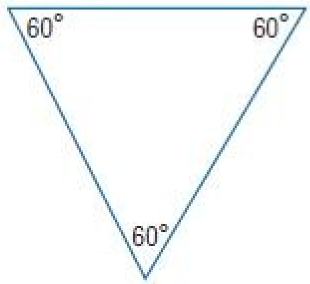
SOLUTION:

One angle of the triangle measures 120, so it is an obtuse angle. Since the triangle has an obtuse angle, it is an obtuse triangle.

ANSWER:

obtuse

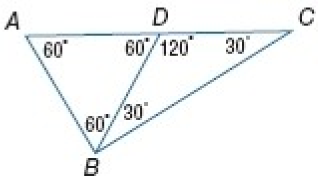
3. **Refer to the figure on page 240.**



SOLUTION:
Since all the angles are congruent, the triangle is equiangular.

ANSWER:
equiangular

Classify each triangle as *acute*, *equiangular*, *obtuse*, or *right*. Explain your reasoning.



4. ΔABD

SOLUTION:
ΔABD is equiangular, since all three angles are congruent.

ANSWER:
equiangular; all three angles are 60°

5. ΔBDC

SOLUTION:
In $\Delta BDC, m\angle BDC = 120$. So, $\angle BDC$ is obtuse. Since the triangle ΔBDC has an obtuse angle, it is obtuse.

ANSWER:
obtuse; $\Delta BDC > 90°$

6. ΔABC

SOLUTION:
In the figure, $m\angle ABC = m\angle ABD + m\angle DBC$.
So by substitution, $m\angle ABC = 60 + 30 = 90.$
ΔABC is a right triangle, since $m\angle ABC = 90.$.

ANSWER:
right; $\Delta ABC = 90°$

CCSS PRECISION Classify each triangle as *equilateral, isosceles,* or *scalene*.

7. 

SOLUTION:

The triangle has two congruent sides. So, it is isosceles.

ANSWER:

isosceles

8. 

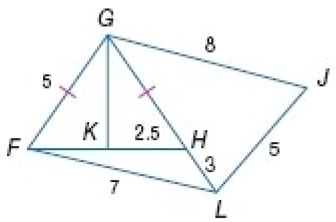
SOLUTION:

No two sides are congruent in the given triangle. So, it is scalene.

ANSWER:

scalene

If point *K* is the midpoint of $\overline{FH}$, classify each triangle in the figure as *equilateral, isosceles,* or *scalene*.



9. ΔFGH

SOLUTION:

In ΔFGH, *K* is the midpoint of $\overline{FH}$. So, $FK = KH$.

$$\begin{aligned} FH &= FK + KH && \text{Segment Addition Postulate} \\ &= KH + KH && \text{Substitution.} \\ &= 2.5 + 2.5 && \text{Substitution.} \\ &= 5 && \text{Add.} \end{aligned}$$

So, all the sides of ΔFGH have equal lengths. Therefore, ΔFGH is equilateral.

ANSWER:

equilateral

10. ΔGJL

SOLUTION:

In ΔGHL , $GL = GH + HL$ by Segment Addition Postulate.

$$\begin{aligned} GL &= GH + HL && \text{Segment Addition Postulate} \\ GL &= 5 + 3 && \text{Substitution.} \\ GL &= 8 && \text{Add.} \end{aligned}$$

ΔGJL has two congruent sides. So, it is isosceles.

ANSWER:

isosceles

11. ΔFHL

SOLUTION:

In ΔFGH , K is the midpoint of $\overline{FH}$. So, $FK = KH$.

$$\begin{aligned} FH &= FK + KH && \text{Segment Addition Postulate} \\ &= KH + KH && \text{Substitution.} \\ &= 2.5 + 2.5 && \text{Substitution.} \\ &= 5 && \text{Add.} \end{aligned}$$

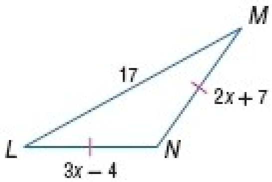
Also, $HL = 3$ and $FL = 7$.

No two sides are congruent in ΔFGH . Therefore, it is scalene.

ANSWER:

scalene

ALGEBRA Find *x* and the measures of the unknown sides of each triangle.

12. 

SOLUTION:

In the figure, $\overline{LN} \cong \overline{MN}$.
So, $3x - 4 = 2x + 7$.

Solve for *x*.

$$\begin{aligned} 3x - 4 &= 2x + 7 \\ 3x - 4 - 2x &= 2x + 7 - 2x && -2x \text{ from each side.} \\ x - 4 &= 7 && \text{Simplify.} \\ x - 4 + 4 &= 7 + 4 && \text{Add 4 to each side.} \\ x &= 11 && \text{Simplify.} \end{aligned}$$

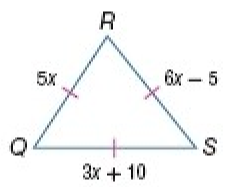
Substitute $x = 11$ in *LN* and *MN*.

$$\begin{aligned} LN &= 3x - 4 && \text{Original equation} \\ &= 3(11) - 4 && x = 11 \\ &= 33 - 4 && \text{Multiply.} \\ &= 29 && \text{Subtract.} \end{aligned}$$

$$\begin{aligned} MN &= 2x + 7 && \text{Original equation} \\ &= 2(11) + 7 && x = 11 \\ &= 22 + 7 && \text{Multiply.} \\ &= 29 && \text{Add.} \end{aligned}$$

ANSWER:

x = 11, *LN* = 29, and *MN* = 29

13. 

SOLUTION:

In the figure, $\overline{RQ} \cong \overline{QS} \cong \overline{RS}$. So any combination of two side measures can be used to find *x*.

$$\begin{aligned} 5x &= 3x + 10 && RQ = QS \\ 5x - 3x &= 3x + 10 - 3x && \text{Subtract } 3x \text{ from each side.} \\ 2x &= 10 && \text{Simplify.} \\ x &= 5 && \text{Divide each side by 2.} \end{aligned}$$

Substitute $x = 5$ in *QR*..

$$\begin{aligned} QR &= 5x \\ &= 5(5) && x = 5 \\ &= 25 && \text{Multiply.} \end{aligned}$$

Since all the sides are congruent, *QR* = *RS* = *QS* = 25.

ANSWER:

x = 5, *QR* = *RS* = *QS* = 25

14. **JEWELRY** Suppose you are bending stainless steel wire to make the earring shown. The triangular portion of the earring is an isosceles triangle. If 1.5 centimeters are needed to make the hook portion of the earring, how many earrings can be made from 45 centimeters of wire? Explain your reasoning.

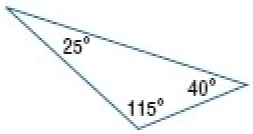
SOLUTION:

4; The total amount of wire needed, including the hook, is 2.1 + 3.2 + 3.2 + 1.5 or 10 cm. 45 cm ÷ 10 cm/earring = 4.5 earrings. There is not enough wire to make 5 earrings, only 4 can be made from 45 cm of wire.

ANSWER:

4; The total amount of wire needed, including the hook, is 2.1 + 3.2 + 3.2 + 1.5 or 10 cm. 45 cm ÷ 10 cm/earring = 4.5 earrings. There is not enough wire to make 5 earrings, only 4 can be made from 45 cm of wire.

Classify each triangle as *acute, equiangular, obtuse,* or *right*.

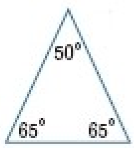
15. 

SOLUTION:

One angle of the triangle measures 115, so it is a obtuse angle. Since the triangle has an obtuse angle, it is an obtuse triangle.

ANSWER:

obtuse

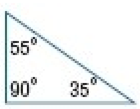
16. 

SOLUTION:

The triangle has three acute angles that are not all equal. It is an acute triangle.

ANSWER:

acute

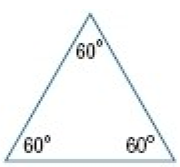
17. 

SOLUTION:

One angle of the triangle measures 90, so it is a right angle. Since the triangle has a right angle, it is a right triangle.

ANSWER:

right

18. 

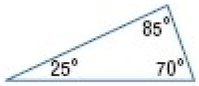
SOLUTION:

Since all the angles are congruent, it is a equiangular triangle.

ANSWER:

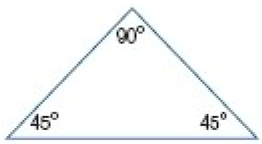
equiangular

4-1 Classifying Triangles

19. 

SOLUTION:
The triangle has three acute angles. It is an acute triangle.

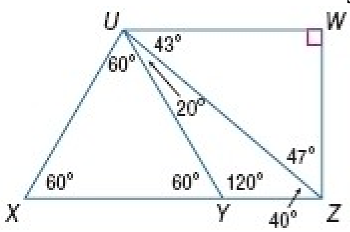
ANSWER:
acute

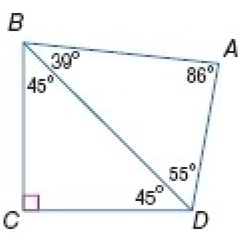
20. 

SOLUTION:
One angle of the triangle measures 90, so it is a right angle. Since the triangle has a right angle, it is a right triangle.

ANSWER:
right

CCSS PRECISION Classify each triangle as *acute, equiangular, obtuse,* or *right*.





21. ΔUYZ

SOLUTION:
In ΔUYZ, $m\angle UYZ = 120$. So, $\angle UYZ$ is obtuse. Since the triangle ΔUYZ has an obtuse angle, it is obtuse.

ANSWER:
obtuse

22. ΔBCD

SOLUTION:
In ΔBCD, $m\angle BCD = 90$. So, $\angle BCD$ is a right angle. Since the triangle ΔBCD has a right angle, it is a right triangle.

ANSWER:
right

23. ΔADB

SOLUTION:

All the angles are acute angles in ΔADB. So, it is an acute triangle.

ANSWER:

acute

24. ΔUXZ

SOLUTION:

All the angles are acute angles in ΔUXZ. So, it is acute triangle.

ANSWER:

acute

25. ΔUWZ

SOLUTION:

In ΔUWZ, $m\angle UWZ = 90$. So, $\angle UWZ$ is a right angle. Since the triangle ΔUWZ has a right angle, it is a right triangle.

ANSWER:

right

26. ΔUXY

SOLUTION:

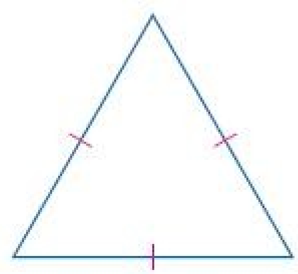
All the angles are congruent in ΔUXY. So, it is equiangular.

ANSWER:

equiangular

Classify each triangle as *equilateral*, *isosceles*, or *scalene*.

27. **Refer to the figure on page 241.**



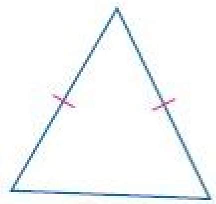
SOLUTION:

Since all the sides are congruent, the triangle is equilateral.

ANSWER:

equilateral

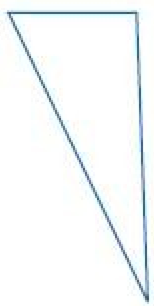
28. **Refer to the figure on page 241.**



SOLUTION:
The triangle has two congruent sides. So, it is isosceles.

ANSWER:
isosceles

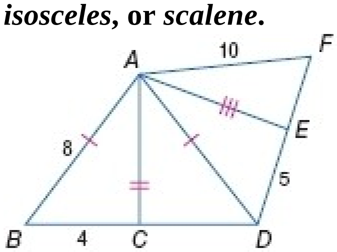
29. **Refer to the figure on page 241.**



SOLUTION:
No two sides are congruent in the given triangle, so it is scalene.

ANSWER:
scalene

If point *C* is the midpoint of $\overline{BD}$ and point *E* is the midpoint of $\overline{DF}$, classify each triangle as *equilateral*, *isosceles*, or *scalene*.



30. ΔABC

SOLUTION:
In ΔABC, all the sides are of different lengths.
So, it is scalene.

ANSWER:
scalene

31. ΔAEF

SOLUTION:

Here, *E* is the midpoint of $\overline{DF}$. So, *DE* = *EF* = 5. In ΔAEF, all the sides are having different lengths. So, it is scalene.

ANSWER:
scalene

32. ΔADF

SOLUTION:

Here, *E* is the midpoint of $\overline{DF}$. So, *DE* = *EF* = 5.
Also by the Segment Addition Postulate, *DF* = *DE* + *EF* = 10.
In ΔADF, *DF* = 10 and *AF* = 10. The triangle ΔADF has two congruent sides. So, it is isosceles.

ANSWER:
isosceles

33. ΔACD

SOLUTION:

No two sides are congruent in ΔACD. So, it is scalene.

ANSWER:
scalene

34. ΔAED

SOLUTION:

No two sides are congruent in ΔAED. So, it is scalene.

ANSWER:
scalene

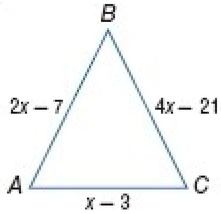
35. ΔABD

SOLUTION:

Here, *C* is the midpoint of $\overline{BD}$. So, *BC* = *CD* = 4.
Also by Segment Addition Postulate, *BC* = *BC* + *CD* = 8.
In ΔABD, *AB* = 8, *BD* = 8, and *AD* = 8. All the sides are congruent in ΔABD. So, it is equilateral.

ANSWER:
equilateral

36. **ALGEBRA** Find x and the length of each side if ΔABC is an isosceles triangle with $\overline{AB} \cong \overline{BC}$.



SOLUTION:

Here, $\overline{AB} \cong \overline{BC}$.
By the definition of congruence, $AB = BC$.

Substitute.

$$\begin{aligned} 2x-7 &= 4x-21 && \text{Substitution.} \\ 2x-7-4x &= 4x-21-4x && -4x \text{ from each side.} \\ -2x-7 &= -21 && \text{Simplify.} \\ -2x-7+7 &= -21+7 && +7 \text{ to each side.} \\ -2x &= -14 && \text{Simplify.} \\ x &= 7 && \text{Divide each side by } -2. \end{aligned}$$

Substitute $x = 7$ in AB, BC, and CA.

$$\begin{aligned} AB &= 2x-7 && \text{Original expression.} \\ &= 2(7)-7 && x = 7 \\ &= 14-7 && \text{Multiply.} \\ &= 7 && \text{Subtract.} \end{aligned}$$

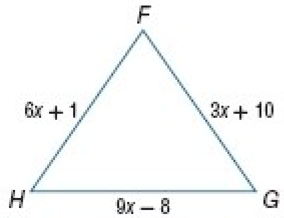
$$\begin{aligned} BC &= 4x-21 && \text{Original expression.} \\ &= 4(7)-21 && x = 7 \\ &= 28-21 && \text{Multiply.} \\ &= 7 && \text{Subtract.} \end{aligned}$$

$$\begin{aligned} CA &= x-3 && \text{Original expression.} \\ &= 7-3 && x = 7 \\ &= 4 && \text{Subtract.} \end{aligned}$$

ANSWER:

$x = 7$; $AB = 7$, $BC = 7$, $CA = 4$

37. **ALGEBRA** Find *x* and the length of each side if ΔFGH is an equilateral triangle.



SOLUTION:

Since ΔFGH is an equilateral triangle, $\overline{FG} \cong \overline{GH} \cong \overline{HF}$. So any combination of sides can be used to find *x*. Let's use $6x + 1 = 3x + 10$.

Solve for *x*.

$$6x + 1 = 3x + 10 \quad FH = FG$$
$$6x + 1 - 3x = 3x + 10 - 3x \quad \text{Subtract } 3x \text{ from each side.}$$
$$3x + 1 = 10 \quad \text{Simplify.}$$
$$3x + 1 - 1 = 10 - 1 \quad \text{Subtract 1 from each side.}$$
$$3x = 9 \quad \text{Simplify.}$$
$$x = 3 \quad \text{Divide each side by 3.}$$

Substitute $x = 3$ in *FG*.

$$FG = 3x + 10 \quad \text{Original equation}$$
$$= 3(3) + 10 \quad x = 3$$
$$= 19 \quad \text{Simplify.}$$

Since all the sides are congruent, *FG* = *GH* = *HF* = 19.

ANSWER:

x = 3; *FG* = *GH* = *HF* = 19

38. **GRAPHIC ART** Classify each numbered triangle in Kat by its angles and by its sides. Use the corner of a sheet of notebook paper to classify angle measures and a ruler to measure sides.
Refer to the figure on page 242.

SOLUTION:

$\triangle$1: right scalene; this has 1 right angle and the lengths of each side are different
$\triangle$2: right scalene; this has 1 right angle and the lengths of each side are different
$\triangle$3: obtuse scalene; this has 1 obtuse angle and the lengths of each side are different
$\triangle$4: acute scalene; all angles are less than 90 and the lengths of each side are different
$\triangle$5: right scalene; this has 1 right angle and the lengths of each side are different
$\triangle$6: obtuse scalene; this has 1 obtuse angle and the lengths of each side are different

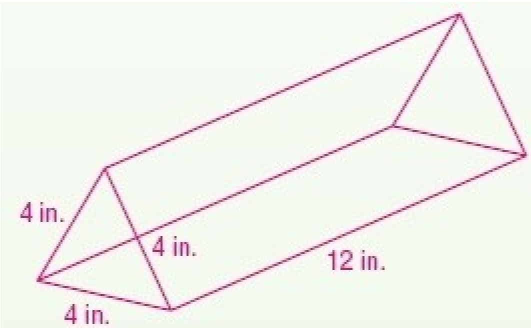
ANSWER:

Δ1: right scalene, Δ2: right scalene, Δ3: obtuse scalene, Δ4: acute isosceles, Δ5: right scalene, Δ6: obtuse scalene

39. **KALEIDOSCOPE** Josh is building a kaleidoscope using PVC pipe, cardboard, bits of colored paper, and a 12-inch square mirror tile. The mirror tile is to be cut into strips and arranged to form an open prism with a base like that of an equilateral triangle. Make a sketch of the prism, giving its dimensions. Explain your reasoning.

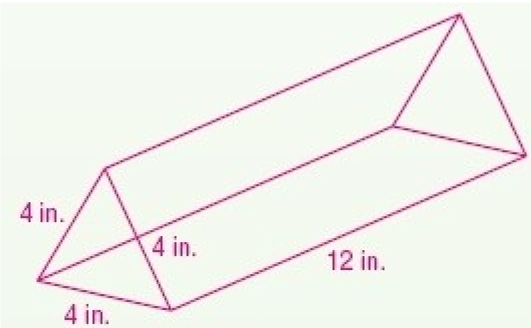
SOLUTION:

Because the base of the prism formed is an equilateral triangle, the mirror tile must be cut into three strips of congruent width. Since the original tile is a 12-inch square, each strip will be 12 inches long by 12 ÷ 3 or 4 inches wide.

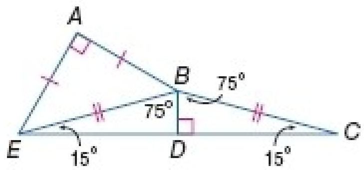


ANSWER:

Because the base of the prism formed is an equilateral triangle, the mirror tile must be cut into three strips of congruent width. Since the original tile is a 12-inch square, each strip will be 12 inches long by 12 ÷ 3 or 4 inches wide.



CCSS PRECISION Classify each triangle in the figure by its angles and sides.



40. $\triangle ABE$

SOLUTION:

$\triangle ABE$ has two congruent sides and a right angle. So, it is an isosceles right triangle.

ANSWER:

isosceles right

41. ΔEBC

SOLUTION:

In the figure, $m\angle EBC = m\angle EBD + m\angle DBC$.

$$\begin{aligned} m\angle EBC &= m\angle EBD + m\angle DBC && \text{Angle Addition Postulate.} \\ &= 75 + 75 && \text{Substitution.} \\ &= 150 && \text{Add.} \end{aligned}$$

ΔEBC has two congruent sides and an obtuse angle. So, it is an isosceles obtuse triangle.

ANSWER:

isosceles obtuse

42. ΔBDC

SOLUTION:

No two sides are congruent in ΔBDC and it has a right angle. So, it is a scalene right triangle.

ANSWER:

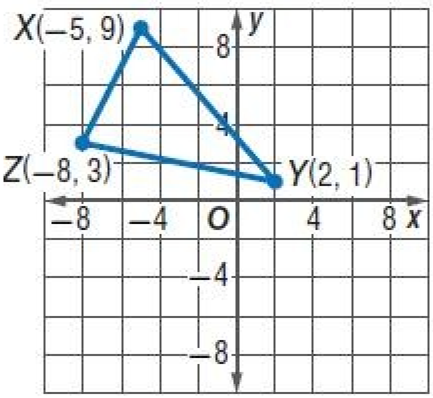
scalene right

COORDINATE GEOMETRY Find the measures of the sides of ΔXYZ and classify each triangle by its sides.

43. $X(–5, 9)$, $Y(2, 1)$, $Z(–8, 3)$

SOLUTION:

Graph the points on a coordinate plane.



Use the Distance Formula to find the lengths of $\overline{XY}, \overline{YZ}$ and $\overline{ZX}$.

$\overline{XY}$ has endpoints $X(–5, 9)$ and $Y(2, 1)$.

$$XY = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Substitute.

$$\begin{aligned} XY &= \sqrt{(2-(-5))^2 + (1-9)^2} \\ &= \sqrt{(7)^2 + (-8)^2} \\ &= \sqrt{49 + 64} \\ &= \sqrt{113} \end{aligned}$$

$\overline{YZ}$ has endpoints $Y(2, 1)$ and $Z(–8, 3)$.

$YZ = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

Substitute.

$$\begin{aligned} YZ &= \sqrt{(-8-2)^2 + (3-1)^2} \\ &= \sqrt{(-10)^2 + (2)^2} \\ &= \sqrt{100+4} \\ &= \sqrt{104} \\ &= 2\sqrt{26} \end{aligned}$$

$\overline{XZ}$ has endpoints $X(–5, 9)$ and $Z(–8, 3)$.

$XZ = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

Substitute.

$$\begin{aligned} XZ &= \sqrt{(-8-(-5))^2 + (3-9)^2} \\ &= \sqrt{(-3)^2 + (-6)^2} \\ &= \sqrt{9+36} \\ &= \sqrt{45} \\ &= 3\sqrt{5} \end{aligned}$$

No two sides are congruent. So, it is scalene.

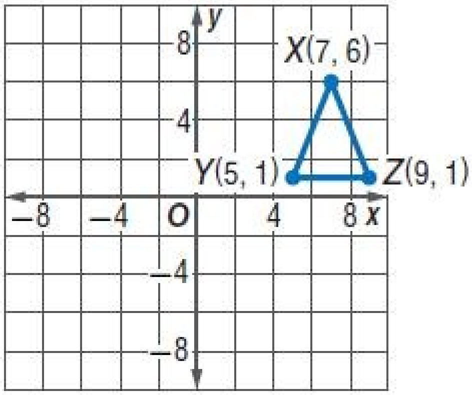
ANSWER:

scalene; $XZ = 3\sqrt{5}$, $XY = \sqrt{113}$, $YZ = 2\sqrt{26}$

44. $X(7, 6)$, $Y(5, 1)$, $Z(9, 1)$

SOLUTION:

Graph the points on a coordinate plane.



Use the Distance Formula to find the lengths of $\overline{XY}, \overline{YZ}$ and $\overline{ZX}$.

$\overline{XY}$ has end points $X(7, 6)$ and $Y(5, 1)$.

$XY = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

Substitute.

$$\begin{aligned} XY &= \sqrt{(5-7)^2 + (1-6)^2} \\ &= \sqrt{(-2)^2 + (-5)^2} \\ &= \sqrt{4+25} \\ &= \sqrt{29} \end{aligned}$$

$\overline{YZ}$ has endpoints $Y(5, 1)$ and $Z(9, 1)$.

$$YZ = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Substitute.

$$\begin{aligned} YZ &= \sqrt{(9-5)^2 + (1-1)^2} \\ &= \sqrt{(4)^2 + (0)^2} \\ &= 4 \end{aligned}$$

$\overline{XZ}$ has endpoints $X(7, 6)$ and $Z(9, 1)$.

$$XZ = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Substitute.

$$\begin{aligned} XZ &= \sqrt{(9-7)^2 + (1-6)^2} \\ &= \sqrt{(2)^2 + (-5)^2} \\ &= \sqrt{4+25} \\ &= \sqrt{29} \end{aligned}$$

$XY = XZ$. This triangle has two congruent sides. So, it is isosceles.

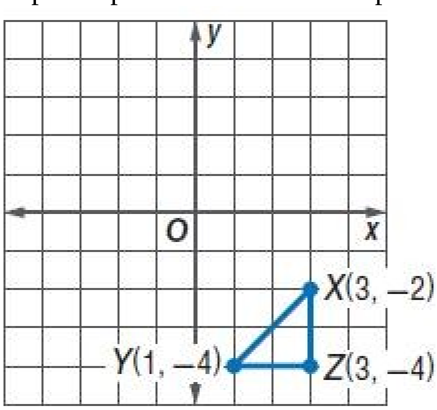
ANSWER:

isosceles; $XZ = \sqrt{29}$, $XY = \sqrt{29}$, $YZ = 4$

45. $X(3, -2)$, $Y(1, -4)$, $Z(3, -4)$

SOLUTION:

Graph the points on a coordinate plane.



Use the Distance Formula to find the lengths of $\overline{XY}$, $\overline{YZ}$ and $\overline{ZX}$.

$\overline{XY}$ has end points $X(3, –2)$ and $Y(1, –4)$.

$$XY = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Substitute.

$$\begin{aligned} XY &= \sqrt{(1-3)^2 + (-4-(-2))^2} \\ &= \sqrt{(-2)^2 + (-2)^2} \\ &= \sqrt{4+4} \\ &= \sqrt{8} \\ &= 2\sqrt{2} \end{aligned}$$

$\overline{YZ}$ has endpoints $Y(1, –4)$ and $Z(3, –4)$.

$$YZ = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Substitute.

$$\begin{aligned} YZ &= \sqrt{(3-1)^2 + (-4-(-4))^2} \\ &= \sqrt{(2)^2 + (0)^2} \\ &= 2 \end{aligned}$$

$\overline{XZ}$ has endpoints $X(3, –2)$ and $Z(3, –4)$.

$$XZ = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Substitute.

$$\begin{aligned} XZ &= \sqrt{(3-3)^2 + (-4-(-2))^2} \\ &= \sqrt{0^2 + (-2)^2} \\ &= 2 \end{aligned}$$

$YZ = XZ = 2$. This triangle has two congruent sides. So, it is isosceles.

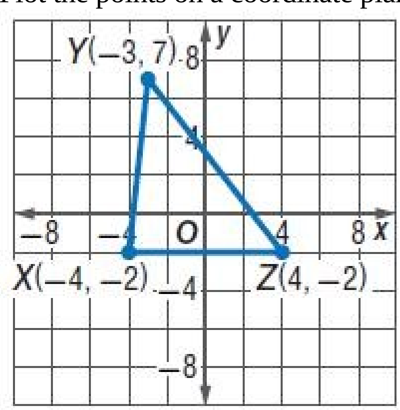
ANSWER:

isosceles; $XZ = 2$, $XY = 2\sqrt{2}$, $YZ = 2$

46. $X(–4, –2)$, $Y(–3, 7)$, $Z(4, –2)$

SOLUTION:

Plot the points on a coordinate plane.



Use the Distance Formula to find the lengths of $\overline{XY}, \overline{YZ}$ and $\overline{ZX}$.

$\overline{XY}$ has endpoints $X(-4, -2)$ and $Y(-3, 7)$.

$$XY = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Substitute.

$$\begin{aligned} XY &= \sqrt{(-3-(-4))^2 + (7-(-2))^2} \\ &= \sqrt{(1)^2 + (9)^2} \\ &= \sqrt{1+81} \\ &= \sqrt{82} \end{aligned}$$

$\overline{YZ}$ has endpoints $Y(-3, 7)$ and $Z(4, -2)$.

$$YZ = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Substitute.

$$\begin{aligned} YZ &= \sqrt{(4-(-3))^2 + (-2-7)^2} \\ &= \sqrt{(7)^2 + (-9)^2} \\ &= \sqrt{49+81} \\ &= \sqrt{130} \end{aligned}$$

$\overline{XZ}$ has endpoints $X(-4, -2)$ and $Z(4, -2)$.

$$XZ = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Substitute.

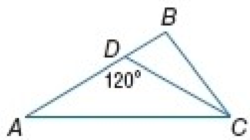
$$\begin{aligned} XZ &= \sqrt{(4-(-4))^2 + (-2-(-2))^2} \\ &= \sqrt{8^2 + (0)^2} \\ &= \sqrt{64} \\ &= 8 \end{aligned}$$

No two sides are congruent. So, it is scalene.

ANSWER:

scalene; $XZ = 8$, $XY = \sqrt{82}$, $YZ = \sqrt{130}$

47. **PROOF** Write a paragraph proof to prove that ΔDBC is an acute triangle if $m\angle ADC = 120$ and ΔABC is acute.



SOLUTION:

Given: $m\angle ADC = 120$

Prove: ΔDBC is acute.

Proof: $\angle ADC$ and $\angle BDC$ form a linear pair. $\angle ADC$ and $\angle BDC$ are supplementary because if two angles form a linear pair, then they are supplementary. So, $m\angle ADC + m\angle BDC = 180$. We know $m\angle ADC = 120$, so by substitution, $120 + m\angle BDC = 180$. Subtract to find that $m\angle BDC = 60$. We already know that $\angle B$ is acute because ΔABC is acute. $\angle BCD$ must also be acute because $\angle C$ is acute and $m\angle C = m\angle ACD + m\angle BCD$. ΔDBC is acute by definition.

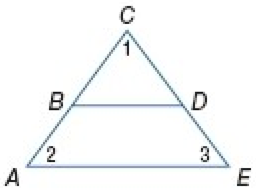
ANSWER:

Given: $m\angle ADC = 120$

Prove: ΔDBC is acute.

Proof: $\angle ADC$ and $\angle BDC$ form a linear pair. $\angle ADC$ and $\angle BDC$ are supplementary because if two angles form a linear pair, then they are supplementary. So, $m\angle ADC + m\angle BDC = 180$. We know $m\angle ADC = 120$, so by substitution, $120 + m\angle BDC = 180$. Subtract to find that $m\angle BDC = 60$. We already know that $\angle B$ is acute because ΔABC is acute. $\angle BCD$ must also be acute because $\angle C$ is acute and $m\angle C = m\angle ACD + m\angle BCD$. ΔDBC is acute by definition.

48. **PROOF** Write a two-column proof to prove that ΔBCD is equiangular if ΔACE is equiangular and $\overline{BD} \parallel \overline{AE}$.



SOLUTION:

Given: ΔACE is equiangular and $\overline{BD} \parallel \overline{AE}$.
Prove: ΔBCD is equiangular.
Proof:
Statements (Reasons)

1. ΔACE is equiangular and $\overline{BD} \parallel \overline{AE}$. (Given)
2. $\angle 1 \cong \angle 2 \cong \angle 3$ (Def. of equiangular Δ)
3. $\angle 2 \cong \angle CBD$ and $\angle 3 \cong \angle CDB$ (Corr. $\angle s$ Post.)
4. $\angle 1 \cong \angle CBD \cong \angle CDB$ (Substitution)
5. ΔBCD is equiangular. (Def. of equiangular Δ)

ANSWER:

Given: ΔACE is equiangular and $\overline{BD} \parallel \overline{AE}$.
Prove: ΔBCD is equiangular.
Proof:
Statements (Reasons)

1. ΔACE is equiangular and $\overline{BD} \parallel \overline{AE}$. (Given)
2. $\angle 1 \cong \angle 2 \cong \angle 3$ (Def. of equiangular Δ)
3. $\angle 2 \cong \angle CBD$ and $\angle 3 \cong \angle CDB$ (Corr. $\angle s$ Post.)
4. $\angle 1 \cong \angle CBD \cong \angle CDB$ (Substitution)
5. ΔBCD is equiangular. (Def. of equiangular Δ)

ALGEBRA For each triangle, find *x* and the measure of each side.

49. ΔFGH is an equilateral triangle with $FG = 3x - 10$, $GH = 2x + 5$, and $HF = x + 20$.

SOLUTION:

Since ΔFGH is equilateral, $FG = GH = HF$.
Consider $FG = GH$.
$3x - 10 = 2x + 5$
$3x - 10 - 2x = 2x + 5 - 2x$
$x - 10 = 5$
$x = 15$
Substitute $x = 15$.

$$\begin{aligned} FG &= 3x - 10 \\ &= 3(15) - 10 \\ &= 35 \end{aligned}$$

Since all the sides are congruent, $FG = GH = HF = 35$.

ANSWER:

$x = 15$; $FG = 35$, $GH = 35$, $HF = 35$

50. ΔJKL is isosceles with $\overline{JK} \cong \overline{KL}$, $JK = 4x - 1$, $KL = 2x + 5$, and $LJ = 2x - 1$.

SOLUTION:

Here, $\overline{JK} \cong \overline{KL}$.
By the definition of congruence, $JK = KL$.

Substitute.

$$\begin{aligned} 4x - 1 &= 2x + 5 && \text{Substitution.} \\ 4x - 1 - 2x &= 2x + 5 - 2x && -2x \text{ from each side.} \\ 2x - 1 &= 5 && \text{Simplify.} \\ 2x - 1 + 1 &= 5 + 1 && +1 \text{ to each side.} \\ 2x &= 6 && \text{Add.} \\ x &= 3 && \div \text{ each side by 2.} \end{aligned}$$

Substitute $x = 3$ in JK, KL, and LJ.

$$\begin{aligned} JK &= 4x - 1 && \text{Original expression} \\ &= 4(3) - 1 && x = 3 \\ &= 12 - 1 && \text{Multiply.} \\ &= 11 && \text{Subtract.} \end{aligned}$$

$$\begin{aligned} KL &= 2x + 5 && \text{Original expression} \\ &= 2(3) + 5 && x = 3 \\ &= 6 + 5 && \text{Multiply.} \\ &= 11 && \text{Add.} \end{aligned}$$

$$\begin{aligned} LJ &= 2x - 1 && \text{Original expression} \\ &= 2(3) - 1 && x = 3 \\ &= 6 - 1 && \text{Multiply.} \\ &= 5 && \text{Subtract.} \end{aligned}$$

ANSWER:

$x = 3$; $JK = 11$, $KL = 11$, $LJ = 5$

51. ΔMNP is isosceles with $\overline{MN} \cong \overline{NP}$. *MN* is two less than five times *x*, *NP* is seven more than two times *x*, and *PM* is two more than three times *x*.

SOLUTION:

Here, $\overline{MN} \cong \overline{NP}$.
By the definition of congruence, $MN = NP$.
$MN = 5x - 2$, $NP = 2x + 7$, $PM = 3x + 2$

Substitute.

$$\begin{aligned} MN &= NP \\ 5x - 2 &= 2x + 7 && \text{Substitution.} \\ 5x - 2 - 2x &= 2x + 7 - 2x && \text{Subtract } 2x \text{ from each side.} \\ 3x - 2 &= 7 && \text{Simplify.} \\ 3x - 2 + 2 &= 7 + 2 && \text{Add 2 to each side.} \\ 3x &= 9 && \text{Simplify.} \\ x &= 3 && \text{Divide each side by 3.} \end{aligned}$$

Substitute $x = 3$ in *MN*, *NP*, and *PM*.

$$\begin{aligned} MN &= 5x - 2 && \text{Original expression} \\ &= 5(3) - 2 && x = 3 \\ &= 15 - 2 && \text{Multiply.} \\ &= 13 && \text{Subtract.} \end{aligned}$$

$$\begin{aligned} NP &= 2x + 7 && \text{Original expression} \\ &= 2(3) + 7 && x = 3 \\ &= 6 + 7 && \text{Multiply.} \\ &= 13 && \text{Add.} \end{aligned}$$

$$\begin{aligned} PM &= 3x + 2 && \text{Original expression} \\ &= 3(3) + 2 && x = 3 \\ &= 9 + 2 && \text{Multiply.} \\ &= 11 && \text{Add.} \end{aligned}$$

ANSWER:

$x = 3$; $MN = 13$, $NP = 13$, $PM = 11$

52. ΔRST is equilateral. *RS* is three more than four times *x*, *ST* is seven more than two times *x*, and *TR* is one more than five times *x*.

SOLUTION:

Since ΔRST is equilateral, $RS = ST = TR$.
$RS = 4x + 3$, $ST = 2x + 7$, $TR = 5x + 1$

$$\begin{aligned} RS &= ST \\ 4x + 3 &= 2x + 7 && \text{Substitution.} \\ 4x + 3 - 2x &= 2x + 7 - 2x && -2x \text{ from each side.} \\ 2x + 3 &= 7 && \text{Simplify.} \\ 2x + 3 - 3 &= 7 - 3 && -3 \text{ to each side.} \\ 2x &= 4 && \text{Simplify.} \\ x &= 2 && \text{Divide each side by 2.} \end{aligned}$$

Substitute $x = 2$.

$$\begin{aligned} RS &= 4x + 3 && \text{Original equation.} \\ &= 4(2) + 3 && x = 2 \\ &= 8 + 3 && \text{Multiply.} \\ &= 11 && \text{Add.} \end{aligned}$$

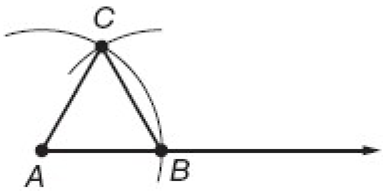
Since all the sides are congruent, $RS = ST = TR = 11$.

ANSWER:

$x = 2$; $RS = ST = TR = 11$

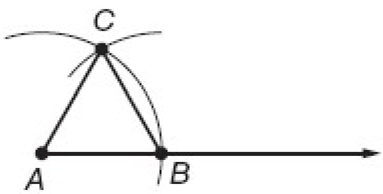
53. **CONSTRUCTION** Construct an equilateral triangle. Verify your construction using measurement and justify it using mathematics. (*Hint*: Use the construction for copying a segment.)

SOLUTION:



Sample answer: In ΔABC $AB = BC = AC = 1.3$ cm. Since all sides have the same length, they are all congruent. Therefore the triangle is equilateral. ΔABC was constructed using AB as the length of each side. Since the arc for each segment is the same, the triangle is equilateral.

ANSWER:

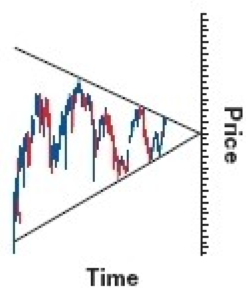


Sample answer: In ΔABC, $AB = BC = AC = 1.3$ cm. Since all sides have the same length, they are all congruent. Therefore the triangle is equilateral. ΔABC was constructed using AB as the length of each side. Since the arc for each segment is the same, the triangle is equilateral.

54. **STOCKS** Technical analysts use charts to identify patterns that can suggest future activity in stock prices. Symmetrical triangle charts are most useful when the fluctuation in the price of a stock is decreasing over time.

a. Classify by its sides and angles the triangle formed if a vertical line is drawn at any point on the graph.

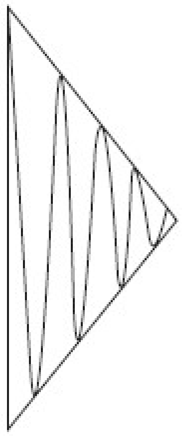
b. How would the price have to fluctuate in order for the data to form an obtuse triangle? Draw an example to support your reasoning.



SOLUTION:

a. The two lines represent the highs and lows of a stock converging on the same price. Since the triangle is symmetrical, the top side would be a reflection of the bottom side. So, the two segments are congruent and the triangle is isosceles.The base angles of an isosceles triangle would be congruent acute angles and the angle formed by the converging prices is acute. Therefore, the triangle is acute.

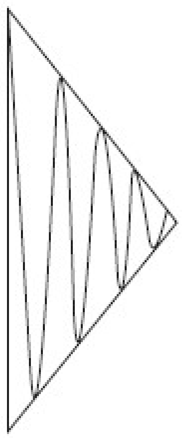
b. Sample answer: The fluctuation (the difference between the high and low prices of the stock) would have to be high and decrease quickly (over a short period of time) in order to form an obtuse triangle. A greater fluctuation will stretch out the angle, making its measure increase.



ANSWER:

a. isosceles; acute

b. Sample answer: The fluctuation would have to be high and decrease quickly in order to form an obtuse triangle.



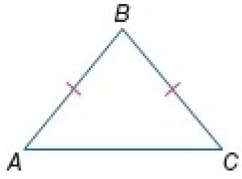
55. **MULTIPLE REPRESENTATIONS** In the diagram, the vertex *opposite* side $\overline{BC}$ is $\angle A$.

a. GEOMETRIC Draw four isosceles triangles, including one acute, one right, and one obtuse isosceles triangle. Label the vertices opposite the congruent sides as *A* and *C*. Label the remaining vertex *B*. Then measure the angles of each triangle and label each angle with its measure.

b. TABULAR Measure all the angles of each triangle. Organize the measures for each triangle into a table. Include a column in your table to record the sum of these measures.

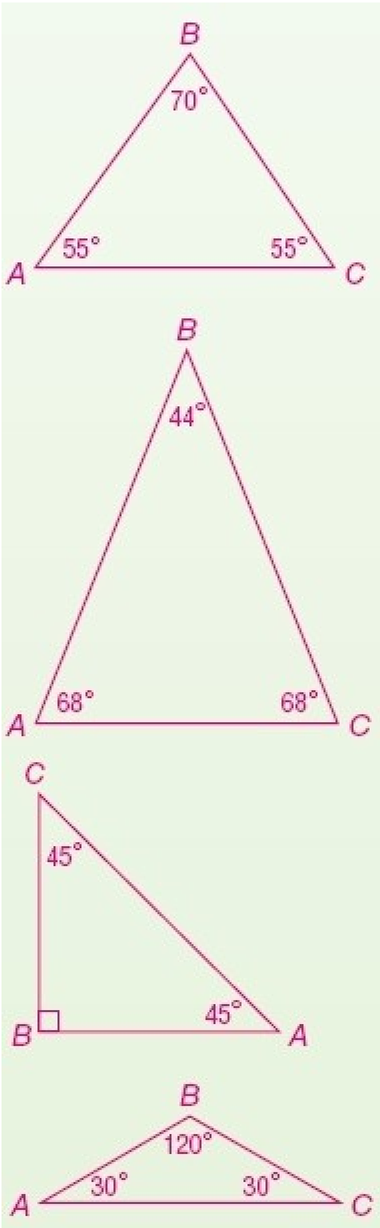
c. VERBAL Make a conjecture about the measures of the angles that are opposite the congruent sides of an isosceles triangle. Then make a conjecture about the sum of the measures of the angles of an isosceles triangle.

d. ALGEBRAIC If *x* is the measure of one of the angles opposite one of the congruent sides in an isosceles triangle, write expressions for the measures of each of the other two angles in the triangle. Explain.



SOLUTION:

a. Sample answer:



b.

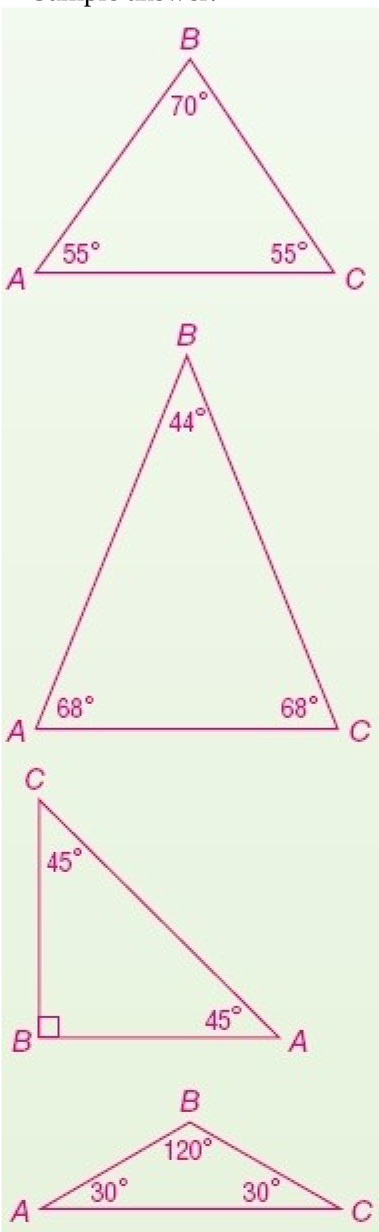
$m\angle A$	$m\angle C$	$m\angle B$	Sum of Angle Measures
55	55	70	180
68	68	44	180
45	45	90	180
30	30	120	180

c. Sample answer: In an isosceles triangle, the angles opposite the congruent sides have the same measure. The sum of the measures of the angles of an isosceles triangle is 180.

d. x and $180 - 2x$; If the measures of the angles opposite the congruent sides of an isosceles triangle have the same measure, then if one angle measures x, then the other angle also measures x. The sum of the measures of the angles of an isosceles triangle is 180, thus the measure of the third angle is 180 – 2*x*.

ANSWER:

a. Sample answer:



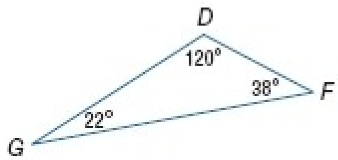
b.

$m\angle A$	$m\angle C$	$m\angle B$	Sum of Angle Measures
55	55	70	180
68	68	44	180
45	45	90	180
30	30	120	180

c. Sample answer: In an isosceles triangle, the angles opposite the congruent sides have the same measure. The sum of the measures of the angles of an isosceles triangle is 180.

d. x and 180 – 2x; If the measures of the angles opposite the congruent sides of an isosceles triangle have the same measure, then if one angle measures x, then the other angle also measures x. The sum of the measures of the angles of an isosceles triangle is 180, thus the measure of the third angle is 180 – 2x.

56. **ERROR ANALYSIS** Elaina says that ΔDFG is obtuse. Ines disagrees, explaining that the triangle has more acute angles than obtuse angles so it must be acute. Is either of them correct? Explain your reasoning.



SOLUTION:

Sample answer: Elaina; all triangles have at least two acute angles, so using Ines' reasoning all triangles would be classified as acute. Instead, triangles are classified by their third angle. If the third angle is also acute, then the triangle is acute. If the third angle is obtuse, as in the triangle shown, the triangle is classified as obtuse.

ANSWER:

Sample answer: Elaina; all triangles have at least two acute angles, so using Ines' reasoning all triangles would be classified as acute. Instead, triangles are classified by their third angle. If the third angle is also acute, then the triangle is acute. If the third angle is obtuse, as in the triangle shown, the triangle is classified as obtuse.

CCSS PRECISION Determine whether the statements below are *sometimes, always,* or *never* true. Explain your reasoning.

57. Equiangular triangles are also right triangles.

SOLUTION:

Never; all equiangular triangles have three 60° angles, so they do not have a 90° angle. Therefore they cannot be right triangles.

ANSWER:

Never; all equiangular triangles have three 60° angles, so they do not have a 90° angle. Therefore they cannot be right triangles.

58. Equilateral triangles are isosceles.

SOLUTION:

Always; all equilateral triangles have three equal sides and isosceles triangles have at least two equal sides, so all triangles with three equal sides are isosceles.

ANSWER:

Always; all equilateral triangles have three equal sides and isosceles triangles have at least two equal sides, so all triangles with three equal sides are isosceles.

59. Right triangles are equilateral.

SOLUTION:

Never; all equilateral triangles are also equiangular, which means all of the angles are 60°. A right triangle has one 90° angle.

ANSWER:

Never; all equilateral triangles are also equiangular, which means all of the angles are 60°. A right triangle has one 90° angle.

60. **CHALLENGE** An equilateral triangle has sides that measure $5x + 3$ units and $7x – 5$ units. What is the perimeter of the triangle? Explain.

SOLUTION:

Sample answer: Since the triangle is equilateral, the sides are equal. Setting $5x + 3$ equal to $7x – 5$ and solving, x is 4. The length of one side is 5(4) + 3 or 23 units. The perimeter of an equilateral triangle is the sum of the three sides or three times one side. The perimeter is 3(23) or 69 units.

ANSWER:

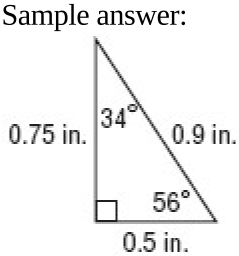
Sample answer: Since the triangle is equilateral, the sides are equal. Setting $5x + 3$ equal to $7x – 5$ and solving, x is 4. The length of one side is 5(4) + 3 or 23 units. The perimeter of an equilateral triangle is the sum of the three sides or three times one side. The perimeter is 3(23) or 69 units.

OPEN ENDED Draw an example of each type of triangle below using a protractor and a ruler. Label the sides and angles of each triangle with their measures. If not possible, explain why not.

61. scalene right

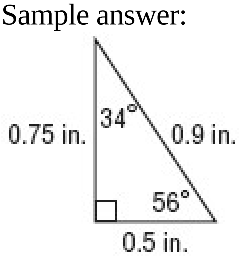
SOLUTION:

Sample answer:



ANSWER:

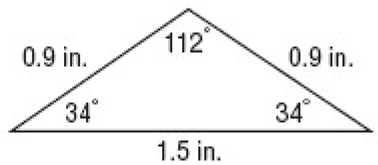
Sample answer:



62. isosceles obtuse

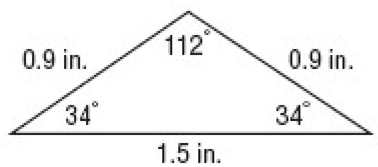
SOLUTION:

Sample answer:



ANSWER:

Sample answer:



63. equilateral obtuse

SOLUTION:

Not possible; all equilateral triangles have three acute angles.

ANSWER:

Not possible; all equilateral triangles have three acute angles.

64. **WRITING IN MATH** Explain why classifying an equiangular triangle as an *acute* equiangular triangle is unnecessary.

SOLUTION:

Sample answer: An acute triangle has three acute angles and an equiangular triangle has three angles that measure 60°. Since an angle that measures 60° is an acute angle, all equiangular triangles are acute. Therefore, acute equiangular is redundant.

ANSWER:

Sample answer: An acute triangle has three acute angles and an equiangular triangle has three angles that measure 60°. Since an angle that measures 60° is an acute angle, all equiangular triangles are acute. Therefore, acute equiangular is redundant.

4-1 Classifying Triangles

65. Which type of triangle can serve as a counterexample to the conjecture below?

If two angles of a triangle are acute, then the measure of the third angle must be greater than or equal to 90.

A equilateral
B obtuse
C right
D scalene

SOLUTION:

An acute angle is less than 90. In an equilateral triangle, all 3 angles are acute. Therefore, an equilateral triangle is a counterexample to the given statement. The correct answer is A.

ANSWER:

A

66. **ALGEBRA** A baseball glove originally cost \$84.50. Kenji bought it at 40% off. How much was deducted from the original price?
F \$50.70
G \$44.50
H \$33.80
J \$32.62

SOLUTION:

We need to find 40% of 84.50 in order to calculate the discount.

$$\begin{aligned}\text{Discount} &= 84.50 \times \frac{40}{100}\\ &= 84.50 \times 0.4\\ &= 33.8\end{aligned}$$

So, \$33.80 was deducted from the original price. The correct option is H.

ANSWER:

H

67. **GRIDDED RESPONSE** Jorge is training for a 20-mile race. Jorge runs 7 miles on Monday, Tuesday, and Friday, and 12 miles on Wednesday and Saturday. After 6 weeks of training, Jorge will have run the equivalent of how many races?

SOLUTION:

Jorge runs 7 miles on Monday, Tuesday, and Friday, and 12 miles on Wednesday and Saturday. Total miles per week = 7 + 7 + 7 +12 + 12 = 45 miles
He will run 45 times 6 or 270 miles in 6 weeks.
Number of races = 270/20 =13.5

ANSWER:

13.5

68. **SAT/ACT** What is the slope of the line determined by the equation $2x + y = 5$?

A $-\frac{5}{2}$
B –2
C –1
D 2
E $\frac{5}{2}$

SOLUTION:

Write the equation $2x + y = 5$ in slope-intercept form.
$y = -2x + 5$
Here, slope is –2.
So, the correct option is B.

ANSWER:

B

Find the distance between each pair of parallel lines with the given equations.

69. $x = -2$
$x = 5$

SOLUTION:

The two lines are of the form $x = a$. So, the slopes are undefined. Therefore, the lines are vertical lines passing through $x = -2$ and $x = 5$ respectively. The perpendicular distance between the two vertical lines is 5 – (–2) = 7 units.

ANSWER:

7

70. $y = -6$
$y = 1$

SOLUTION:

The two lines have the coefficient of x, zero. So, the slopes are zero. Therefore, the lines are horizontal lines passing through $y = -6$ and $y = 1$ respectively. The perpendicular distance between the two horizontal lines is 1 – (–6) = 7 units.

ANSWER:

7

71. $y = 2x + 3$
$y = 2x - 7$

SOLUTION:

The slope of a line perpendicular to both the lines will be $-\frac{1}{2}$. Consider the y-intercept of any of the two lines and write the equation of the perpendicular line through it. The y-intercept of the line $y = 2x - 7$ is (0, –7). So, the equation of a line with slope $-\frac{1}{2}$ and a y-intercept of –7 is $y = -\frac{1}{2}x - 7$.

The perpendicular meets the line $y = 2x - 7$ at (0, –7). To find the point of intersection of the perpendicular and the other line, solve the two equations.
The left sides of the equations are the same. So, equate the right sides and solve for x.

$$2x + 3 = -\frac{1}{2}x - 7$$
$$\frac{5}{2}x = -10 \quad \text{Combine like terms.}$$
$$x = -4 \quad \text{Multiply each side by } \frac{2}{5}.$$

Use the value of x to find the value of y.

$$\begin{aligned} y &= 2x + 3 && \text{Original equation} \\ &= 2(-4) + 3 && x = -4 \\ &= -8 + 3 && \text{Multiply.} \\ &= -5 && \text{Add.} \end{aligned}$$

So, the point of intersection is (–4, –5).

Use the Distance Formula to find the distance between the points (–4, –5) and (0, –7).

$$\begin{aligned} d &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} && \text{Distance Formula} \\ &= \sqrt{(0 - (-4))^2 + (-7 - (-5))^2} && \text{Substitution.} \\ &= \sqrt{(0 - (-4))^2 + (-7 + 5)^2} && \text{Simplify.} \\ &= \sqrt{16 + 4} && \text{Simplify.} \\ &= \sqrt{20} && \text{Add.} \\ &= 2\sqrt{5} && \text{Simplify.} \end{aligned}$$

Therefore, the distance between the two lines is $2\sqrt{5}$ units.

ANSWER:

$2\sqrt{5}$ units.

72. $y = x + 2$
$y = x - 4$

SOLUTION:

The slope of a line perpendicular to both the lines will be –1. Consider the *y*-intercept of any of the two lines and write the equation of the perpendicular line through it. The *y*-intercept of the line *y* = *x* – 4 is (0, –4). So, the equation of a line with slope –1 and a *y*-intercept of –4 is $y = -x - 4$.
The perpendicular meets the line *y* = *x* – 4 at (0, –4). To find the point of intersection of the perpendicular and the other line, solve the two equations.

The left sides of the equations are the same. So, equate the right sides and solve for *x*.

$$\begin{aligned} x + 2 &= -x - 4 \\ 2x &= -6 && \text{Combine like terms.} \\ x &= -3 && \text{Divide each side by 2.} \end{aligned}$$

Use the value of *x* to find the value of *y*.

$$\begin{aligned} y &= x + 2 && \text{Original equation} \\ &= -3 + 2 && x = -3 \\ &= -1 && \text{Add.} \end{aligned}$$

So, the point of intersection is (–3, –1).

Use the Distance Formula to find the distance between the points (–3, –1) and (0, –4).

$$\begin{aligned} d &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} && \text{Distance Formula} \\ &= \sqrt{(0 - (-3))^2 + (-4 - (-1))^2} && \text{Substitution.} \\ &= \sqrt{9 + 9} && \text{Simplify.} \\ &= \sqrt{18} && \text{Add.} \\ &= 3\sqrt{2} && \text{Simplify.} \end{aligned}$$

Therefore, the distance between the two lines is $3\sqrt{2}$ units.

ANSWER:

$3\sqrt{2}$ units.

73. **FOOTBALL** When striping the practice football field, Mr. Hawkins first painted the sidelines. Next he marked off 10-yard increments on one sideline. He then constructed lines perpendicular to the sidelines at each 10-yard mark. Why does this guarantee that the 10-yard lines will be parallel?

SOLUTION:

The lines at each 10-yard mark are perpendicular to the sideline. Since two lines perpendicular to the same line are parallel, Mr. Hawkins can be certain that the 10-yard lines are parallel.

ANSWER:

Two lines in a plane that are perpendicular to the same line are parallel.

Identify the hypothesis and conclusion of each conditional statement.

74. If three points lie on a line, then they are collinear.

SOLUTION:

H: three points lie on a line; C: the points are collinear

ANSWER:

H: three points lie on a line; C: the points are collinear

75. If you are a teenager, then you are at least 13 years old.

SOLUTION:

H: you are a teenager; C: you are at least 13 years old

ANSWER:

H: you are a teenager; C: you are at least 13 years old

76. If $2x + 6 = 10$, then $x = 2$.

SOLUTION:

H: $2x + 6 = 10$; C: $x = 2$

ANSWER:

H: $2x + 6 = 10$; C: $x = 2$

77. If you have a driver's license, then you are at least 16 years old.

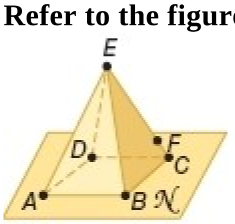
SOLUTION:

H: you have a driver's license; C: you are at least 16 years old

ANSWER:

H: you have a driver's license; C: you are at least 16 years old

Refer to the figure.



78. How many planes appear in this figure?

SOLUTION:

The planes in the figure are:
Plane *AEB*, plane *BEC*, plane *CED*, plane *AED*, and plane *N*. So, there are 5 planes appear in this figure.

ANSWER:

5

79. Name the intersection of plane *AEB* with plane *N*.

SOLUTION:

Plane *AEB* intersects with plane *N* in $\overline{AB}$.

ANSWER:

Plane *AEB* intersects with plane *N* in $\overline{AB}$.

80. Name three points that are collinear.

SOLUTION:

Points *E*, *F*, and *C* lie in the same line. Thus they are collinear.

ANSWER:

E, *F*, *C*

81. Are points *D*, *E*, *C*, and *B* coplanar?

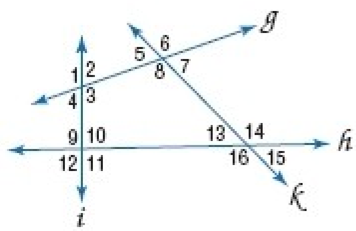
SOLUTION:

Points *D*, *C*, and *B* lie in plane *N*, but point *E* does not lie in plane *N*. Thus, they are not coplanar.

ANSWER:

Points *D*, *C*, and *B* lie in plane *N*, but point *E* does not lie in plane *N*. Thus, they are not coplanar.

Identify each pair of angles as *alternate interior, alternate exterior, corresponding,* or *consecutive interior angles*.



82. $\angle 5$ and $\angle 3$

SOLUTION:

Alternate interior angles

ANSWER:

alt. int.

83. $\angle 9$ and $\angle 4$

SOLUTION:

Consecutive interior angles

ANSWER:

cons. int.

84. $\angle 11$ and $\angle 13$

SOLUTION:
Alternate interior angles

ANSWER:
alt. int.

85. $\angle 1$ and $\angle 11$

SOLUTION:
Alternate exterior angles

ANSWER:
alt. ext.