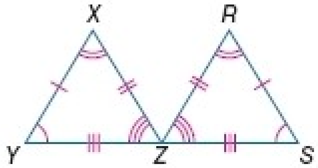
4-3 Congruent Triangles

Show that polygons are congruent by identifying all congruent corresponding parts. Then write a congruence statement.

1. 

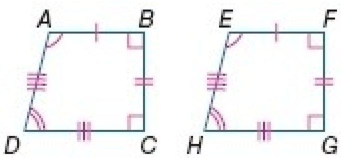
SOLUTION:

$\angle Y \cong \angle S, \angle X \cong \angle R, \angle XYZ \cong \angle RZS,$

$\overline{YX} \cong \overline{SR}, \overline{YZ} \cong \overline{SZ}, \overline{XZ} \cong \overline{RZ};$ All corresponding parts of the two triangles are congruent. Therefore, $\Delta YXZ \cong \Delta SRZ$.

ANSWER:

$\angle Y \cong \angle S$, $\angle X \cong \angle R$, $\angle XZY \cong \angle RZS$, $\overline{YX} \cong \overline{SR}, \overline{YZ} \cong \overline{SZ}, \overline{XZ} \cong \overline{RZ};$ $\Delta YXZ \cong \Delta SRZ$

2. 

SOLUTION:

$\angle A \cong \angle E, \angle B \cong \angle F, \angle C \cong \angle G, \angle D \cong \angle H,$

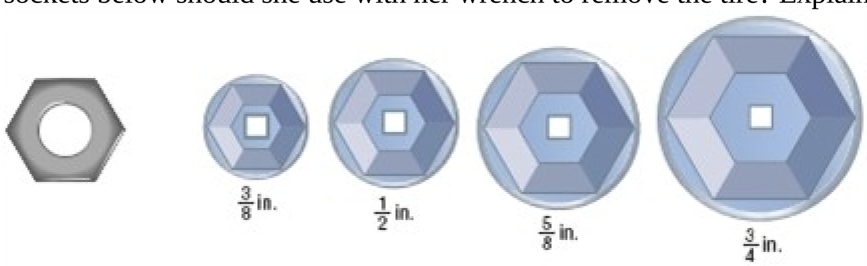
$\overline{AB} \cong \overline{EF}, \overline{CD} \cong \overline{GH}, \overline{AD} \cong \overline{EH}, \overline{BC} \cong \overline{FG};$

All corresponding parts of the two polygons are congruent. Therefore,
polygon $ABCD$ $\cong$ polygon $EFGH$.

ANSWER:

$\angle A \cong \angle E$, $\angle B \cong \angle F$, $\angle C \cong \angle G$, $\angle D \cong \angle H$, $\overline{AB} \cong \overline{EF}, \overline{CD} \cong \overline{GH}, \overline{AD} \cong \overline{EH}, \overline{BC} \cong \overline{FG};$ polygon $ABCD$ $\cong$ polygon $EFGH$

3. **TOOLS** Sareeta is changing the tire on her bike and the nut securing the tire looks like the one shown. Which of the sockets below should she use with her wrench to remove the tire? Explain your reasoning.



SOLUTION:

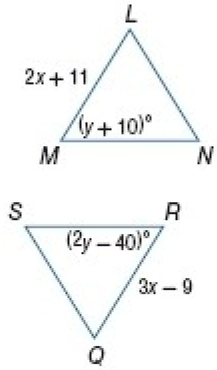
$\frac{1}{2}$ in.; Sample answer: The nut is congruent to the opening for the $\frac{1}{2}$ in. socket.

ANSWER:

$\frac{1}{2}$ in.; Sample answer: The nut is congruent to the opening for the $\frac{1}{2}$ in. socket.

4-3 Congruent Triangles

In the figure, $\Delta LMN \cong \Delta QRS$.



4. Find x.

SOLUTION:

By CPCTC, $\overline{QR} \cong \overline{LM}$.
By the definition of congruence, $QR = LM$.

Substitute.

$$\begin{aligned} 3x - 9 &= 2x + 11 && \text{Substitute.} \\ 3x - 9 - 2x &= 2x + 11 - 2x && -2x \text{ from each side.} \\ x - 9 &= 11 && \text{Simplify.} \\ x - 9 + 9 &= 11 + 9 && +9 \text{ to each side.} \\ x &= 20 && \text{Simplify.} \end{aligned}$$

ANSWER:

20

5. Find y.

SOLUTION:

By CPCTC, $\angle R \cong \angle M$.
By the definition of congruence, $m\angle R = m\angle M$.

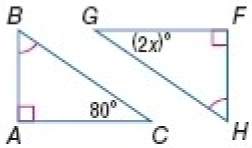
Substitute.

$$\begin{aligned} 2y - 40 &= y + 10 && \text{Substitute.} \\ 2y - 40 - y &= y + 10 - y && -y \text{ from each side.} \\ y - 40 &= 10 && \text{Simplify.} \\ y - 40 + 40 &= 10 + 40 && +40 \text{ to each side.} \\ y &= 50 && \text{Simplify.} \end{aligned}$$

ANSWER:

50

CCSS REGULARITY Find *x*. Explain your reasoning.

6. 

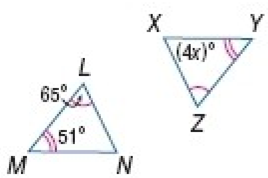
SOLUTION:

Since $\angle A \cong \angle F$ and $\angle B \cong \angle H$, $\angle G$ corresponds to $\angle C$.

$$\begin{aligned} m\angle C &= m\angle G && \text{CPCTC.} \\ 2x &= 80 && \text{Subsitution.} \\ x &= 40 && \text{Divide each side by 2.} \end{aligned}$$

ANSWER:

40; $\angle G$ corresponds to $\angle C$, so $2x = 80$.

7. 

SOLUTION:

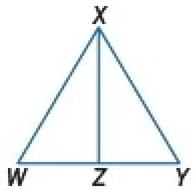
Since $\angle M \cong \angle Y$ and $\angle L \cong \angle Z$, $\angle N$ corresponds to $\angle X$. By the Third Angles Theorem, $m\angle N = 64$.

$$\begin{aligned} m\angle N &= m\angle X && \text{CPCTC.} \\ 4x &= 64 && \text{Subsitution.} \\ x &= 16 && \text{Divide each side by 4.} \end{aligned}$$

ANSWER:

16; $\angle N$ corresponds to $\angle X$. By the Third Angles Theorem, $m\angle N = 64$, so $4x = 64$.

8. **PROOF** Write a paragraph proof.
Given: $\angle WXZ \cong \angle YXZ$, $\angle XZW \cong \angle XZY$,
$\overline{WX} \cong \overline{YX}$, $\overline{WZ} \cong \overline{YZ}$
Prove: $\Delta WXZ \cong \Delta YXZ$



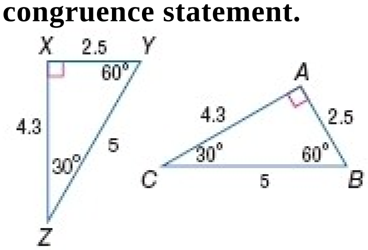
SOLUTION:

We know that $\overline{WX} \cong \overline{YX}$, $\overline{WZ} \cong \overline{YZ}$, also $\overline{XZ} \cong \overline{XZ}$ by the Reflexive Property. We also know $\angle WXZ \cong \angle YXZ, \angle XZW \cong \angle XZY$ and by the Third Angles Theorem, $\angle W \cong \angle Y$ .So, $\Delta WXZ \cong \Delta YXZ$ by the definition of congruent polygons.

ANSWER:

We know that $\overline{WX} \cong \overline{YX}$, $\overline{WZ} \cong \overline{YZ}$,$\overline{XZ} \cong \overline{XZ}$ by the Reflexive Property. We also know $\angle WXZ \cong \angle YXZ$, $\angle XZW \cong \angle XZY$ and by the Third Angles Theorem, $\angle W \cong \angle Y$. So, $\Delta WXZ \cong \Delta YXZ$ by the definition of congruent polygons.

Show that polygons are congruent by identifying all congruent corresponding parts. Then write a congruence statement.

9. 

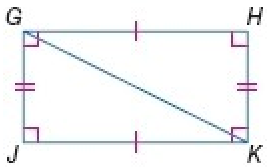
SOLUTION:

$\angle X \cong \angle A$, $\angle Y \cong \angle B$, $\angle Z \cong \angle C$, $\overline{XY} \cong \overline{AB}$, $\overline{XZ} \cong \overline{AC}$, $\overline{YZ} \cong \overline{BC}$; $\Delta XYZ \cong \Delta ABC$ All corresponding parts of the two triangles are congruent.

ANSWER:

$\angle X \cong \angle A$, $\angle Y \cong \angle B$, $\angle Z \cong \angle C$, $\overline{XY} \cong \overline{AB}$, $\overline{XZ} \cong \overline{AC}$, $\overline{YZ} \cong \overline{BC}$; $\Delta XYZ \cong \Delta ABC$

4-3 Congruent Triangles

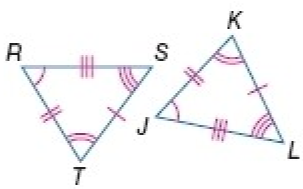
10. 

SOLUTION:

$\angle J \cong \angle H$, $\angle JGK \cong \angle HKJ$, $\angle KGH \cong \angle GKJ$, $\overline{GJ} \cong \overline{KH}$, $\overline{JK} \cong \overline{HG}$, $\overline{GK} \cong \overline{GK}$; $\Delta GJK \cong \Delta KHG$ All corresponding parts of the two triangles are congruent.

ANSWER:

$\angle J \cong \angle H$, $\angle JGK \cong \angle HKG$, $\angle KGH \cong \angle GKJ$, $\overline{GJ} \cong \overline{KH}$, $\overline{JK} \cong \overline{HG}$, $\overline{GK} \cong \overline{GK}$; $\Delta GJK \cong \Delta KHG$

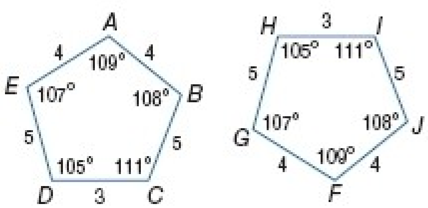
11. 

SOLUTION:

$\angle R \cong \angle J, \angle T \cong \angle K, \angle S \cong \angle L$, $\overline{RT} \cong \overline{JK}, \overline{TS} \cong \overline{KL}$, $\overline{RS} \cong \overline{JL}$; $\Delta RTS \cong \Delta JKL$ All corresponding parts of the two triangles are congruent.

ANSWER:

$\angle R \cong \angle J$, $\angle T \cong \angle K$, $\angle S \cong \angle L$, $\overline{RT} \cong \overline{JK}, \overline{TS} \cong \overline{KL}$, $\overline{RS} \cong \overline{JL}$; $\Delta RTS \cong \Delta JKL$

12. 

SOLUTION:

$\angle A \cong \angle F, \angle B \cong \angle J, \angle C \cong \angle I, \angle D \cong \angle H, \angle E \cong \angle G,$

$\overline{AB} \cong \overline{FJ}, \overline{BC} \cong \overline{JI}, \overline{CD} \cong \overline{IH}, \overline{DE} \cong \overline{HG}, \overline{AE} \cong \overline{FG};$

All corresponding parts of the two polygons are congruent. polygon $ABCDE \cong$ polygon $FJIHG$

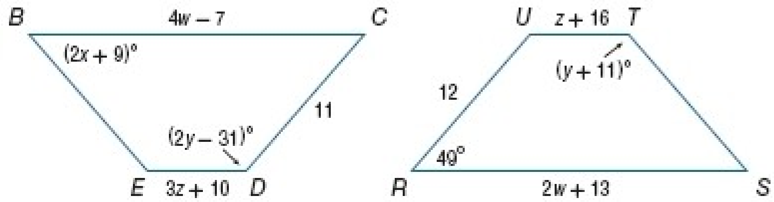
ANSWER:

$\angle A \cong \angle F$, $\angle B \cong \angle J$, $\angle C \cong \angle I$, $\angle D \cong \angle H$, $\angle E \cong \angle G$,

$\overline{AB} \cong \overline{FJ}, \overline{BC} \cong \overline{JI}, \overline{CD} \cong \overline{IH}, \overline{DE} \cong \overline{HG}, \overline{AE} \cong \overline{FG};$ polygon $ABCDE \cong$ polygon $FJIHG$

4-3 Congruent Triangles

Polygon *BCDE* ≅ polygon *RSTU*. Find each value.



13. x

SOLUTION:

By CPCTC, $\angle B \cong \angle R$.
By the definition of congruence, $m\angle B = m\angle R$.

Substitute.

$$\begin{aligned} m\angle B &= m\angle R && \text{CPCTC.} \\ 2x + 9 &= 49 && \text{Substitute.} \\ 2x + 9 - 9 &= 49 - 9 && -9 \text{ from each side.} \\ 2x &= 40 && \text{Simplify.} \\ x &= 20 && \div \text{ each side by 2.} \end{aligned}$$

ANSWER:

20

14. y

SOLUTION:

By CPCTC, $\angle D \cong \angle T$.
By the definition of congruence, $m\angle D = m\angle T$.

Substitute.

$$\begin{aligned} m\angle D &= m\angle T && \\ 2y - 31 &= y + 11 && \text{Substitute.} \\ 2y - 31 + 31 &= y + 11 + 31 && +31 \text{ to each side.} \\ 2y &= y + 42 && \text{Simplify.} \\ 2y - y &= y + 42 - y && -y \text{ from each side.} \\ y &= 42 && \text{Simplify.} \end{aligned}$$

ANSWER:

42

15. *z*

SOLUTION:

By CPCTC, $\overline{ED} \cong \overline{UT}$.
By the definition of congruence, *ED* = *UT*.

Substitute.

$$
\begin{aligned}
ED &= UT \\
3z + 10 &= z + 16 && \text{Substitute.} \\
3z + 10 - 10 &= z + 16 - 10 && -10 \text{ from each side.} \\
2z &= 6 && \text{Simplify.} \\
z &= 3 && \div \text{ each side by 2.}
\end{aligned}
$$

ANSWER:

3

16. *w*

SOLUTION:

By CPCTC, $\overline{BC} \cong \overline{RS}$.
By the definition of congruence, *BC* = *RS*.

Substitute.

$$
\begin{aligned}
BC &= RS \\
4w - 7 &= 2w + 13 && \text{Substitute.} \\
4w - 7 - 2w &= 2w + 13 - 2w && -2w \text{ from each side.} \\
2w - 7 &= 13 && \text{Simplify.} \\
2w - 7 + 7 &= 13 + 7 && +7 \text{ to each side.} \\
2w &= 20 && \text{Simplify.} \\
w &= 10 && \div \text{ each side by 2.}
\end{aligned}
$$

ANSWER:

10

17. **SAILING** To ensure that sailboat races are fair, the boats and their sails are required to be the same size and shape. Refer to the figure on page 260.
a. Write a congruence statement relating the triangles in the photo.
b. Name six pairs of congruent segments.
c. Name six pairs of congruent angles.

SOLUTION:

a. $\triangle ABC \cong \triangle MNO,\ \triangle DEF \cong \triangle PQR$

b. $\overline{AB} \cong \overline{MN},\ \overline{BC} \cong \overline{NO},\ \overline{AC} \cong \overline{MO},\ \overline{DE} \cong \overline{PQ},$
$\overline{EF} \cong \overline{QR},\ \overline{DF} \cong \overline{PR}$

c. $\angle A \cong \angle M,\ \angle B \cong \angle N,\ \angle C \cong \angle O,$
$\angle D \cong \angle P,\ \angle E \cong \angle Q,\ \angle F \cong \angle R$

ANSWER:

a. $\triangle ABC \cong \triangle MNO,\ \triangle DEF \cong \triangle PQR$
b. $\overline{AB} \cong \overline{MN},\ \overline{BC} \cong \overline{NO},\ \overline{AC} \cong \overline{MO},\ \overline{DE} \cong \overline{PQ},$
$\overline{EF} \cong \overline{QR},\ \overline{DF} \cong \overline{PR}$
c. $\angle A \cong \angle M,\ \angle B \cong \angle N,\ \angle C \cong \angle O,$
$\angle D \cong \angle P,\ \angle E \cong \angle Q,\ \angle F \cong \angle R$

Find *x* and *y*.

18. 

SOLUTION:

Since vertical angles are congruent, $y = 40$. The sum of the measures of the angles of a triangle is 180. So, $2x + 2x + 40 = 180$.

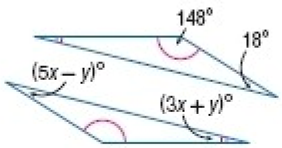
Solve for x.

$$\begin{aligned} 2x + 2x + 40 &= 180 && \text{Triangle Angle-Sum Thm.} \\ 4x + 40 &= 180 && \text{Addition.} \\ 4x + 40 - 40 &= 180 - 40 && -40 \text{ from each side.} \\ 4x &= 140 && \text{Simplify.} \\ x &= 35 && \div \text{ each side by 4.} \end{aligned}$$

ANSWER:

$y = 40$; $x = 35$

4-3 Congruent Triangles

19. 

SOLUTION:

Let p be the measure of an unknown angle in the upper triangle. So, $p+148+18=180.$
Solve for p.

$$\begin{aligned} p+148+18&=180 && \text{Triangle Angle-Sum Thm.}\\ p+166&=180 && \text{Simplify.}\\ p+166-166&=180-166 && -138 \text{ from each side.}\\ p&=14 && \text{Simplify.} \end{aligned}$$

Since the corresponding angles are congruent, the triangles are congruent.

$$5x-y=18$$
$$3x+y=14$$

Add the above equations.

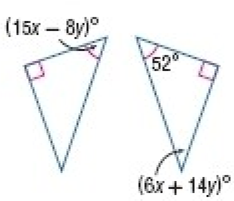
$$\begin{aligned} 5x-y&=18 && \text{Equation 1}\\ (+)\ \underline{3x+y=14} & && \text{Equation 2}\\ 8x&=32 && \text{Addition.}\\ x&=4 && \div \text{ each side by 8.} \end{aligned}$$

Substitute x = 4 in $5x-y=18$.

$$\begin{aligned} 5(4)-y&=18 && \text{Substitute.}\\ 20-y&=18 && \text{Simplify.}\\ y&=2 && \text{Simplify.} \end{aligned}$$

ANSWER:

x = 4; y = 2

20. 

SOLUTION:

The given triangles are similar by AA, so $15x-8y=52.$
Consider the triangle at right. In that triangle, by the Triangle Angle-Sum Theorem, $52+6x+14y+90=180.$

Simplify.

$$
\begin{aligned}
52+6x+14y+90&=180 && \text{Triangle Angle-Sum Thm.}\\
6x+14y+142&=180 && \text{Simplify.}\\
6x+14y+142-142&=180-140 && -142 \text{ from each side.}\\
6x+14y&=38 && \text{Simplify.}
\end{aligned}
$$

Solve the equation $15x-8y=52$ for y.

$$
\begin{aligned}
52&=15x-8y && \text{CPCTC.}\\
8y+52&=15x && +8\text{y to each side.}\\
8y&=15x-52 && -52 \text{ from each side.}\\
y&=\frac{15x-52}{8} && \div \text{ each side by 8.}
\end{aligned}
$$

To solve for *x*, substitute $y=\frac{15x-52}{8}$ in $6x+14y=38$.

$$
\begin{aligned}
6x+14\left(\frac{15x-52}{8}\right)&=38 && \text{Substitute.}\\
6x+7\left(\frac{15x-52}{4}\right)&=38 && \text{Simplify.}\\
6x+\frac{105x-364}{4}&=38 && \text{Distributive Property}\\
6x+\frac{105x}{4}-91&=38 && \text{Simplify.}\\
6x+\frac{105x}{4}&=129 && +91 \text{ to each side.}\\
\frac{129x}{4}&=129 && \text{Combine like terms.}\\
129x&=(129)(4) && \text{Multiply each side by 4.}\\
x&=4 && \text{Simplify.}
\end{aligned}
$$

To solve for y, substitute $x=4$ in $15x-8y=52$.

$$
\begin{aligned}
15(4)-8y&=52 && \text{Substitute.}\\
60-8y&=52 && \text{Simplify.}\\
8y&=8 && \text{Simplify.}\\
y&=1 && \text{Simplify.}
\end{aligned}
$$

ANSWER:

$x=4; y=1$

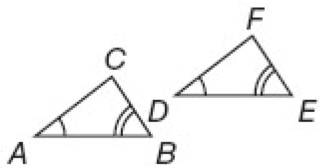
21. **PROOF** Write a two-column proof of Theorem 4.3.

SOLUTION:

Given: $\angle A \cong \angle D$

$\angle B \cong \angle E$

Prove: $\angle C \cong \angle F$



Proof:

Statements (Reasons)

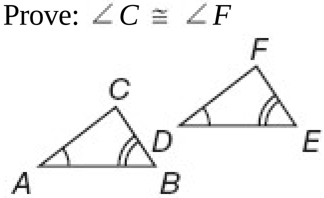
1. $\angle A \cong \angle D$ and $\angle B \cong \angle E$ (Given)
2. $m\angle A = m\angle D$ and $m\angle B = m\angle E$ (Def. of $\cong \angle s$)
   $m\angle A + m\angle B + m\angle C = 180$ and
3. $m\angle D + m\angle E + m\angle F = 180$ ($\angle$ Sum Theorem)
4. $m\angle A + m\angle B + m\angle C = m\angle D + m\angle E + m\angle F$ (Trans. Prop.)
5. $m\angle D + m\angle E + m\angle C = m\angle D + m\angle E + m\angle F$ (Subst.)
6. $m\angle C = m\angle F$ *F* (Subt. Prop.)
7. $\angle C \cong \angle F$ (Def. of $\cong \angle s$)

ANSWER:

Given: $\angle A \cong \angle D$

$\angle B \cong \angle E$

Prove: $\angle C \cong \angle F$



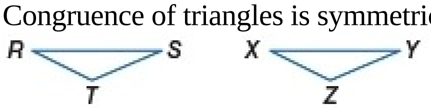
Proof:

Statements (Reasons)

1. $\angle A \cong \angle D$, $\angle B \cong \angle E$ (Given)
2. $m\angle A = m\angle D$, $m\angle B = m\angle E$ (Def. of $\cong \angle$s)
3. $m\angle A + m\angle B + m\angle C = 180$, $m\angle D + m\angle E + m\angle F = 180$ ($\angle$ Sum Theorem)
4. $m\angle A + m\angle B + m\angle C = m\angle D + m\angle E + m\angle F$ (Trans. Prop.)
5. $m\angle D + m\angle E + m\angle C = m\angle D + m\angle E + m\angle F$ (Subst.)
6. $m\angle C = m\angle F$ (Subt. Prop.)
7. $\angle C \cong \angle F$ (Def. of $\cong \angle$s)

22. **PROOF** Put the statements used to prove the statement below in the correct order. Provide the reasons for each statement.

Congruence of triangles is symmetric. (Theorem 4.4)



Given: $\Delta RST \cong \Delta XYZ$

Prove: $\Delta XYZ \cong \Delta RST$

Proof:

$\angle X \cong \angle R$, $\angle Y \cong \angle S$, $\angle Z \cong \angle T$, $\overline{XY} \cong \overline{RS}$, $\overline{YZ} \cong \overline{ST}$, $\overline{XZ} \cong \overline{RT}$

?

$\angle R \cong \angle X$, $\angle S \cong \angle Y$, $\angle T \cong \angle Z$, $\overline{RS} \cong \overline{XY}$, $\overline{ST} \cong \overline{YZ}$, $\overline{RT} \cong \overline{XZ}$

?

$\triangle RST \cong \triangle XYZ$

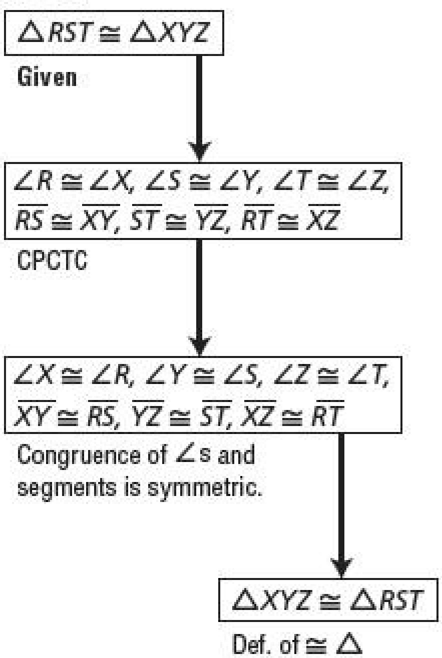
?

$\triangle XYZ \cong \triangle RST$

?

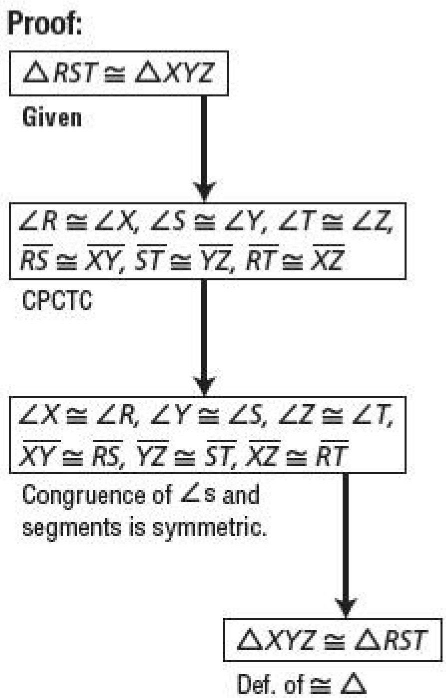
SOLUTION:

Proof:



ANSWER:

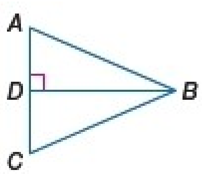
Proof:



CCSS ARGUMENTS Write a two-column proof.

23. **Given:** $\overline{BD}$ bisects $\angle B$.
$\overline{BD} \perp \overline{AC}$

Prove: $\angle A \cong \angle C$



SOLUTION:

Proof:

Statements (Reasons)

1. $\overline{BD}$ bisects $\angle B$, $\overline{BD} \perp \overline{AC}$. (Given)
2. $\angle ABD \cong \angle DBC$ (Def. of angle bisector)
3. $\angle ADB$ and $\angle BDC$ are right angles. ($\perp$ lines form rt. $\angle s$.)
4. $\angle ADB \cong \angle BDC$ (All rt. $\angle s$ are $\cong$.)
5. $\angle A \cong \angle C$ (Third $\angle$ Thm.)

ANSWER:

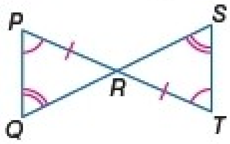
Proof:

Statements (Reasons)

1. $\overline{BD}$ bisects $\angle B$, $\overline{BD} \perp \overline{AC}$ (Given)
2. $\angle ABD \cong \angle DBC$ (Def. of angle bisector)
3. $\angle ADB$ and $\angle BDC$ are right angles. ($\perp$ lines form rt. $\angle$s.)
4. $\angle ADB \cong \angle BDC$ (All rt. $\angle$s are $\cong$.)
5. $\angle A \cong \angle C$ (Third $\angle$s Thm.)

24. **Given:** $\angle P \cong \angle T$, $\angle S \cong \angle Q$
$\overline{TR} \cong \overline{PR}$, $\overline{RP} \cong \overline{RQ}$,
$\overline{RT} \cong \overline{RS}$
$\overline{PQ} \cong \overline{TS}$

Prove: $\Delta PRQ \cong \Delta TRS$



SOLUTION:

Proof:

Statements (Reasons)

1. $\angle P \cong \angle T$, $\angle S \cong \angle Q$,$\overline{TR} \cong \overline{PR}$, $\overline{RP} \cong \overline{RQ}$, $\overline{RT} \cong \overline{RS}$, $\overline{PQ} \cong \overline{TS}$ (Given)
2. $\overline{PR} \cong \overline{QR}$, $\overline{TR} \cong \overline{SR}$ (Symm. Prop.)
3. $\overline{TR} \cong \overline{QR}$ (Trans. Prop)
4. $\overline{QR} \cong \overline{TR}$ (Symm. Prop.)
5. $\overline{QR} \cong \overline{SR}$ (Trans. Prop.)
6. $\angle PRQ \cong \angle TRS$ (Vert. $\angle s$ are $\cong$.)
7. $\Delta PRQ \cong \Delta TRS$ (Def. of $\cong$ Δ s)

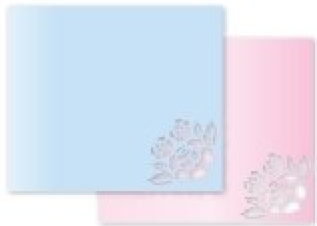
ANSWER:

Proof:

Statements (Reasons)

1. $\angle P \cong \angle T$, $\angle S \cong \angle Q$, $\overline{TR} \cong \overline{PR}$, $\overline{RP} \cong \overline{RQ}$, $\overline{RT} \cong \overline{RS}$, $\overline{PQ} \cong \overline{TS}$ (Given)
2. $\overline{PR} \cong \overline{QR}$, $\overline{TR} \cong \overline{SR}$ (Symm. Prop.)
3. $\overline{TR} \cong \overline{QR}$ (Trans. Prop)
4. $\overline{QR} \cong \overline{TR}$ (Symm. Prop.)
5. $\overline{QR} \cong \overline{SR}$ (Trans. Prop.)
6. $\angle PRQ \cong \angle TRS$ (Vert. $\angle$s are $\cong$.)
7. $\Delta PRQ \cong \Delta TRS$ (Def. of $\cong$ Δ s)

25. **SCRAPBOOKING** Lanie is using a flower-shaped corner decoration punch for a scrapbook she is working on. If she punches the corners of two pages as shown, what property guarantees that the punched designs are congruent? Explain.



SOLUTION:

Sample answer: Both of the punched flowers are congruent to the flower on the stamp, because it was used to create the images. According to the Transitive Property of Polygon Congruence, the two stamped images are congruent to each other because they are both congruent to the flowers on the punch.

ANSWER:

Sample answer: Both of the punched flowers are congruent to the flower on the stamp, because it was used to create the images. According to the Transitive Property of Polygon Congruence, the two stamped images are congruent to each other because they are both congruent to the flowers on the punch.

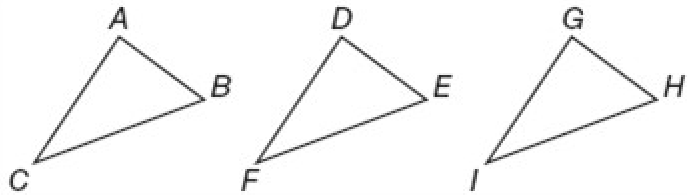
PROOF Write the specified type of proof of the indicated part of Theorem 4.4.

26. Congruence of triangles is transitive. (paragraph proof)

SOLUTION:

Given: $\Delta ABC \cong \Delta DEF$, $\Delta DEF \cong \Delta GHI$

Prove: $\Delta ABC \cong \Delta GHI$



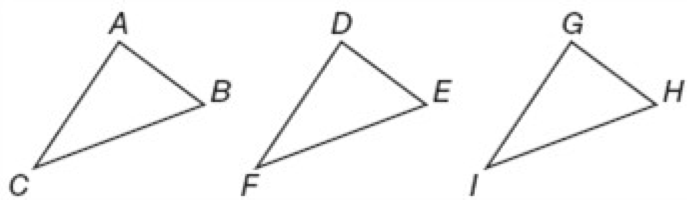
Proof:

We know that $\Delta ABC \cong \Delta DEF$. Because corresponding parts of congruent triangles are congruent, $\angle A \cong \angle D, \angle B \cong \angle E, \angle C \cong \angle F$, $\overline{AB} \cong \overline{DE}$, $\overline{BC} \cong \overline{EF}$, $\overline{AC} \cong \overline{DF}$. We also know that $\Delta DEF \cong \Delta GHI$. So $\angle D \cong \angle G, \angle E \cong \angle H, \angle F \cong \angle I$, $\overline{DE} \cong \overline{GH}$, $\overline{EF} \cong \overline{HI}$, $\overline{DF} \cong \overline{GI}$, by CPCTC. Therefore, $\angle A \cong \angle G, \angle B \cong \angle H, \angle C \cong \angle I$ $\overline{AB} \cong \overline{GH}$, $\overline{BC} \cong \overline{HI}$, $\overline{AC} \cong \overline{GI}$ because congruence of angles and segments is transitive. Thus, $\Delta ABC \cong \Delta GHI$ by the definition of congruent triangles.

ANSWER:

Given: $\boldsymbol{\Delta ABC \cong \Delta DEF}$, $\boldsymbol{\Delta DEF \cong \Delta GHI}$

Prove: $\boldsymbol{\Delta ABC \cong \Delta GHI}$



Proof:

We know that $\boldsymbol{\Delta ABC \cong \Delta DEF}$. Because corresponding parts of congruent triangles are congruent, $\angle A \cong \angle D$, $\angle B \cong \angle E$, $\angle C \cong \angle F$, $\overline{AB} \cong \overline{DE}$, $\overline{BC} \cong \overline{EF}$, $\overline{AC} \cong \overline{DF}$. We also know that $\boldsymbol{\Delta DEF \cong \Delta GHI}$. So $\angle D \cong \angle G$, $\angle E \cong \angle H$, $\angle F \cong \angle I$, $\overline{DE} \cong \overline{GH}$, $\overline{EF} \cong \overline{HI}$, $\overline{DF} \cong \overline{GI}$, by CPCTC. Therefore, $\angle A \cong \angle G$, $\angle B \cong \angle H$, $\angle C \cong \angle I$, $\overline{AB} \cong \overline{GH}$, $\overline{BC} \cong \overline{HI}$, $\overline{AC} \cong \overline{GI}$ because congruence of angles and segments is transitive. Thus, $\boldsymbol{\Delta ABC \cong \Delta GHI}$ by the definition of congruent triangles.

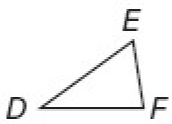
4-3 Congruent Triangles

27. Congruence of triangles is reflexive. (flow proof)

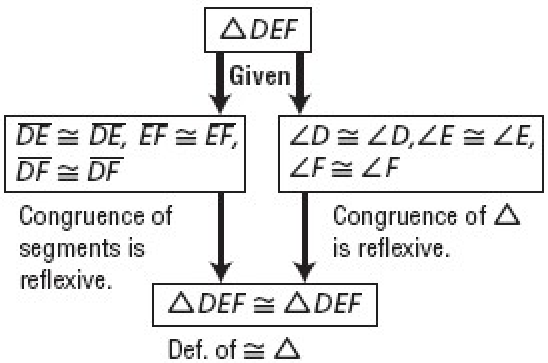
SOLUTION:

Given: ΔDEF

Prove: $\Delta DEF \cong \Delta DEF$



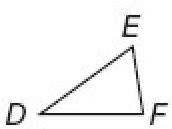
Proof:



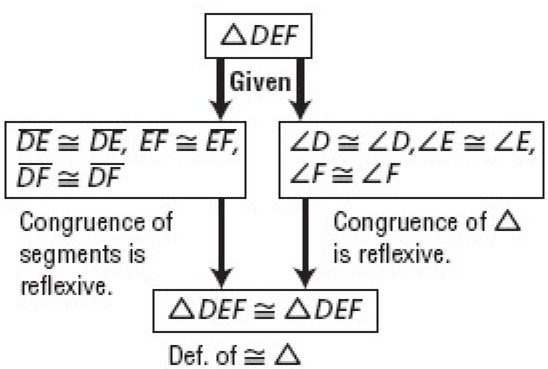
ANSWER:

Given: ΔDEF

Prove: $\Delta DEF \cong \Delta DEF$



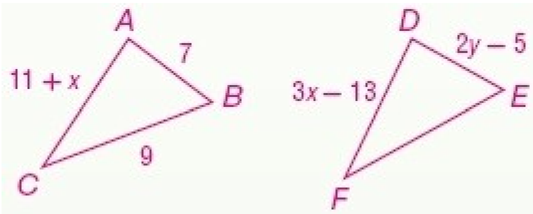
Proof:



ALGEBRA Draw and label a figure to represent the congruent triangles. Then find *x* and *y*.

28. $\Delta ABC \cong \Delta DEF$, $AB = 7$, $BC = 9$, $AC = 11 + x$, $DF = 3x - 13$, and $DE = 2y - 5$

SOLUTION:



Since the triangles are congruent, the corresponding sides are congruent.

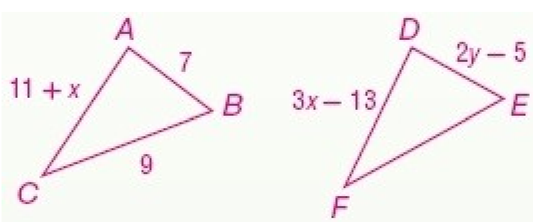
$$
\begin{aligned}
AC &= DF && \text{CPCTC.}\\
11 + x &= 3x - 13 && \text{Substitution.}\\
11 + x - 3x &= 3x - 3x - 13 && -3x \text{ from each side.}\\
11 - 2x &= -13 && \text{Simplfy.}\\
11 - 11 - 2x &= -13 - 11 && -11 \text{ from each side.}\\
-2x &= -24 && \text{Simplify.}\\
x &= 1 && \div \text{ each side by } -2.
\end{aligned}
$$

Similarly, $AB = DE$.

$$
\begin{aligned}
AB &= DE && \text{CPCTC.}\\
7 &= 2y - 5 && \text{Substitution.}\\
7 + 5 &= 2y - 5 + 5 && +5 \text{ to each side.}\\
12 &= 2y && \text{Simplfy.}\\
6 &= y && \div \text{ each side by } 2.
\end{aligned}
$$

That is, $y = 6$.

ANSWER:

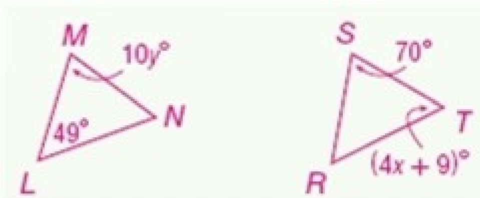


$x = 12$; $y = 6$

4-3 Congruent Triangles

29. $\Delta LMN \cong \Delta RST$, $m\angle L = 49$, $m\angle M = 10y$, $m\angle S = 70$, and $m\angle T = 4x + 9$

SOLUTION:



Since the triangles are congruent, the corresponding angles are congruent.
$\angle M \cong \angle S$ and $\angle N \cong \angle T$.
By the definition of congruence $m\angle M = m\angle S$.

Substitute.

$$\begin{aligned} m\angle M &= m\angle S && \text{Def. of congruence.} \\ 10y &= 70 && \text{Substitute.} \\ y &= 7 && \div \text{ each side by 10.} \end{aligned}$$

So, $m\angle M \cong 70$.

Use the Triangle Angle Sum Theorem in $\triangle LMN$.

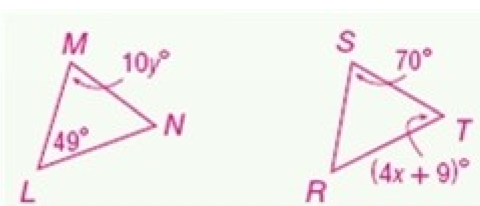
$$\begin{aligned} m\angle L + m\angle M + m\angle N &= 180 && \text{Triangle Angle-Sum Thm.} \\ 49 + 70 + m\angle N &= 180 && \text{Substitute.} \\ 119 + m\angle N &= 180 && \text{Simplify.} \\ m\angle N &= 61 && -119 \text{ from each side.} \end{aligned}$$

By the definition of congruence $m\angle N \cong m\angle T$.
Substitute.

$$\begin{aligned} m\angle N &= m\angle N && \text{Def. of congruence.} \\ 61 &= 4x + 9 && \text{Substitute.} \\ 61 - 9 &= 4x + 9 - 9 && -9 \text{ from each side.} \\ 52 &= 4x && \text{Simplify.} \\ 13 &= x && \div \text{ each side by 4.} \end{aligned}$$

That is, $x = 13$.

ANSWER:

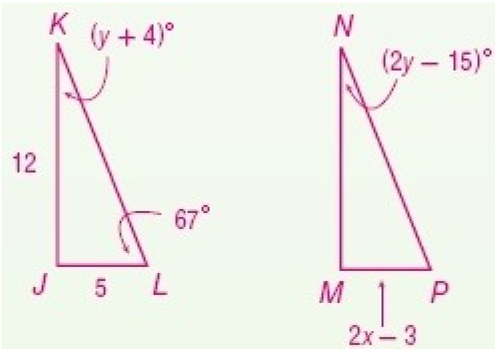


$x = 13; y = 7$

4-3 Congruent Triangles

30. $\Delta JKL \cong \Delta MNP$, $JK = 12$, $LJ = 5$, $PM = 2x - 3$, $m\angle L = 67$, $m\angle K = y + 4$ and $m\angle N = 2y - 15$

SOLUTION:

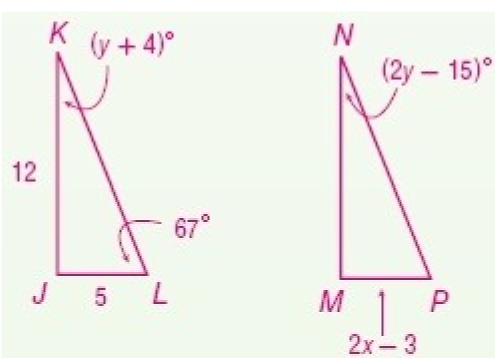


Since the triangles are congruent, the corresponding angles and corresponding sides are congruent.

$$\begin{aligned} \angle K &\cong \angle N && \text{CPCTC} \\ m\angle K &= m\angle N && \text{Def.of congruence} \\ y + 4 &= 2y - 15 && \text{Substitute.} \\ y + 4 - 2y &= 2y - 15 - 2y && -2y \text{ from each side.} \\ -y + 4 &= -15 && \text{Simplify.} \\ -y &= -19 && -4 \text{ from each side.} \\ y &= 19 && \times \text{ each side by } -1. \end{aligned}$$

$$\begin{aligned} \overline{MP} &\cong \overline{JL} && \text{CPCTC} \\ MP &= JL && \text{Def.of congruence} \\ 2x - 3 &= 5 && \text{Substitute.} \\ 2x - 3 + 3 &= 5 + 3 && +3 \text{ to each side.} \\ 2x &= 8 && \text{Simplify.} \\ x &= 4 && \div \text{ each side by } 2. \end{aligned}$$

ANSWER:



$x = 4$; $y = 19$

31. **PENNANTS** Scott is in charge of roping off an area of 100 square feet for the band to use during a pep rally. He is using a string of pennants that are congruent isosceles triangles.
Refer to the figure on page 259.
a. List seven pairs of congruent segments in the photo.
b. If the area he ropes off for the band is a square, how long will the pennant string need to be?
c. How many pennants will be on the string?

SOLUTION:

a. $\overline{AB} \cong \overline{CB}, \overline{AB} \cong \overline{DE}, \overline{AB} \cong \overline{FE}, \overline{CB} \cong \overline{DE}, \overline{CB} \cong \overline{FE}, \overline{DE} \cong \overline{FE}, \overline{AC} \cong \overline{DF}$

b. Substitute the value of area in the formula for area of a square and solve for its side.

$$A = s^2 \quad \text{Area formula}$$
$$100 = s^2 \quad \text{Area is 100}$$
$$10 = s \quad \text{take the square root}$$

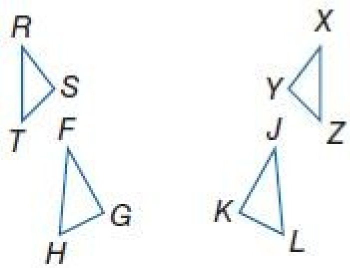
The length of each side is 10 ft. Since the perimeter of a square is 4s, the perimeter is 4(10) or 40. So 40 ft of pennants strings needed.

c. Each pennant is 4 inches wide and they are placed 2 inches apart. So, there are 2 pennants for each foot of rope. So, 40 feet × 2 pennants per foot means that 80 pennants will fit on the string.

ANSWER:

a. $\overline{AB} \cong \overline{CB}, \overline{AB} \cong \overline{DE}, \overline{AB} \cong \overline{FE}, \overline{CB} \cong \overline{DE}, \overline{CB} \cong \overline{FE}, \overline{DE} \cong \overline{FE}, \overline{AC} \cong \overline{DF}$
b. 40 ft
c. 80

32. **CCSS SENSE-MAKING** In the photo of New York City's Chrysler Building, $\overline{TS} \cong \overline{ZY}$, $\overline{XY} \cong \overline{RS}$, $\overline{TR} \cong \overline{ZX}$, $\angle X \cong \angle R$, $\angle T \cong \angle Z$, $\angle Y \cong \angle S$, and $\Delta HGF \cong \Delta LKJ$.
Refer to the figure on page 261.



a. Which triangle, if any, is congruent to ΔYXZ? Explain your reasoning.
b. Which side(s) are congruent to $\overline{JL}$? Explain your reasoning.
c. Which angle(s) are congruent to $\angle G$? Explain your reasoning.

SOLUTION:

a. ΔSRT is congruent to ΔYXZ. The corresponding parts of the triangles are congruent, therefore the triangles are congruent.

b. $\overline{FH}$ is congruent to $\overline{JL}$. We are given that $\Delta HGF \cong \Delta LKJ$, and $\overline{JL}$ corresponds with $\overline{FH}$. Since corresponding parts of congruent triangles are congruent, $\overline{JL} \cong \overline{FH}$.

c. $\angle K$ is congruent to $\angle G$. We are given that $\Delta HGF \cong \Delta LKJ$, and $\angle G$ corresponds with $\angle K$. Since corresponding parts of congruent triangles are congruent, $\angle G \cong \angle K$.

ANSWER:

a. ΔSRT The corresponding parts of the triangles are congruent, therefore the triangles are congruent.
b. $\overline{FH}$ We are given that $\Delta HGF \cong \Delta LKJ$ and $\overline{JL}$ corresponds with $\overline{FH}$ Since corresponding parts of congruent triangles are congruent, $\overline{JL} \cong \overline{FH}$.
c. $\angle K$; we are given that $\Delta HGF \cong \Delta LKJ$ and $\angle G$ corresponds with $\angle K$. Since corresponding parts of congruent triangles are congruent, $\angle G \cong \angle K$.

33. **MULTIPLE REPRESENTATIONS** In this problem, you will explore the following statement.
The areas of congruent triangles are equal.
a. VERBAL Write a conditional statement to represent the relationship between the areas of a pair of congruent triangles.
b. VERBAL Write the converse of your conditional statement. Is the converse *true* or *false*? Explain your reasoning.
c. GEOMETRIC If possible, draw two equilateral triangles that have the same area but are not congruent. If not possible, explain why not.
d. GEOMETRIC If possible, draw two rectangles that have the same area but are not congruent. If not possible, explain why not.
e. GEOMETRIC If possible, draw two squares that have the same area but are not congruent. If not possible, explain why not.
f. VERBAL For which polygons will the following conditional and its converse both be true? Explain your reasoning.
If a pair of ___________ are congruent, then they have the same area.

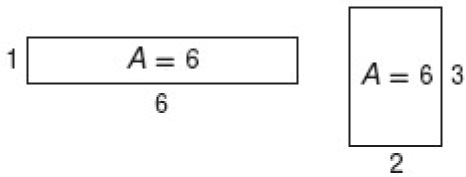
SOLUTION:

a. If two triangles are congruent, then their areas are equal.

b. If the areas of a pair of triangles are equal, then the triangles are congruent; false; If one triangle has a base of 2 and a height of 6 and a second triangle has a base of 3 and a height of 4, then their areas are equal, but they are not congruent.

c. No; sample answer: Any pair of equilateral triangles that have the same base also have the same height, so it is not possible to draw a pair of equilateral triangles with the same area that are not congruent.

d. yes; sample answer:



e. No; any pair of squares that have the same area have the same side length, which is the square root of the area. If their areas are equal, they are congruent.

f. Regular *n*-gons; If two regular *n*-gons are congruent, then they have the same area. All regular *n*-gons have the same shape, but may have different sizes. If two regular *n*-gons have the same area, then they not only have the same shape but also the same size. Therefore, they are congruent.

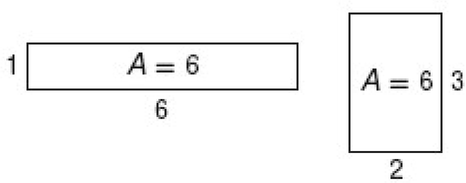
ANSWER:

a. If two triangles are congruent, then their areas are equal.

b. If the areas of a pair of triangles are equal, then the triangles are congruent; false; If one triangle has a base of 2 and a height of 6 and a second triangle has a base of 3 and a height of 4, then their areas are equal, but they are not congruent.

c. No; sample answer: Any pair of equilateral triangles that have the same base also have the same height, so it is not possible to draw a pair of equilateral triangles with the same area that are not congruent.

d. yes; sample answer:

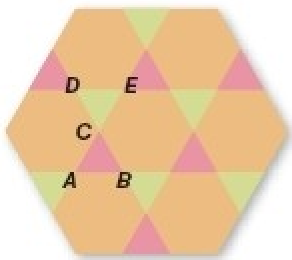


e. No; any pair of squares that have the same area have the same side length, which is the square root of the area. If their areas are equal, they are congruent.

f. Regular *n*-gons; If two regular *n*-gons are congruent, then they have the same area. All regular *n*-gons have the same shape, but may have different sizes. If two regular *n*-gons have the same area, then they not only have the same shape but also the same size. Therefore, they are congruent.

34. **PATTERNS** The pattern shown is created using regular polygons.

a. What two polygons are used to create the pattern?
b. Name a pair of congruent triangles.
c. Name a pair of corresponding angles.
d. If CB = 2 inches, what is AE? Explain.
e. What is the measure of $\angle D$? Explain.



SOLUTION:

a. Hexagons and triangles are used to create the pattern.
b. Sample answer: $\Delta ABC \cong \Delta DEC$
c. Sample answer: $\angle B$ and $\angle E$ are corresponding angles.
d. AE is 4 in. Sample answer: Because the polygons that make the pattern are regular, all of the sides of the triangles must be equal, so the triangles are equilateral. That means that CB is equal to AC and CE, so AE is 2(CB), or 4 inches.
e. $m\angle D$ is 60. Sample answer: Because the triangles are regular, they must be equilateral, and all of the angles of an equilateral triangle are 60.

ANSWER:

a. hexagons and triangles
b. Sample answer: $\Delta ABC \cong \Delta DEC$
c. Sample answer: $\angle B$ and $\angle E$
d. 4 in.; Sample answer: Because the polygons that make the pattern are regular, all of the sides of the triangles must be equal, so the triangles are equilateral. That means that CB is equal to AC and CE, so AE is 2(CB), or 4 inches.
e. 60°; Sample answer: Because the triangles are regular, they must be equilateral, and all of the angles of an equilateral triangle are 60°.

35. **FITNESS** A fitness instructor is starting a new aerobics class using fitness hoops. She wants to confirm that all of the hoops are the same size. What measure(s) can she use to prove that all of the hoops are congruent? Explain your reasoning.

SOLUTION:

To prove that all of the hoops are congruent, use the diameter, radius, or circumference. Sample answer: Two circles are the same size if they have the same diameter, radius, or circumference, so she can determine if the hoops are congruent if she measures any of them.

ANSWER:

diameter, radius, or circumference; Sample answer: Two circles are the same size if they have the same diameter, radius, or circumference, so she can determine if the hoops are congruent if she measures any of them.

36. **WRITING IN MATH** Explain why the order of the vertices is important when naming congruent triangles. Give an example to support your answer.

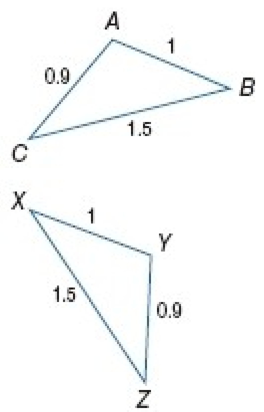
SOLUTION:

Sample answer: When naming congruent triangles, it is important that the corresponding vertices be in the same location for both triangles because the location indicates congruence. For example if ΔABC is congruent to ΔDEF, then $\angle A \cong \angle D$, $\angle B \cong \angle E$, and $\angle C \cong \angle F$.

ANSWER:

Sample answer: When naming congruent triangles, it is important that the corresponding vertices be in the same location for both triangles because the location indicates congruence. For example if ΔABC is congruent to ΔDEF, then $\angle A \cong \angle D$, $\angle B \cong \angle E$, and $\angle C \cong \angle F$.

37. **ERROR ANALYSIS** Jasmine and Will are evaluating the congruent figures below. Jasmine says that $\Delta CAB \cong \Delta ZYX$ and Will says that $\Delta ABC \cong \Delta YXZ$. Is either of them correct? Explain.



SOLUTION:

Both; Sample answer: $\angle A$ corresponds with $\angle Y$, $\angle B$ corresponds with $\angle X$, and $\angle C$ corresponds with $\angle Z$. ΔCAB is the same triangle as ΔABC and ΔZXY is the same triangle as ΔXYZ.

ANSWER:

Both; Sample answer: $\angle A$ corresponds with $\angle Y$, $\angle B$ corresponds with $\angle X$, and $\angle C$ corresponds with $\angle Z$. ΔCAB is the same triangle as ΔABC and ΔZXY is the same triangle as ΔXYZ.

38. **WRITE A QUESTION** A classmate is using the Third Angles Theorem to show that if 2 corresponding pairs of the angles of two triangles are congruent, then the third pair is also congruent. Write a question to help him decide if he can use the same strategy for quadrilaterals.

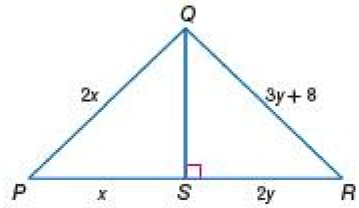
SOLUTION:

Sample answer: Do you think that the sum of the angles of a quadrilateral is constant? If so, do you think that the final pair of corresponding angles will be congruent if three other pairs of corresponding angles are congruent for a pair of quadrilaterals?

ANSWER:

Sample answer: Do you think that the sum of the angles of a quadrilateral is constant? If so, do you think that the final pair of corresponding angles will be congruent if three other pairs of corresponding angles are congruent for a pair of quadrilaterals?

39. **CHALLENGE** Find x and y if $\Delta PQS \cong \Delta RQS$.



SOLUTION:

If two triangles are congruent, then their corresponding sides are congruent.

$PQ = QR$

$PS = SR$

Substitute.

$PG = QR$ CPCTC

$2x = 3y + 8$ Substitute.

$PS = SR$ CPCTC

$x = 2y$ Substitute.

Substitute $x = 2y$ in $2x = 3y + 8$.

$2x = 3y + 8$ CPCTC

$2(2y) = 3y + 8$ Substitute.

$4y = 3y + 8$ Multiply.

$4y - 3y = 3y + 8 - 3y$ $-3y$ from each side.

$y = 8$ Simplify.

Substitute $y = 8$ in $x = 2y$.

$x = 2y$

$x = 2(8)$

$x = 16$

ANSWER:

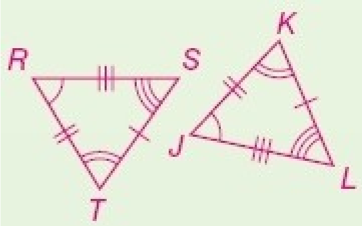
$x = 16, y = 8$

CCSS ARGUMENTS Determine whether each statement is *true* or *false*. If false, give a counterexample. If true, explain your reasoning.

40. Two triangles with two pairs of congruent corresponding angles and three pairs of congruent corresponding sides are congruent.

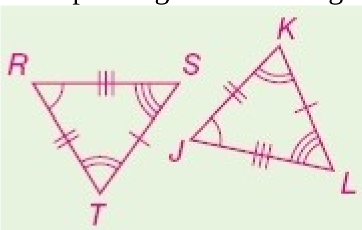
SOLUTION:

True; Sample answer: Using the Third Angles Theorem, the third pair of angles is also congruent and all corresponding sides are congruent, so since CPCTC, the triangles are congruent.



ANSWER:

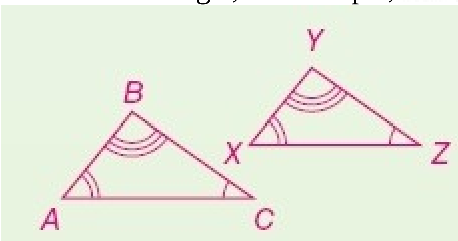
True; Sample answer: Using the Third Angles Theorem, the third pair of angles is also congruent and all corresponding sides are congruent, so since CPCTC, the triangles are congruent.



41. Two triangles with three pairs of corresponding congruent angles are congruent.

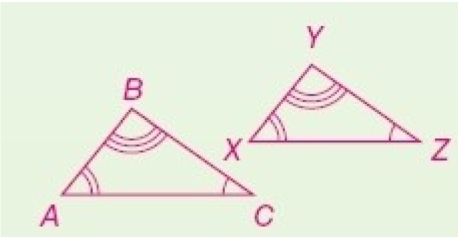
SOLUTION:

False; a pair of triangles can have corresponding angles congruent with the sides of one triangle longer than the sides of the other triangle; for example; $\angle A \cong \angle X$, $\angle B \cong \angle Y$, $\angle C \cong \angle Z$, but corresponding sides are not congruent.

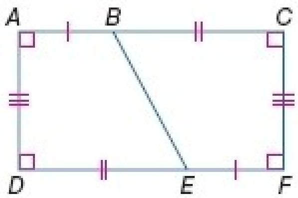


ANSWER:

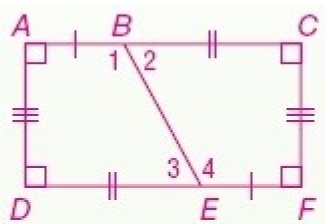
False; $\angle A \cong \angle X$, $\angle B \cong \angle Y$, $\angle C \cong \angle Z$, but corresponding sides are not congruent.



42. **CHALLENGE** Write a paragraph proof to prove polygon *ABED* ≅ polygon *FEBC*.

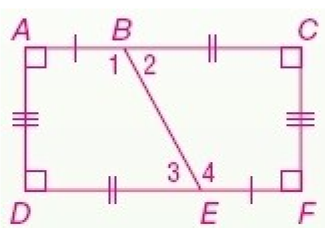


SOLUTION:



We know that $\overline{AB} \cong \overline{FE}$, $\overline{ED} \cong \overline{BC}$, and $\overline{AD} \cong \overline{FC}$. By the reflexive property, $\overline{BE} \cong \overline{EB}$. $\angle A \cong \angle F$ and $\angle D \cong \angle C$ since all right angles are congruent. Since $\overline{AC}$ and $\overline{DF}$ are both perpendicular to $\overline{CF}$, $\overline{AC} \parallel \overline{DF}$ (Theorem 3.8). $\angle 1 \cong \angle 4$ and $\angle 2 \cong \angle 3$ because alternate interior angles are congruent to each other. Since all corresponding parts are congruent, polygon *ABED* ≅ polygon *FEBC*.

ANSWER:



We know that $\overline{AB} \cong \overline{FE}$, $\overline{ED} \cong \overline{BC}$, and $\overline{AD} \cong \overline{FC}$. By the reflexive property, $\overline{BE} \cong \overline{EB}$. $\angle A \cong \angle F$ and $\angle D \cong \angle C$ since all right angles are congruent. Since $\overline{AC}$ and $\overline{DF}$ are both perpendicular to $\overline{CF}$, $\overline{AC} \parallel \overline{DF}$ (Theorem 3.8). $\angle 1 \cong \angle 4$ and $\angle 2 \cong \angle 3$ because alternate interior angles are congruent to each other. Since all corresponding parts are congruent, polygon *ABED* ≅ polygon *FEBC*.

43. **WRITING IN MATH** Determine whether the following statement is *always, sometimes,* or *never* true. Explain your reasoning.
Equilateral triangles are congruent.

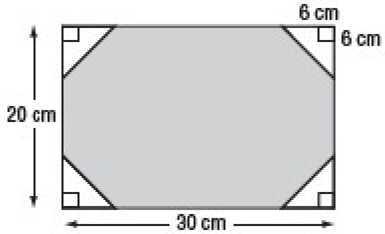
SOLUTION:

Sometimes; while equilateral triangles are equiangular, the corresponding sides may not be congruent. Equilateral triangles will be congruent if one pair of corresponding sides are congruent.

ANSWER:

Sometimes; Equilateral triangles will be congruent if one pair of corresponding sides are congruent.

44. Barrington cut four congruent triangles off the corners of a rectangle to make an octagon as shown below. What is the area of the octagon?



A 456 cm^2

B 528 cm^2

C 552 cm^2

D 564 cm^2

SOLUTION:

First find the area of the rectangle. Then find the area of one triangle that was cut from the rectangle. Subtracting the areas of the 4 triangles from the area of the rectangle will give the area of the octagon.

$$A = 30(20) \quad \text{Area of Rectangle}$$
$$= 600\text{cm}^2 \quad \text{Simplify.}$$

$$A = \frac{1}{2}(6)(6) \quad \text{Area of one triangle}$$
$$= 18\text{cm}^2 \quad \text{Simplify.}$$

The area of four congruent triangle is 4(18) or 72 cm^2, since four triangles are congruent.

$$A_{\text{octagon}} = A_{\text{rectangle}} - A_{\text{4triangles}}$$
$$= 600 - 72 \quad \text{Substitution.}$$
$$= 528\text{cm}^2 \quad \text{Subtraction.}$$

So, the correct choice is B.

ANSWER:

B

45. **GRIDDED RESPONSE** Triangle *ABC* is congruent to ΔHIJ. The vertices of ΔABC are *A*(–1, 2), *B*(0, 3) and *C* (2, –2). What is the measure of side $\overline{HJ}$?

SOLUTION:

Triangle *ABC* is congruent to ΔHIJ. Since the corresponding parts are congruent, $\overline{AC} \cong \overline{FJ}$.
$\overline{AC}$ has end points *A*(–1, 2) and *C*(2, –2).
$AC = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

Substitute.

$$\begin{aligned} AC &= \sqrt{(2-(-1))^2 + (-2-2)^2} && \text{Substitute.} \\ &= \sqrt{(3)^2 + (-4)^2} && \text{Subtraction.} \\ &= \sqrt{9+16} && \text{Square terms.} \\ &= \sqrt{25} && \text{Addition.} \\ &= 5 && \text{Take the square root.} \end{aligned}$$

ANSWER:
5

46. **ALGEBRA** Which is a factor of $x^2 + 19x - 42$?
F $x + 14$
G $x + 2$
H $x - 2$
J $x - 14$

SOLUTION:
Find the factors of –42 that have a sum of 19.

Factors of -42	Sum
1, –42	–40
–2, 21	19
2, –21	–19
–6, 7	1

$$\begin{aligned} x^2 + 19x - 42 &= (x+m)(x+p) && \text{Write the pattern.} \\ &= (x-2)(x+21) && m = -2, p = 21 \end{aligned}$$

So, the correct choice is H.

ANSWER:
H

47. **SAT/ACT** Mitsu travels a certain distance at 30 miles per hour and returns the same route at 65 miles per hour. What is his average speed in miles per hour for the round trip?
A 32.5
B 35.0
C 41.0
D 47.5
E 55.3

SOLUTION:
To find the average speed in mph, we need to know the total distance traveled and the total time it took to travel it. Since we are not given a specific distance, assume that he travels 390 miles (390 is the LCM of 30 and 65). Then the time for onward journey is $\frac{390}{30}$ or 13 hours and the time for the return journey is $\frac{390}{65}$ or 6 hours.

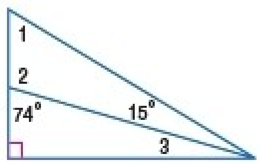
So, he traveled 780 miles in 19 hours.

$$\begin{aligned}\text{Average speed} &= \frac{\text{Total Distance}}{\text{Total Time}} \\ &= \frac{780}{19} && \text{Substitution.} \\ &\approx 41.05 && \text{Division.}\end{aligned}$$

The correct choice is C.

ANSWER:
C

Find each measure.



48. $m\angle 2$

SOLUTION:
Here, $m\angle 2 + 74 = 180$.
Solve.

$$\begin{aligned}m\angle 2 + 74 &= 180 && \text{Def. of Linear pair} \\ m\angle 2 + 74 - 74 &= 180 - 74 && -74 \text{ from each side.} \\ m\angle 2 &= 106 && \text{Simplify.}\end{aligned}$$

ANSWER:
106

49. $m\angle 1$

SOLUTION:

There are two different ways to find $m\angle 1$.

Method 1 Exterior Angles

By the Exterior Angle Theorem,

$$\begin{aligned} m\angle 1 + 15 &= 74 && \text{Exterior Angle Thm.} \\ m\angle 1 &= 59 && -15 \text{ from each side.} \end{aligned}$$

Method 2 Triangle Angle-Sum

In the figure, $m\angle 1 + m\angle 2 + 15 = 180$.

Here, $m\angle 2 + 74 = 180$.

Solve.

$$\begin{aligned} m\angle 2 + 74 &= 180 && \text{Def. of Linear Pair} \\ m\angle 2 + 74 - 74 &= 180 - 74 && -72 \text{ from each side} \\ m\angle 2 &= 106 && \text{Simplify.} \end{aligned}$$

Substitute.

$$\begin{aligned} m\angle 1 + m\angle 2 + 15 &= 180 && \text{Triangle Angle-Sum Thm.} \\ m\angle 1 + 106 + 15 &= 180 && \text{Substitute.} \\ m\angle 1 + 121 &= 180 && \text{Simplify.} \\ m\angle 1 + 121 - 121 &= 180 - 121 && -121 \text{ from each side.} \\ m\angle 1 &= 59 && \text{Simplify.} \end{aligned}$$

ANSWER:

59

50. $m\angle 3$

SOLUTION:

In the figure, $90 + 74 + m\angle 3 = 180$.

$$\begin{aligned} 90 + 74 + m\angle 3 &= 180 && \text{Triangle Angle-Sum Thm.} \\ 164 + m\angle 3 &= 180 && \text{Simplify.} \\ 164 + m\angle 3 - 164 &= 180 - 164 && -164 \text{ from each side.} \\ m\angle 3 &= 16 && \text{Simplify.} \end{aligned}$$

ANSWER:

16

COORDINATE GEOMETRY Find the measures of the sides of ΔJKL and classify each triangle by the measures of its sides.

51. $J(-7, 10)$, $K(15, 0)$, $L(-2, -1)$

SOLUTION:

Use the Distance Formula to find the lengths of $\overline{JK}, \overline{KL}$ and $\overline{LJ}$.

$\overline{JK}$ has end points $J(-7, 10)$ and $K(15, 0)$.

$JK = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

Substitute.

$$\begin{aligned} JK &= \sqrt{(15-(-7))^2 + (0-10)^2} && \text{Substitute.} \\ &= \sqrt{(22)^2 + (-10)^2} && \text{Subtraction.} \\ &= \sqrt{484 + 100} && \text{Square terms.} \\ &= \sqrt{584} && \text{Addition.} \\ &= 2\sqrt{146} && \text{Take the square root.} \end{aligned}$$

$\overline{KL}$ has end points $K(15, 0)$ and $L(-2, -1)$.

$KL = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

Substitute.

$$\begin{aligned} KL &= \sqrt{(-2-15)^2 + (-1-0)^2} && \text{Substitution.} \\ &= \sqrt{(-17)^2 + (-1)^2} && \text{Subtraction.} \\ &= \sqrt{289 + 1} && \text{Sqaure terms.} \\ &= \sqrt{290} && \text{Addition.} \end{aligned}$$

$\overline{LJ}$ has end points $L(-2, -1)$ and $J(-7,10)$.

$LJ = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

Substitute.

$$\begin{aligned} LJ &= \sqrt{(-7-(-2))^2 + (10-(-1))^2} && \text{Substitute.} \\ &= \sqrt{(-5)^2 + (11)^2} && \text{Subtraction.} \\ &= \sqrt{25 + 121} && \text{Square terms.} \\ &= \sqrt{146} && \text{Addition.} \end{aligned}$$

No two sides are congruent. So, it is scalene.

ANSWER:

$JK = 2\sqrt{146}$, $KL = \sqrt{290}$, $JL = \sqrt{146}$; scalene

52. $J(9, 9)$, $K(12, 14)$, $L(14, 6)$

SOLUTION:

Use the Distance Formula to find the lengths of $\overline{JK}, \overline{KL}$ and $\overline{LJ}$.

$\overline{JK}$ has end points $J(9, 9)$ and $K(12, 14)$.

$JK = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

Substitute.

$$\begin{aligned} JK &= \sqrt{(12-9)^2 + (14-9)^2} && \text{Substitute.} \\ &= \sqrt{(3)^2 + (5)^2} && \text{Subtraction.} \\ &= \sqrt{9+25} && \text{Square terms.} \\ &= \sqrt{34} && \text{Addition.} \end{aligned}$$

$\overline{KL}$ has end points $K(12, 14)$ and $L(14, 6)$.

$KL = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

Substitute.

$$\begin{aligned} KL &= \sqrt{(14-12)^2 + (6-14)^2} && \text{Substitute.} \\ &= \sqrt{(2)^2 + (-8)^2} && \text{Subtraction.} \\ &= \sqrt{4+64} && \text{Square terms.} \\ &= \sqrt{68} && \text{Addition.} \\ &= 2\sqrt{17} && \text{Simplify.} \end{aligned}$$

$\overline{LJ}$ has end points $L(14, 6)$ and $J(9,9)$.

$LJ = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

Substitute.

$$\begin{aligned} LJ &= \sqrt{(9-14)^2 + (9-6)^2} && \text{Substitute.} \\ &= \sqrt{(-5)^2 + (3)^2} && \text{Subtraction.} \\ &= \sqrt{25+9} && \text{Square terms.} \\ &= \sqrt{34} && \text{Addition.} \end{aligned}$$

Here, $JK = LJ$. This triangle has two congruent sides. So, it is isosceles.

ANSWER:

$JK = \sqrt{34}$, $KL = 2\sqrt{17}$, $JL = \sqrt{34}$; isosceles

53. $J(4, 6)$, $K(4, 11)$, $L(9, 6)$

SOLUTION:

Use the Distance Formula to find the lengths of $\overline{JK}, \overline{KL}$ and $\overline{LJ}$.

$\overline{JK}$ has end points *J*(4, 6) and *K*(4, 11).

$$JK = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Substitute.

$$\begin{aligned} JK &= \sqrt{(4-4)^2 + (11-6)^2} && \text{Substitute.} \\ &= \sqrt{(0)^2 + (5)^2} && \text{Subtraction.} \\ &= \sqrt{0+25} && \text{Square terms.} \\ &= \sqrt{25} && \text{Addition.} \\ &= 5 && \text{Simplify.} \end{aligned}$$

$\overline{KL}$ has end points *K*(4, 11) and *L*(9, 6).

$$KL = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Substitute.

$$\begin{aligned} KL &= \sqrt{(9-4)^2 + (6-11)^2} && \text{Substitute.} \\ &= \sqrt{(5)^2 + (-5)^2} && \text{Subtraction.} \\ &= \sqrt{25+25} && \text{Square terms.} \\ &= \sqrt{50} && \text{Addition.} \\ &= 5\sqrt{2} && \text{Simplify.} \end{aligned}$$

$\overline{LJ}$ has end points *L*(9, 6) and *J*(4,6).

$$LJ = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Substitute.

$$\begin{aligned} LJ &= \sqrt{(4-9)^2 + (6-6)^2} && \text{Substitute.} \\ &= \sqrt{(-5)^2 + (0)^2} && \text{Subtraction.} \\ &= \sqrt{25+0} && \text{Square terms.} \\ &= \sqrt{25} && \text{Addition.} \\ &= 5 && \text{Simplify.} \end{aligned}$$

Here, *JK* = *LJ*. This triangle has two congruent sides. So, it is isosceles.

ANSWER:

JK = 5, *KL* = $5\sqrt{2}$, *JL* = 5; isosceles

54. $J(16, 14)$, $K(7, 6)$, $L(-5, -14)$

SOLUTION:

Use the Distance Formula to find the lengths of $\overline{JK}, \overline{KL}$ and $\overline{LJ}$.

$\overline{JK}$ has end points $J(16, 14)$ and $K(7, 6)$.

$$JK = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Substitute.

$$\begin{aligned} JK &= \sqrt{(7-16)^2 + (6-14)^2} && \text{Substitute.} \\ &= \sqrt{(-9)^2 + (-8)^2} && \text{Subtraction.} \\ &= \sqrt{81 + 64} && \text{Square terms.} \\ &= \sqrt{145} && \text{Addition.} \end{aligned}$$

$\overline{KL}$ has end points $K(7, 6)$ and $L(-5, -14)$.

$$KL = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Substitute.

$$\begin{aligned} KL &= \sqrt{(-5-7)^2 + (-14-6)^2} && \text{Substitute.} \\ &= \sqrt{(-12)^2 + (-20)^2} && \text{Subtraction.} \\ &= \sqrt{144 + 400} && \text{Square terms.} \\ &= \sqrt{544} && \text{Addition.} \\ &= 4\sqrt{34} && \text{Simplify.} \end{aligned}$$

$\overline{LJ}$ has end points $L(-5, -14)$ and $J(16,14)$.

$$LJ = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Substitute.

$$\begin{aligned} LJ &= \sqrt{(16-(-5))^2 + (14-(-14))^2} && \text{Substitute.} \\ &= \sqrt{(21)^2 + (28)^2} && \text{Subtraction.} \\ &= \sqrt{441 + 784} && \text{Square terms.} \\ &= \sqrt{1225} && \text{Addition.} \\ &= 35 && \text{Simplify.} \end{aligned}$$

No two sides are congruent. So, it is scalene.

ANSWER:

$JK = \sqrt{145}$, $KL = 4\sqrt{34}$, $JL = 35$; scalene

Determine whether each statement is *always, sometimes,* or *never* true.

55. Two angles that form a linear pair are supplementary.

SOLUTION:

Always; a linear pair is a pair of adjacent angles with noncommon sides of opposite rays.

ANSWER:

always

56. If two angles are supplementary, then one of the angles is obtuse.

SOLUTION:

Sometimes; two angles that each measure 90 are supplementary and neither angle is obtuse.

ANSWER:

sometimes

57. **CARPENTRY** A carpenter must cut two pieces of wood at angles so that they fit together to form the corner of a picture frame. What type of angles must he use to make sure that a 90° corner results?

SOLUTION:

The corner of a picture frame is 90 degrees. Complementary angles are two angles with a sum of 90 degrees, so the carpenter should use complementary angles.

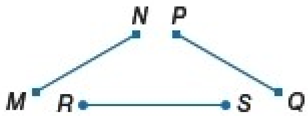
ANSWER:

complementary angles

58. Copy and complete the proof.

Given: $\overline{MN} \cong \overline{PQ}$, $\overline{PQ} \cong \overline{RS}$

Prove: $\overline{MN} \cong \overline{RS}$



Proof:

Statements	Reasons
a. ?	**a.** Given
b. $MN = PQ$, $PQ = RS$	**b.** ?
c. ?	**c.** ?
d. $\overline{MN} \cong \overline{RS}$	**d.** Definition of congruent segments

SOLUTION:

Statements	Reasons
a. $\overline{MN} \cong \overline{PQ}$, $\overline{PQ} \cong \overline{RS}$	**a.** Given
b. $MN = PQ$, $PQ = RS$	**b. Def. $\cong$ segments**
c. $MN = RS$	**c. Transitive Prop. (=)**
d. $\overline{MN} \cong \overline{RS}$	**d.** Definition of congruent segments

ANSWER:

Statements	Reasons
a. $\overline{MN} \cong \overline{PQ}$, $\overline{PQ} \cong \overline{RS}$	**a.** Given
b. $MN = PQ$, $PQ = RS$	**b. Def. $\cong$ segments**
c. $MN = RS$	**c. Transitive Prop. (=)**
d. $\overline{MN} \cong \overline{RS}$	**d.** Definition of congruent segments