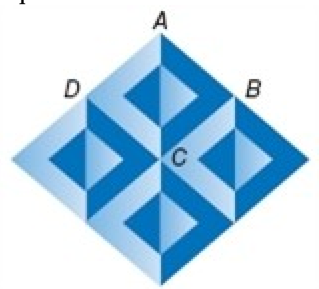
4-4 Proving Triangles Congruent-SSS, SAS

1. **OPTICAL ILLUSION** The figure below is a pattern formed using four large congruent squares and four small congruent squares.



a. How many different-sized triangles are used to create the illusion?
b. Use the Side-Side-Side Congruence Postulate to prove that $\Delta ABC \cong \Delta CDA$.
c. What is the relationship between $\overleftrightarrow{AB}$ and $\overleftrightarrow{CD}$? Explain your reasoning.

SOLUTION:

a. There are two differently sized triangles in the figure: ΔABC and the smaller triangle in the interior of ΔABC.

b. Given: $ABCD$ is a square
Prove: $\Delta ABC \cong \Delta CDA$
Proof:
Statements (Reasons)
1. $ABCD$ is a square (Given)
2. $\overline{AB} \cong \overline{CD}, \overline{BC} \cong \overline{DA}$ (Def. of a square)
3. $\overline{AC} \cong \overline{CA}$ (Reflex. Prop. $\cong$)
4. $\Delta ABC \cong \Delta CDA$. (SSS)

c. Sample answer: $\overleftrightarrow{AB} \parallel \overleftrightarrow{CD}$; $\overleftrightarrow{AC}$ is a transversal to $\overleftrightarrow{AB}$ and $\overleftrightarrow{CD}$, so $\angle CAB$ and $\angle ACD$ are alternate interior angles. Since $\Delta ABC \cong \Delta CDA$, $\angle CAB$ and $\angle ACD$ are congruent corresponding angles. Therefore, the lines are parallel

ANSWER:

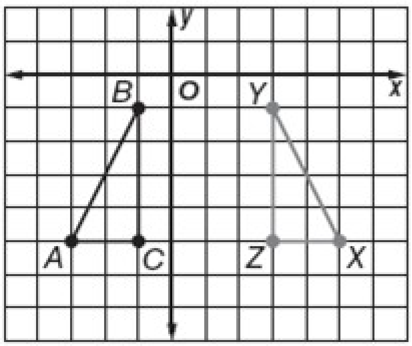
a. two
b. Given: $ABCD$ is a square
Prove: $\Delta ABC \cong \Delta CDA$
Proof:
Statements (Reasons)
1. $ABCD$ is a square (Given)
2. $\overline{AB} \cong \overline{CD}, \overline{BC} \cong \overline{DA}$ (Def. of a square)
3. $\overline{AC} \cong \overline{CA}$ (Reflex. Prop. $\cong$)
4. $\Delta ABC \cong \Delta CDA$. (SSS)

c. Sample answer: $\overleftrightarrow{AB} \parallel \overleftrightarrow{CD}$; $\overleftrightarrow{AC}$ is a transversal to $\overleftrightarrow{AB}$ and $\overleftrightarrow{CD}$, so $\angle CAB$ and $\angle ACD$ are alternate interior angles. Since $\Delta ABC \cong \Delta CDA$, $\angle CAB$ and $\angle ACD$ are congruent corresponding angles. Therefore, the lines are parallel

2. **EXTENDED RESPONSE** Triangle *ABC* has vertices *A*(–3, –5), *B*(–1, –1), and *C*(–1, –5). Triangle *XYZ* has vertices *X*(5, –5), *Y*(3, –1), and *Z*(3, –5).
a. Graph both triangles on the same coordinate plane.
b. Use your graph to make a conjecture as to whether the triangles are congruent. Explain your reasoning.
c. Write a logical argument using coordinate geometry to support your conjecture.

SOLUTION:

a.



b. The triangles look the same size and shape so we can conjecture that they are congruent.

c. Observe the graph, the triangles are right triangles. In triangle *ABC*, *AC* = 2 and *BC* = 4. Use the Pythagorean Theorem to find *AB*.

$$AB^2 = AC^2 + BC^2$$

$$AB = \sqrt{AC^2 + BC^2} \quad \text{Take the square root.}$$

$$= \sqrt{2^2 + 4^2} \quad \text{Substitute.}$$

$$= \sqrt{4 + 16} \quad \text{Square terms.}$$

$$= \sqrt{20} \quad \text{Addition.}$$

Similarly In triangle *XYZ*, *ZX* = 2 and *ZY* = 4. Use the Pythagorean Theorem to find *XY*.

$$XY^2 = XZ^2 + ZY^2$$

$$XY = \sqrt{XZ^2 + ZY^2} \quad \text{Take the square root.}$$

$$= \sqrt{2^2 + 4^2} \quad \text{Substitute.}$$

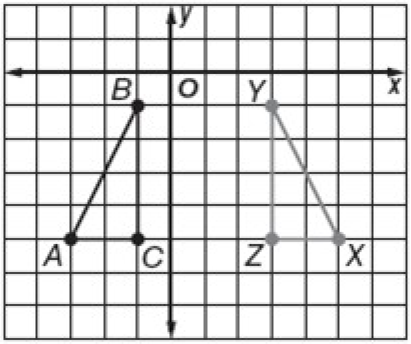
$$= \sqrt{4 + 16} \quad \text{Square terms.}$$

$$= \sqrt{20} \quad \text{Addition.}$$

The corresponding sides have the same measure and are congruent. So, $\Delta ABC \cong \Delta XYZ$ by SSS.

ANSWER:

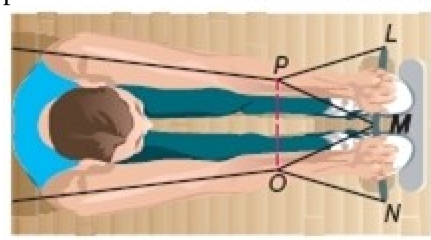
a.



b. The triangles look the same size and shape so we can conjecture that they are congruent.

c. $AB = \sqrt{20}$, $XY = \sqrt{20}$, $BC = 4$, $YZ = 4$, $AC = 2$, and $XZ = 2$. The corresponding sides have the same measure and are congruent. So, $\Delta ABC \cong \Delta XYZ$ by SSS.

3. **EXERCISE** In the exercise diagram, if $\overline{LP} \cong \overline{NO}$, $\angle LPM \cong \angle NOM$, and ΔMOP is equilateral, write a paragraph proof to show that $\Delta LMP \cong \Delta NMO$.



SOLUTION:

Sample answer: We are given that $\overline{LP} \cong \overline{NO}$ and $\angle LPM \cong \angle NOM$. Since ΔMOP is equilateral, $\overline{MO} \cong \overline{MP}$ by the definition of an equilateral triangle. Therefore, ΔLMP is congruent to ΔNMO by the Side-Angle-Side Congruent Postulate.

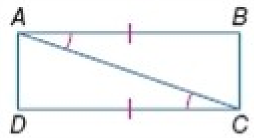
ANSWER:

Sample answer: We are given that $\overline{LP} \cong \overline{NO}$ and $\angle LPM \cong \angle NOM$. Since ΔMOP is equilateral, $\overline{MO} \cong \overline{MP}$ by the definition of an equilateral triangle. Therefore, ΔLMP is congruent to ΔNMO by the Side-Angle-Side Congruent Postulate.

Write a two-column proof.

4. **Given:** $\overline{BA} \cong \overline{DC}$, $\angle BAC \cong \angle DCA$

Prove: $\overline{BC} \cong \overline{DA}$



SOLUTION:

Proof:

Statements (Reasons)

1. $\overline{BA} \cong \overline{DC}$, $\angle BAC \cong \angle DCA$ (Given)
2. $\overline{AC} \cong \overline{CA}$ (Reflex. Prop. $\cong$)
3. $\Delta BCA \cong \Delta DAC$ (SAS)
4. $\overline{BC} \cong \overline{DA}$ (CPCTC)

ANSWER:

Proof:

Statements (Reasons)

1. $\overline{BA} \cong \overline{DC}$, $\angle BAC \cong \angle DCA$ (Given)
2. $\overline{AC} \cong \overline{CA}$ (Reflex. Prop. $\cong$)
3. $\Delta BCA \cong \Delta DAC$ (SAS)
4. $\overline{BC} \cong \overline{DA}$ (CPCTC)

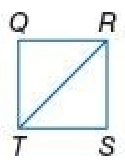
PROOF Write the specified type of proof.

5. paragraph proof

Given: $\overline{QR} \cong \overline{SR}$,

$\overline{ST} \cong \overline{QT}$

Prove: $\Delta QRT \cong \Delta SRT$



SOLUTION:

Proof: We know that $\overline{QR} \cong \overline{SR}$ and $\overline{ST} \cong \overline{QT}$. $\overline{RT} \cong \overline{RT}$ by the Reflexive Property. Since $\overline{QR} \cong \overline{SR}$, $\overline{ST} \cong \overline{QT}$, and $\overline{RT} \cong \overline{RT}$, $\Delta QRT \cong \Delta SRT$ by SSS.

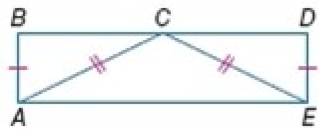
ANSWER:

Proof: We know that $\overline{QR} \cong \overline{SR}$ and $\overline{ST} \cong \overline{QT}$. $\overline{RT} \cong \overline{RT}$ by the Reflexive Property. Since $\overline{QR} \cong \overline{SR}$, $\overline{ST} \cong \overline{QT}$, and $\overline{RT} \cong \overline{RT}$, $\Delta QRT \cong \Delta SRT$ by SSS.

6. two-column proof

Given: $\overline{AB} \cong \overline{ED}$, $\overline{CA} \cong \overline{CE}$; $\overline{AC}$ bisects $\overline{BD}$.

Prove: $\triangle ABC \cong \triangle EDC$



SOLUTION:

Proof:

Statements (Reasons)

1. $\overline{AB} \cong \overline{ED}$, $\overline{CA} \cong \overline{CE}$, and $\overline{AC}$ bisects $\overline{BD}$ (Given)
2. *C* is the midpoint of $\overline{BD}$ (Def. of Segment Bisectors)
3. $\overline{BC} \cong \overline{CD}$ (Midpoint Thm.)
4. $\triangle ABC \cong \triangle EDC$ (SSS)

ANSWER:

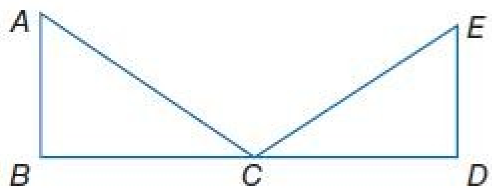
Proof:

Statements (Reasons)

1. $\overline{AB} \cong \overline{ED}$, $\overline{CA} \cong \overline{CE}$, and $\overline{AC}$ bisects $\overline{BD}$ (Given)
2. *C* is the midpoint of $\overline{BD}$ (Def. of Segment Bisectors)
3. $\overline{BC} \cong \overline{CD}$ (Midpoint Thm.)
4. $\triangle ABC \cong \triangle EDC$ (SSS)

7. **BRIDGES** The Sunshine Skyway Bridge in Florida is the world's longest cable-stayed bridge, spanning 4.1 miles of Tampa Bay. It is supported using steel cables suspended from two concrete supports. If the supports are the same height above the roadway and perpendicular to the roadway, and the topmost cables meet at a point midway between the supports, prove that the two triangles shown in the photo are congruent.

Refer to the figure on page 269.



SOLUTION:

Given: $\overline{AB} \cong \overline{ED}$, $\angle ABC$ and $\angle EDC$ are right angles, and C is the midpoint of $\overline{BD}$.
Prove: $\Delta ABC \cong \Delta EDC$
Proof:
Statements (Reasons)
1. $\overline{AB} \cong \overline{ED}$, $\angle ABC$ and $\angle EDC$ are right angles, and C is the midpoint of $\overline{BD}$. (Given)
2. $\angle ABC \cong \angle EDC$ (All rt. $\angle$s $\cong$)
3. $\overline{BC} \cong \overline{DC}$ (Midpoint Thm.)
4. $\Delta ABC \cong \Delta EDC$ (SAS)

ANSWER:

Given: $\overline{AB} \cong \overline{ED}$, $\angle ABC$ and $\angle EDC$ are right angles, and C is the midpoint of $\overline{BD}$.
Prove: $\Delta ABC \cong \Delta EDC$
Proof:
Statements (Reasons)
1. $\overline{AB} \cong \overline{ED}$, $\angle ABC$ and $\angle EDC$ are right angles, and C is the midpoint of $\overline{BD}$. (Given)
2. $\angle ABC \cong \angle EDC$ (All rt. $\angle$s $\cong$)
3. $\overline{BC} \cong \overline{DC}$ (Midpoint Thm.)
4. $\Delta ABC \cong \Delta EDC$ (SAS)

CCSS SENSE-MAKING Determine whether $\Delta MNO \cong \Delta QRS$. Explain.

8. $M(2, 5)$, $N(5, 2)$, $O(1, 1)$, $Q(-4, 4)$, $R(-7, 1)$, $S(-3, 0)$

SOLUTION:

Use the Distance Formula to find the lengths of $\overline{MN}, \overline{NO}$ and $\overline{OM}$.

$\overline{MN}$ has end points $M(2, 5)$ and $N(5, 2)$.

$$MN = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Substitute.

$$\begin{aligned} MN &= \sqrt{(5-2)^2+(2-5)^2} && \text{Substitute.} \\ &= \sqrt{(3)^2+(-3)^2} && \text{Subtraction.} \\ &= \sqrt{9+9} && \text{Square terms.} \\ &= \sqrt{18} && \text{Addition.} \end{aligned}$$

$\overline{NO}$ has end points $N(5, 2)$ and $O(1, 1)$.

$$NO = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Substitute.

$$\begin{aligned} NO &= \sqrt{(1-5)^2+(1-2)^2} && \text{Substitute.} \\ &= \sqrt{(-4)^2+(-1)^2} && \text{Subtraction.} \\ &= \sqrt{16+1} && \text{Square terms.} \\ &= \sqrt{17} && \text{Addition.} \end{aligned}$$

$\overline{OM}$ has end points $O(1, 1)$ and $M(2, 5)$.

$$OM = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Substitute.

$$\begin{aligned} OM &= \sqrt{(2-1)^2+(5-1)^2} && \text{Substitute.} \\ &= \sqrt{(1)^2+(4)^2} && \text{Subtraction.} \\ &= \sqrt{1+16} && \text{Square terms.} \\ &= \sqrt{17} && \text{Addition.} \end{aligned}$$

Similarly, find the lengths of $\overline{QR}, \overline{RS}$ and $\overline{SQ}$.

$\overline{QR}$ has end points $Q(–4, 4)$ and $R(–7, 1)$.

$$QR = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Substitute.

$$\begin{aligned} QR &= \sqrt{(-7-(-4))^2+(1-4)^2} && \text{Substitute.} \\ &= \sqrt{(-3)^2+(-3)^2} && \text{Subtraction.} \\ &= \sqrt{9+9} && \text{Square terms.} \\ &= \sqrt{18} && \text{Addition.} \end{aligned}$$

$\overline{RS}$ has end points $R(–7, 1)$ and $S(–3, 0)$.

$RS = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

Substitute.

$$\begin{aligned} RS &= \sqrt{(-3-(-7))^2 + (0-1)^2} && \text{Substitute.} \\ &= \sqrt{(4)^2 + (-1)^2} && \text{Subtraction.} \\ &= \sqrt{16+1} && \text{Square terms.} \\ &= \sqrt{17} && \text{Addition.} \end{aligned}$$

$\overline{SQ}$ has end points $S(–3, 0)$ and $Q(–4, 4)$.

$SQ = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

Substitute.

$$\begin{aligned} SQ &= \sqrt{(-4-(-3))^2 + (4-0)^2} && \text{Substitute.} \\ &= \sqrt{(-1)^2 + (4)^2} && \text{Subtraction.} \\ &= \sqrt{1+16} && \text{Square terms.} \\ &= \sqrt{17} && \text{Addition.} \end{aligned}$$

So, $\overline{MN} \cong \overline{QR}, \overline{NO} \cong \overline{RS}$ and $\overline{OM} \cong \overline{SQ}$.

Each pair of corresponding sides has the same measure so they are congruent. $\Delta MNO \cong \Delta QRS$ by SSS.

ANSWER:

$MN = \sqrt{18}$, $NO = \sqrt{17}$, $MO = \sqrt{17}$, $QR = \sqrt{18}$, $RS = \sqrt{17}$, and $QS = \sqrt{17}$. Each pair of corresponding sides has the same measure so they are congruent. $\Delta MNO \cong \Delta QRS$ by SSS.

9. $M(0, –1)$, $N(–1, –4)$, $O(–4, –3)$, $Q(3, –3)$, $R(4, –4)$, $S(3, 3)$

SOLUTION:

Use the Distance Formula to find the lengths of $\overline{MN}, \overline{NO}$ and $\overline{OM}$.

$\overline{MN}$ has end points $M(0, –1)$ and $N(–1, –4)$.

$MN = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

Substitute.

$$\begin{aligned} MN &= \sqrt{(-1-0)^2 + (-4-(-1))^2} && \text{Substitute.} \\ &= \sqrt{(-1)^2 + (-3)^2} && \text{Subtraction.} \\ &= \sqrt{1+9} && \text{Square terms.} \\ &= \sqrt{10} && \text{Addition.} \end{aligned}$$

$\overline{NO}$ has end points $N(-1, -4)$ and $O(-4, -3)$.

$NO = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

Substitute.

$$\begin{aligned} NO &= \sqrt{(-4-(-1))^2 + (-3-(-4))^2} && \text{Substitute.} \\ &= \sqrt{(-3)^2 + (1)^2} && \text{Subtraction.} \\ &= \sqrt{9+1} && \text{Square terms.} \\ &= \sqrt{10} && \text{Addition.} \end{aligned}$$

$\overline{OM}$ has end points $O(-4, -3)$ and $M(0, -1)$.

$OM = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

Substitute.

$$\begin{aligned} OM &= \sqrt{(0-(-4))^2 + (-1-(-3))^2} && \text{Substitute.} \\ &= \sqrt{(4)^2 + (2)^2} && \text{Subtraction.} \\ &= \sqrt{16+4} && \text{Square terms.} \\ &= \sqrt{20} && \text{Addition.} \end{aligned}$$

Similarly, find the lengths of $\overline{QR}, \overline{RS}$ and $\overline{SQ}$.

$\overline{QR}$ has end points $Q(3,-3)$ and $R(4, -4)$.

$QR = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

Substitute.

$$\begin{aligned} QR &= \sqrt{(4-3)^2 + (-4-(-3))^2} && \text{Substitute.} \\ &= \sqrt{(-1)^2 + (-1)^2} && \text{Subtraction.} \\ &= \sqrt{1+1} && \text{Square terms.} \\ &= \sqrt{2} && \text{Addition.} \end{aligned}$$

$\overline{RS}$ has end points $R(4, -4)$ and $S(3, 3)$.

$RS = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

Substitute.

$$\begin{aligned} RS &= \sqrt{(3-4)^2 + (3-(-4))^2} && \text{Substitute.} \\ &= \sqrt{(-1)^2 + (7)^2} && \text{Subtraction.} \\ &= \sqrt{1+49} && \text{Square terms.} \\ &= \sqrt{50} && \text{Addition.} \end{aligned}$$

$\overline{SQ}$ has end points $S(3, 3)$ and $Q(3, -3)$.

$$SQ = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Substitute.

$$\begin{aligned} SQ &= \sqrt{(3-3)^2 + (-3-3)^2} && \text{Substitute.} \\ &= \sqrt{(0)^2 + (-6)^2} && \text{Subtraction.} \\ &= \sqrt{0+36} && \text{Square terms.} \\ &= 6 && \text{Addition.} \end{aligned}$$

The corresponding sides are not congruent, so the triangles are not congruent.

ANSWER:

$MN = \sqrt{10},\ NO = \sqrt{10},\ MO = \sqrt{20},\ QR = \sqrt{2},\ RS = \sqrt{50},$ and $QS = 6$. The corresponding sides are not congruent, so the triangles are not congruent.

10. $M(0, -3)$, $N(1, 4)$, $O(3, 1)$, $Q(4, -1)$, $R(6, 1)$, $S(9, -1)$

SOLUTION:

Use the Distance Formula to find the lengths of $\overline{MN}, \overline{NO}$ and $\overline{OM}$.

$\overline{MN}$ has end points $M(0, -3)$ and $N(1, 4)$.

$$MN = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Substitute.

$$\begin{aligned} MN &= \sqrt{(1-0)^2 + (4-(-3))^2} && \text{Substitute.} \\ &= \sqrt{(1)^2 + (7)^2} && \text{Subtraction.} \\ &= \sqrt{1+49} && \text{Square terms.} \\ &= \sqrt{50} && \text{Addition.} \end{aligned}$$

$\overline{NO}$ has end points $N(1, 4)$ and $O(3, 1)$.

$$NO = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Substitute.

$$\begin{aligned} NO &= \sqrt{(3-1)^2+(1-4)^2} && \text{Substitute.} \\ &= \sqrt{(2)^2+(-3)^2} && \text{Subtraction.} \\ &= \sqrt{4+9} && \text{Square terms.} \\ &= \sqrt{13} && \text{Addition.} \end{aligned}$$

$\overline{OM}$ has end points $O(3, 1)$ and $M(0, -3)$.

$OM = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$

Substitute.

$$\begin{aligned} OM &= \sqrt{(0-3)^2+(-3-1)^2} && \text{Substitute.} \\ &= \sqrt{(-3)^2+(-4)^2} && \text{Subtraction.} \\ &= \sqrt{9+16} && \text{Square terms.} \\ &= \sqrt{25} && \text{Addition.} \\ &= 5 && \text{Simplify.} \end{aligned}$$

Similarly, find the lengths of $\overline{QR}, \overline{RS}$ and $\overline{SQ}$.

$\overline{QR}$ has end points $Q(4,-1)$ and $R(6, 1)$.

$QR = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$

Substitute.

$$\begin{aligned} QR &= \sqrt{(6-4)^2+(1-(-1))^2} && \text{Substitute.} \\ &= \sqrt{(2)^2+(2)^2} && \text{Subtraction.} \\ &= \sqrt{4+4} && \text{Square terms.} \\ &= \sqrt{8} && \text{Addition.} \end{aligned}$$

$\overline{RS}$ has end points $R(6, 1)$ and $S(9, -1)$.

$RS = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$

Substitute.

$$\begin{aligned} RS &= \sqrt{(9-6)^2+(-1-1)^2} && \text{Substitute.} \\ &= \sqrt{(3)^2+(-2)^2} && \text{Subtraction.} \\ &= \sqrt{9+4} && \text{Square terms.} \\ &= \sqrt{13} && \text{Addition.} \end{aligned}$$

$\overline{SQ}$ has end points $S(9, -1)$ and $Q(4, -1)$.

$SQ = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

Substitute.

$$\begin{aligned} SQ &= \sqrt{(4-9)^2 + (-1-(-1))^2} && \text{Substitute.} \\ &= \sqrt{(-5)^2 + (0)^2} && \text{Subtraction.} \\ &= \sqrt{25} && \text{Square terms.} \\ &= 5 && \text{Simplify.} \end{aligned}$$

The corresponding sides are not congruent, so the triangles are not congruent.

ANSWER:

$MN = \sqrt{50}$, $NO = \sqrt{13}$, $MO = 5$, $QR = \sqrt{8}$, $RS = \sqrt{13}$, and $QS = 5$. The corresponding sides are not congruent, so the triangles are not congruent

11. $M(4, 7)$, $N(5, 4)$, $O(2, 3)$, $Q(2, 5)$, $R(3, 2)$, $S(0, 1)$

SOLUTION:

Use the Distance Formula to find the lengths of $\overline{MN}, \overline{NO}$ and $\overline{OM}$.

$\overline{MN}$ has end points $M(4, 7)$ and $N(5, 4)$.

$MN = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

Substitute.

$$\begin{aligned} MN &= \sqrt{(5-4)^2 + (4-7)^2} && \text{Substitute.} \\ &= \sqrt{(1)^2 + (-3)^2} && \text{Subtraction.} \\ &= \sqrt{1+9} && \text{Square terms.} \\ &= \sqrt{10} && \text{Addition.} \end{aligned}$$

$\overline{NO}$ has end points $N(5, 4)$ and $O(2, 3)$.

$NO = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

Substitute.

$$\begin{aligned} NO &= \sqrt{(2-5)^2 + (3-4)^2} && \text{Substitute.} \\ &= \sqrt{(-3)^2 + (-1)^2} && \text{Subtraction.} \\ &= \sqrt{9+1} && \text{Square terms.} \\ &= \sqrt{10} && \text{Addition.} \end{aligned}$$

$\overline{OM}$ has end points $O(2, 3)$ and $M(4,7)$.

$OM = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

Substitute.

$$\begin{aligned} OM &= \sqrt{(4-2)^2 + (7-3)^2} && \text{Substitute.} \\ &= \sqrt{(2)^2 + (4)^2} && \text{Subtraction.} \\ &= \sqrt{4+16} && \text{Square terms.} \\ &= \sqrt{20} && \text{Addition.} \end{aligned}$$

Similarly, find the lengths of $\overline{QR}, \overline{RS}$ and $\overline{SQ}$.

$\overline{QR}$ has end points $Q(2, 5)$ and $R(3, 2)$.

$QR = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

Substitute.

$$\begin{aligned} QR &= \sqrt{(3-2)^2 + (2-5)^2} && \text{Substitute.} \\ &= \sqrt{(1)^2 + (-3)^2} && \text{Subtraction.} \\ &= \sqrt{1+9} && \text{Square terms.} \\ &= \sqrt{10} && \text{Addition.} \end{aligned}$$

$\overline{RS}$ has end points $R(3, 2)$ and $S(0, 1)$.

$RS = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

Substitute.

$$\begin{aligned} RS &= \sqrt{(0-3)^2 + (1-2)^2} && \text{Substitute.} \\ &= \sqrt{(-3)^2 + (-1)^2} && \text{Subtraction.} \\ &= \sqrt{9+1} && \text{Square terms.} \\ &= \sqrt{10} && \text{Addition.} \end{aligned}$$

$\overline{SQ}$ has end points $S(0, 1)$ and $Q(2, 5)$.

$SQ = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

Substitute.

$$\begin{aligned} SQ &= \sqrt{(2-0)^2+(5-1)^2} && \text{Substitute.} \\ &= \sqrt{(2)^2+(4)^2} && \text{Subtraction.} \\ &= \sqrt{4+16} && \text{Square terms.} \\ &= \sqrt{20} && \text{Addition.} \end{aligned}$$

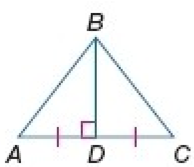
So, $\overline{MN} \cong \overline{QR}, \overline{NO} \cong \overline{RS}$ and $\overline{OM} \cong \overline{SQ}$.
Each pair of corresponding sides has the same measure so they are congruent. $\Delta MNO \cong \Delta QRS$ by SSS.

ANSWER:

$MN = \sqrt{10}$, $NO = \sqrt{10}$, $MO = \sqrt{20}$, $QR = \sqrt{10}$, $RS = \sqrt{10}$, and $QS = \sqrt{20}$. Each pair of corresponding sides has the same measure, so they are congruent. $\Delta MNO \cong \Delta QRS$ by SSS.

PROOF Write the specified type of proof.

12. two-column proof
Given: $\overline{BD} \perp \overline{AC}$,
$\overline{BD}$ bisects $\overline{AC}$.
Prove: $\Delta ABD \cong \Delta CBD$



SOLUTION:

Proof: Statements (Reasons)

1. $\overline{BD} \perp \overline{AC}$, $\overline{BD}$ bisects $\overline{AC}$. (Given)
2. $\angle BDA$ and $\angle BDC$ are right angles. (Def. of $\perp$)
3. $\angle BDA \cong \angle BDC$ (all right angles are $\cong$)
4. $\overline{AD} \cong \overline{DC}$ (Def. of bisects)
5. $\overline{BD} \cong \overline{BD}$ (Ref. Prop.)
6. $\Delta ABD \cong \Delta CBD$ (SAS)

ANSWER:

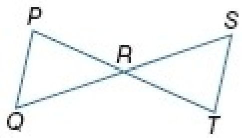
Proof: Statements (Reasons)

1. $\overline{BD} \perp \overline{AC}$, $\overline{BD}$ bisects $\overline{AC}$. (Given)
2. $\angle BDA$ and $\angle BDC$ are right angles. (Def. of $\perp$)
3. $\angle BDA \cong \angle BDC$ (all right angles are $\cong$)
4. $\overline{AD} \cong \overline{DC}$ (Def. of bisects)
5. $\overline{BD} \cong \overline{BD}$ (Ref. Prop.)
6. $\Delta ABD \cong \Delta CBD$ (SAS)

13. paragraph proof

Given: *R* is the midpoint of $\overline{QS}$ and $\overline{PT}$.

Prove: $\Delta PRQ \cong \Delta TRS$



SOLUTION:

Since *R* is the midpoint of $\overline{QS}$ and $\overline{PT}, \overline{PR} \cong \overline{RT}$ and $\overline{RQ} \cong \overline{RS}$ by definition of a midpoint. . $\angle PRQ \cong \angle TRS$ by the Vertical Angles Theorem.
So, $\Delta PRQ \cong \Delta TRS$ by SAS.

ANSWER:

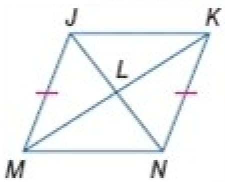
Since *R* is the midpoint of $\overline{QS}$ and $\overline{PT}, \overline{PR} \cong \overline{RT}$ and $\overline{RQ} \cong \overline{RS}$ by definition of a midpoint. $\angle PRQ \cong \angle TRS$ by the Vertical Angles Theorem.
So, $\Delta PRQ \cong \Delta TRS$ by SAS.

PROOF Write the specified type of proof.

14. flow proof

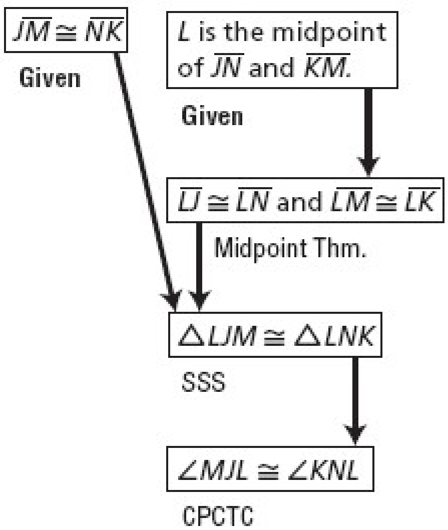
Given: $\overline{JM} \cong \overline{NK}$; L is the midpoint of $\overline{JN}$ and $\overline{KM}$.

Prove: $\angle MJL \cong \angle KNL$



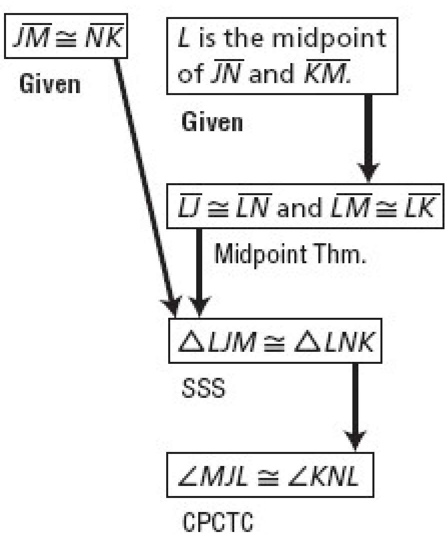
SOLUTION:

Proof:



ANSWER:

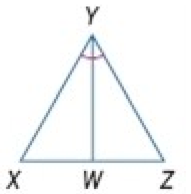
Proof:



15. paragraph proof

Given: ΔXYZ is equilateral. $\overline{WY}$ bisects $\angle Y$.

Proof: $\overline{XW} \cong \overline{ZW}$



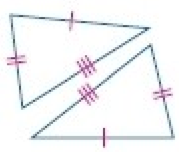
SOLUTION:

Proof: We know that $\overline{WY}$ bisects $\angle Y$, so $\angle XYW \cong \angle ZYW$. Also, $\overline{YW} \cong \overline{YW}$ by the Reflexive Property. Since ΔXYZ is equilateral it is a special type of isosceles triangle, so $\overline{XY} \cong \overline{ZY}$. By the Side-Angle-Side Congruence Postulate, $\Delta XYW \cong \Delta ZYW$. By CPCTC, $\overline{XW} \cong \overline{ZW}$.

ANSWER:

Proof: We know that $\overline{WY}$ bisects $\angle Y$, so $\angle XYW \cong \angle ZYW$. Also, $\overline{YW} \cong \overline{YW}$ by the Reflexive Property. Since ΔXYZ is equilateral it is a special type of isosceles triangle, so $\overline{XY} \cong \overline{ZY}$. By the Side-Angle-Side Congruence Postulate, $\Delta XYW \cong \Delta ZYW$ By CPCTC, $\overline{XW} \cong \overline{ZW}$

CCSS ARGUMENTS Determine which postulate can be used to prove that the triangles are congruent. If it is not possible to prove congruence, write *not possible*.

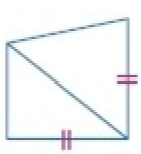
16. 

SOLUTION:

The corresponding sides are congruent; SSS.

ANSWER:

SSS

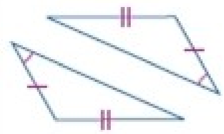
17. 

SOLUTION:

The triangles have two corresponding sides congruent but no information is given about the included angle or the third pair of sides; not possible.

ANSWER:

not possible

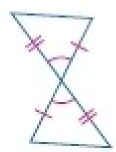
18. 

SOLUTION:

Information is given about two pairs of corresponding sides and one pair of corresponding angles but the congruent angles are not in the interior of the congruent sides; not possible.

ANSWER:

not possible

19. 

SOLUTION:

The triangles have two pairs of corresponding sides congruent and the interior angles are congruent; SAS.

ANSWER:

SAS

20. **SIGNS** Refer to the diagram.
a. Identify the three-dimensional figure represented by the wet floor sign.
b. If $\overline{AB} \cong \overline{AD}$ and $\overline{CB} \cong \overline{DC}$, prove that $\Delta ACB \cong \Delta ACD$.
c. Why do the triangles not look congruent in the diagram?

SOLUTION:
a. triangular pyramid
b. Given: $\overline{AB} \cong \overline{AD}$ and $\overline{CB} \cong \overline{DC}$
Prove: $\Delta ACB \cong \Delta ACD$.
Proof:
Statements (Reasons)
1. $\overline{AB} \cong \overline{AD}$ and $\overline{CB} \cong \overline{DC}$ (Given)
2. $\overline{AC} \cong \overline{AC}$ (Refl. Prop.)
3. $\Delta ACB \cong \Delta ACD$. (SSS)

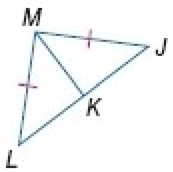
c. Sample answer: The object is three-dimensional, so when it is viewed in two dimensions, the perspective makes it look like the triangles are differently shaped.

ANSWER:
a. triangular pyramid
b. Given: $\overline{AB} \cong \overline{AD}$ and $\overline{CB} \cong \overline{DC}$
Prove: $\boldsymbol{\Delta ACB \cong \Delta ACD}$
Proof:
Statements (Reasons)
1. $\overline{AB} \cong \overline{AD}$ and $\overline{CB} \cong \overline{DC}$ (Given)
2. $\overline{AC} \cong \overline{AC}$ (Refl. Prop.)
3. $\Delta ACB \cong \Delta ACD$. (SSS)

c. Sample answer: The object is three-dimensional, so when it is viewed in two dimensions, the perspective makes it look like the triangles are differently shaped.

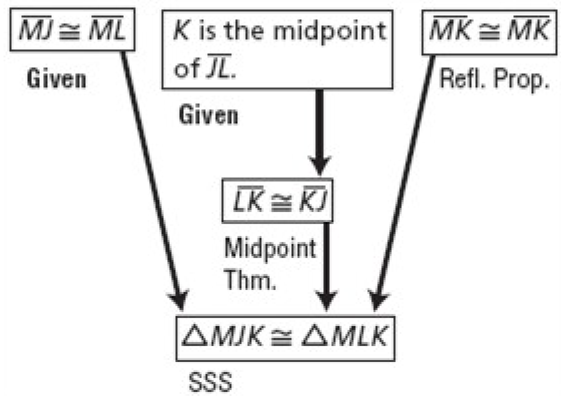
PROOF Write a flow proof.

21. **Given:** $\overline{MJ} \cong \overline{ML}$; K is the midpoint of $\overline{JL}$.
Prove: $\Delta MJK \cong \Delta MLK$



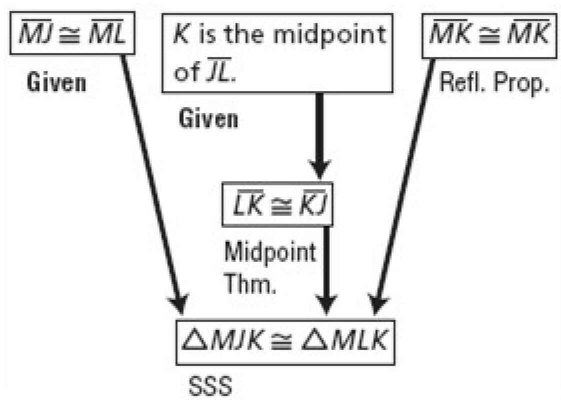
SOLUTION:

Proof:



ANSWER:

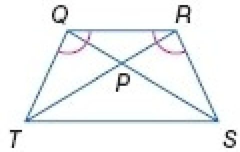
Proof:



22. **Given:** $\Delta TPQ \cong \Delta SPR$

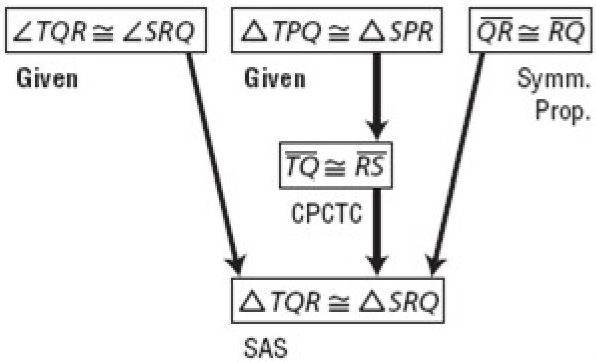
$\angle TQR \cong \angle SRQ$

Prove: $\Delta TQR \cong \Delta SRQ$



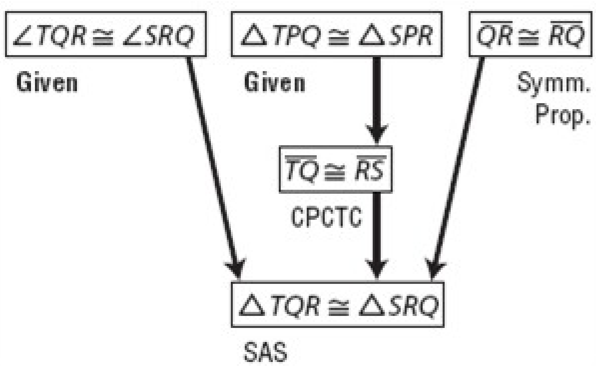
SOLUTION:

Proof:



ANSWER:

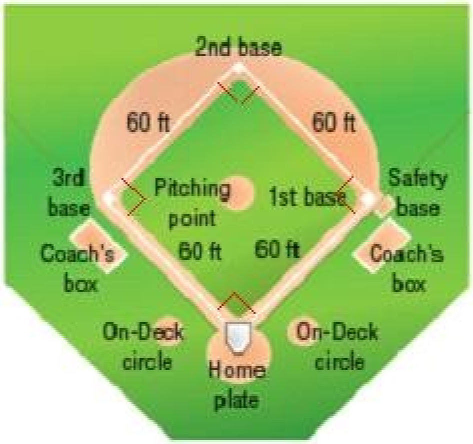
Proof:



23. **SOFTBALL** Use the diagram of a fast-pitch softball diamond shown. Let F = first base, S = second base, T = third base, P = pitcher's point, and R = home plate.

a. Write a two-column proof to prove that the distance from first base to third base is the same as the distance from home plate to second base.

b. Write a two-column proof to prove that the angle formed between second base, home plate, and third base is the same as the angle formed between second base, home plate, and first base.

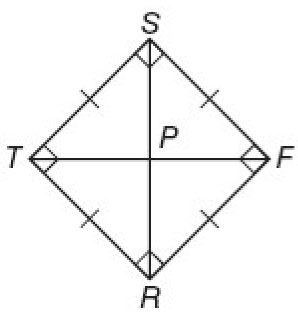


SOLUTION:

a. Given: $\overline{TS} \cong \overline{SF} \cong \overline{FR} \cong \overline{RT}$

$\angle TSF$, $\angle SFR$, $\angle FRT$, and $\angle RTS$ are right angles.

Prove: $\overline{RS} \cong \overline{TF}$



Proof:

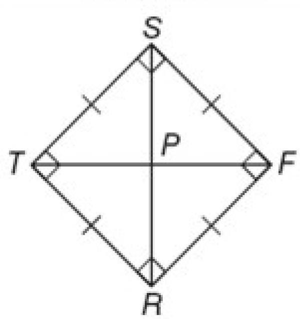
Statements (Reasons)

1. $\overline{TS} \cong \overline{SF} \cong \overline{FR} \cong \overline{RT}$ (Given)
2. $\angle TSF$, $\angle SFR$, $\angle FRT$, and $\angle RTS$ are right angles (Given)
3. $\angle STR \cong \angle TRF$ (All rt $\angle$s are $\cong$.)
4. $\Delta STR \cong \Delta TRF$ (SAS)
5. $\overline{RS} \cong \overline{TF}$ (CPCTC)

b. Given: $\overline{TS} \cong \overline{SF} \cong \overline{FR} \cong \overline{RT}$

$\angle TSF$, $\angle SFR$, $\angle FRT$, and $\angle RTS$ are right angles.

Prove: $\angle SRT \cong \angle SRF$



Proof:

Statements (Reasons)

4-4 Proving Triangles Congruent-SSS, SAS

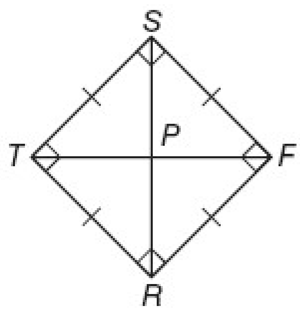
1. $\overline{TS} \cong \overline{SF} \cong \overline{FR} \cong \overline{RT}$ (Given)
2. $\angle TSF$, $\angle SFR$, $\angle FRT$, and $\angle RTS$ are right angles (Given)
3. $\angle STR \cong \angle SFR$ (All rt $\angle$s are $\cong$.)
4. $\Delta STR \cong \Delta SFR$ (SAS)
5. $\angle SRT \cong \angle SRF$ (CPCTC)

ANSWER:

a. Given: $\overline{TS} \cong \overline{SF} \cong \overline{FR} \cong \overline{RT}$

$\angle TSF$, $\angle SFR$, $\angle FRT$, and $\angle RTS$ are right angles.

Prove: $\overline{RS} \cong \overline{TF}$



Proof:

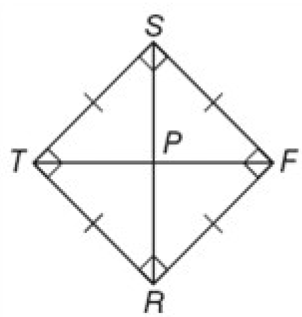
Statements (Reasons)

1. $\overline{TS} \cong \overline{SF} \cong \overline{FR} \cong \overline{RT}$ (Given)
2. $\angle TSF$, $\angle SFR$, $\angle FRT$, and $\angle RTS$ are right angles. (Given)
3. $\angle STR \cong \angle TRF$ (All rt $\angle$s are $\cong$.)
4. $\Delta STR \cong \Delta TRF$ (SAS)
5. $\overline{RS} \cong \overline{TF}$ (CPCTC)

b. Given: $\overline{TS} \cong \overline{SF} \cong \overline{FR} \cong \overline{RT}$

$\angle TSF$, $\angle SFH$, $\angle FHT$, and $\angle HTS$ are right $\angle$s.

Prove: $\angle SRT \cong \angle SRF$



Proof:

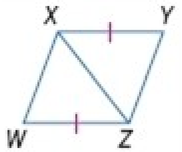
Statements (Reasons)

1. $\overline{TS} \cong \overline{SF} \cong \overline{FR} \cong \overline{RT}$ (Given)
2. $\angle TSF$, $\angle SFR$, $\angle FRT$, and $\angle RTS$ are right angles. (Given)
3. $\angle STR \cong \angle SFR$ (All rt $\angle$s are $\cong$.)
4. $\Delta STR \cong \Delta SFR$ (SAS)
5. $\angle SRT \cong \angle SRF$ (CPCTC)

PROOF Write a two-column proof.

24. **Given:** $\overline{YX} \cong \overline{WZ}$, $\overline{YX} \parallel \overline{ZW}$

Prove: $\Delta YXZ \cong \Delta WZX$



SOLUTION:

Proof:

Statements (Reasons)

1. $\overline{YX} \cong \overline{WZ}$, $\overline{YX} \parallel \overline{ZW}$ (Given)
2. $\angle YXZ \cong \angle WZX$ (Alt. Int. $\angle$s)
3. $\overline{XZ} \cong \overline{ZX}$ (Reflex. Prop.)
4. $\Delta YXZ \cong \Delta WZX$ (SAS)

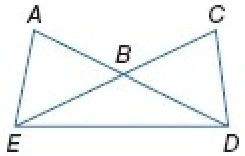
ANSWER:

Proof:

Statements (Reasons)

1. $\overline{YX} \cong \overline{WZ}$, $\overline{YX} \parallel \overline{ZW}$ (Given)
2. $\angle YXZ \cong \angle WZX$ (Alt. In/t. $\angle$s)
3. $\overline{XZ} \cong \overline{ZX}$ (Reflex. Prop.)
4. $\Delta YXZ \cong \Delta WZX$ (SAS)

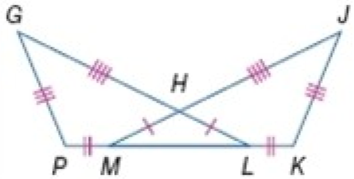
25. **Given:** $\Delta EAB \cong \Delta DCB$
Prove: $\Delta EAD \cong \Delta DCE$



SOLUTION:
Proof:
Statements (Reasons)
1. $\Delta EAB \cong \Delta DCB$ (Given)
2. $\overline{EA} \cong \overline{DC}$ (CPCTC)
3. $\overline{ED} \cong \overline{DE}$ (Reflex. Prop.)
4. $\overline{AB} \cong \overline{CB}$ (CPCTC)
5. $\overline{DB} \cong \overline{EB}$ (CPCTC)
6. $AB = CB$, $DB = EB$ (Def. $\cong$ segments)
7. $AB + DB = CB + EB$ (Add. Prop. =)
8. $AD = AB + DB$, $CE = CB + EB$ (Segment addition)
9. $AD = CE$ (Subst. Prop. =)
10. $\overline{AD} \cong \overline{CE}$ (Def. $\cong$ segments)
11. $\Delta EAD \cong \Delta DCE$ (SSS)

ANSWER:
Proof:
Statements (Reasons)
1. $\Delta EAB \cong \Delta DCB$ (Given)
2. $\overline{EA} \cong \overline{DC}$ (CPCTC)
3. $\overline{ED} \cong \overline{DE}$ (Reflex. Prop.)
4. $\overline{AB} \cong \overline{CB}$ (CPCTC)
5. $\overline{DB} \cong \overline{EB}$ (CPCTC)
6. $AB = CB$, $DB = EB$ (Def. $\cong$ segments)
7. $AB + DB = CB + EB$ (Add. Prop. =)
8. $AD = AB + DB$, $CE = CB + EB$ (Segment addition)
9. $AD = CE$ (Subst. Prop. =)
10. $\overline{AD} \cong \overline{CE}$ (Def. $\cong$ segments)
11. $\Delta EAD \cong \Delta DCE$ (SSS)

26. **CCSS ARGUMENTS** Write a paragraph proof.
Given: $\overline{HL} \cong \overline{HM}, \overline{PM} \cong \overline{KL}, \overline{PG} \cong \overline{KJ}, \overline{GH} \cong \overline{JH}$
Prove: $\angle G \cong \angle J$



SOLUTION:

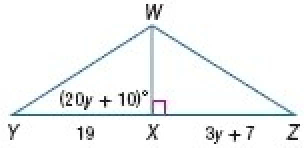
Proof: $\overline{GH} \cong \overline{JH}$ and $\overline{HL} \cong \overline{HM}$ so by the definition of congruence, $GH = JH$ and $HL = HM$. By the Segment Addition Postulate, $GL = GH + HL$ and $JM = JH + HM$. By substitution, $GL = JH + HM$ and $GL = JM$. By the definition of congruence, $\overline{GL} \cong \overline{JM}$. $\overline{PM} \cong \overline{KL},$ so by the definition of congruence, $PM = KL$. $\overline{ML} \cong \overline{LM}$ by the Reflexive Property of Congruence, so by the definition of congruence, $ML = LM$. By the Segment Addition Postulate, $PL = PM + ML$ and $KM = KL + LM$. By substitution, $PL = KL + LM$ and $PL = KM$. By the definition of congruence, $\overline{PL} \cong \overline{KM}$. $\overline{PG} \cong \overline{KJ},$ so by SSS, $\Delta GPL \cong \Delta JKM$. By CPCTC, $\angle G \cong \angle J$.

ANSWER:

Proof: $\overline{GH} \cong \overline{JH}$ and $\overline{HL} \cong \overline{HM}$ so by the definition of congruence, $GH = JH$ and $HL = HM$. By the Segment Addition Postulate, $GL = GH + HL$ and $JM = JH + HM$. By substitution, $GL = JH + HM$ and $GL = JM$. By the definition of congruence, $\overline{GL} \cong \overline{JM}$. $\overline{PM} \cong \overline{KL},$ so by the definition of congruence, $PM = KL$. $\overline{ML} \cong \overline{LM}$ by the Reflexive Property of Congruence, so by the definition of congruence, $ML = LM$. By the Segment Addition Postulate, $PL = PM + ML$ and $KM = KL + LM$. By substitution, $PL = KL + LM$ and $PL = KM$. By the definition of congruence, $\overline{PL} \cong \overline{KM}$. $\overline{PG} \cong \overline{KJ},$ so by SSS, $\Delta GPL \cong \Delta JKM$. By CPCTC, $\angle G \cong \angle J$.

ALGEBRA Find the value of the variable that yields congruent triangles. Explain.

27. $\Delta WXY \cong \Delta WXZ$



SOLUTION:

Given that $\Delta WXY \cong \Delta WXZ$, by CPCTC $\angle WXZ \cong \angle WXY$ and $\overline{XY} \cong \overline{XZ}$.

The value of y can be found using the sides or the angles.

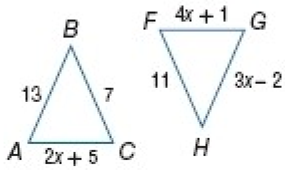
$$\begin{aligned} \angle WXZ &\cong \angle WXY && \text{CPCTC.} \\ m\angle WXZ &= m\angle WXY && \text{Def. of congruence.} \\ 90 &= 20y + 10 && \text{Substitution.} \\ 80 &= 20y && -10 \text{ from each side.} \\ 4 &= y && \div \text{ each side by } 20. \end{aligned}$$

$$\begin{aligned} \overline{XY} &\cong \overline{XZ} && \text{CPCTC.} \\ XY &= XZ && \text{Def. of congruence.} \\ 3y + 7 &= 19 && \text{Substitution.} \\ 3y &= 12 && -12 \text{ from each side.} \\ y &= 4 && \div \text{ each side by } 3. \end{aligned}$$

ANSWER:

y = 4; By CPCTC, $\angle WXZ \cong \angle WXY$, and $\overline{YX} \cong \overline{ZX}$.

28. $\Delta ABC \cong \Delta FGH$



SOLUTION:

$\Delta ABC \cong \Delta FGH$

By CPCTC, $\overline{AC} \cong \overline{FH}$.

By the definition of congruence, $AC = FH$.

Substitute.

$$\begin{aligned} 2x+5&=11 && AC=FH \\ 2x+5-5&=11-5 && -5 \text{ from each side.} \\ 2x&=6 && \text{Simplify.} \\ x&=3 && \div \text{ each side by 2.} \end{aligned}$$

By CPCTC, $\overline{BC} \cong \overline{GH}$.

By the definition of congruence, $BC = GH$.

Substitute.

$$\begin{aligned} 7&=3x-2 && BC=GH \\ 7+2&=3x-2+2 && +2 \text{ to each side.} \\ 9&=3x && \text{Simplify.} \\ 3&=x && \div \text{ each side by 3.} \end{aligned}$$

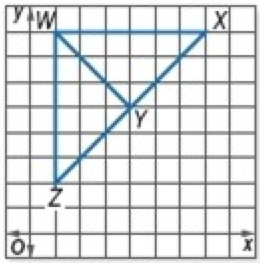
ANSWER:

x = 3; By CPCTC, $\overline{AC} \cong \overline{FH}$, and $\overline{BC} \cong \overline{GH}$.

29. **CHALLENGE** Refer to the graph shown.

a. Describe two methods you could use to prove that ΔWYZ is congruent to ΔWYX. You may not use a ruler or a protractor. Which method do you think is more efficient? Explain.

b. Are ΔWYZ and ΔWYX congruent? Explain your reasoning.



SOLUTION:

a. Sample answer:
Method 1: You could use the Distance Formula to find the length of each of the sides, and then use the Side-Side-Side Congruence Postulate to prove the triangles congruent.

Method 2 : You could find the slopes of $\overline{ZX}$ and $\overline{WY}$ to prove that they are perpendicular and that $\angle WYZ$ and $\angle WYX$ are both right angles. You can use the Distance Formula to prove that $\overline{XY}$ is congruent to $\overline{ZY}$. Since the triangles share the leg $\overline{WY}$, you can use the Side-Angle-Side Congruence Postulate; Sample answer: I think that method 2 is more efficient, because you only have two steps instead of three.

b. Sample answer: Yes; the slope of $\overline{WY}$ is –1 and the slope of $\overline{ZX}$ is 1, and –1 and 1 are opposite reciprocals, so $\overline{WY}$ is perpendicular to $\overline{ZX}$. Since they are perpendicular, $\angle WYZ$ and $\angle WYX$ are both 90°. Using the Distance Formula, the length of $\overline{ZY}$ is $\sqrt{(4-1)^2+(5-2)^2}$ or $3\sqrt{2}$, and the length of $\overline{XY}$ is $\sqrt{(7-4)^2+(8-5)^2}$ or $3\sqrt{2}$. Since $\overline{WY}$ is congruent to $\overline{WY}, \Delta WYZ$ is congruent to ΔWYX by the Side-Angle-Side Congruent Postulate.

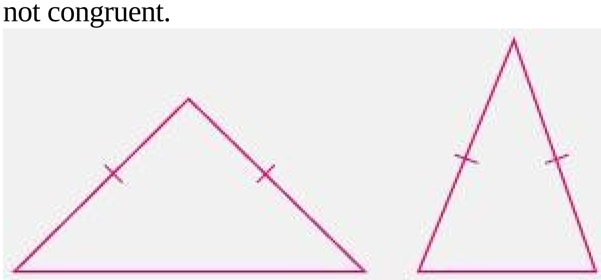
ANSWER:

a. Sample answer: Method 1: You could use the Distance Formula to find the length of each of the sides, and then use the Side-Side-Side Congruence Postulate to prove the triangles congruent. Method 2 : You could find the slopes of $\overline{ZX}$ and $\overline{WY}$ to prove that they are perpendicular and that $\angle WYZ$ and $\angle WYX$ are both right angles. You can use the Distance Formula to prove that $\overline{XY}$ is congruent to $\overline{ZY}$. Since the triangles share the leg $\overline{WY}$ you can use the Side-Angle-Side Congruence Postulate; Sample answer: I think that method 2 is more efficient, because you only have two steps instead of three.

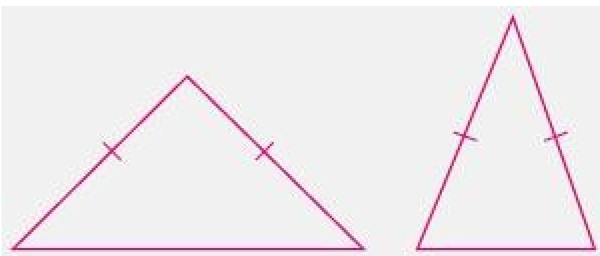
b. Sample answer: Yes; the slope of $\overline{WY}$ is –1 and the slope of $\overline{ZX}$ is 1, and −1 and 1 are opposite reciprocals, so $\overline{WY}$ is perpendicular to $\overline{ZX}$. Since they are perpendicular, $\angle WYZ$ and $\angle WYX$ are both 90°. Using the Distance Formula, the length of $\overline{ZY}$. is $\sqrt{(4-1)^2+(5-2)^2}$ or $3\sqrt{2}$, and the length of $\overline{XY}$ is $\sqrt{(7-4)^2+(8-5)^2}$ or $3\sqrt{2}$. Since $\overline{WY}$ is congruent to $\overline{WY}, \Delta WYZ$ is congruent to ΔWYX by the Side-Angle-Side Congruent Postulate.

30. **REASONING** Determine whether the following statement is *true* or *false*. If true, explain your reasoning. If *false*, provide a counterexample.
If the congruent sides in one isosceles triangle have the same measure as the congruent sides in another isosceles triangle, then the triangles are congruent.

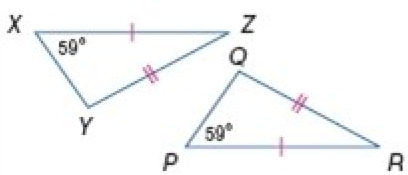
SOLUTION:
False, the triangles shown below are isosceles triangles with two pairs of sides congruent and the third pair of sides not congruent.



ANSWER:
false



31. **ERROR ANALYSIS** Bonnie says that $\triangle PQR \cong \triangle XYZ$ by SAS. Shada disagrees. She says that there is not enough information to prove that the two triangles are congruent. Is either of them correct? Explain.



SOLUTION:
Shada; for SAS the angle must be the included angle $(\angle R$ and $\angle Z)$ and here it is not included.

ANSWER:
Shada; for SAS the angle must be the included angle and here it is not included.

32. **OPEN ENDED** Use a straightedge to draw obtuse triangle *ABC*. Then construct $\triangle XYZ$ so that it is congruent to $\triangle ABC$ using either SSS or SAS. Justify your construction mathematically and verify it using measurement.

SOLUTION:
Sample answer: Using a ruler I measured all of the sides and they are congruent so the triangles are congruent by SSS.

ANSWER:
Sample answer: Using a ruler I measured all of the sides and they are congruent so the triangles are congruent by SSS.

33. **WRITING IN MATH** Two pairs of corresponding sides of two right triangles are congruent. Are the triangles congruent? Explain your reasoning.

SOLUTION:

Case 1: The two pairs of congruent angles includes the right angle. You know the hypotenuses are congruent and one of the legs are congruent. Then the Pythagorean Theorem says that the other legs are congruent so the triangles are congruent by SSS.
Case 2: The two pairs of congruent angles do not include the right angle. You know the legs are congruent and the right angles are the included angles and all right angles are congruent, then the triangles are congruent by SAS.

ANSWER:

Case 1: You know the hypotenuses are congruent and one of the legs are congruent. Then the Pythagorean Theorem says that the other legs are congruent so the triangles are congruent by SSS.
Case 2: You know the legs are congruent and the right angles are congruent, then the triangles are congruent by SAS.

34. **ALGEBRA** The Ross Family drove 300 miles to visit their grandparents. Mrs. Ross drove 70 miles per hour for 65% of the trip and 35 miles per hour or less for 20% of the trip that was left. Assuming that Mrs. Ross never went over 70 miles per hour, how many miles did she travel at a speed between 35 and 70 miles per hour?

A 195
B 84
C 21
D 18

SOLUTION:

For the 65% of the distance, she drove 70 miles per hour. So she covered $\frac{65}{100} \times 300$ or 195 miles.

The remaining distance is 300 – 195 = 105 miles.

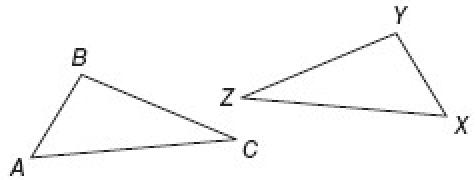
For the 20% of the remaining distance, she drove less than 35 miles per hour. So she covered $\frac{20}{100} \times 105$ or 21 miles.

So, totally she covered 195 + 21 or 216 miles at the speed 70 miles per hour and less than 30 miles per hour. Bu the trip has 300 miles, so the remaining distance is 300 – 216 or 84 miles. She has traveled 84 miles at a speed between 35 and 70 miles per hour. So, the correct option is B.

ANSWER:

B

35. In the figure, $\angle C \cong \angle Z$ and $\overline{AC} \cong \overline{XZ}$.



What additional information could be used to prove that $\Delta ABC \cong \Delta XYZ$?

F $\overline{BC} \cong \overline{YZ}$

G $\overline{AB} \cong \overline{XY}$

H $\overline{BC} \cong \overline{XZ}$

J $\overline{XZ} \cong \overline{XY}$

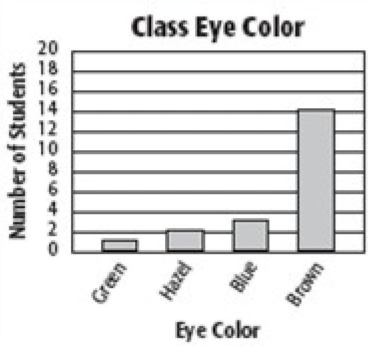
SOLUTION:

We can prove $\Delta ABC \cong \Delta XYZ$ by SAS if we knew that $\overline{BC} \cong \overline{YZ}$. So, the needed information is $\overline{BC} \cong \overline{YZ}$. The correct option is F.

ANSWER:

F

36. **EXTENDED RESPONSE** The graph below shows the eye colors of all of the students in a class. What is the probability that a student chosen at random from this class will have blue eyes? Explain your reasoning.



SOLUTION:

$\frac{3}{20}$; First you have to find how many students there are in the class. There are 1 + 2 + 3 + 14 or 20. Then the probability of randomly choosing a student with blue eyes is the number of students with blue eyes divided by 20. Since there are 3 students with blue eyes, the probability is $\frac{3}{20}$.

ANSWER:

$\frac{3}{20}$; First you have to find how many students there are in the class. There are 1 + 2 + 3 + 14 or 20. Then the probability of randomly choosing a student with blue eyes is the number of students with blue eyes divided by 20. Since there are 3 students with blue eyes, the probability is $\frac{3}{20}$;

37. **SAT/ACT** If $4a + 6b = 6$ and $-2a + b = -7$, what is the value of a?

A -2
B -1
C 2
D 3
E 4

SOLUTION:

Solve the equation $-2a + b = -7$ for b.

$$
\begin{aligned}
-2a + b &= -7 && \text{Original equation} \\
-2a + 2a + b &= 2a - 7 && +2a \text{ to each side.} \\
b &= 2a - 7 && \text{Simplify.}
\end{aligned}
$$

Substitute $b = 2a - 7$ in $4a + 6b = 6$.

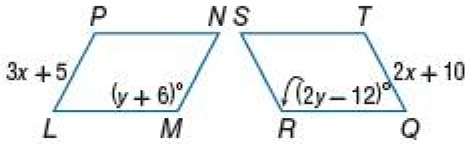
$$
\begin{aligned}
4a + 6b &= 6 && \text{Original equation} \\
4a + 6(2a - 7) &= 6 && \text{Substitution.} \\
4a + 12a - 42 &= 6 && \text{Distributive Property} \\
16a - 42 &= 6 && \text{Simplify.} \\
16a - 42 + 42 &= 6 + 42 && +42 \text{ to each side.} \\
16a &= 48 && \text{Simplify.} \\
a &= 3 && \text{Divide each side by 16.}
\end{aligned}
$$

So, the correct option is D.

ANSWER:

D

In the diagram, $\square LMNP \cong \square QRST$.



38. Find x.

SOLUTION:

By CPCTC, $\overline{PL} \cong \overline{TQ}$.
By the definition of congruence, $PL = TQ$.
Substitute.

$$\begin{aligned} PL &= TQ && \text{CPCTC} \\ 3x+5 &= 2x+10 && \text{Substitution.} \\ 3x+5-2x &= 2x+10-2x && -2x \text{ from each side.} \\ x+5 &= 10 && \text{Simplify.} \\ x+5-5 &= 10-5 && -5 \text{ from each side.} \\ x &= 5 && \text{Simplify.} \end{aligned}$$

ANSWER:

5

39. Find y.

SOLUTION:

By CPCTC, $\angle R \cong \angle M$.
By the definition of congruence, $m\angle R = m\angle M$. Substitute.

$$\begin{aligned} m\angle R &= m\angle M && \text{CPCTC.} \\ 2y-12 &= y+6 && \text{Substitution.} \\ 2y-12-y &= y+6-y && -y \text{ from each side.} \\ y-12 &= 6 && \text{Simplify.} \\ y-12+12 &= 6+12 && +12 \text{ to each side.} \\ y &= 18 && \text{Simplify.} \end{aligned}$$

ANSWER:

18

40. **ASTRONOMY** The Big Dipper is a part of the larger constellation Ursa Major. Three of the brighter stars in the constellation form ΔRSA. If $m\angle R = 41$ and $m\angle S = 109$, find $m\angle A$.

SOLUTION:

The sum of the measures of the angles of a triangle is 180. So, $m\angle R + m\angle S + m\angle A = 180$.

$$\begin{aligned} m\angle R + m\angle S + m\angle A &= 180 && \text{Triangle Angle-Sum Thm.} \\ 41 + 109 + m\angle A &= 180 && \text{Substitute.} \\ 150 + m\angle A &= 180 && \text{Addition.} \\ 150 + m\angle A - 150 &= 180 - 150 && -150 \text{ from each side.} \\ m\angle A &= 30 && \text{Simplify.} \end{aligned}$$

ANSWER:

30

Write an equation in slope-intercept form for each line.

41. (–5, –3) and (10, –6)

SOLUTION:

Substitute the values in the slope formula.

$$\begin{aligned} m &= \frac{y_2 - y_1}{x_2 - x_1} && \text{Slope Formula} \\ &= \frac{-6-(-3)}{10-(-5)} && \text{Substitute.} \\ &= \frac{-3}{15} && \text{Subtraction.} \\ &= -\frac{1}{5} && \text{Simplify.} \end{aligned}$$

Therefore, the slope of the line is $-\frac{1}{5}$.

Use the slope and one of the points to write the equation of the line in point-slope form.
The point-slope form of a line is $y - y_1 = m(x - x_1)$ where m is the slope and (x_1, y_1) is a point on the line.

Here, $m = -\frac{1}{5}$ and $(x_1, y_1) = (-5, -3)$.

So, the equation of the line is

$$\begin{aligned} y-(-3) &= -\frac{1}{5}(x-(-5)). && \text{Substitution} \\ y+3 &= -\frac{1}{5}x-1 && \text{Simplify.} \\ y+3-3 &= -\frac{1}{5}x-1-3 && -3 \text{ from each side.} \\ y &= -\frac{1}{5}x-4 && \text{Subtraction.} \end{aligned}$$

ANSWER:

$y = -\frac{1}{5}x - 4$

42. (4, –1) and (–2, –1)

SOLUTION:

Substitute the values in the slope formula.

$$\begin{aligned} m &= \frac{y_2 - y_1}{x_2 - x_1} && \text{Slope Formula} \\ &= \frac{-1-(-1)}{-2-4} && \text{Substitute.} \\ &= 0 && \text{Simplify.} \end{aligned}$$

Therefore, the slope of the line is 0.

Use the slope and one of the points to write the equation of the line in point-slope form.

The point-slope form of a line is $y - y_1 = m(x - x_1)$ where m is the slope and (x_1, y_1) is a point on the line.

Here, $m = 0$ and $(x_1, y_1) = (4, -1)$.

So, the equation of the line is

$$\begin{aligned} y-(-1) &= 0(x-4) \\ y+1 &= 0 \\ y &= -1 \end{aligned}$$

ANSWER:

$y = -1$

43. (–4, –1) and (–8, –5)

SOLUTION:

Substitute the values in the slope formula.

$$
\begin{aligned}
m &= \frac{y_2 - y_1}{x_2 - x_1} && \text{Slope Formula} \\
&= \frac{-5-(-1)}{-8-(-4)} && \text{Substitute.} \\
&= \frac{-4}{-4} && \text{Subtraction.} \\
&= 1 && \text{Simplify.}
\end{aligned}
$$

Therefore, the slope of the line is 1.
Use the slope and one of the points to write the equation of the line in point-slope form.
The point-slope form of a line is $y - y_1 = m(x - x_1)$ where m is the slope and (x_1, y_1) is a point on the line.

Here, $m = 1$ and $(x_1, y_1) = (-4, -1)$.
So, the equation of the line is

$$
\begin{aligned}
y-(-1) &= 1(x-(-4)) \\
y+1 &= x+4 \\
y &= x+3
\end{aligned}
$$

ANSWER:

$y = x + 3$

Determine the truth value of each conditional statement. If *true*, explain your reasoning. If *false*, give a counterexample.

44. If $x^2 = 25$, then $x = 5$.

SOLUTION:

The conditional statement "If $x^2 = 25$, then $x = 5$." is false. A counterexample is " If $x = -5$, $(-5)^2 = 25$". The hypothesis of the conditional is true, but the conclusion is false. This counterexample shows that the conditional statement is false.

ANSWER:

False; if $x = -5$, $(-5)^2 = 25$. The hypothesis of the conditional is true, but the conclusion is false. This counterexample shows that the conditional statement is false.

45. If you are 16, you are a junior in high school.

SOLUTION:

The conditional statement "If you are 16, you are a junior in high school." is false. A counterexample is " a 16-year-old could be a freshman, sophomore, junior, or senior". The hypothesis of the conditional is true, but the conclusion is false. This counterexample shows that the conditional statement is false.

ANSWER:

False; a 16-year-old could be a freshman, sophomore, junior, or senior. The hypothesis of the conditional is true, but the conclusion is false. This counterexample shows that the conditional statement is false.

State the property that justifies each statement.

46. $AB = AB$

SOLUTION:
Reflexive Prop.

ANSWER:
Reflexive Prop.

47. If $EF = GH$ and $GH = JK$, then $EF = JK$.

SOLUTION:
Transitive Prop.

ANSWER:
Transitive Prop.

48. If $a^2 = b^2 - c^2$, then $b^2 - c^2 = a^2$.

SOLUTION:
Symmetric Prop.

ANSWER:
Symmetric Prop.

49. If $XY + 20 = YW$ and $XY + 20 = DT$, then $YW = DT$.

SOLUTION:
Substitution Prop.

ANSWER:
Substitution Prop.