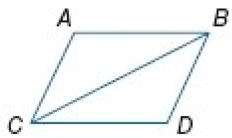
4-5 Proving Triangles Congruent - ASA, AAS

PROOF Write the specified type of proof.

1. two-column proof

Given: $\overline{CB}$ bisects $\angle ABD$ and $\angle ACD$.

Prove: $\Delta ABC \cong \Delta DBC$



SOLUTION:

Proof:

Statements (Reasons)

1. $\overline{CB}$ bisects $\angle ABD$ and $\angle ACD$. (Given)
2. $\angle$ABC $\cong$ $\angle$DBC (Definition of angular bisector)
3. $\overline{BC} \cong \overline{BC}$ (Reflexive Property)
4. $\angle ACB \cong \angle DCB$ (Definition of angular bisector)
5. $\Delta ABC \cong \Delta DBC$ (ASA)

ANSWER:

Proof:

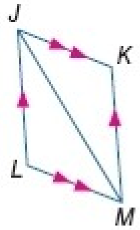
Statements (Reasons)

1. $\overline{CB}$ bisects $\angle ABD$ and $\angle ACD$. (Given)
2. $\angle$ABC $\cong$ $\angle$DBC (Def. of $\angle$ bisector)
3. $\overline{BC} \cong \overline{BC}$ (Refl. Prop.)
4. $\angle ACB \cong \angle DCB$ (Def. of $\angle$ bisector)
5. $\Delta ABC \cong \Delta DBC$ (ASA)

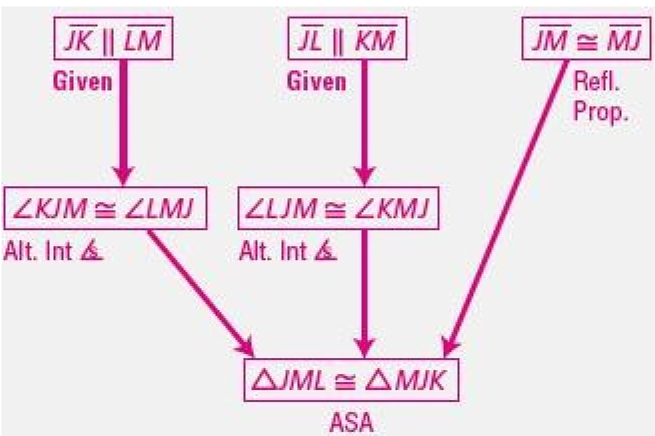
2. flow proof

Given: $\overline{JK} \parallel \overline{LM}$, $\overline{JL} \parallel \overline{KM}$

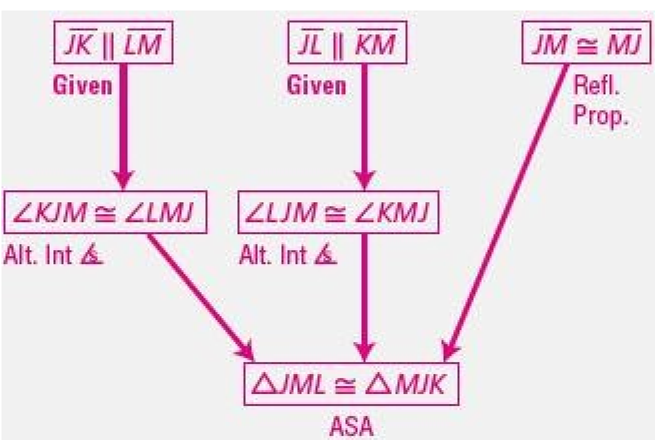
Prove: $\triangle JML \cong \triangle MJK$



SOLUTION:



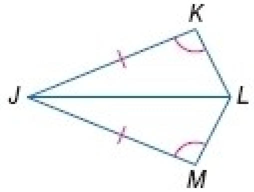
ANSWER:



3. paragraph proof

Given: $\angle K \cong \angle M$, $\overline{JK} \cong \overline{JM}$, $\overline{JL}$ bisects $\angle KLM$.

Prove: $\Delta JKL \cong \Delta JML$



SOLUTION:

Proof: We are given $\angle K \cong \angle M$, $\overline{JK} \cong \overline{JM}$, and $\overline{JL}$ bisects $\angle KLM$. Since $\overline{JL}$ bisects $\angle KLM$, we know $\angle KLJ \cong \angle MLJ$. So, $\Delta JKL \cong \Delta JML$ by the Angle-Angle-Side Congruence Theorem.

ANSWER:

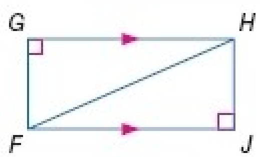
Proof: We are given $\angle K \cong \angle M$, $\overline{JK} \cong \overline{JM}$, and $\overline{JL}$ bisects $\angle KLM$. Since $\overline{JL}$ bisects $\angle KLM$, we know $\angle KLJ \cong \angle MLJ$. So, $\Delta JKL \cong \Delta JML$ by the Angle-Angle-Side Congruence Theorem.

4. two-column proof

Given: $\overline{GH} \parallel \overline{FJ}$

$m\angle G = m\angle J = 90$

Prove: $\Delta HJF \cong \Delta FGH$



SOLUTION:

Proof:

Statements (Reasons)

1. $\overline{GH} \parallel \overline{FJ}$, $m\angle G = m\angle J = 90$ (Given)
2. $\angle G \cong \angle J$ (Definition of congruent angles)
3. $\angle GHF \cong \angle JFH$ (Alternate Interior angles are congruent.)
4. $\overline{HF} \cong \overline{FH}$ (Reflexive Prop.)
5. $\Delta HJF \cong \Delta FGH$ (AAS)

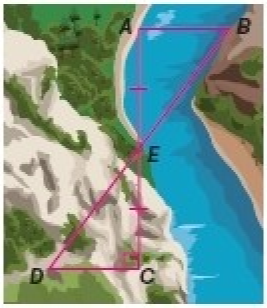
ANSWER:

Proof:

Statements (Reasons)

1. $\overline{GH} \parallel \overline{FJ}$, $m\angle G = m\angle J = 90$ (Given)
2. $\angle G \cong \angle J$ (Def. of $\cong$ $\angle$s.)
3. $\angle GHF \cong \angle JFH$ (Alt. Int. $\angle$s are $\cong$.)
4. $\overline{HF} \cong \overline{FH}$ (Ref. Prop.)
5. $\Delta HJF \cong \Delta FGH$ (AAS)

5. **BRIDGE BUILDING** A surveyor needs to find the distance from point *A* to point *B* across a canyon. She places a stake at *A*, and a coworker places a stake at *B* on the other side of the canyon. The surveyor then locates *C* on the same side of the canyon as *A* such that $\overline{CA} \perp \overline{AB}$. A fourth stake is placed at *E*, the midpoint of $\overline{CA}$. Finally, a stake is placed at *D* such that $\overline{CD} \perp \overline{CA}$ and *D*, *E*, and *B* are sited as lying along the same line.
a. Explain how the surveyor can use the triangles formed to find *AB*.
b. If *AC* = 1300 meters, *DC* = 550 meters, and *DE* = 851.5 meters, what is *AB*? Explain your reasoning.



SOLUTION:

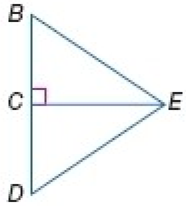
a. We know $\angle BAE$ and $\angle DCE$ are congruent because they are both right angles. $\overline{AE}$ is congruent to $\overline{EC}$ by the Midpoint Theorem. From the Vertical Angles Theorem, $\angle DEC \cong \angle BEA$. By ASA, the surveyor knows that $\Delta DCE \cong \Delta BAE$. By CPCTC, $\overline{DC} \cong \overline{AB}$, so the surveyor can measure $\overline{DC}$ and know the distance between *A* and *B*.
b. Since *DC* = 550 m and $\overline{DC} \cong \overline{AB}$, then by the definition of congruence, *AB* = 550 m.

ANSWER:

a. We know $\angle BAE$ and $\angle DCE$ are congruent because they are both right angles. $\overline{AE}$ is congruent to $\overline{EC}$ by the Midpoint Theorem. From the Vertical Angles Theorem, $\angle DEC \cong \angle BEA$. By ASA, the surveyor knows that $\Delta DCE \cong \Delta BAE$. By CPCTC, $\overline{DC} \cong \overline{AB}$, so the surveyor can measure $\overline{DC}$ and know the distance between *A* and *B*.
b. 550 m; Since *DC* = 550 m and $\overline{DC} \cong \overline{AB}$, then by the definition of congruence, *AB* = 550 m.

PROOF Write a paragraph proof.

6. **Given:** $\overline{CE}$ bisects *BED*; $\angle BCE$ and $\angle ECD$ are right angles.
Prove: $\Delta ECB \cong \Delta ECD$



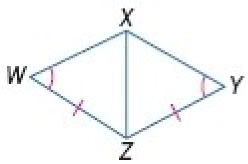
SOLUTION:

Proof: We are given that $\overline{CE}$ bisects $\angle BED$ and $\angle BCE$ and $\angle ECD$ are right angles.
Since all right angles are congruent, $\angle BCE \cong \angle ECD$. By the definition of angle bisector, $\angle BEC \cong \angle DEC$. The Reflexive Property tells us that $\overline{EC} \cong \overline{EC}$.
By Angle-Side-Angle Congruence Postulate, $\Delta ECB \cong \Delta ECD$.

ANSWER:

Proof: We are given that $\overline{CE}$ bisects $\angle BED$ and $\angle BCE$ and $\angle ECD$ are right angles. Since all right angles are congruent, $\angle BCE \cong \angle ECD$. By the definition of angle bisector, $\angle BEC \cong \angle DEC$. The Reflexive Property tells us that $\overline{EC} \cong \overline{EC}$. By Angle-Side-Angle Congruence Postulate, $\Delta ECB \cong \Delta ECD$.

7. **Given:** $\angle W \cong \angle Y$, $\overline{WZ} \cong \overline{YZ}$, $\overline{XZ}$ bisects $\angle WZY$.
Prove: $\Delta XWZ \cong \Delta XYZ$



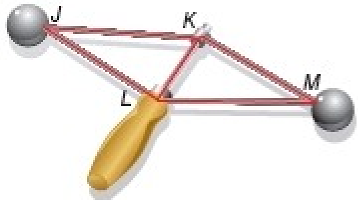
SOLUTION:

Proof: It is given that $\angle W \cong \angle Y$, $\overline{WZ} \cong \overline{YZ}$, and $\overline{XZ}$ bisects $\angle WZY$. By the definition of angle bisector, $\angle WZX \cong \angle YZX$.
The Angle-Side-Angle Congruence Postulate tells us that $\Delta XWZ \cong \Delta XYZ$.

ANSWER:

Proof: It is given that $\angle W \cong \angle Y$, $\overline{WZ} \cong \overline{YZ}$, and $\overline{XZ}$ bisects $\angle WZY$. By the definition of angle bisector, $\angle WZX \cong \angle YZX$. The Angle-Side-Angle Congruence Postulate tells us that $\Delta XWZ \cong \Delta XYZ$.

8. **TOYS** The object of the toy shown is to make the two spheres meet and strike each other repeatedly on one side of the wand and then again on the other side. If $\angle JKL \cong \angle MLK$ and $\angle JLK \cong \angle MKL$, prove that $\overline{JK} \cong \overline{ML}$.



SOLUTION:

Proof:

Statements (Reasons)

1. $\angle JKL \cong \angle MLK$, $\angle JLK \cong \angle MKL$ (Given)
2. $\overline{KL} \cong \overline{KL}$ (Reflexive Prop.)
3. $\Delta KLM \cong \Delta LKJ$ (ASA)
4. $\overline{JK} \cong \overline{ML}$ (CPCTC)

ANSWER:

Proof:

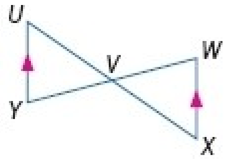
Statements (Reasons)

1. $\angle JKL \cong \angle MLK$, $\angle JLK \cong \angle MKL$ (Given)
2. $\overline{KL} \cong \overline{KL}$ (Refl. Prop.)
3. $\Delta KLM \cong \Delta LKJ$ (ASA)
4. $\overline{JK} \cong \overline{ML}$ (CPCTC)

PROOF Write a two-column proof.

9. **Given:** *V* is the midpoint of $\overline{YW}$;
$\overline{UY} \parallel \overline{XW}$.

Prove: $\Delta UVY \cong \Delta XVW$



SOLUTION:

Proof:

Statements (Reasons)

1. *V* is the midpoint of $\overline{YW}$; $\overline{UY} \parallel \overline{XW}$. (Given)
2. $\overline{YV} \cong \overline{VW}$ (Midpoint Theorem)
3. $\angle VWX \cong \angle VYU$ (Alternate Interior Angles Theorem)
4. $\angle VUY \cong \angle VXW$ (Alternate Interior Angles Theorem)
5. $\Delta UVY \cong \Delta XVW$ (AAS)

ANSWER:

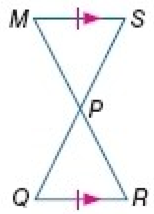
Proof:

Statements (Reasons)

1. *V* is the midpoint of $\overline{YW}$; $\overline{UY} \parallel \overline{XW}$. (Given)
2. $\overline{YV} \cong \overline{VW}$ (Midpoint Theorem)
3. $\angle VWX \cong \angle VYU$ (Alt. Int. $\angle$ Thm.)
4. $\angle VUY \cong \angle VXW$ (Alt. Int. $\angle$ Thm.)
5. $\Delta UVY \cong \Delta XVW$ (AAS)

10. **Given:** $\overline{MS} \cong \overline{RQ}, \overline{MS} \parallel \overline{RQ}$

Prove: $\Delta MSP \cong \Delta RQP$



SOLUTION:

Proof:

Statements (Reasons)

1. $\overline{MS} \cong \overline{RQ}, \overline{MS} \parallel \overline{RQ}$ (Given)
2. $\angle SPM \cong \angle QPR$ (Vertical angles are congruent.)
3. $\angle SMP \cong \angle QRP$ (Alternate Interior Angle Theorem)
4. $\Delta MSP \cong \Delta RQP$ (AAS)

ANSWER:

Proof:

Statements (Reasons)

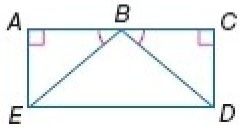
1. $\overline{MS} \cong \overline{RQ}, \overline{MS} \parallel \overline{RQ}$ (Given)
2. $\angle SPM \cong \angle QPR$ (Vert. $\angle$s are $\cong$.)
3. $\angle SMP \cong \angle QRP$ (Alt. Int. $\angle$ Thm.)
4. $\Delta MSP \cong \Delta RQP$ (AAS)

11. **CCSS ARGUMENTS** Write a flow proof.

Given: $\angle A$ and $\angle C$ are right angles.

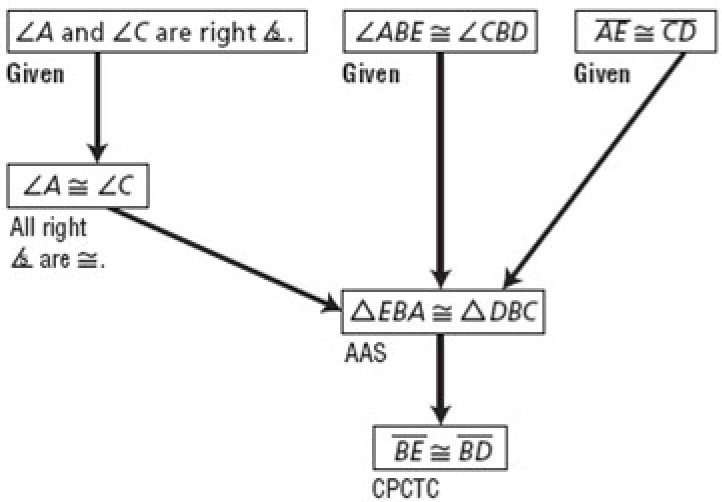
$\angle ABE \cong \angle CBD$, $\overline{AE} \cong \overline{CD}$

Prove: $\overline{BE} \cong \overline{BD}$



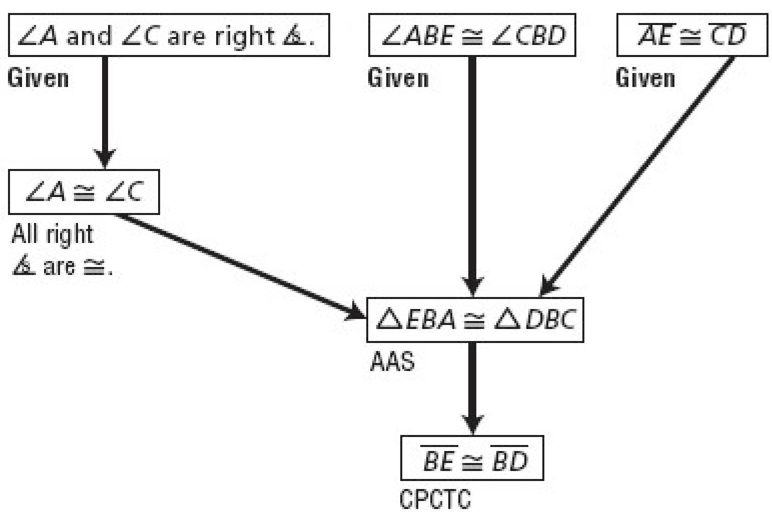
SOLUTION:

Proof:



ANSWER:

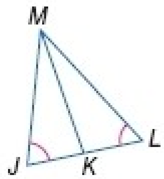
Proof:



12. **PROOF** Write a flow proof.

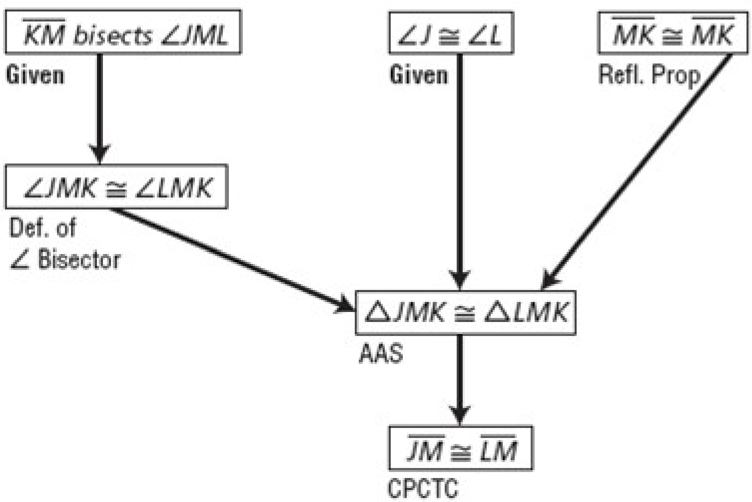
Given: $\overline{KM}$ bisects $\angle JML$; $\angle J \cong \angle L$.

Prove: $\overline{JM} \cong \overline{LM}$



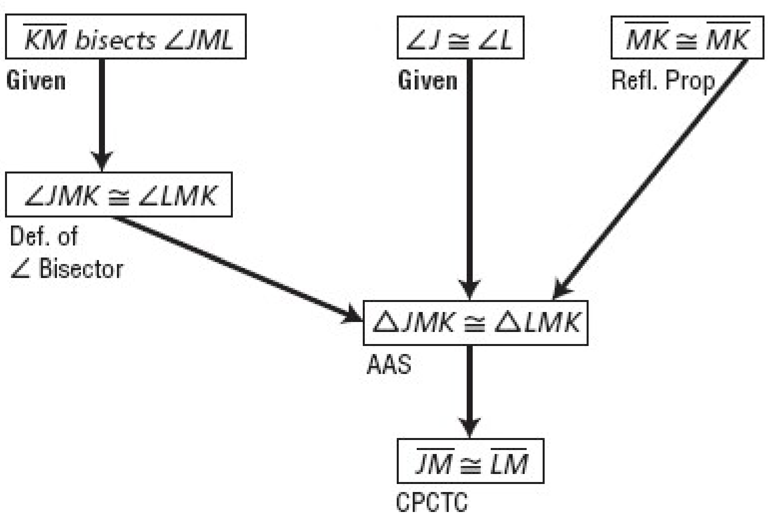
SOLUTION:

Proof:

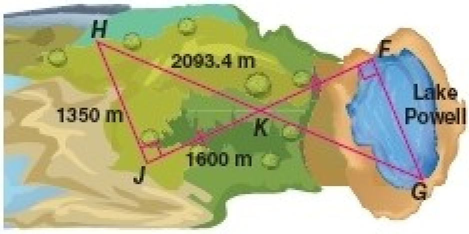


ANSWER:

Proof:



13. **CCSS MODELING** A high school wants to hold a 1500-meter regatta on Lake Powell but is unsure if the lake is long enough. To measure the distance across the lake, the crew members locate the vertices of the triangles below and find the measures of the lengths of ΔHJK as shown below.



a. Explain how the crew team can use the triangles formed to estimate the distance FG across the lake.
b. Using the measures given, is the lake long enough for the team to use as the location for their regatta? Explain your reasoning.

SOLUTION:

a. $\angle HJK \cong \angle GFK$ since all right angles are congruent.

We are given that $\overline{JK} \cong \overline{KF}$. $\angle HKJ$ and $\angle FKG$ are vertical angles, so $\angle HKJ \cong \angle FKG$ by the Vertical Angles Theorem.

By ASA, $\Delta HJK \cong \Delta GFK$, so $\overline{FG} \cong \overline{HJ}$ by CPCTC.

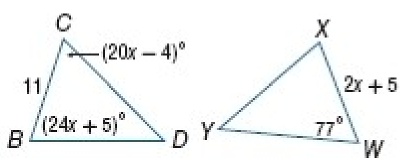
b. Since $\overline{FG} \cong \overline{HJ}$ and $HJ = 1350$, $FG = 1350$.
If the regatta is to be 1500 m, the lake is not long enough, since 1350 < 1500.

ANSWER:

a. $\angle HJK \cong \angle GFK$ since all right angles are congruent. We are given that $\overline{JK} \cong \overline{KF}$. $\angle HKJ$ and $\angle FKG$ are vertical angles, so $\angle HKJ \cong \angle FKG$ by the Vertical Angles Theorem. By ASA, $\Delta HJK \cong \Delta GFK$, so $\overline{FG} \cong \overline{HJ}$ by CPCTC.
b. No; $HJ = 1350$ m, so $FG = 1350$ m. If the regatta is to be 1500 m, the lake is not long enough, since 1350 < 1500**.**

ALGEBRA Find the value of the variable that yields congruent triangles.

14. $\Delta BCD \cong \Delta WXY$



SOLUTION:

Since $\Delta BCD \cong \Delta WXY$, the corresponding sides are congruent. Therefore, $\overline{BC} \cong \overline{WX}$. By the definition of congruence, $BC = WX$.

$$11 = 2x + 5 \quad BC = WX$$

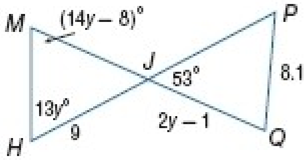
$$2x = 6 \quad \text{Simplify.}$$

$$x = 3 \quad \text{Divide each side by 2.}$$

ANSWER:

$x = 3$

15. $\Delta MHJ \cong \Delta PQJ$



SOLUTION:

Since $\Delta MHJ \cong \Delta PQJ$, the corresponding sides are congruent. Therefore, $\overline{HJ} \cong \overline{QJ}$. By the definition of congruence, *HJ* = *QJ*.

$9 = 2y - 1$ $HJ = QJ$

$2y = 10$ Simplify.

$y = 5$ Divide each side by 2.

ANSWER:

y = 5

16. **THEATER DESIGN** The trusses of the roof of the outdoor theater shown below appear to be several different pairs of congruent triangles. Assume that trusses that appear to lie on the same line actually lie on the same line. **Refer to the figure on page 280.**

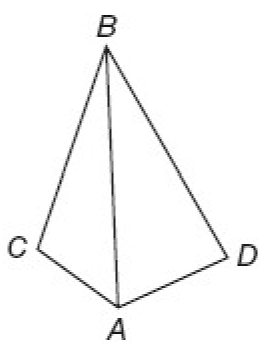
a. If $\overline{AB}$ bisects $\angle CBD$ and $\angle CAD$, prove that $\Delta ABC \cong \Delta ABD$.

b. If $\Delta ABC \cong \Delta ABD$ and $\angle$FCA $\cong$ $\angle$EDA, prove that $\Delta CAF \cong \Delta DAE$.

c. If $\overline{HB} \cong \overline{EB}$, $\angle BHG \cong \angle BEA$, $\angle HGJ \cong \angle EAD$, and $\angle JGB \cong \angle DAB$, prove that $\Delta BHG \cong \Delta BEA$.

SOLUTION:

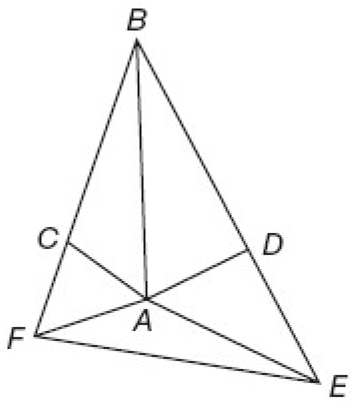
a. Given: $\overline{AB}$ bisects $\angle CBD$ and $\angle CAD$.
Prove: $\Delta ABC \cong \Delta ABD$.



Proof:
<u>Statements (Reasons)</u>

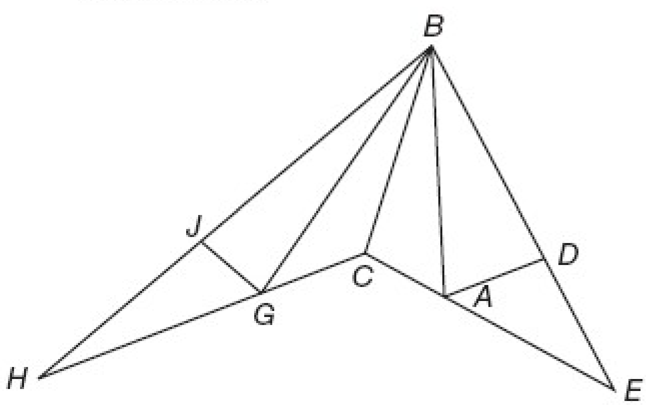
1. $\overline{AB}$ bisects $\angle CBD$ and $\angle CAD$. (Given)
2. $\angle ABC \cong \angle ABD$, $\angle CAB \cong \angle DAB$ (Definition of bisect)
3. $\overline{AB} \cong \overline{AB}$ (Reflexive Property)
4. $\Delta ABC \cong \Delta ABD$ (ASA)

b. Given: $\Delta ABC \cong \Delta ABD$, $\angle FCA \cong \angle EDA$
Prove: $\Delta CAF \cong \Delta DAE$.



Proof:
Statements (Reasons)
1. $\triangle ABC \cong \triangle ABD$, $\angle FCA \cong \angle EDA$ (Given)
2. $\overline{CA} \cong \overline{DA}$ (CPCTC)
3. $\angle CAF \cong \angle DAE$ (Vertically opposite angles are congruent.)
4. $\triangle CAF \cong \triangle DAE$ (ASA)

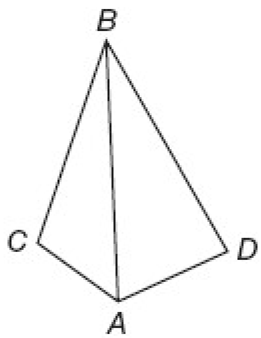
c. Given: $\overline{HB} \cong \overline{EB}$, $\angle BHG \cong \angle BEA$, $\angle HGJ \cong \angle EAD$, $\angle JGB \cong \angle DAB$
Prove: $\triangle BHG \cong \triangle BEA$



Proof:
Statements (Reasons)
1. $\overline{HB} \cong \overline{EB}$, $\angle BHG \cong \angle BEA$, $\angle HGJ \cong \angle EAD$, $\angle JGB \cong \angle DAB$ (Given)
2. $m\angle HGJ = m\angle EAD$, $m\angle JGB = m\angle DAB$ (Definition of Congruence)
3. $m\angle HGJ + m\angle JGB = m\angle HGB$, $m\angle EAD + m\angle DAB = m\angle EAB$ (Addition Property of Equality)
4. $m\angle EAD + m\angle DAB = m\angle HGB$, $m\angle EAD + m\angle DAB = m\angle EAB$ (Angle Addition Postulate)
5. $m\angle HGB = m\angle EAB$ (Substitution)
6. $\angle HGB \cong \angle EAB$ (Definition of congruence)
7. $\triangle BHG \cong \triangle BEA$ (AAS)

ANSWER:

a. Given: $\overline{AB}$ bisects $\angle CBD$ and $\angle CAD$.
Prove: $\triangle ABC \cong \triangle ABD$.



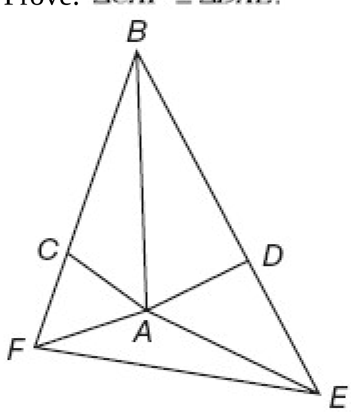
Proof:

Statements (Reasons)

1. $\overline{AB}$ bisects $\angle CBD$ and $\angle CAD$. (Given)
2. $\angle ABC \cong \angle ABD$, $\angle CAB \cong \angle DAB$ (Def. of bisect)
3. $\overline{AB}$ (Refl. Prop.)
4. $\Delta ABC \cong \Delta ABD$ (ASA**)**

b. Given: $\Delta ABC \cong \Delta ABD$, $\angle FCA \cong \angle EDA$

Prove: $\Delta CAF \cong \Delta DAE$.



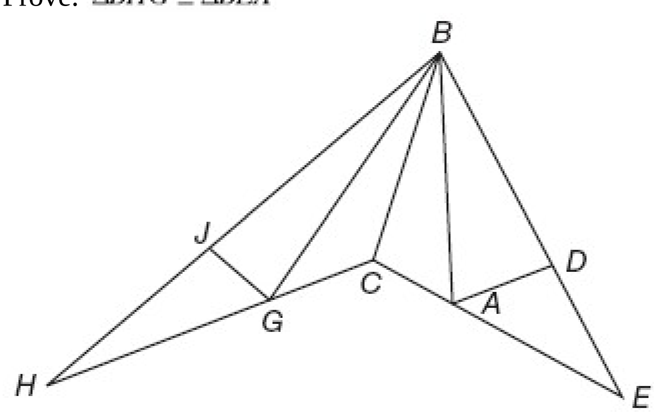
Proof:

Statements (Reasons)

1. $\Delta ABC \cong \Delta ABD$, $\angle FCA \cong \angle EDA$ (Given)
2. $\overline{CA} \cong \overline{DA}$ (CPCTC)
3. $\angle CAF \cong \angle DAE$ (Vert. $\angle$s are $\cong$.)
4. $\Delta CAF \cong \Delta DAE$ (ASA)

c. Given: $\overline{HB} \cong \overline{EB}$, $\angle BHG \cong \angle BEA$, $\angle HGJ \cong \angle EAD$, $\angle JGB \cong \angle DAB$

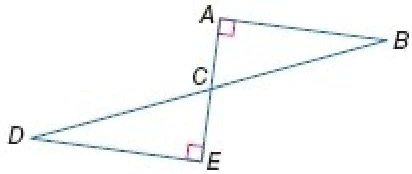
Prove: $\Delta BHG \cong \Delta BEA$



Proof:
Statements (Reasons)
1. $\overline{HB} \cong \overline{EB}$, $\angle BHG \cong \angle BEA$, $\angle HGJ \cong \angle EAD$, $\angle JGB \cong \angle DAB$ (Given)
2. $m\angle HGJ = m\angle EAD$, $m\angle JGB = m\angle DAB$ (Def. of $\cong$)
3. $m\angle HGJ + m\angle JGB = m\angle HGB$, $m\angle EAD + m\angle DAB = m\angle EAB$ (Add Prop of =)
4. $m\angle EAD + m\angle DAB = m\angle HGB$, $m\angle EAD + m\angle DAB = m\angle EAB$ (Angle Add. Post.)
5. $m\angle HGB = m\angle EAB$ (Subst.)
6. $\angle HGB \cong \angle EAB$ (Def. of congruence)
7. $\Delta BHG \cong \Delta BEA$ (AAS)

PROOF Write a paragraph proof.

17. **Given:** $\overline{AE} \perp \overline{DE}, \overline{EA} \perp \overline{AB}$, *C* is the midpoint of $\overline{AE}$.
Prove: $\overline{CD} \cong \overline{CB}$



SOLUTION:

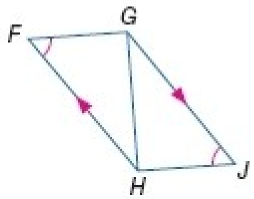
Proof: We are given that $\overline{AE}$ is perpendicular to $\overline{DE}$, $\overline{EA}$ is perpendicular to $\overline{AB}$, and *C* is the midpoint of $\overline{AE}$. Since $\overline{AE}$ is perpendicular to $\overline{DE}$, $m\angle CED = 90$. Since $\overline{EA}$ is perpendicular to $\overline{AB}$, $m\angle BAC = 90$. $\angle CED \cong \angle BAC$ because all right angles are congruent. $\overline{AC} \cong \overline{CE}$ from the Midpoint Theorem. $\angle ECD \cong \angle ACB$ because they are vertical angles. Angle-Side-Angle postulate gives us that $\Delta CED \cong \Delta CAB$. $\overline{CD} \cong \overline{CB}$ because corresponding parts of congruent triangles are congruent.

ANSWER:

Proof: We are given that $\overline{AE}$ is perpendicular to $\overline{DE}$, $\overline{EA}$ is perpendicular to $\overline{AB}$, and *C* is the midpoint of $\overline{AE}$ Since $\overline{AE}$ is perpendicular to $\overline{DE}$, $m\angle CED = 90$. Since $\overline{EA}$ is perpendicular to $\overline{AB}$, $m\angle BAC = 90$. $\angle CED \cong \angle BAC$ because all right angles are congruent. $\overline{AC} \cong \overline{CE}$ from the Midpt. Thm. $\angle ECD \cong \angle ACB$ because they are vertical angles. Angle-Side-Angle gives us that $\Delta CED \cong \Delta CAB$. $\overline{CD} \cong \overline{CB}$ because corresponding parts of congruent triangles are congruent.

18. **Given:** $\angle F \cong \angle J$, $\overline{FH} \parallel \overline{GJ}$

Prove: $\overline{FH} \cong \overline{JG}$



SOLUTION:

Proof: $\angle F \cong \angle J$ and $\overline{FH} \parallel \overline{GJ}$ because it is given.

$\angle FHG \cong \angle JGH$ because they are alternate interior angles.

By the Reflexive Property, $\overline{GH} \cong \overline{GH}$.

So $\Delta GHJ \cong \Delta HGF$ by the Angle-Angle-Side postulate. Then $\overline{FH} \cong \overline{JG}$ since corresponding parts of congruent triangles are congruent.

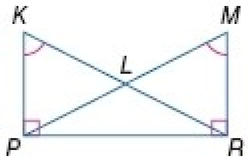
ANSWER:

Proof: $\angle F \cong \angle J$ and $\overline{FH} \parallel \overline{GJ}$ because it is given. $\angle FHG \cong \angle JGH$ because they are alternate interior angles. By the Reflexive Property, $\overline{GH} \cong \overline{GH}$. So $\Delta GHJ \cong \Delta HGF$ by the Angle-Angle-Side postulate. Then $\overline{FH} \cong \overline{JG}$ since corresponding parts of congruent triangles are congruent.

PROOF Write a two-column proof.

19. **Given:** $\angle K \cong \angle M$, $\overline{KP} \perp \overline{PR}$, $\overline{MR} \perp \overline{PR}$

Prove: $\angle KPL \cong \angle MRL$



SOLUTION:

Proof:

Statements (Reasons)

1. $\angle K \cong \angle M$, $\overline{KP} \perp \overline{PR}$, $\overline{MR} \perp \overline{PR}$ (Given)
2. $\angle KPR$ and $\angle MRP$ are both right angles. (Definition of Perpendicular lines.)
3. $\angle KPR \cong \angle MRP$ (All right angles are congruent.)
4. $\overline{PR} \cong \overline{PR}$ (Reflexive Prop.)
5. $\Delta KPR \cong \Delta MRP$ (AAS)
6. $\overline{KP} \cong \overline{MR}$ (CPCTC)
7. $\angle KLP \cong \angle MLR$ (Vertical angles are congruent.)
8. $\Delta KLP \cong \Delta MLR$ (AAS)
9. $\angle KPL \cong \angle MRL$ (CPCTC)

ANSWER:

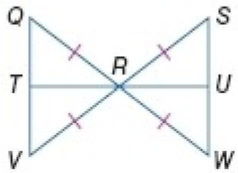
Proof:

Statements (Reasons)

1. $\angle K \cong \angle M$, $\overline{KP} \perp \overline{PR}$, $\overline{MR} \perp \overline{PR}$ (Given)
2. $\angle KPR$ and $\angle MRP$ are both right angles. (Def. of $\perp$)
3. $\angle KPR \cong \angle MRP$ (All rt. $\angle$s are congruent.)
4. $\overline{PR} \cong \overline{PR}$ (Refl. Prop.)
5. $\Delta KPR \cong \Delta MRP$ (AAS)
6. $\overline{KP} \cong \overline{MR}$ (CPCTC)
7. $\angle KLP \cong \angle MLR$ (Vertical angles are $\cong$.)
8. $\Delta KLP \cong \Delta MLR$ (AAS)
9. $\angle KPL \cong \angle MRL$ (CPCTC)

20. **Given:** $\overline{QR} \cong \overline{SR} \cong \overline{WR} \cong \overline{VR}$

Prove: $\overline{QT} \cong \overline{WU}$



SOLUTION:

Proof:

Statements (Reasons)

1. $\overline{QR} \cong \overline{SR} \cong \overline{WR} \cong \overline{VR}$ (Given)
2. $\angle QRV \cong \angle SRW$ (Vertical angles are congruent.)
3. $\Delta VRQ \cong \Delta SRW$ (SAS)
4. $\angle VQR \cong \angle SWR$ (CPCTC)
5. $\angle QRT \cong \angle URW$ (Vertical angles are congruent.)
6. $\Delta URW \cong \Delta TRQ$ (ASA)
7. $\overline{QT} \cong \overline{WU}$ (CPCTC)

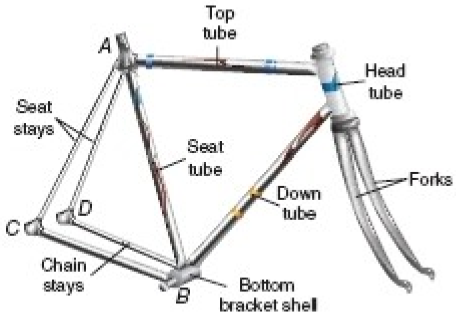
ANSWER:

Proof:

Statements (Reasons)

1. $\overline{QR} \cong \overline{SR} \cong \overline{WR} \cong \overline{VR}$ (Given)
2. $\angle QRV \cong \angle SRW$ (Vert. $\angle$s are $\cong$.)
3. $\Delta VRQ \cong \Delta SRW$ (SAS)
4. $\angle VQR \cong \angle SWR$ (CPCTC)
5. $\angle QRT \cong \angle URW$ (Vert. $\angle$s are $\cong$.)
6. $\Delta URW \cong \Delta TRQ$ (ASA)
7. $\overline{QT} \cong \overline{WU}$ (CPCTC)

21. **FITNESS** The seat tube of a bicycle forms a triangle with each seat and chain stay as shown. If each seat stay makes a 44° angle with its corresponding chain stay and each chain stay makes a 68° angle with the seat tube, show that the two seat stays are the same length.



SOLUTION:

Proof:

Statements (Reasons)

1. $m\angle ACB = 44$, $m\angle ADB = 44$, $m\angle CBA = 68$, $m\angle DBA = 68$ (Given)
2. $m\angle ACB = m\angle ADB$, $m\angle CBA = m\angle DBA$ (Substitution)
3. $\angle ACB \cong \angle ADB$, $\angle CBA \cong \angle DBA$ (Definition of congruence)
4. $\overline{AB} \cong \overline{AB}$ (Reflexive Property)
5. $\Delta ADB \cong \Delta ACB$ (AAS)
6. $\overline{AC} \cong \overline{AD}$ (CPCTC)

ANSWER:

Proof:

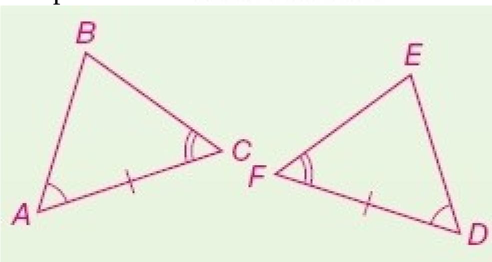
Statements (Reasons)

1. $m\angle ACB = 44$, $m\angle ADB = 44$, $m\angle CBA = 68$, $m\angle DBA = 68$ (Given)
2. $m\angle ACB = m\angle ADB$, $m\angle CBA = m\angle DBA$ (Subst.)
3. $\angle ACB \cong \angle ADB$, $\angle CBA \cong \angle DBA$ (Def. of $\cong$)
4. $\overline{AB} \cong \overline{AB}$ (Refl. Prop.)
5. $\Delta ADB \cong \Delta ACB$ (AAS)
6. $\overline{AC} \cong \overline{AD}$ (CPCTC)

22. **OPEN ENDED** Draw and label two triangles that could be proved congruent by ASA.

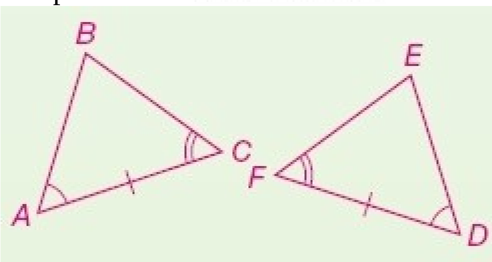
SOLUTION:

Sample answer: $\Delta ABC \cong \Delta DEF$

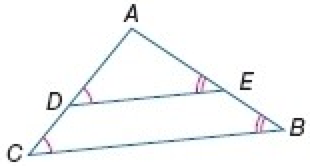


ANSWER:

Sample answer: $\Delta ABC \cong \Delta DEF$



23. **CCSS CRITIQUE** Tyrone says it is not possible to show that $\Delta ADE \cong \Delta ACB$. Lorenzo disagrees, explaining that since $\angle ADE \cong \angle ACB$, and $\angle A \cong \angle A$ by the Reflexive Property, $\Delta ADE \cong \Delta ACB$. Is either of them correct? Explain.



SOLUTION:

Tyrone is correct.
Lorenzo showed that all three corresponding angles were congruent, but AAA is not a proof of triangle congruence.

ANSWER:

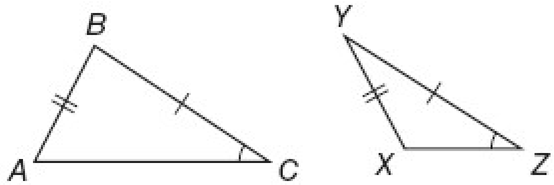
Tyrone; Lorenzo showed that all three corresponding angles were congruent, but AAA is not a proof of triangle congruence.

24. **REASONING** Find a counterexample to show why SSA (Side-Side-Angle) cannot be used to prove the congruence of two triangles.

SOLUTION:

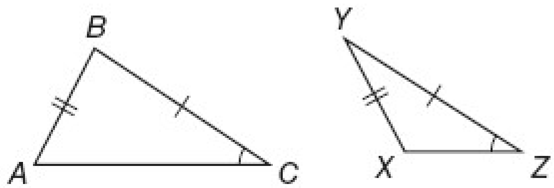
Sample answer: To find a counterexample, show a set of triangles with corresponding SSA congruence and then show that at least one pair of the other 2 corresponding angles are not congruent. If SSA was a valid congruence theorem, then each pair of corresponding angles would be congruent.

Consider triangles *ABC* and *XYZ*. $\overline{AB} \cong \overline{XY}$, $\overline{BC} \cong \overline{YZ}$, $\angle C \cong \angle Z$. However, $\angle X \ncong \angle A$. $\angle X$ is obtuse while $\angle A$ is acute so $\Delta ABC \ncong \Delta XYZ$.

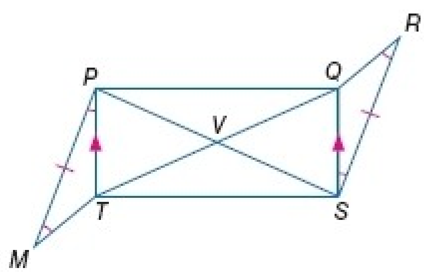


ANSWER:

Sample answer: $\overline{AB} \cong \overline{XY}$, $\overline{BC} \cong \overline{YZ}$, $\angle C \cong \angle Z$. $\Delta ABC \ncong \Delta XYZ$.

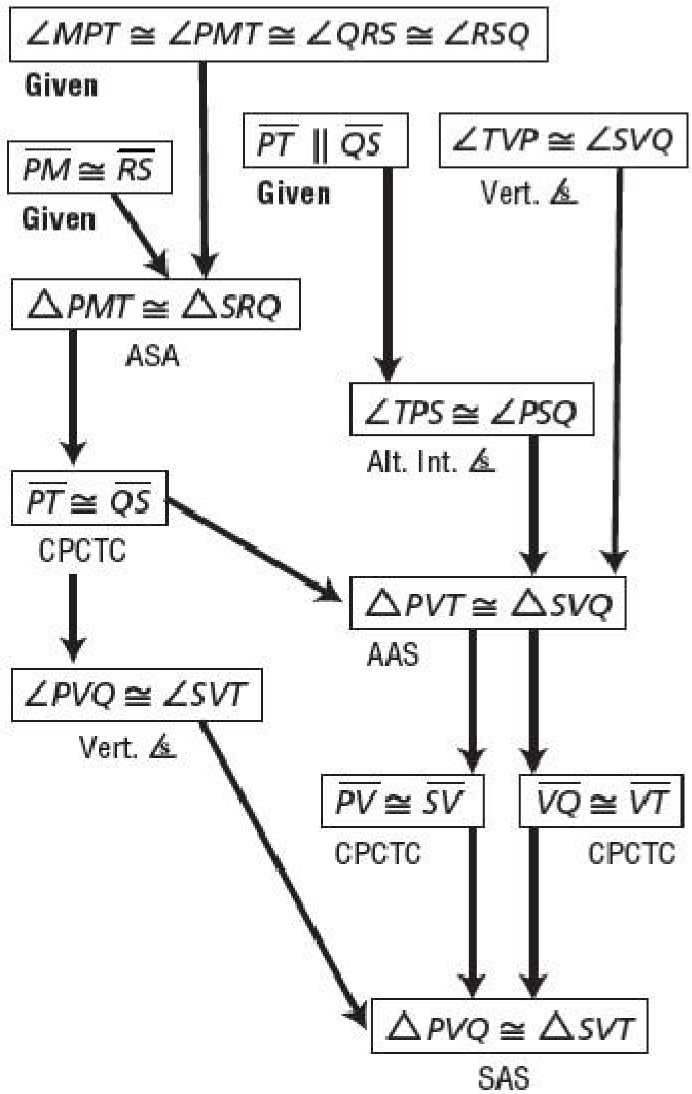


25. **CHALLENGE** Using the information given in the diagram, write a flow proof to show that $\Delta PVQ \cong \Delta SVT$.

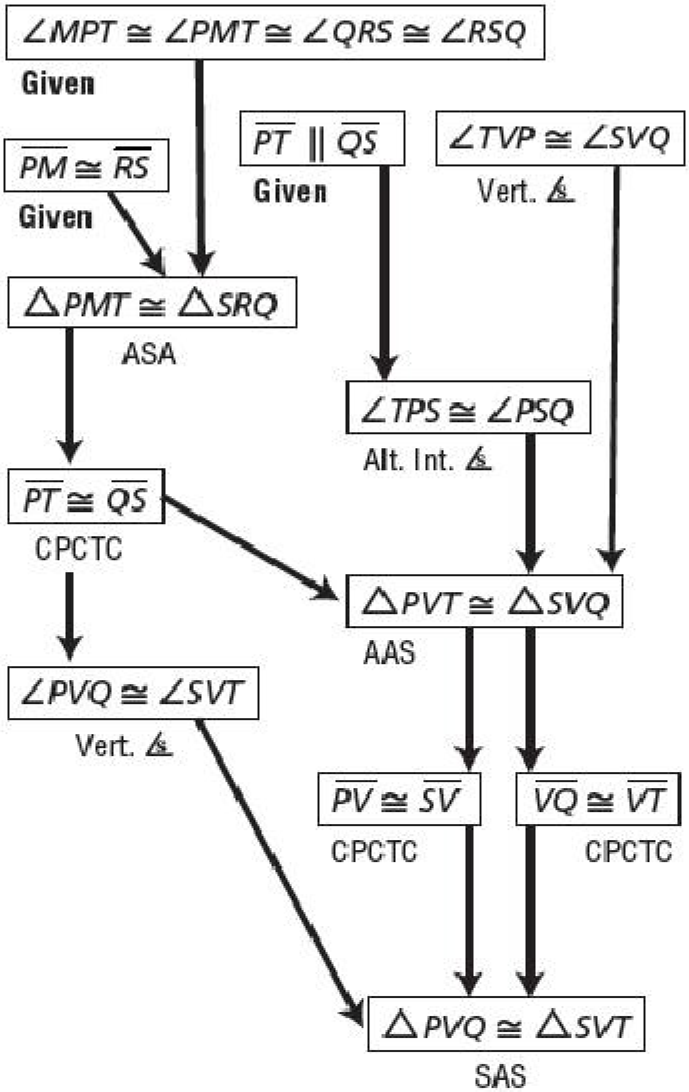


SOLUTION:

Proof:



ANSWER:
Proof:



26. **WRITING IN MATH** Summarize the methods described in Lessons 4-3, 4-4, and 4-5 for proving triangle congruence into a chart that explains when to use each method.

SOLUTION:

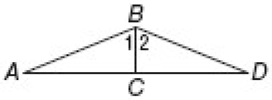
4-5 Proving Triangles Congruent - ASA, AAS

Method	Use when…
Definition of Congruent Triangles	All corresponding parts of one triangle are congruent to the corresponding parts of the other triangle.
SSS	The three sides of one triangle must be congruent to the three sides of the other triangle.
SAS	Two sides and the included angle of one triangle must be congruent to two sides and the included angle of the other triangle.
ASA	Two angles and the included side of one triangle must be congruent to two angles and the included side of the other triangle.
AAS	Two angles and a nonincluded side of one triangle must be congruent to two angles and the corresponding nonincluded side of the other triangle.

ANSWER:

Method	Use when…
Definition of Congruent Triangles	All corresponding parts of one triangle are congruent to the corresponding parts of the other triangle.
SSS	The three sides of one triangle must be congruent to the three sides of the other triangle.
SAS	Two sides and the included angle of one triangle must be congruent to two sides and the included angle of the other triangle.
ASA	Two angles and the included side of one triangle must be congruent to two angles and the included side of the other triangle.
AAS	Two angles and a nonincluded side of one triangle must be congruent to two angles and the corresponding nonincluded side of the other triangle.

27. Given: $\overline{BC}$ is perpendicular to $\overline{AD}$; $\angle 1 \cong \angle 2$.



Which theorem or postulate could be used to prove $\Delta ABC \cong \Delta DBC$?

A AAS
B ASA
C SAS
D SSS

SOLUTION:

Given: $\angle 1 \cong \angle 2; \overline{BC} \perp \overline{AD}.$

By the definition of perpendicular lines, $m\angle BCA = m\angle BCD = 90.$ That is, $\angle BCA \cong \angle BCD.$

And $\overline{BC} \cong \overline{BC},$ by the Reflexive property.

Therefore, by ASA postulate, $\Delta ABC \cong \Delta DBC.$
The correct choice is B.

ANSWER:

B

28. **SHORT RESPONSE** Write an expression that can be used to find the values of $s(n)$ in the table.

n	−8	−4	−1	0	1
$s(n)$	1.00	2.00	2.75	3.00	3.25

SOLUTION:

Is the expression linear? To check, confirm if there is a constant rate of increase (slope).

As the input value n increases from -1 to 0, the output value $s(n)$ increases from 2.75 to 3.00.
As the input value n increases from 0 to , the output value $s(n)$ increases from 3.00 to 3.25.

We see a constant increase of 0.25 for every increase in input by 1. The same constant can be found when testing other values of n.

This constant increase of 0.25 is equivalent to a slope m of $\frac{1}{4}$. We now know the expression is linear. The y-intercept is at (0, 3.00), so the value of b in $mx + b$ is 3

Therefore, the expression for $s(n)$ could be $\frac{1}{4}n + 3.$

ANSWER:

$\frac{1}{4}n + 3.$

29. **ALGEBRA** If –7 is multiplied by a number greater than 1, which of the following describes the result?
F a number greater than 7
G a number between –7 and 7
H a number greater than –7
J a number less than –7

SOLUTION:
When a number greater than 1 is multiplied with –7, we get a negative number less than –7.

For example:

$4 \times -7 = -28$
$-28 < -7$

$2 \times -7 = -14$
$-14 < -7$

$1.1 \times -7 = -7.7$
$-7.7 < -7$
The correct choice is J.

ANSWER:
J

30. **SAT/ACT** $\sqrt{121+104} = ?$
A 15
B 21
C 25
D 125
E 225

SOLUTION:
$$\sqrt{121+104} = \sqrt{225} \quad \text{Simplify.}$$
$$= 15 \quad \text{Take the square root.}$$
The correct choice is A.

ANSWER:
A

Determine whether $\triangle ABC \cong \triangle XYZ$. Explain.

31. *A*(6, 4), *B*(1, –6), *C*(–9, 5), *X*(0, 7), *Y*(5, –3), *Z*(15, 8)

SOLUTION:

Use the distance formula to find the length of each side of the triangles.
The side lengths of the triangle *ABC* are:

$AB = \sqrt{(1-6)^2 + (-6-4)^2} = \sqrt{125};$

$BC = \sqrt{(-9-1)^2 + (5+6)^2} = \sqrt{221};$

$CA = \sqrt{(-9-6)^2 + (5-4)^2} = \sqrt{226}$

The side lengths of the triangle *XYZ* are:

$XY = \sqrt{(5-0)^2 + (-3-7)^2} = \sqrt{125};$

$YZ = \sqrt{(15-5)^2 + (8+3)^2} = \sqrt{221};$

$ZX = \sqrt{(0-15)^2 + (7-8)^2} = \sqrt{226}$

$\overline{AB} \cong \overline{XY}; \overline{BC} \cong \overline{YZ}; AC \cong \overline{ZX}$

The corresponding sides have the same measure and are congruent. $\triangle ABC \cong \triangle XYZ$ by SSS.

ANSWER:

AB = $\sqrt{125}$, *BC* = $\sqrt{221}$, *AC* = $\sqrt{226}$, *XY* = $\sqrt{125}$, *YZ* = $\sqrt{221}$, *XZ* = $\sqrt{226}$. The corresponding sides have the same measure and are congruent. $\triangle ABC \cong \triangle XYZ$ by SSS.

32. $A(0, 5)$, $B(0, 0)$, $C(–2, 0)$, $X(4, 8)$, $Y(4, 3)$, $Z(6, 3)$

SOLUTION:

Use the distance formula to find the length of each side of the triangles.
The side lengths of the triangle *ABC* are:

$$AB = \sqrt{(0-0)^2 + (0-5)^2} = 5;$$
$$BC = \sqrt{(-2-0)^2 + (0-0)^2} = 2;$$
$$CA = \sqrt{(-2-0)^2 + (0-5)^2} = \sqrt{29};$$

The side lengths of the triangle *XYZ* are:

$$XY = \sqrt{(4-4)^2 + (3-8)^2} = 5;$$
$$YZ = \sqrt{(6-4)^2 + (3-3)^2} = 2;$$
$$ZX = \sqrt{(6-4)^2 + (3-8)^2} = \sqrt{29};$$

$$\overline{AB} \cong \overline{XY}; \overline{BC} \cong \overline{YZ}; AC \cong \overline{ZX};$$

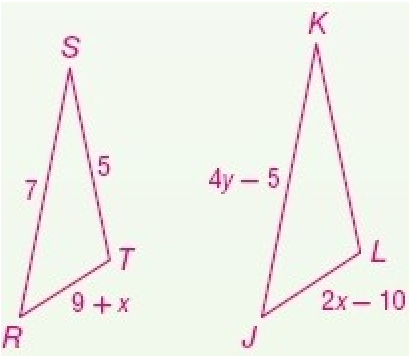
The corresponding sides have the same measure and are congruent. $\Delta ABC \cong \Delta XYZ$ by SSS.

ANSWER:

$AB = 5$, $BC = 2$, $AC = \sqrt{29}$, $XY = 5$, $YZ = 2$, $XZ = \sqrt{29}$; the corresponding sides have the same measure and are congruent. $\Delta ABC \cong \Delta XYZ$ by SSS.

33. **ALGEBRA** If $\Delta RST \cong \Delta JKL$, $RS = 7$, $ST = 5$, $RT = 9 + x$, $JL = 2x - 10$, and $JK = 4y - 5$, draw and label a figure to represent the congruent triangles. Then find x and y.

SOLUTION:



Corresponding sides of triangles *RST* and *JKL* are congruent.

Since $\overline{RS} \cong \overline{JK}$, $RS = JK$.

Solve the equation for y.

$$\begin{aligned} 7 &= 4y - 5 && RS = JK \\ 12 &= 4y && \text{Add 5 to each side.} \\ y &= 3 && \text{Divide each side by 4.} \end{aligned}$$

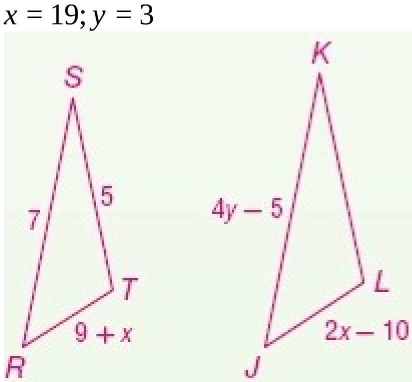
Since $\overline{RT} \cong \overline{JL}$, RT = JL.

Solve for x.

$$\begin{aligned} 9 + x &= 2x - 10 && \text{RT = JL} \\ 9 + 10 &= 2x - x && \text{Simplify.} \\ x &= 19 && \text{Simplify.} \end{aligned}$$

ANSWER:

$x = 19$; $y = 3$



34. **BUSINESS** Maxine charges $5 to paint a mailbox and $4 per hour to mow a lawn. Write an equation to represent the amount of money Maxine can earn from a homeowner who has his or her mailbox painted and lawn mowed.

SOLUTION:

Assume that x represents the time Maxine spends mowing a lawn and let y be the amount of money Maxine can earn. The expression that represents the amount of money Maxine can earn could be: $y = 4(x) + 5$.

ANSWER:

$y = 4x + 5$

Copy and complete each truth table.

35.

p	q	~p	~p ∨ q
F	T		
T	T		
F	F		
T	F		

SOLUTION:

p	q	~p	~p ∨ q
F	T	T	T
T	T	F	T
F	F	T	T
T	F	F	F

ANSWER:

p	q	~p	~p ∨ q
F	T	T	T
T	T	F	T
F	F	T	T
T	F	F	F

36.

p	q	~q	~q ∧ p
F		F	
T		T	
T		F	
F		T	

SOLUTION:

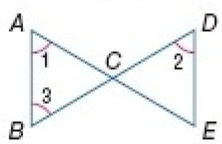
p	q	~q	~q ∧ p
F	T	F	F
T	F	T	T
T	T	F	F
F	F	T	T

ANSWER:

p	q	~q	~q ∧ p
F	T	F	F
T	F	T	T
T	T	F	F
F	F	T	T

PROOF Write a two-column proof for each of the following.

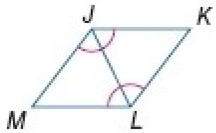
37. **Given:** $\angle 2 \cong \angle 1$
$\angle 1 \cong \angle 3$
Prove: $\overline{AB} \parallel \overline{DE}$



SOLUTION:
Proof:
Statements (Reasons)
1. $\angle 2 \cong \angle 1$, $\angle 1 \cong \angle 3$ (Given)
2. $\angle 2 \cong \angle 3$ (Transitive Property)
3. $\overline{AB} \parallel \overline{DE}$ (If alternative interior angles are congruent, then the lines are parallel.)

ANSWER:
Proof:
Statements (Reasons)
1. $\angle 2 \cong \angle 1$, $\angle 1 \cong \angle 3$ (Given)
2. $\angle 2 \cong \angle 3$ (Trans. Prop.)
3. $\overline{AB} \parallel \overline{DE}$ (If alt. int. $\angle$s are $\cong$, lines are $\parallel$.)

38. **Given:** $\angle MJK \cong \angle KLM$
$\angle LMJ$ and $\angle KLM$ are supplementary.
Prove: $\overline{KJ} \parallel \overline{LM}$



SOLUTION:
Proof:
Statements (Reasons)
1. $\angle MJK \cong \angle KLM$, $\angle LMJ$ and $\angle KLM$ are supplementary. (Given)
2. $m\angle MJK = m\angle KLM$ (Definition of congruent angles.)
3. $m\angle LMJ + m\angle KLM = 180$ (Definition of supplementary angles)
4. $m\angle LMJ + m\angle MJK = 180$ (Substitution)
5. $\angle LMJ$ and $\angle MJK$ are supplementary. (Definition of supplementary angles)
6. $\overline{KJ} \parallel \overline{LM}$ (If consecutive interior angles are supplementary, then the lines are parallel.)

ANSWER:
Proof:
Statements (Reasons)
1. $\angle MJK \cong \angle KLM$, $\angle LMJ$ and $\angle KLM$ are suppl. (Given)
2. $m\angle MJK = m\angle KLM$ (Def. of $\cong$ $\angle$s)
3. $m\angle LMJ + m\angle KLM = 180$ (Def. of $\cong$ suppl. $\angle$s)
4. $m\angle LMJ + m\angle MJK = 180$ (Subst.)
5. $\angle LMJ$ and $\angle MJK$ are suppl. (Def. of suppl. $\angle$s)
6. $\overline{KJ} \parallel \overline{LM}$ (If cons. int. $\angle$s are suppl., lines are $\parallel$.)