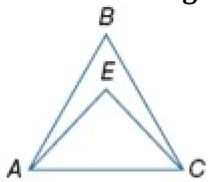
# 4-6 Isosceles and Equilateral Triangles

**Refer to the figure.**




1. If $\overline{AB} \cong \overline{CB}$, name two congruent angles.

*SOLUTION:*

Isosceles Triangle Theorem states that if two sides of the triangle are congruent, then the angles opposite those sides are congruent.

Therefore In triangle *ABC*, $\angle BAC \cong \angle BCA$.

*ANSWER:*

$\angle BAC$ and $\angle BCA$

2. If $\angle EAC \cong \angle ECA$, name two congruent segments.

*SOLUTION:*

Converse of Isosceles Triangle Theorem states that if two angles of a triangle congruent, then the sides opposite those angles are congruent.

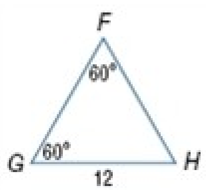
Therefore, in triangle *EAC*, $\overline{EA} \cong \overline{EC}$.

*ANSWER:*

$\overline{EA} \cong \overline{EC}$

**Find each measure.**

3. *FH*




*SOLUTION:*

By the Triangle Sum Theorem,

$$
\begin{aligned}
m\angle F + m\angle G + m\angle H &= 180 && \text{Triangle Angle Sum Thm.}\\
60 + 60 + m\angle H &= 180 && \text{Substitute.}\\
m\angle H &= 60 && \text{Simplify.}
\end{aligned}
$$

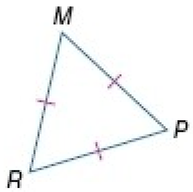
Since the measures of all the three angles are 60°; the triangle must be an equiangular.

All the equiangular triangles are equilateral. Therefore, *FH* = *GH* = 12.

*ANSWER:*

12

4. $m\angle MRP$



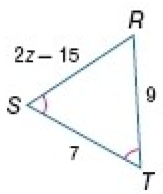

*SOLUTION:*

Since all the sides are congruent, $\Delta MRP$ is an equilateral triangle.
Each angle of an equilateral triangle measures $60°$.
Therefore, $m\angle MRP = 60°$.

*ANSWER:*

60

**CCSS SENSE-MAKING Find the value of each variable.**

5. 

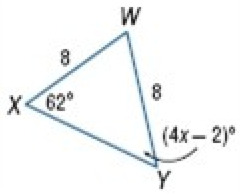

*SOLUTION:*

Here $\angle S \cong \angle T$. Therefore, the triangle *RST* is an Isosceles triangle.
By the Converse of Isosceles Triangle Theorem, $\overline{RS} \cong \overline{RT}$.
That is, $RS \cong RT$.

$$2z - 15 = 9 \quad RS = RT$$
$$2z = 24 \quad \text{Simplify.}$$
$$z = 12$$

*ANSWER:*

12

6. 


*SOLUTION:*

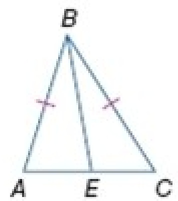
Here $\overline{WX} \cong \overline{WY}$. Therefore, the triangle *WXY* is an Isosceles triangle.
By the Isosceles Triangle Theorem, $\angle X \cong \angle Y$.

$$4x - 2 = 62 \quad m\angle X = m\angle Y$$
$$4x = 64 \quad \text{Simplify.}$$
$$x = 16$$

*ANSWER:*

16

7. **PROOF** Write a two-column proof.
**Given:** $\Delta ABC$ is isosceles; $\overline{EB}$ bisects $\angle ABC$.
**Prove:** $\Delta ABE \cong \Delta CBE$




*SOLUTION:*

Proof:
Statements (Reasons)

1. $\Delta ABC$ is isosceles; $\overline{EB}$ bisects $\angle ABC$. (Given)
2. $\overline{AB} \cong \overline{BC}$ (Definition of isosceles)
3. $\angle ABE \cong \angle CBE$ (Definition of angle bisector)
4. $\overline{BE} \cong \overline{BE}$ (Reflection Property)
5. $\Delta ABE \cong \Delta CBE$ (SAS)

*ANSWER:*

Proof:
Statements (Reasons)

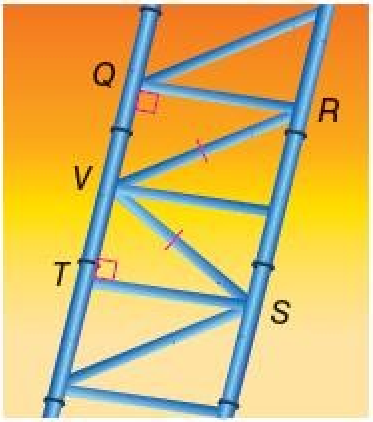
1. $\Delta ABC$ is isosceles; $\overline{EB}$ bisects $\angle ABC$. (Given)
2. $\overline{AB} \cong \overline{BC}$ (Def. of isosceles)
3. $\angle ABE \cong \angle CBE$ (Def. of $\angle$ bisector)
4. $\overline{BE} \cong \overline{BE}$ (Refl. Prop.)
5. $\Delta ABE \cong \Delta CBE$ (SAS)

8. **ROLLER COASTERS** A roller coaster track appears to be composed of congruent triangles. A portion of the track is shown.

**a.** If $\overline{QR}$ and $\overline{ST}$ are perpendicular to $\overline{QT}$, $\Delta VSR$ is isosceles with base $\overline{SR}$, and $\overline{QT} \parallel \overline{SR}$, prove that

$\Delta RQV \cong \Delta STV$.

**b.** If *VR* = 2.5 meters and *QR* = 2 meters, find the distance between $\overline{QR}$ and $\overline{ST}$. Explain your reasoning.




*SOLUTION:*

**a.** Given: $\overline{QR}$ and $\overline{ST}$ are perpendicular to $\overline{QT}$, $\Delta VSR$ is isosceles with base $\overline{SR}$, and $\overline{QT} \parallel \overline{SR}$.
Prove: $\Delta RQV \cong \Delta STV$
Proof:
Statements (Reasons)
1. $\overline{QR}$ and $\overline{ST}$ are perpendicular to $\overline{QT}$, $\Delta VSR$ is isosceles with base $\overline{SR}$, and $\overline{QT} \parallel \overline{SR}$. (Given)
2. $\angle RQV$ and $\angle STV$ are right angles. (Definition of the perpendicular line)
3. $\angle RQV \cong \angle STV$ (All the right angles are congruent)
4. $\overline{VR} \cong \overline{VS}$ (Definition of isosceles)
5. $\angle VSR \cong \angle VRS$ (Isosceles Triangle Theorem.)
6. $\angle QVR \cong \angle VRS$
$\angle TVS \cong \angle VSR$ (Alternative Interior Angle Theorem)
7. $\angle TVS \cong \angle QVR$ (Transitive Property)
8. $\Delta RQV \cong \Delta STV$ (AAS)

**b.** Use the Pythagorean Theorem in the triangle *RQV* to find *QV*.

$$QV = \sqrt{2.5^2 - 2^2}$$
$$= 1.5 \text{ m}$$

By CPCTC we know that *VT* = 1.5 m.
The Segment Addition Postulate says *QV* + *VT* = *QT*.
By substitution, we have 1.5 + 1.5 = *QT*. So *QT* = 3 m.

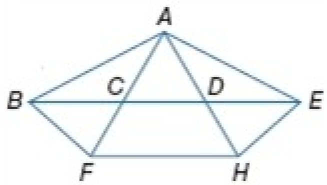
*ANSWER:*

**a.** Given: $\overline{QR}$ and $\overline{ST}$ are perpendicular to $\overline{QT}$, $\Delta VSR$ is isosceles with base $\overline{SR}$, and $\overline{QT} \parallel \overline{SR}$.
Prove: $\Delta RQV \cong \Delta STV$
Proof:
Statements (Reasons)
1. $\overline{QR}$ and $\overline{ST}$ are perpendicular to $\overline{QT}$, $\Delta VSR$ is isosceles with base $\overline{SR}$, and $\overline{QT} \parallel \overline{SR}$. (Given)
2. $\angle RQV$ and $\angle STV$ are right angles. (Def. of $\perp$)
3. $\angle RQV \cong \angle STV$ (All rt. $\angle$s are $\cong$.)
4. $\overline{VR} \cong \overline{VS}$ (Def. of isosceles)
5. $\angle VSR \cong \angle VRS$ (Isos. $\Delta$ Thm.)

6. $\angle QVR \cong \angle VRS$
$\angle TVS \cong \angle VSR$ (Alt. Int. $\angle$s Thm.)
7. $\angle TVS \cong \angle QVR$ (Trans. Property)
8. $\Delta RQV \cong \Delta STV$ (AAS)

**b.** The Pythagorean Theorem tells us that $QV = \sqrt{2.5^2 - 2^2}$ or 1.5 m. By CPCTC we know that *VT* = 1.5 m. The Seg. Add. Post. says *QV* + *VT* = *QT*. By substitution, we have 1.5 + 1.5 = *QT*. So *QT* = 3 m.

**Refer to the figure.**




9. If $\overline{AB} \cong \overline{AE}$, name two congruent angles.

*SOLUTION:*
By the Isosceles Triangle Theorem, In triangle *ABE*, $\angle ABE \cong \angle AEB$.

*ANSWER:*
$\angle ABE$ and $\angle AEB$

10. If $\angle ABF \cong \angle AFB$, name two congruent segments.

*SOLUTION:*
By the Converse of Isosceles Triangle Theorem, In triangle *ABF*, $\overline{AB} \cong \overline{AF}$.

*ANSWER:*
$\overline{AB}$ and $\overline{AF}$

11. If $\overline{CA} \cong \overline{DA}$, name two congruent angles.

*SOLUTION:*
By the Isosceles Triangle Theorem, In triangle *ACD*, $\angle ACD \cong \angle ADC$.

*ANSWER:*
$\angle ACD$ and $\angle ADC$

12. If $\angle DAE \cong \angle DEA$, name two congruent segments.

*SOLUTION:*
By the Converse of Isosceles Triangle Theorem, In triangle *ADE*, $\overline{AD} \cong \overline{DE}$.

*ANSWER:*
$\overline{AD}$ and $\overline{DE}$

13. If $\angle BCF \cong \angle BFC$, name two congruent segments.

*SOLUTION:*

By the Converse of Isosceles Triangle Theorem, In triangle *BCF*, $\overline{BF} \cong \overline{BC}$.

*ANSWER:*

$\overline{BF}$ and $\overline{BC}$

14. If $\overline{FA} \cong \overline{AH}$, name two congruent angles.

*SOLUTION:*

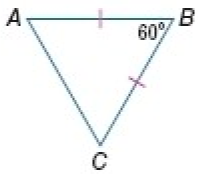
By the Isosceles Triangle Theorem, In triangle *AFH*, $\angle AFH \cong \angle AHF$.

*ANSWER:*

$\angle AFH$ and $\angle AHF$

**Find each measure.**

15. $m\angle BAC$




*SOLUTION:*

Here $\overline{AB} = \overline{BC}$.

By Isosceles Triangle Theorem, $\angle BCA \cong \angle BAC$.

Apply the Triangle Sum Theorem.

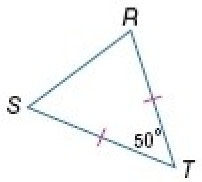
$$
\begin{aligned}
m\angle BAC + m\angle BCA + m\angle ABC &= 180 && \text{Triangle Angle-Sum Thm.} \\
m\angle BAC + m\angle BAC + 60 &= 180 && \text{Substitute.} \\
2m\angle BAC + 60 &= 180 && \text{Addition.} \\
2m\angle BAC &= 120 && -60 \text{ from each side.} \\
m\angle BAC &= 60 && \div \text{ each side by } 60.
\end{aligned}
$$

*ANSWER:*

60

## 4-6 Isosceles and Equilateral Triangles

16. $m\angle SRT$




*SOLUTION:*

Given: $\overline{ST} = \overline{RT}$.

By Isosceles Triangle Theorem, $\angle RST \cong \angle SRT$.

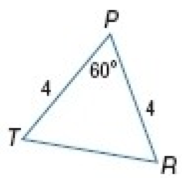
Apply Triangle Sum Theorem.

$$
\begin{aligned}
m\angle SRT + m\angle RST + m\angle STR &= 180 && \text{Triangle Angle Sum Theorem} \\
m\angle SRT + m\angle SRT + 50 &= 180 && \text{Substitute.} \\
2m\angle SRT + 50 &= 180 && \text{Addition.} \\
2m\angle SRT &= 130 && \text{–50 from each side.} \\
m\angle SRT &= 65 && \div \text{ each side by 2.}
\end{aligned}
$$

*ANSWER:*

65

17. $TR$




*SOLUTION:*

Since the triangle is Isosceles, $\angle PTR \cong \angle PRT$.

Therefore,

$$
\begin{aligned}
m\angle PRT + m\angle PTR + m\angle RPT &= 180 && \text{Triangle Angle-Sum Thm.} \\
m\angle PRT + m\angle PTR + 60 &= 180 && \text{Substitute.} \\
2m\angle PRT + 60 &= 180 && \text{Addition.} \\
2m\angle PRT &= 120 && \text{–60 from each side.} \\
m\angle PRT &= 60 && \div \text{ each side by 2.}
\end{aligned}
$$

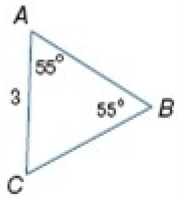
All the angles are congruent. Therefore it is an equiangular triangle.

Since the equiangular triangle is an equilateral, $TR = 4$.

*ANSWER:*

4

18. *CB*



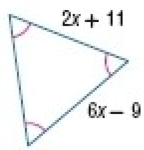

*SOLUTION:*

By the Converse of Isosceles Triangle Theorem, In triangle *ABC*, $\overline{AC} \cong \overline{CB}$.
That is, *CB* = 3.

*ANSWER:*

3

**CCSS REGULARITY Find the value of each variable.**

19. 

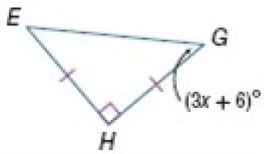

*SOLUTION:*

Since all the angles are congruent, the sides are also congruent to each other.
Therefore, $2x+11=6x-9$.
Solve for *x*.

$2x+11=6x-9$.
$4x=20$ Combine like terms.
$x=5$ Simplify.

*ANSWER:*

$x = 5$

20. 


*SOLUTION:*

Given: $\overline{GH} \cong \overline{EH}$.
By the Isosceles Triangle Theorem,

$$m\angle HEG = m\angle HGE$$
$$= 3x + 6$$

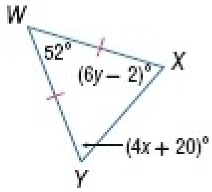
And $m\angle EHG = 90$.

We know that,

$$m\angle HEG + m\angle HGE + m\angle EHG = 180 \quad \text{Triangle Angle Sum Theorem}$$
$$(3x+6) + (3x+6) + 90 = 180 \quad \text{Substitute.}$$
$$6x + 102 = 180 \quad \text{Simplify.}$$
$$6x = 78$$
$$x = 13$$

*ANSWER:*

$x = 13$

21. 


*SOLUTION:*

Given: $\overline{WX} \cong \overline{WY}$.
By the Isosceles Triangle Theorem,
$\angle WYX \cong \angle WXY$.
That is, $4x + 20 = 6y - 2$.

Then, let Equation 1 be: $4x - 6y = -22$.

By the Triangle Sum Theorem, we can find Equation 2:

$$(4x + 20) + (6y - 2) + 52 = 180 \quad \text{Substitute.}$$
$$4x + 6y = 110 \quad \text{Simplify.}$$

Add the equations 1 and 2.

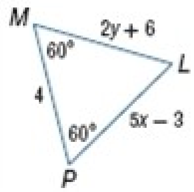
$$4x - 6y = -22 \quad \text{Equation 1}$$
$$\underline{4x + 6y = 110} \quad \text{Equation 2}$$
$$8x = 88 \quad \text{Add equations.}$$
$$x = 11 \quad \div \text{ each side by 8.}$$

Substitute the value of *x* in one of the two equations to find the value of *y*.

$$4x + 6y = 110 \quad \text{Equation 2}$$
$$4(11) + 6y = 110 \quad \text{Substitution}$$
$$44 + 6y = 110 \quad \text{Multiply.}$$
$$6y = 66 \quad -44 \text{ from each side.}$$
$$y = 11 \quad \div \text{ each side by 6.}$$

*ANSWER:*

$x = 11, y = 11$

22. 


*SOLUTION:*

Given: $m\angle PML = m\angle MPL = 60.$
By the Triangle Sum Theorem, $m\angle MLP = 60.$

So, the triangle is an equilateral triangle.
Therefore, $2y + 6 = 5x - 3 = 4.$

Set up two equations to solve for $x$ and $y$.

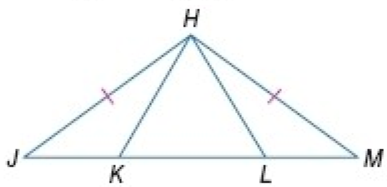
$$2y + 6 = 4; \quad 5x - 3 = 4$$
$$2y = -2 \text{ and } 5x = 7$$
$$y = -1 \quad x = 1.4$$

*ANSWER:*

$x = 1.4, y = -1$

**PROOF Write a paragraph proof.**

23. Given: $\Delta HJM$ is isosceles and $\Delta HKL$ is equilateral. $\angle JKH$ and $\angle HKL$ are supplementary and $\angle HLK$ and $\angle MLH$ are supplementary.
Prove: $\angle JHK \cong \angle MHL$




*SOLUTION:*

Proof: We are given that $\Delta HJM$ is an isosceles triangle and $\Delta HKL$ is an equilateral triangle, $\angle JKH$ and $\angle HKL$ are supplementary and $\angle HLK$ and $\angle MLH$ are supplementary. From the Isosceles Triangle Theorem, we know that $\angle HJK \cong \angle HML$. Since $\Delta HKL$ is an equilateral triangle, we know $\angle HLK \cong \angle LKH \cong \angle KHL$ and $\overline{HL} \cong \overline{KL} \cong \overline{HK}$. $\angle JKH$, $\angle HKL$ and $\angle HLK$, $\angle MLH$ are supplementary, and $\angle HKL \cong \angle HLK$, we know $\angle JKH \cong \angle MLH$ by the Congruent Supplements Theorem. By AAS, $\Delta JHK \cong \Delta MLH$. By CPCTC, $\angle JHK \cong \angle MHL$.

*ANSWER:*

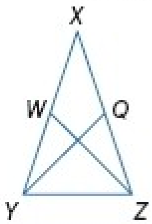
Proof: We are given that $\Delta HJM$ is an isosceles triangle and $\Delta HKL$ is an equilateral triangle, $\angle JKH$ and $\angle HKL$ are supplementary and $\angle HLK$ and $\angle MLH$ are supplementary. From the Isosceles Triangle Theorem, we know that $\angle HJK \cong \angle HML$. Since $\Delta HKL$ is an equilateral triangle, we know $\angle HLK \cong \angle LKH \cong \angle KHL$ and $\overline{HL} \cong \overline{KL} \cong \overline{HK}$. $\angle JKH$, $\angle HKL$ and $\angle HLK$, $\angle MLH$ are supplementary, and $\angle HKL \cong \angle HLK$, we know $\angle JKH \cong \angle MLH$ by the Congruent Supplements Theorem. By AAS, $\Delta JHK \cong \Delta MLH$. By CPCTC, $\angle JHK \cong \angle MHL$.

24. Given: $\overline{XY} \cong \overline{XZ}$

$W$ is the midpoint of $\overline{XY}$.

$Q$ is the midpoint of $\overline{XZ}$.

Prove: $\overline{WZ} \cong \overline{QY}$




*SOLUTION:*

Proof: We are given $\overline{XY} \cong \overline{XZ}$, $W$ is the midpoint of $\overline{XY}$, and $Q$ is the midpoint of $\overline{XZ}$.
Since $W$ is the midpoint of $\overline{XY}$, we know that $\overline{XW} \cong \overline{WY}$. Similarly, since $Q$ is the midpoint of $\overline{XZ}, \overline{XQ} \cong \overline{QZ}$.
The Segment Addition Postulate gives us $XW + WY = XY$ and $XQ + QZ = XZ$. Substitution gives
$XW + WY = XQ + QZ$
$WY + WY = QZ + QZ$.
So, $2WY = 2QZ$.
If we divide each side by 2, we have $WY = QZ$.
The Isosceles Triangle Theorem says $\angle XYZ \cong \angle XZY$. $\overline{YZ} \cong \overline{ZY}$ by the Reflexive Property.
By SAS, $\Delta WYZ \cong \Delta QZY$.
So, $\overline{WZ} \cong \overline{QY}$ by CPCTC.

*ANSWER:*

Proof: We are given $\overline{XY} \cong \overline{XZ}$, $W$ is the midpoint of $\overline{XY}$, and $Q$ is the midpoint of $\overline{XZ}$. Since $W$ is the midpoint of $\overline{XY}$, we know that $\overline{XW} \cong \overline{WY}$. Similarly, since $Q$ is the midpoint of $\overline{XZ}, \overline{XQ} \cong \overline{QZ}$. The Segment Addition Postulate gives us $XW + WY = XY$ and $XQ + QZ = XZ$. Substitution gives $XW + WY = XQ + QZ$ and $WY + WY = QZ + QZ$. So, $2WY = 2QZ$. If we divide each side by 2, we have $WY = QZ$. The Isosceles Triangle Theorem says $\angle XYZ \cong \angle XZY$. $\overline{YZ} \cong \overline{ZY}$ by the Reflexive Property. By SAS, $\Delta WYZ \cong \Delta QZY$. So, $\overline{WZ} \cong \overline{QY}$ by CPCTC.

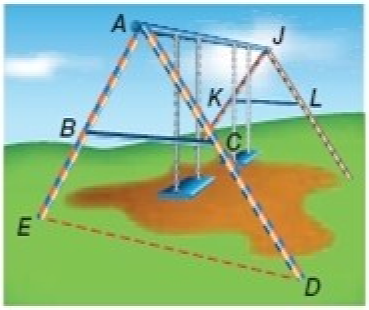
25. **BABYSITTING** While babysitting her neighbor's children, Elisa observes that the supports on either side of a park swing set form two sets of triangles. Using a jump rope to measure, Elisa is able to determine that $\overline{AB} \cong \overline{AC}$, but $\overline{BC} \not\cong \overline{AB}$.

**a.** Elisa estimates $m\angle BAC$ to be 50. Based on this estimate, what is $m\angle ABC$? Explain.

**b.** If $\overline{BE} \cong \overline{CD}$, show that $\Delta AED$ is isosceles.

**c.** If $\overline{BC} \parallel \overline{ED}$ and $\overline{ED} \cong \overline{AD}$, show that $\Delta AED$ is equilateral.

**d.** If $\Delta JKL$ is isosceles, what is the minimum information needed to prove that $\Delta ABC \cong \Delta JKL$? Explain your reasoning.




*SOLUTION:*

**a.** Given: $\overline{AB} = \overline{BC}$.
By Isosceles Triangle Theorem, $\angle ABC \cong \angle ACB$.
Apply Triangle Sum Theorem.

$m\angle ABC + m\angle ACB + m\angle BAC = 180$ Triangle Angle Sum Thm.
$m\angle ABC + m\angle ABC + 50 = 180$ Substitute 50 for $m\angle BAC$.
$2m\angle ABC = 130$ Add.
$m\angle ABC = 65$ Divide each side by 2.

**b. Proof:**
**Statements (Reasons)**
1. $\overline{AB} \cong \overline{AC}$, $\overline{BE} \cong \overline{CD}$ (Given)
2. $AB = AC$, $BE = CD$ (Definition of Congruency)
3. $AB + BE = AE$, $AC + CD = AD$ (Segment Addition Postulate)
4. $AB + BE = AC + CD$ (Addition Property of Equality)
5. $AE = AD$ (Substitution)
6. $\overline{AE} \cong \overline{AD}$ (Definition of Congruency)
7. $\Delta AED$ is isosceles. (Definition of isosceles)

**c. Proof:**
**Statements (Reasons)**
1. $\overline{AB} \cong \overline{AC}$, $\overline{BC} \parallel \overline{ED}$, and $\overline{ED} \cong \overline{AD}$ (Given)
2. $\angle ABC \cong \angle ACB$ (Isosceles Triangle Theorem)
3. $m\angle ABC = m\angle ACB$ (Definition of Congruent Angles )
4. $\angle ABC \cong \angle AED$, $\angle ACB \cong \angle ADE$ (Corresponding Angles )
5. $m\angle ABC = m\angle AED$, $m\angle ACB = m\angle ADE$
(Definition of Congruent angles)
6. $m\angle AED = m\angle ACB$ (Substitution)
7. $m\angle AED = m\angle ADE$ (Substitution)
8. $\angle AED \cong \angle ADE$ (Definition of congruent Angles )
9. $\overline{AD} \cong \overline{AE}$ (Converse of Isosceles Triangle Theorem)
10. $\Delta ADE$ is equilateral. (Definition of Equilateral Triangle)

**d.** One pair of congruent corresponding sides and one pair of congruent corresponding angles; since you know that the triangle is isosceles, if one leg is congruent to a leg of $\Delta ABC$, then you know that both pairs of legs are congruent. Because the base angles of an isosceles triangle are congruent, if you know that $\angle K \cong \angle B$ you know that $\angle K \cong \angle L$, $\angle B \cong \angle C$, and $\angle C \cong \angle L$. Therefore, with one pair of congruent corresponding sides and one pair of congruent corresponding angles, the triangles can be proved congruent using either ASA or SAS.

*ANSWER:*

**a.** 65°; Since $\triangle ABC$ is isosceles, $\angle ABC \cong \angle ACB$, so 180 – 50 = 130 and 130 ÷ 2 = 65.

**b. Proof:**

**Statements (Reasons)**

1. $\overline{AB} \cong \overline{AC}$, $\overline{BE} \cong \overline{CD}$ (Given)
2. $AB = AC$, $BE = CD$ (Def. of $\cong$)
3. $AB + BE = AE$, $AC + CD = AD$ (Seg. Add. Post.)
4. $AB + BE = AC + CD$ (Add. Prop. =)
5. $AE = AD$ (Subst.)
6. $\overline{AE} \cong \overline{AD}$ (Def. of $\cong$)
7. $\triangle AED$ is isosceles. (Def. of isosceles)

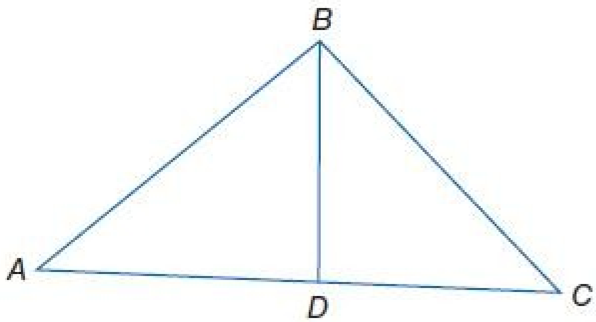
**c. Proof:**

**Statements (Reasons)**

1. $\overline{AB} \cong \overline{AC}$, $\overline{BC} \parallel \overline{ED}$, and $\overline{ED} \cong \overline{AD}$ (Given)
2. $\angle ABC \cong \angle ACB$ (Isos. $\Delta$ Thm.)
3. $m\angle ABC = m\angle ACB$ (Def. of $\cong \angle$s )
4. $\angle ABC \cong \angle AED$, $\angle ACB \cong \angle ADE$ (Corr. $\angle$s )
5. $m\angle ABC = m\angle AED$, $m\angle ACB = m\angle ADE$ (Def. of $\cong \angle$s )
6. $m\angle AED = m\angle ACB$ (Subst.)
7. $m\angle AED = m\angle ADE$ (Subst.)
8. $\angle AED \cong \angle ADE$ (Def. of $\cong \angle$s )
9. $\overline{AD} \cong \overline{AE}$ (Conv. of Isos. $\Delta$ Thm.)
10. $\triangle ADE$ is equilateral. (Def. of equilateral $\Delta$)

**d.** One pair of congruent corresponding sides and one pair of congruent corresponding angles; since you know that the triangle is isosceles, if one leg is congruent to a leg of $\triangle ABC$, then you know that both pairs of legs are congruent. Because the base angles of an isosceles triangle are congruent, if you know that $\angle K \cong \angle B$ you know that $\angle K \cong \angle L$, $\angle B \cong \angle C$, and $\angle C \cong \angle L$. Therefore, with one pair of congruent corresponding sides and one pair of congruent corresponding angles, the triangles can be proved congruent using either ASA or SAS.

26. **CHIMNEYS** In the picture, $\overline{BD} \perp \overline{AC}$ and $\Delta ABC$ is an isosceles triangle with base $\overline{AC}$. Show that the chimney of the house, represented by $\overline{BD}$, bisects the angle formed by the sloped sides of the roof, $\angle ABC$.
**Refer to the figure on page 291.**




*SOLUTION:*

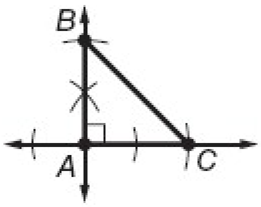
Given: $\overline{BD} \perp \overline{AC}$ and $\Delta ABC$ is an isosceles with base $\overline{AC}$.
Prove: $\overline{BD}$ bisects the angle formed by the sloped sides of the roof, $\angle ABC$.
Proof:
Statements (Reasons)
1. $\overline{BD} \perp \overline{AC}$ and $\Delta ABC$ is an isosceles with base $\overline{AC}$. (Given)
2. $\angle BDA$ and $\angle BDC$ are right angles. (Definition of right angle)
3. $\angle BDA \cong \angle BDC$ (All right angles are congruent)
4. $\overline{AB} \cong \overline{BC}$ (Definition of Isosceles triangle)
5. $\angle BAD \cong \angle BCD$ (Isosceles Triangle Theorem)
6. $\Delta BAD \cong \Delta BCD$ (AAS)
7. $\angle ABD \cong \angle CBD$ (CPCTC)
8. $\overline{BD}$ bisects the angle formed by the sloped sides of the roof, $\angle ABC$. (Definition of angular bisector)

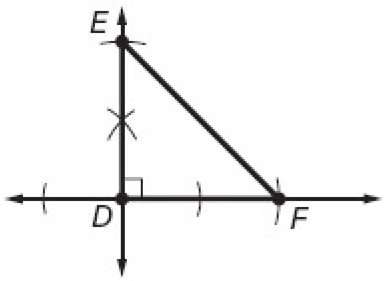
*ANSWER:*

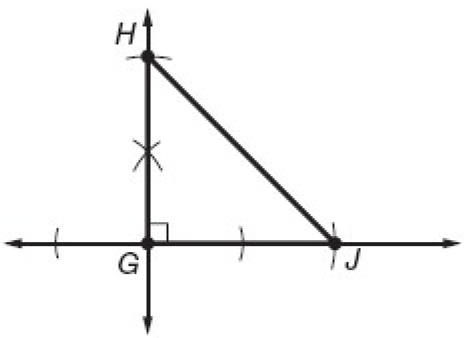
Given: $\overline{BD} \perp \overline{AC}$ and $\Delta ABC$ is an isosceles with base $\overline{AC}$.
Prove: $\overline{BD}$ bisects the angle formed by the sloped sides of the roof, $\angle ABC$.
Proof:
Statements (Reasons)
1. $\overline{BD} \perp \overline{AC}$ and $\Delta ABC$ is an isosceles with base $\overline{AC}$. (Given)
2. $\angle BDA$ and $\angle BDC$ are rt. $\angle$s. (Def. of $\perp$)
3. $\angle BDA \cong \angle BDC$ (All rt. $\angle$s are $\cong$.)
4. $\overline{AB} \cong \overline{BC}$ (Def. of Isos.)
5. $\angle BAD \cong \angle BCD$ (Isos. $\Delta$ Thm.)
6. $\Delta BAD \cong \Delta BCD$ (AAS)
7. $\angle ABD \cong \angle CBD$ (CPCTC)
8. $\overline{BD}$ bisects the angle formed by the sloped sides of the roof, $\angle ABC$. (Def. of $\angle$ bisector)

27. **CONSTRUCTION** Construct three different isosceles right triangles. Explain your method. Then verify your constructions using measurement and mathematics.

*SOLUTION:*

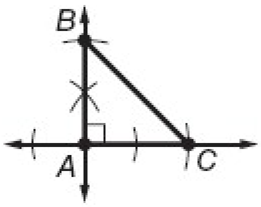


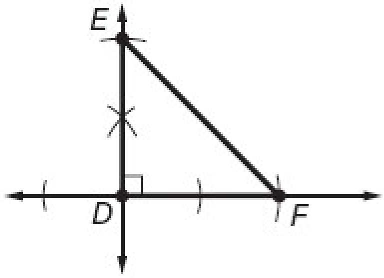



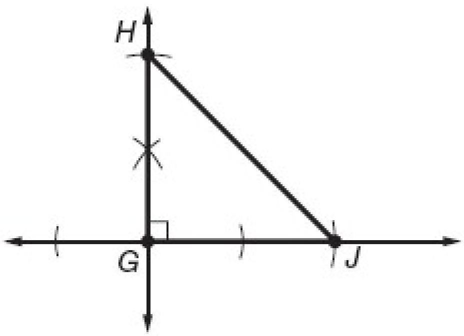




Sample answer: I constructed a pair of perpendicular segments and then used the same compass setting to mark points equidistant from their intersection. I measured both legs for each triangle.
When using the student edition, the measurements are:
*AB* = *AC* = 1.3 cm, *DE* = *DF* = 1.9 cm, and *GH* = *GJ* = 2.3 cm, the triangles are isosceles. I used a protractor to confirm that $\angle A$, $\angle D$, and $\angle G$ are all right angles. For other forms of media, answer will vary.

*ANSWER:*




Sample answer: I constructed a pair of perpendicular segments and then used the same compass setting to mark points equidistant from their intersection. I measured both legs for each triangle. Since $AB = AC = 1.3$ cm, $DE = DF = 1.9$ cm, and $GH = GJ = 2.3$ cm, the triangles are isosceles. I used a protractor to confirm that $\angle A$, $\angle D$, and $\angle G$ are all right angles.

28. **PROOF** Based on your construction in Exercise 27, make and prove a conjecture about the relationship between the base angles of an isosceles right triangle.

*SOLUTION:*

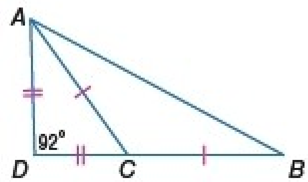
Conjecture: The measures of the base angles of an isosceles right triangle are 45.
Proof: The base angles are congruent because it is an isosceles triangle. Let the measure of each acute angle be *x*. The acute angles of a right triangle are complementary, $x + x = 90$. Solve for *x*

$$2x = 90$$
$$x = 45$$

*ANSWER:*

Conjecture: The measures of the base angles of an isosceles right triangle are 45.
Proof: The base angles are congruent because it is an isosceles triangle. Let the measure of each acute angle be *x*. The acute angles of a right triangle are complementary, so $x + x = 90$ and $x = 45$.

**CCSS REGULARITY Find each measure.**




29. $m\angle CAD$

*SOLUTION:*

From the figure, $\overline{AD} \cong \overline{DC}$.
Therefore, $\Delta ADC$ is Isosceles triangle.
$\Rightarrow m\angle ACD = m\angle CAD.$

By the Triangle Sum Theorem,

$$m\angle CAD + m\angle ACD + m\angle ADC = 180 \quad \text{Triangle Angle-Sum Thm.}$$
$$m\angle CAD + m\angle CAD + 92 = 180 \quad \text{Substitute.}$$
$$2m\angle CAD = 88 \quad \text{Simplify.}$$
$$m\angle CAD = 44 \quad \div \text{ each side by 2.}$$

*ANSWER:*

44

30. $m\angle ACD$

*SOLUTION:*

We know that $m\angle ACD = m\angle CAD$ and $m\angle CAD = 44.$ Therefore, $m\angle ACD = 44.$

*ANSWER:*

44

31. $m\angle ACB$

*SOLUTION:*

The angles in a straight line add to 180°.
Therefore, $m\angle ACD + m\angle ACB = 180.$

We know that $m\angle ACD = 44.$

$$m\angle ACD + m\angle ACB = 180 \quad \text{Def. of linear pair}$$
$$44 + m\angle ACB = 180 \quad \text{Substitute.}$$
$$m\angle ACB = 136 \quad \text{Simplify.}$$

*ANSWER:*

136

# 4-6 Isosceles and Equilateral Triangles

32. $m\angle ABC$

*SOLUTION:*

From the figure, $\overline{AC} \cong \overline{BC}$. So, $\Delta ABC$ is Isosceles triangle. Therefore, $m\angle BAC = m\angle ABC$.

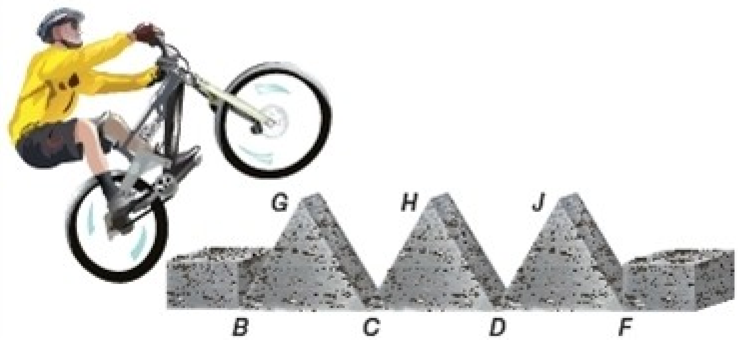
By the Triangle Angle Sum Theorem,

$$\begin{aligned} m\angle ABC + m\angle BAC + m\angle ACB &= 180 && \text{Triangle Angle-Sum Thm.} \\ m\angle ABC + m\angle ABC + 136 &= 180 && \text{Substitute.} \\ 2m\angle ABC &= 44 && \text{Simplify.} \\ m\angle ABC &= 22 && \div \text{ each side by 2.} \end{aligned}$$

*ANSWER:*

22

33. **FITNESS** In the diagram, the rider will use his bike to hop across the tops of each of the concrete solids shown. If each triangle is isosceles with vertex angles *G*, *H*, and *J*, and $\overline{BG} \cong \overline{HC}$, $\overline{HD} \cong \overline{JF}$, $\angle G \cong \angle H$, and $\angle H \cong \angle J$, show that the distance from *B* to *F* is three times the distance from *D* to *F*.




*SOLUTION:*

Given: Each triangle is isosceles, $\overline{BG} \cong \overline{HC}$, $\overline{HD} \cong \overline{JF}$, $\angle G \cong \angle H$, and $\angle H \cong \angle J$.
Prove: The distance from *B* to *F* is three times the distance from *D* to *F*.
Proof:
Statements (Reasons)

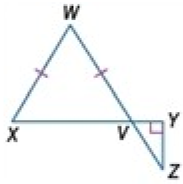
1. Each triangle is isosceles, $\overline{BG} \cong \overline{HC}$, $\overline{HD} \cong \overline{JF}$, $\angle G \cong \angle H$, and $\angle H \cong \angle J$. (Given)
2. $\angle G \cong \angle J$ (Transitive Property)
3. $\overline{BG} \cong \overline{CG}$, $\overline{HC} \cong \overline{HD}$, $\overline{JD} \cong \overline{JF}$ (Definition of Isosceles Triangle)
4. $\overline{BG} \cong \overline{JD}$ (Transitive Property)
5. $\overline{HC} \cong \overline{JD}$ (Transitive Property)
6. $\overline{CG} \cong \overline{JF}$ (Transitive Property.)
7. $\Delta BCG \cong \Delta CDH \cong \Delta DFJ$ (SAS)
8. $\overline{BC} \cong \overline{CD} \cong \overline{DF}$ (CPCTC)
9. *BC* = *CD* = *DF* (Definition of congruence)
10. *BC* + *CD* + *DF* = *BF* (Segment Addition Postulate)
11. *DF* + *DF* + *DF* = *BF* (Substitution.)
12. 3*DF* = *BF* (Addition)

*ANSWER:*

Given: Each triangle is isosceles, $\overline{BG} \cong \overline{HC}$, $\overline{HD} \cong \overline{JF}$, $\angle G \cong \angle H$, and $\angle H \cong \angle J$.
Prove: The distance from *B* to *F* is three times the distance from *D* to *F*.
Proof:
Statements (Reasons)
1. Each triangle is isosceles, $\overline{BG} \cong \overline{HC}$, $\overline{HD} \cong \overline{JF}$, $\angle G \cong \angle H$, and $\angle H \cong \angle J$. (Given)
2. $\angle G \cong \angle J$ (Trans. Prop.)
3. $\overline{BG} \cong \overline{CG}$, $\overline{HC} \cong \overline{HD}$, $\overline{JD} \cong \overline{JF}$ (Def. of Isosceles)
4. $\overline{BG} \cong \overline{JD}$ (Trans. Prop.)
5. $\overline{HC} \cong \overline{JD}$ (Trans. Prop.)
6. $\overline{CG} \cong \overline{JF}$ (Trans. Prop.)
7. $\Delta BCG \cong \Delta CDH \cong \Delta DFJ$ (SAS)
8. $\overline{BC} \cong \overline{CD} \cong \overline{DF}$ (CPCTC)
9. $BC = CD = DF$ (Def. of congruence)
10. $BC + CD + DF = BF$ (Seg. Add. Post.)
11. $DF + DF + DF = BF$ (Subst.)
12. $3DF = BF$ (Addition)

34. Given: $\Delta XWV$ is isosceles; $\overline{ZY} \perp \overline{YV}$.

Prove: $\angle X$ and $\angle YZV$ are complementary.




*SOLUTION:*

Proof:

Statements (Reasons)

1. $\Delta XWV$ is isosceles; $\overline{ZY} \perp \overline{YV}$. (Given)
2. $\angle X \cong \angle WVX$ (Isosceles Triangle Theorem)
3. $\angle WVX \cong \angle YVZ$ (Vertical angles are congruent)
4. $\angle X \cong \angle YVZ$ (Transitive Property)
5. $m\angle X = m\angle YVZ$ (Definition of Congruent angles)
6. $m\angle VYZ = 90$ (Perpendicular lines form right angles)
7. $\Delta ZVY$ is a right triangle. (Definition of right triangle)
8. $\angle YZV$ and $\angle YVZ$ are complementary. (The acute angles of a right triangle are complementary)
9. $m\angle YZV + m\angle YVZ = 90$ (Definition of Complementary angles)
10. $m\angle YZV + m\angle X = 90$ (Substitutition)
11. $\angle X$ and $\angle YZV$ are complementary (Definition of Complementary angles)

*ANSWER:*

Proof:

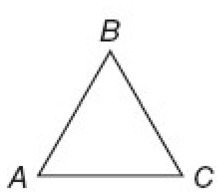
Statements (Reasons)

1. $\Delta XWV$ is isosceles; $\overline{ZY} \perp \overline{YV}$. (Given)
2. $\angle X \cong \angle WVX$ (Isos. $\Delta$ Thm.)
3. $\angle WVX \cong \angle YVZ$ (Vert $\angle$s are $\cong$.)
4. $\angle X \cong \angle YVZ$ (Trans. Prop.)
5. $m\angle X = m\angle YVZ$ (Def. of $\cong$ $\angle$s)
6. $m\angle VYZ = 90$ ($\perp$ lines form rt $\angle$s.)
7. $\Delta ZVY$ is a right triangle. (Def of rt. $\Delta$)
8. $\angle YZV$ and $\angle YVZ$ are complementary. (The acute $\angle$s of a rt. $\Delta$ are comp.)
9. $m\angle YZV + m\angle YVZ = 90$ (Def. Of Compl. $\angle$s)
10. $m\angle YZV + m\angle X = 90$ (Subst.)
11. $\angle X$ and $\angle YZV$ are complementary (Def. of Compl. $\angle$s)

**PROOF Write a two-column proof of each corollary or theorem.**

35. Corollary 4.3

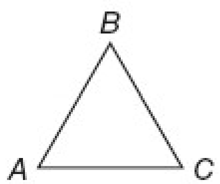
*SOLUTION:*




Case I

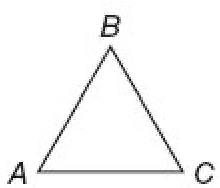
Given: $\Delta ABC$ is an equilateral triangle.
Prove: $\Delta ABC$ is an equiangular triangle.
Proof:
Statements (Reasons)
1. $\Delta ABC$ is an equilateral triangle. (Given)
2. $\overline{AB} \cong \overline{AC} \cong \overline{BC}$ (Def. of equilateral $\Delta$)
3. $\angle A \cong \angle B \cong \angle C$ (Isosceles $\Delta$ Th.)
4. $\Delta ABC$ is an equiangular triangle. (Def. of equiangular)

Case II
Given: $\Delta ABC$ is an equiangular triangle.
Prove: $\Delta ABC$ is an equilateral triangle.



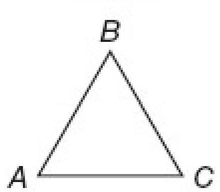

Proof:
Statements (Reasons)
1. $\Delta ABC$ is an equiangular triangle. (Given)
2. $\angle A \cong \angle B \cong \angle C$ (Def. of equiangular $\Delta$)
3. $\overline{AB} \cong \overline{AC} \cong \overline{BC}$ (If 2 $\angle$s of a $\Delta$ are $\cong$ then the sides opp. those $\angle$s are $\cong$.)
4. $\Delta ABC$ is an equilateral triangle. (Def. of equilateral)

*ANSWER:*




Case I
Given: $\Delta ABC$ is an equilateral triangle.
Prove: $\Delta ABC$ is an equiangular triangle.
Proof:
Statements (Reasons)
1. $\Delta ABC$ is an equilateral triangle. (Given)
2. $\overline{AB} \cong \overline{AC} \cong \overline{BC}$ (Def. of equilateral $\Delta$)
3. $\angle A \cong \angle B \cong \angle C$ (Isosceles $\Delta$ Th.)
4. $\Delta ABC$ is an equiangular triangle. (Def. of equiangular)

Case II
Given: $\Delta ABC$ is an equiangular triangle.
Prove: $\Delta ABC$ is an equilateral triangle.



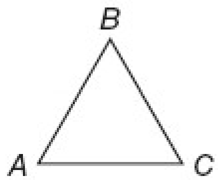

Proof:

Statements (Reasons)
1. $\Delta ABC$ is an equiangular triangle. (Given)
2. $\angle A \cong \angle B \cong \angle C$ (Def. of equiangular $\Delta$)
3. $\overline{AB} \cong \overline{AC} \cong \overline{BC}$ (If 2 $\angle$s of a $\Delta$ are $\cong$ then the sides opp. those $\angle$s are $\cong$.)
4. $\Delta ABC$ is an equilateral triangle. (Def. of equilateral)

36. Corollary 4.4

*SOLUTION:*

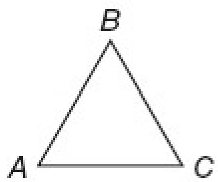
Given: $\Delta ABC$ is an equilateral triangle.
Prove: $m\angle A = m\angle B = m\angle C = 60$




Proof:
Statements (Reasons)
1. $\Delta ABC$ is an equilateral triangle. (Given)
2. $\overline{AB} \cong \overline{AC} \cong \overline{BC}$ (Def. of equilateral $\Delta$)
3. $\angle A \cong \angle B \cong \angle C$ (Isosceles $\Delta$ Thm.)
4. $m\angle A = m\angle B = m\angle C$ (Def. of $\cong$ $\angle$s)
5. $m\angle A + m\angle B + m\angle C = 180$ (Triangle Angle Sum Thm.)
6. $3m\angle A = 180$ (Subst.)
7. $m\angle A = 60$ (Div. Prop.)
8. $m\angle A = m\angle B = m\angle C = 60$ (Subst.)

*ANSWER:*

Given: $\Delta ABC$ is an equilateral triangle.
Prove: $m\angle A = m\angle B = m\angle C = 60$



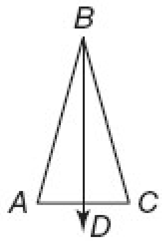

Proof:
Statements (Reasons)
1. $\Delta ABC$ is an equilateral triangle. (Given)
2. $\overline{AB} \cong \overline{AC} \cong \overline{BC}$ (Def. of equilateral $\Delta$)
3. $\angle A \cong \angle B \cong \angle C$ (Isosceles $\Delta$ Thm.)
4. $m\angle A = m\angle B = m\angle C$ (Def. of $\cong$ $\angle$s)
5. $m\angle A + m\angle B + m\angle C = 180$ (Triangle Angle Sum Thm.)
6. $3m\angle A = 180$ (Subst.)
7. $m\angle A = 60$ (Div. Prop.)
8. $m\angle A = m\angle B = m\angle C = 60$ (Subst.)

37. Theorem 4.11

*SOLUTION:*

Given: $\Delta ABC$; $\angle A \cong \angle C$

Prove: $\overline{AB} \cong \overline{CB}$




Proof:

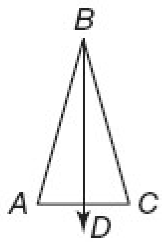
Statements (Reasons)

1. Let $\overrightarrow{BD}$ bisect $\angle ABC$. (Protractor Post.)
2. $\angle ABD \cong \angle CBD$ (Def. of $\angle$ bisector)
3. $\angle A \cong \angle C$ (Given)
4. $\overline{BD} \cong \overline{BD}$ (Refl. Prop.)
5. $\Delta ABD \cong \Delta CBD$ (AAS)
6. $\overline{AB} \cong \overline{CB}$ (CPCTC)

*ANSWER:*

Given: $\Delta ABC$; $\angle A \cong \angle C$

Prove: $\overline{AB} \cong \overline{CB}$

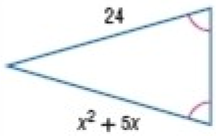



Proof:

Statements (Reasons)

1. Let $\overrightarrow{BD}$ bisect $\angle ABC$. (Protractor Post.)
2. $\angle ABD \cong \angle CBD$ (Def. of $\angle$ bisector)
3. $\angle A \cong \angle C$ (Given)
4. $\overline{BD} \cong \overline{BD}$ (Refl. Prop.)
5. $\Delta ABD \cong \Delta CBD$ (AAS)
6. $\overline{AB} \cong \overline{CB}$ (CPCTC)

**Find the value of each variable.**

38. 


*SOLUTION:*

By the converse of Isosceles Triangle theorem,

$$x^2 + 5x = 24$$

$$x^2 + 5x - 24 = 0$$

Solve the equation for *x*.

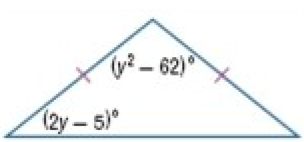
$(x - 3)(x + 8) = 0$ Factor.

$x = 3$ or $-8$ Solve for *x*.

Note that *x* can equal –8 here because $(-8)^2 + 5(-8) = 24$.

*ANSWER:*

3

39. 


*SOLUTION:*

By the Isosceles Triangle Theorem, the third angle is equal to $(2y - 5)°$.

The interior angles of a triangle add up to $180°$.

$(y^2 - 62) + (2y - 5) + (2y - 5) = 180$ Isosceles Triangle Thm.

$y^2 + 4y - 72 = 180$ Simplify.

$y^2 + 4y - 252 = 0$ –180 from each side.

$(y + 18)(y - 14) = 0$ Factor.

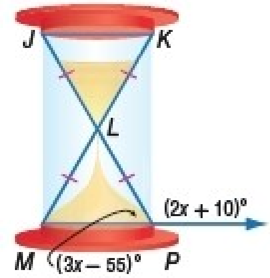
$y = -18$ or $14$. Solve for *y*.

The measure of an angle cannot be negative, and 2(–18) – 5 = –41, so *y* = 14.

*ANSWER:*

14

**GAMES Use the diagram of a game timer shown to find each measure.**




40. $m\angle LPM$

*SOLUTION:*

Angles at a point in a straight line add up to $180°$.

$$\begin{aligned}(3x-55)+(2x+10)&=180 && \text{Def. of linear pair}\\ 5x-45&=180 && \text{Simplify.}\\ 5x&=225 && +45 \text{ to each side.}\\ x&=45 && \div \text{ each side by } 5.\end{aligned}$$

Substitute $x$ = 45 in (3$x$ – 55) to find $m\angle LPM$.

$$\begin{aligned}m\angle LPM&=3(45)-55 && x=45\\ &=135-55 && \text{Multiply.}\\ &=80 && \text{Subtract.}\end{aligned}$$

*ANSWER:*

80

41. $m\angle LMP$

*SOLUTION:*

Since the triangle *LMP* is isosceles, $\angle LMP \cong \angle LPM$. Angles at a point in a straight line add up to $180°$.

$$\begin{aligned}(3x-55)+(2x+10)&=180 && \text{Def. of linear pair}\\ 5x-45&=180 && \text{Simplify.}\\ 5x&=225 && +45 \text{ to each side.}\\ x&=45 && \div \text{ each side by } 5.\end{aligned}$$

Substitute $x$ = 45 in (3$x$ – 55) to find $m\angle LPM$.

$$\begin{aligned}m\angle LPM&=3(45)-55 && x=45\\ &=135-55 && \text{Multiply.}\\ &=80 && \text{Subtract.}\end{aligned}$$

Therefore, $m\angle LMP = 80$.

*ANSWER:*

80

42. $m\angle JLK$

*SOLUTION:*

Vertical angles are congruent. Therefore, $\angle JLK \cong \angle PLM$.

First we need to find $m\angle PLM$.

We know that, $m\angle PLM + m\angle LMP + m\angle LPM = 180$.

$$m\angle PLM + 80 + 80 = 180 \quad \text{Substitute.}$$
$$m\angle PLM = 20 \quad \text{Simplify.}$$

So, $\angle JLK \cong 20$.

*ANSWER:*

20

43. $m\angle JKL$

*SOLUTION:*

Vertical angles are congruent. Therefore, $\angle KLJ \cong \angle PLM$.

First we need to find $m\angle PLM$.

We know that, $m\angle PLM + m\angle LMP + m\angle LPM = 180$.

$$m\angle PLM + 80 + 80 = 180 \quad \text{Substitute.}$$
$$m\angle PLM = 20 \quad \text{Simplify.}$$

So, $\angle KLJ \cong 20$.

In $\Delta JKL$, $m\angle JKL + m\angle KJL + m\angle KLJ = 180$.

Since the triangle *JKL* is isosceles, $\angle KJL \cong \angle JKL$.

$$m\angle JKL + m\angle JKL + m\angle KLJ = 180 \quad \text{Substitute.}$$
$$2m\angle JKL + 20 = 180 \quad \text{Simplify.}$$
$$2m\angle JKL = 160 \quad -20 \text{ from each side.}$$
$$m\angle JKL = 80 \quad \div \text{ each side by 2.}$$

*ANSWER:*

80

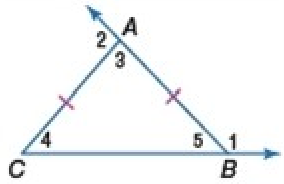
44. **MULTIPLE REPRESENTATIONS** In this problem, you will explore possible measures of the interior angles of an isosceles triangle given the measure of one exterior angle.

**a. GEOMETRIC** Use a ruler and a protractor to draw three different isosceles triangles, extending one of the sides adjacent to the vertex angle and to one of the base angles, and labeling as shown.

**b. TABULAR** Use a protractor to measure and record $m\angle 1$ for each triangle. Use $m\angle 1$ to calculate the measures of $\angle 3$, $\angle 4$, and $\angle 5$. Then find and record $m\angle 2$ and use it to calculate these same measures. Organize your results in two tables.

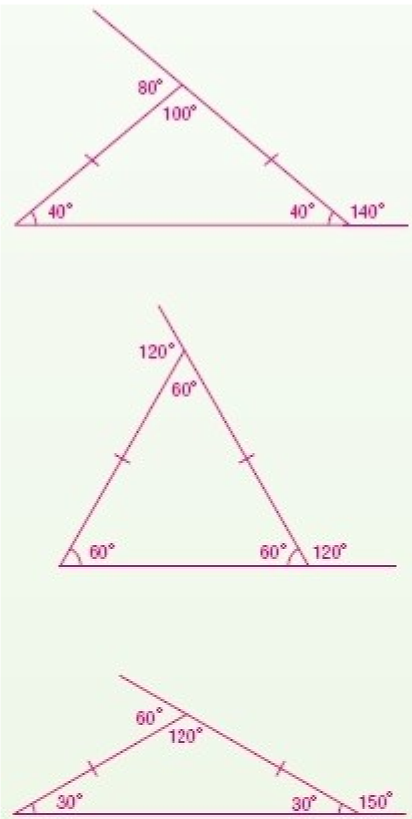
**c. VERBAL** Explain how you used $m\angle 1$ to find the measures of $\angle 3$, $\angle 4$, and $\angle 5$. Then explain how you used $m\angle 2$ to find these same measures.

**d. ALGEBRAIC** If $m\angle 1 = x$, write an expression for the measures of $\angle 3$, $\angle 4$, and $\angle 5$. Likewise, if $m\angle 2 = x$, write an expression for these same angle measures.




*SOLUTION:*

**a.**




**b.**

| $m\angle 1$ | $m\angle 3$ | $m\angle 4$ | $m\angle 5$ |
|---|---|---|---|
| 140 | 100 | 40 | 40 |
| 120 | 60 | 60 | 60 |
| 150 | 120 | 30 | 30 |

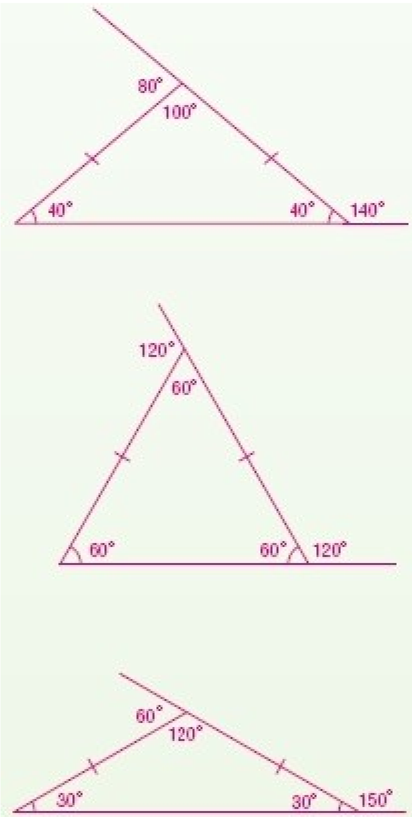
| $m\angle 2$ | $m\angle 3$ | $m\angle 4$ | $m\angle 5$ |
|---|---|---|---|
| 80 | 100 | 40 | 40 |
| 120 | 60 | 60 | 60 |
| 60 | 120 | 30 | 30 |

**c.** $\angle 5$ is supplementary to $\angle 1$, so $m\angle 5 = 180 - m\angle 1$. $\angle 4 \cong \angle 5$, so $m\angle 4 = m\angle 5$. The sum of the angle measures in a triangle must be 180, so $m\angle 3 = 180 - m\angle 4 - \angle 5$. $\angle 2$ is supplementary to $\angle 3$, so $m\angle 3 = 180 - m\angle 2$. $m\angle 2$ is twice as much as $m\angle 4$ and $m\angle 5$, so $m\angle 4 = m\angle 5 = \frac{m\angle 2}{2}$.

**d.** $m\angle 5 = 180 - x$, $m\angle 4 = 180 - x$, $m\angle 3 = 2x - 180$; $m\angle 3 = 180 - x$, $m\angle 4 = \frac{x}{2}$; $m\angle 5 = \frac{x}{2}$

*ANSWER:*

**a.**




**b.**

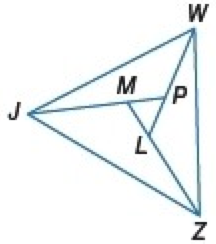
| $m\angle 1$ | $m\angle 3$ | $m\angle 4$ | $m\angle 5$ |
|---|---|---|---|
| 140 | 100 | 40 | 40 |
| 120 | 60 | 60 | 60 |
| 150 | 120 | 30 | 30 |

| $m\angle 2$ | $m\angle 3$ | $m\angle 4$ | $m\angle 5$ |
|---|---|---|---|
| 80 | 100 | 40 | 40 |
| 120 | 60 | 60 | 60 |
| 60 | 120 | 30 | 30 |

**c.** $\angle 5$ is supplementary to $\angle 1$, so $m\angle 5 = 180 - m\angle 1$. $\angle 4 \cong \angle 5$, so $m\angle 4 = m\angle 5$. The sum of the angle measures in a triangle must be 180, so $m\angle 3 = 180 - m\angle 4 - \angle 5$. $\angle 2$ is supplementary to $\angle 3$, so $m\angle 3 = 180 - m\angle 2$. $m\angle 2$ is twice as much as $m\angle 4$ and $m\angle 5$, so $m\angle 4 = m\angle 5 = \frac{m\angle 2}{2}$.

**d.** $m\angle 5 = 180 - x$, $m\angle 4 = 180 - x$, $m\angle 3 = 2x - 180$; $m\angle 3 = 180 - x$, $m\angle 4 = \frac{x}{2}$; $m\angle 5 = \frac{x}{2}$

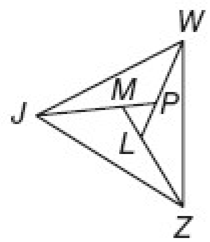
45. **CHALLENGE** In the figure, if $\Delta WJZ$ is equilateral and $\angle ZWP \cong \angle WJM \cong \angle JZL$, prove that $\overline{WP} \cong \overline{ZL} \cong \overline{JM}$.




*SOLUTION:*

Given: $\Delta WJZ$ is equilateral, and $\angle ZWP \cong \angle WJM \cong \angle JZL$.

Prove: $\overline{WP} \cong \overline{ZL} \cong \overline{JM}$



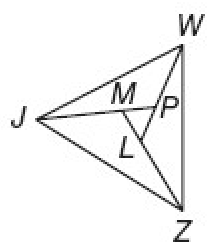

Proof:

We know that $\Delta WJZ$ is equilateral, since an equilateral $\Delta$ is equiangular, $\angle ZWJ \cong \angle WJZ \cong \angle JZW$. So, $m\angle ZWJ = m\angle WJZ = m\angle JZW$, by the definition of congruence. Since $\angle ZWP \cong \angle WJM \cong \angle JZL$, $m\angle ZWP = m\angle WJM = m\angle JZL$, by the definition of congruence. By the Angle Addition Postulate, $m\angle ZWJ = m\angle ZWP + m\angle PWJ$, $m\angle WJZ = m\angle WJM + m\angle MJZ$, $m\angle JZW = m\angle JZL + m\angle LZW$. By substitution, $m\angle ZWP + m\angle PWJ = m\angle WJM + m\angle MJZ = m\angle JZL + m\angle LZW$. Again by substitution, $m\angle ZWP + m\angle PWJ = m\angle ZWP + m\angle PJZ = m\angle ZWP + m\angle LZW$. By the Subtraction Property, $m\angle PWJ = m\angle PJZ = m\angle LZW$. By the definition of congruence, $\angle PWJ \cong \angle PJZ \cong \angle LZW$. So, by ASA, $\Delta WZL \cong \Delta ZJM \cong \Delta JWP$. By CPCTC, $\overline{WP} \cong \overline{ZL} \cong \overline{JM}$.

*ANSWER:*

Given: $\Delta WJZ$ is equilateral, and $\angle ZWP \cong \angle WJM \cong \angle JZL$.

Prove: $\overline{WP} \cong \overline{ZL} \cong \overline{JM}$




Proof:

We know that $\Delta WJZ$ is equilateral, since an equilateral $\Delta$ is equiangular, $\angle ZWJ \cong \angle WJZ \cong \angle JZW$. So, $m\angle ZWJ = m\angle WJZ = m\angle JZW$, by the definition of congruence. Since $\angle ZWP \cong \angle WJM \cong \angle JZL$, $m\angle ZWP = m\angle WJM = m\angle JZL$, by the definition of congruence. By the Angle Addition Postulate, $m\angle ZWJ = m\angle ZWP + m\angle PWJ$, $m\angle WJZ = m\angle WJM + m\angle MJZ$, $m\angle JZW = m\angle JZL + m\angle LZW$. By substitution, $m\angle ZWP + m\angle PWJ = m\angle WJM + m\angle MJZ = m\angle JZL + m\angle LZW$. Again by substitution, $m\angle ZWP + m\angle PWJ = m\angle ZWP + m\angle PJZ = m\angle ZWP + m\angle LZW$. By the Subtraction Property, $m\angle PWJ = m\angle PJZ = m\angle LZW$. By the definition of congruence, $\angle PWJ \cong \angle PJZ \cong \angle LZW$. So, by ASA, $\Delta WZL \cong \Delta ZJM \cong \Delta JWP$. By CPCTC, $\overline{WP} \cong \overline{ZL} \cong \overline{JM}$.

**CCSS PRECISION Determine whether the following statements are *sometimes*, *always*, or *never* true. Explain.**

46. If the measure of the vertex angle of an isosceles triangle is an integer, then the measure of each base angle is an integer.

*SOLUTION:*

Sometimes; only if the measure of the vertex angle is even. For example,
vertex angle = 50, base angles = 65;
vertex angle = 55, base angles = 62.5.

*ANSWER:*

Sometimes; only if the measure of the vertex angle is even.

47. If the measures of the base angles of an isosceles triangle are integers, then the measure of its vertex angle is odd.

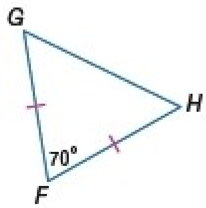
*SOLUTION:*

Never; the measure of the vertex angle will be 180 – 2(measure of the base angle) so if the base angles are integers, then 2(measure of the base angle) will be even and 180 – 2(measure of the base angle) will be even.

*ANSWER:*

Never; the measure of the vertex angle will be 180 – 2(measure of the base angle) so if the base angles are integers, then 2(measure of the base angle) will be even and 180 – 2(measure of the base angle) will be even.

48. **ERROR ANALYSIS** Alexis and Miguela are finding $m\angle G$ in the figure shown. Alexis says that $m\angle G = 35$, while Miguela says that $m\angle G = 60$. Is either of them correct? Explain your reasoning.




*SOLUTION:*

Neither of them is correct. This is an isosceles triangle with a vertex angle of 70. Since this is an isosceles triangle, the base angles are congruent.

$$\begin{aligned} m\angle F + m\angle G + m\angle H &= 180 && \text{Triangle Angle-Sum Thm.} \\ 70 + m\angle G + m\angle H &= 180 && \text{Substitution.} \\ m\angle G + m\angle H &= 110 && -70 \text{ from each side} \\ m\angle G + m\angle G &= 110 && \text{Substitution.} \\ 2m\angle G &= 110 && \text{Addition.} \\ \frac{2m\angle G}{2} &= \frac{110}{2} && \div \text{ each side by 2} \\ &= 55 && \text{Simplify.} \end{aligned}$$

*ANSWER:*

Neither; $m\angle G = \frac{180 - 70}{2}$ or 55.

49. **OPEN ENDED** If possible, draw an isosceles triangle with base angles that are obtuse. If it is not possible, explain why not.

*SOLUTION:*

It is not possible because a triangle cannot have more than one obtuse angle.

*ANSWER:*

It is not possible because a triangle cannot have more than one obtuse angle.

50. **REASONING** In isosceles $\Delta ABC$, $m\angle B = 90$. Draw the triangle. Indicate the congruent sides and label each angle with its measure.

*SOLUTION:*

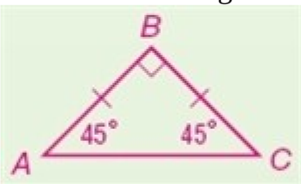
The sum of the angle measures in a triangle must be 180, $m\angle A + m\angle B + m\angle C = 180.$

Since $\Delta ABC$ is isosceles, $m\angle A = m\angle C.$
And given that $m\angle B = 90$.
Therefore,

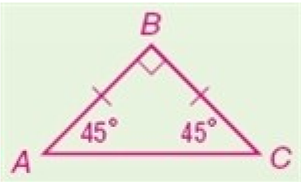
$$m\angle A + 90 + m\angle A = 180$$
$$m\angle A = 45.$$

And $m\angle C = 45$, since $m\angle C = m\angle A.$

Construct the triangle with the angles measures 90, 45, 45.




*ANSWER:*




51. **WRITING IN MATH** How can triangle classifications help you prove triangle congruence?

*SOLUTION:*

Sample answer: If a triangle is already classified, you can use the previously proven properties of that type of triangle in the proof. For example, if you know that a triangle is an equilateral triangle, you can use Corollary 4.3 and 4.4 in the proof. Doing this can save you steps when writing the proof.

*ANSWER:*

Sample answer: If a triangle is already classified, you can use the previously proven properties of that type of triangle in the proof. Doing this can save you steps when writing the proof.

52. **ALGEBRA** What quantity should be added to both sides of this equation to complete the square?

$x^2 - 10x = 3$

**A** –25

**B** –5

**C** 5

**D** 25

*SOLUTION:*

The quantity that should be added to both sides of the equation is:

$$\left(\frac{10}{2}\right)^2 = 25$$

The correct choice is D.

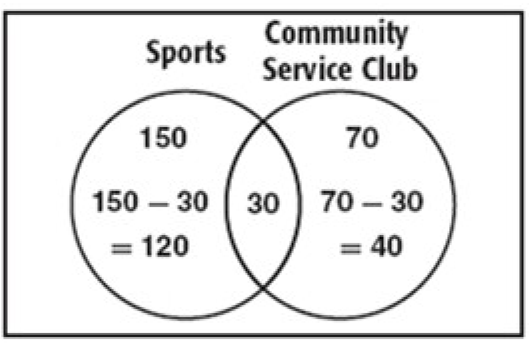
*ANSWER:*

D

53. **SHORT RESPONSE** In a school of 375 students, 150 students play sports and 70 students are involved in the community service club. 30 students play sports and are involved in the community service club. How many students are not involved in either sports or the community service club?

*SOLUTION:*

Use a Venn diagram. Since 150 students play sports, 70 are in community service, and 30 are in both, determine how many students are in sports and how many are in community service.

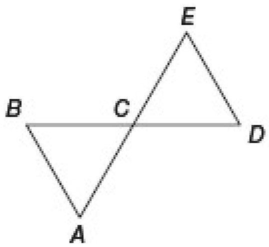



Given:
Total number of students in the class = 375
Number of students involved in sports = 150 – 30 = 120
Number of students involved in community service club = 70 – 30 = 40
Number of students not involved in either club = 375 – (120 + 30 + 40) = 185.

*ANSWER:*

185

54. In the figure below, $\overline{AE}$ and $\overline{BD}$ bisect each other at point *C*.




Which additional piece of information would be enough to prove that $\overline{DE} \cong \overline{DC}$?

**F** $\angle A \cong \angle BCA$
**G** $\angle B \cong \angle D$
**H** $\angle ACB \cong \angle EDC$
**J** $\angle A \cong \angle B$

*SOLUTION:*

Given: $\overline{AE}$ and $\overline{BD}$ bisect each other at point *C*.
Therefore, $\overline{BC} \cong \overline{CD}$ and $\overline{AC} \cong \overline{CE}$.

$\angle BCA \cong \angle DCE$ because vertical angles are congruent.

By SAS postulate $\Delta BCA \cong \Delta DCE$.

To prove $\overline{DE} \cong \overline{DC}$, we need to prove that $\angle E \cong \angle DCE$. Once we know this, $\overline{DE} \cong \overline{DC}$ because $\Delta DCE$ is isosceles and the corresponding angles are congruent.

$\angle E \cong \angle DCE$ if $\angle A \cong \angle BCA$ due to CPCTC.

Therefore, the additional statement $\angle A \cong \angle BCA$ is required.

The correct choice is F.

*ANSWER:*
F

55. **SAT/ACT** If $x = -3$, then $4x^2 - 7x + 5 =$
**A** 2
**B** 14
**C** 20
**D** 42
**E** 62

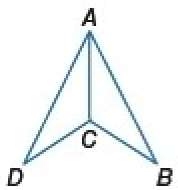
*SOLUTION:*
Substitute -3 for $x$ in the equation and solve.

$$4(-3)^2 - 7(-3) + 5 = 36 + 21 + 5$$
$$= 62$$

The correct choice is E.

*ANSWER:*
E

56. If $m\angle ADC = 35$, $m\angle ABC = 35$, $m\angle DAC = 26$, and $m\angle BAC = 26$, determine whether $\Delta ADC \cong \Delta ABC$.




*SOLUTION:*
Given: $m\angle ADC = 35$, $m\angle ABC = 35$, $m\angle DAC = 26$, and $m\angle BAC = 26$, which means that $\angle ADC \cong \angle ABC$ and $\angle DAC \cong \angle BAC$. By the Reflexive Property, $\overline{AC} \cong \overline{AC}$. Therefore, by AAS congruence, $\Delta ADC \cong \Delta ABC$.

*ANSWER:*
$\Delta ADC \cong \Delta ABC$. Since $\overline{AC} \cong \overline{AC}$, the two triangles are congruent by AAS.

**Determine whether $\triangle STU \cong \triangle XYZ$. Explain.**

57. *S*(0, 5), *T*(0, 0), *U*(1, 1), *X*(4, 8), *Y*(4, 3), *Z*(6, 3)

*SOLUTION:*

Use the distance formula to find the length of each side of the triangles.

The side lengths of the triangle *STU* are:

$$ST = \sqrt{(0-0)^2 + (0-5)^2} = 5;$$

$$TU = \sqrt{(1-0)^2 + (1-0)^2} = \sqrt{2};$$

$$SU = \sqrt{(1-0)^2 + (1-5)^2} = \sqrt{17}$$

The side lengths of the triangle *XYZ* are:

$$XY = \sqrt{(4-4)^2 + (3-8)^2} = 5;$$

$$YZ = \sqrt{(6-4)^2 + (3-3)^2} = 2;$$

$$ZX = \sqrt{(6-4)^2 + (3-8)^2} = \sqrt{29}$$

The corresponding sides are not congruent.
Therefore, the triangles are not congruent.

*ANSWER:*

$SU = \sqrt{17}$, $TU = \sqrt{2}$, $ST = 5$, $XZ = \sqrt{29}$, $YZ = 2$, $XY = 5$; the corresponding sides are not congruent; the triangles are not congruent.

58. $S(2, 2)$, $T(4, 6)$, $U(3, 1)$, $X(-2, -2)$, $Y(-4, 6)$, $Z(-3, 1)$

*SOLUTION:*

Use the distance formula to find the length of each side of the triangles.

The side lengths of the triangle *STU* are:

$$ST = \sqrt{(4-2)^2 + (6-2)^2} = \sqrt{20};$$

$$TU = \sqrt{(3-4)^2 + (1-6)^2} = \sqrt{26};$$

$$SU = \sqrt{(3-2)^2 + (1-2)^2} = \sqrt{2}$$

The side lengths of the triangle *XYZ* are:

$$XY = \sqrt{(-4+2)^2 + (6+2)^2} = \sqrt{68};$$

$$YZ = \sqrt{(-3+4)^2 + (1-6)^2} = \sqrt{26};$$

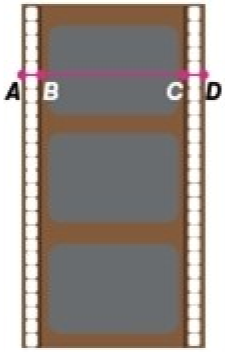
$$ZX = \sqrt{(-3+2)^2 + (1+2)^2} = \sqrt{10}$$

The corresponding sides are not congruent.
Therefore, the triangles are not congruent.

*ANSWER:*

$SU = \sqrt{2}$, $TU = \sqrt{26}$, $ST = \sqrt{20}$, $XZ = \sqrt{10}$, $YZ = \sqrt{26}$, $XY = \sqrt{68}$; the corresponding sides are not congruent; the triangles are not congruent.

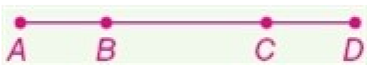
59. **PHOTOGRAPHY** Film is fed through a traditional camera by gears that catch the perforation in the film. The distance from $A$ to $C$ is the same as the distance from $B$ to $D$. Show that the two perforated strips are the same width.




*SOLUTION:*

Given: $AC = BD$

Prove: $AB = CD$




Proof:

Statement (Reasons)

1. $AC = BD$ (Given)
2. $AC = AB + BC$

$BD = BC + CD$ (Segment Addition Postulate)

3. $AB + BC = BC + CD$ (Substitution)
4. $\overline{BC} \cong \overline{BC}$ (Reflexive)
5. $BC = BC$ (Definition of congruent segments)
6. $AB = CD$ (Subt. Prop)

*ANSWER:*

Given: $AC = BD$

Prove: $AB = CD$


Proof:

Statement (Reasons)

1. $AC = BD$ (Given)
2. $AC = AB + BC$

$BD = BC + CD$ (Seg. Add. Post.)

3. $AB + BC = BC + CD$ (Subst.)
4. $\overline{BC} \cong \overline{BC}$ (Reflexive)
5. $BC = BC$ (Def. of $\cong$ Segs.)
6. $AB = CD$ (Subt. Prop)

**State the property that justifies each statement.**

60. If $x(y + z) = a$, then $xy + xz = a$.

*SOLUTION:*

Distributive Property

*ANSWER:*

Dist. Prop.

61. If $n - 17 = 39$, then $n = 56$.

*SOLUTION:*

Addition Property

*ANSWER:*

Addition Property

62. If $m\angle P + m\angle Q = 110$ and $m\angle R = 110$, then $m\angle P + m\angle Q = m\angle R$.

*SOLUTION:*

Substitution Property.

*ANSWER:*

Substitution

63. If $cv = md$ and $md = 15$, then $cv = 15$.

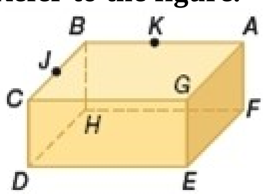
*SOLUTION:*

Transitive Property.

*ANSWER:*

Trans. Prop.

**Refer to the figure.**




64. How many planes appear in this figure?

*SOLUTION:*

The planes in the figure are:
*CDGE*, *GEFA*, *ABFH*, *CDBH*, *ABCG*, *DEFH*.
So, there are 6 planes.

*ANSWER:*

6

65. Name three points that are collinear.

*SOLUTION:*
*A*, *K*, *B* or *B*, *J*, *C* lie on a straight line. Therefore, they are collinear.

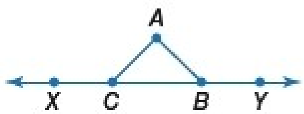
*ANSWER:*
*A*, *K*, *B* or *B*, *J*, *C*

66. Are points *A*, *C*, *D*, and *J* coplanar?

*SOLUTION:*
No; *A*, *C*, and *J* lie in plane *ABC*, but *D* does not.

*ANSWER:*
No; *A*, *C*, and *J* lie in plane *ABC*, but *D* does not.

67. **PROOF** If $\angle ACB \cong \angle ABC$, then $\angle XCA \cong \angle YBA$.




*SOLUTION:*
**Proof:**
**Statement (Reasons)**
1. $\angle ACB \cong \angle ABC$ (Given)
2. $\angle XCA$ and $\angle ACB$ are a linear pair. $\angle ABC$ and $\angle ABY$ are a linear pair. (Definition of Linear Pair)
3. $\angle XCA$, $\angle ACB$ and $\angle ABC$, $\angle ABY$ are supplementary. (Supplementary Theorem)
4. $\angle XCA \cong \angle YBA$ (Angles supplementary to congruent angles are congruent)

*ANSWER:*
**Proof:**
**Statement (Reasons)**
1. $\angle ACB \cong \angle ABC$ (Given)
2. $\angle XCA$ and $\angle ACB$ are a linear pair. $\angle ABC$ and $\angle ABY$ are a linear pair. (Def. of Linear Pair)
3. $\angle XCA$, $\angle ACB$ and $\angle ABC$, $\angle ABY$ are suppl. (Suppl. Thm.)
4. $\angle XCA \cong \angle YBA$ ($\angle$s suppl. to $\cong \angle$s are $\cong$.)