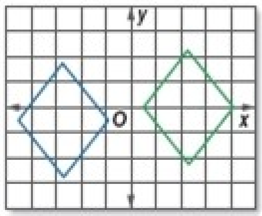
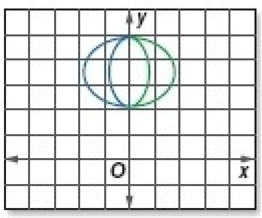
4-7 Congruence Transformations

Identify the type of congruence transformation shown as a *reflection, translation,* or *rotation*.

1. 

SOLUTION:
Each vertex and its image are in the same position, just about 5 units right and about 0.5 unit up. This is a translation.

ANSWER:
translation

2. 

SOLUTION:
The edges of the curve and its image are the same distance from the y-axis. This is a reflection.

ANSWER:
reflection

3. **Refer to the figure on page 299.**

SOLUTION:
The tops of each tree are the same distance from the horizontal line of the water. This is a reflection.

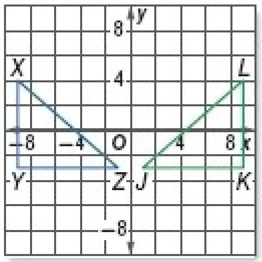
ANSWER:
reflection

4. **Refer to the figure on page 299.**

SOLUTION:
Each animal on the merry-go-round is the same distance from the center as the other animals so each animal can be an image of another animal. Each car on the Ferris wheel is the same distance from the center of the Ferris wheel. Therefore, each car on the Ferris wheel could be an image of another car.
The congruence transformation shown is rotation.

ANSWER:
rotation

COORDINATE GEOMETRY Identify each transformation and verify that it is a congruence transformation.

5. 

SOLUTION:

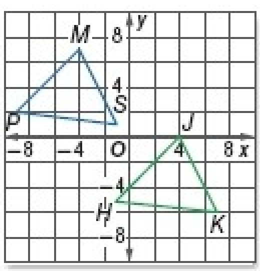
Each vertex and its image are the same distance from the y-axis. ΔLKJ is a reflection of ΔXYZ.

$XY = 7$, $YZ = 8$, by the Pythagorean Theorem, $XZ = \sqrt{8^2 + 7^2}$ or $\sqrt{113}$

$K = 7$, $KJ = 8$, by the Pythagorean Theorem, $LJ = \sqrt{8^2 + 7^2}$ or $\sqrt{113}$. Therefore $\Delta XYZ \cong \Delta LKJ$ by SSS.

ANSWER:

ΔLKJ is a reflection of ΔXYZ. $XY = 7$, $YZ = 8$, $XZ = \sqrt{113}$ $KJ = 8$, $LJ = \sqrt{113}$ $LK = 7$. $\Delta XYZ \cong \Delta LKJ$ by SSS.

6. 

SOLUTION:

Each vertex and its image are in the same position, just 8 units right and 7 units down. This is a translation. That is, ΔJHK is a translation of ΔMPS.

Use the distance formula. $\overline{MP}$ has endpoints $M(–4,7)$ and $P(–9, 2)$.

$$\begin{aligned} MP &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} && \text{Distance Formula} \\ &= \sqrt{(-9-(-4))^2 + (2-7)^2} && \text{Substitute.} \\ &= \sqrt{(-5)^2 + (-5)^2} && \text{Subtraction.} \\ &= \sqrt{25+25} && \text{Square terms.} \\ &= \sqrt{50} && \text{Addition.} \end{aligned}$$

$\overline{PS}$ has endpoints $P(–9, 2)$ and $S(–1, 1)$.

$$\begin{aligned} PS &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} && \text{Distance Formula} \\ &= \sqrt{(-1-(-9))^2 + (1-2)^2} && \text{Substitute.} \\ &= \sqrt{(8)^2 + (-1)^2} && \text{Subtraction.} \\ &= \sqrt{64+1} && \text{Square terms.} \\ &= \sqrt{65} && \text{Addition.} \end{aligned}$$

$\overline{SM}$ has endpoints $S(-1, 1)$ and $M(-4, 7)$.

$$\begin{aligned} SM &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} && \text{Distance Formula} \\ &= \sqrt{(-4-(-1))^2 + (7-1)^2} && \text{Substitute.} \\ &= \sqrt{(-3)^2 + (6)^2} && \text{Subtraction.} \\ &= \sqrt{9+36} && \text{Square terms.} \\ &= \sqrt{45} && \text{Addition.} \end{aligned}$$

Similarly, find the lengths of $\overline{JH}$, $\overline{HK}$ and $\overline{KJ}$. $\overline{JH}$ has end points $J(4,0)$ and $H(-1, -5)$.

$$\begin{aligned} JH &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} && \text{Distance Formula} \\ &= \sqrt{(-1-4)^2 + (-5-0)^2} && \text{Substitute.} \\ &= \sqrt{(-5)^2 + (-5)^2} && \text{Subtraction.} \\ &= \sqrt{25+25} && \text{Square terms.} \\ &= \sqrt{50} && \text{Addition.} \end{aligned}$$

$\overline{HK}$ has endpoints $H(-1, -5)$ and $K(7, -6)$.

$$\begin{aligned} HK &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} && \text{Distance Formula} \\ &= \sqrt{(7-(-1))^2 + (-6-(-5))^2} && \text{Substitute.} \\ &= \sqrt{8^2 + (-1)^2} && \text{Subtraction.} \\ &= \sqrt{64+1} && \text{Square terms.} \\ &= \sqrt{65} && \text{Addition.} \end{aligned}$$

$\overline{KJ}$ has endpoints $K(7, -6)$ and $J(4, 0)$.

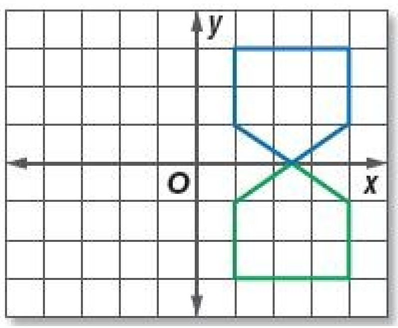
$$\begin{aligned} KJ &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} && \text{Distance Formula} \\ &= \sqrt{(4-7)^2 + (0-(-6))^2} && \text{Substitute.} \\ &= \sqrt{(-3)^2 + (6)^2} && \text{Subtraction.} \\ &= \sqrt{9+36} && \text{Square terms.} \\ &= \sqrt{45} && \text{Addition.} \end{aligned}$$

So, $\overline{MP} \cong \overline{JH}$, $\overline{PS} \cong \overline{HK}$ and $\overline{SM} \cong \overline{KJ}$.
Each pair of corresponding sides has the same measure so they are congruent. $\Delta MPS \cong \Delta JHK$ by SSS.

ANSWER:

ΔJHK is a translation of ΔMPS $MP = \sqrt{50}$, $PS = \sqrt{65}$, $SM = \sqrt{45}$, $JH = \sqrt{50}$, $JK = \sqrt{45}$, $HK = \sqrt{65}$ $\Delta MPS \cong \Delta JHK$ by SSS.

CCSS STRUCTURE Identify the type of congruence transformation shown as a *reflection, translation,* or *rotation*.

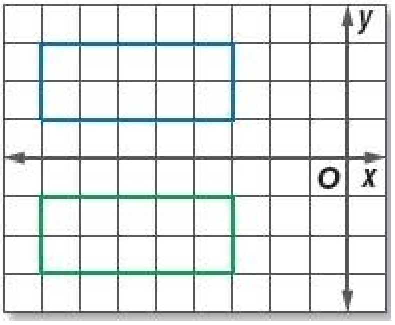
7. 

SOLUTION:

Each vertex and its image are the same distance from the *x*-axis. This is a reflection.

ANSWER:

reflection

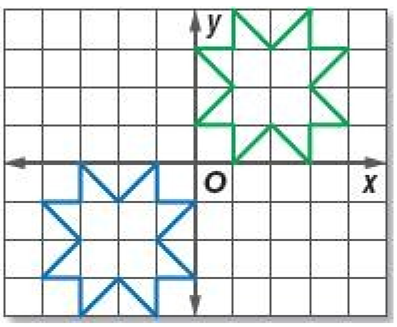
8. 

SOLUTION:

Each vertex and its image are in the same position, just 4 units down so this is a translation. Since each vertex and its image are the same distance from the *x*-axis this is also a reflection.

ANSWER:

translation or reflection

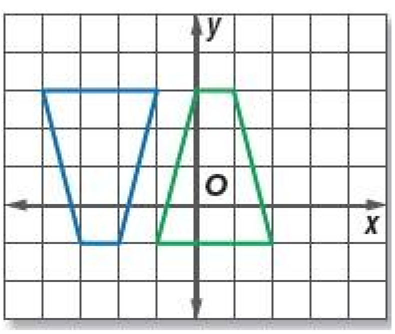
9. 

SOLUTION:

The image in green can be found by translating the blue figure to the right and up, by reflecting the figure in the line $y = -x$, or by rotating the figure 180 degrees in either direction.

ANSWER:

translation, reflection, or rotation

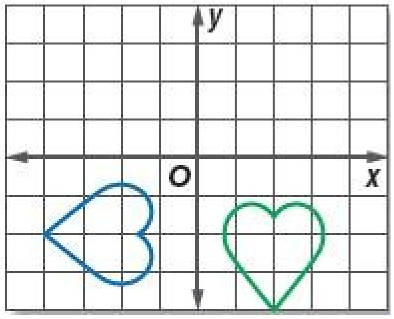
10. 

SOLUTION:

The image in green can be found by rotating the figure 180 degrees. Each vertex and its image are the same distance from (–1, 1). The angles formed by each pair of corresponding points and the origin are congruent. This is a rotation.

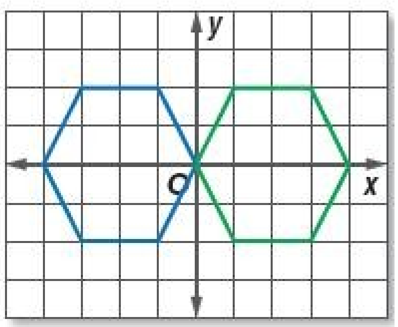
ANSWER:

rotation

11. 

SOLUTION:
Each vertex and its image are the same distance from (0.5, 0). The angles formed by each pair of corresponding points and the origin are congruent. This is a rotation.

ANSWER:
rotation

12. 

SOLUTION:
The image in green can be found by translating the blue figure to the right 4 units, by reflecting the figure in the *y*-axis, or by rotating the figure 180 degrees. Since each vertex and its image are the same distance from the *y*-axis it is a reflection. This is a translation because each vertex and its image are in the same position just 4 units to the right. Each vertex and its image are the same distance from the origin. The angles formed by each pair of corresponding points and the origin are congruent so this is a rotation.

ANSWER:
reflection, rotation, or translation

Identify the type of congruence transformation shown in each picture as a *reflection, translation,* or *rotation*.

13. **Refer to the figure on page 300.**

SOLUTION:
Moving a chess piece on the board is a translation.

ANSWER:
translation

4-7 Congruence Transformations

14. **Refer to the figure on page 300.**

SOLUTION:
The building is reflected in the water so this is a reflection.

ANSWER:
reflection

15. **Refer to the figure on page 300.**

SOLUTION:
As the steering wheel turns, each spoke will rotate about the center. This is a rotation.

ANSWER:
rotation

16. **Refer to the figure on page 300.**

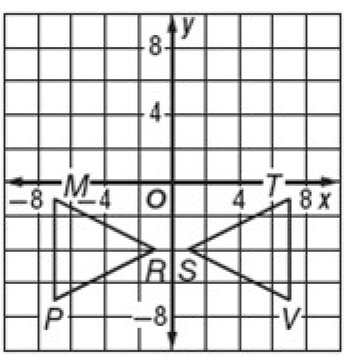
SOLUTION:
The train cars moving along the track is a translation.

ANSWER:
translation

COORDINATE GEOMETRY Graph each pair of triangles with the given vertices. Then, identify the transformation, and verify that it is a congruence transformation.

17. *M*(–7, –1), *P*(–7, –7), *R*(–1, –4); *T*(7, –1), *V*(7, –7), *S*(1, –4)

SOLUTION:



Each vertex and its image are the same distance from the y-axis. This is a reflection. That is, ΔTVS is a reflection of ΔMPR.

Use the distance formula. $\overline{MP}$ has endpoints *M*(–7,–1) and *P*(–7, –7).

$$
\begin{aligned}
MP &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} && \text{Distance Formula} \\
&= \sqrt{(-7-(-7))^2 + (-7-(-1))^2} && \text{Substitute.} \\
&= \sqrt{(0)^2 + (-6)^2} && \text{Subtraction.} \\
&= \sqrt{36} && \text{Square terms.} \\
&= 6 && \text{Addition.}
\end{aligned}
$$

$\overline{PR}$ has endpoints $P(–7, –7)$ and $R(–1, –4)$.

$$\begin{aligned} PR &= \sqrt{(-1-(-7))^2+(-4-(-7))^2} && \text{Substitute.} \\ &= \sqrt{(6)^2+(3)^2} && \text{Simplify.} \\ &= \sqrt{36+9} \\ &= \sqrt{45} \end{aligned}$$

$\overline{RM}$ has endpoints $R(–1, –4)$ and $M(–7, –1)$.

$$\begin{aligned} RM &= \sqrt{(-7-(-1))^2+(-1-(-4))^2} && \text{Substitute.} \\ &= \sqrt{(-6)^2+(3)^2} && \text{Simplify.} \\ &= \sqrt{36+9} \\ &= \sqrt{45} \end{aligned}$$

Similarly, find the lengths of $\overline{TV}, \overline{VS}$ and $\overline{ST}$. $\overline{TV}$ has endpoints $T(7,–1)$ and $V(7, –7)$.

$$\begin{aligned} TV &= \sqrt{(7-7)^2+(-7-(-1))^2} && \text{Substitute.} \\ &= \sqrt{(0)^2+(-6)^2} && \text{Simplify.} \\ &= \sqrt{36} \\ &= 6 \end{aligned}$$

$\overline{VS}$ has endpoints $H(7, –7)$ and $S(1, –4)$.

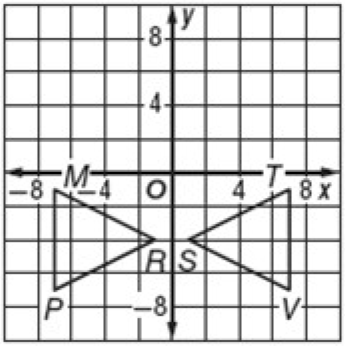
$$\begin{aligned} VS &= \sqrt{(1-7)^2+(-4-(-7))^2} && \text{Substitute.} \\ &= \sqrt{(-6)^2+(3)^2} && \text{Simplify.} \\ &= \sqrt{36+9} \\ &= \sqrt{45} \end{aligned}$$

$\overline{ST}$ has endpoints $S(1, –4)$ and $T(7, –1)$.

$$\begin{aligned} ST &= \sqrt{(7-1)^2+(-1-(-4))^2} && \text{Substitute.} \\ &= \sqrt{(6)^2+(3)^2} && \text{Simplify.} \\ &= \sqrt{36+9} \\ &= \sqrt{45} \end{aligned}$$

So, $\overline{MP} \cong \overline{TV}, \overline{PR} \cong \overline{VS}$ and $\overline{RP} \cong \overline{ST}$.
Each pair of corresponding sides has the same measure so they are congruent. $\Delta MPR \cong \Delta TVS$ by SSS.

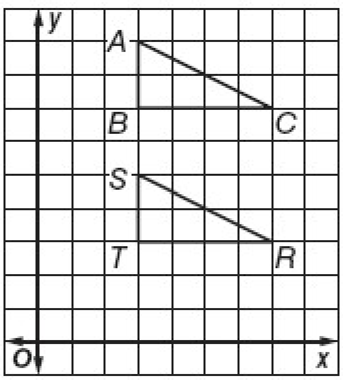
ANSWER:



ΔTVS is a reflection of ΔMPR. MP = 6, $PR = \sqrt{45}, MR = \sqrt{45}$, TV = 6, $VS = \sqrt{45}, ST = \sqrt{45}$. $\Delta MPR \cong \Delta TVS$ by SSS.

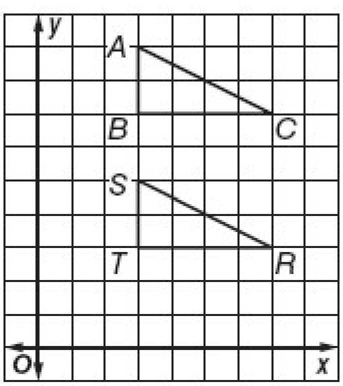
18. $A(3, 9)$, $B(3, 7)$, $C(7, 7)$; $S(3, 5)$, $T(3, 3)$, $R(7, 3)$

SOLUTION:



Each vertex and its image are in the same position, just 2 units up. This is a translation. That is, ΔABC is a translation of ΔSTR. Here, AB = 2, BC = 4, by the Pythagorean Theorem, $CA = \sqrt{2^2 + 4^2}$ or $\sqrt{20}$. Similarly ST = 2, TR = 4, so by the Pythagorean Theorem, $RS = \sqrt{2^2 + 4^2}$ or $\sqrt{20}$. So, $\Delta ABC \cong \Delta STR$ by SSS.

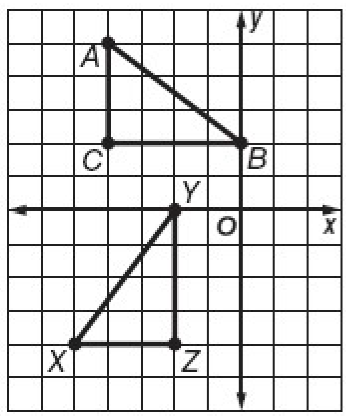
ANSWER:



ΔABC is a translation of ΔSTR. AB = 2, BC = 4, $BC = 4, CA = \sqrt{20}$, ST = 2, $TR = 4, SR = \sqrt{20}$. $\Delta ABC \cong \Delta STR$ by SSS.

19. $A(-4, 5)$, $B(0, 2)$, $C(-4, 2)$; $X(-5, -4)$, $Y(-2, 0)$, $Z(-2, -4)$

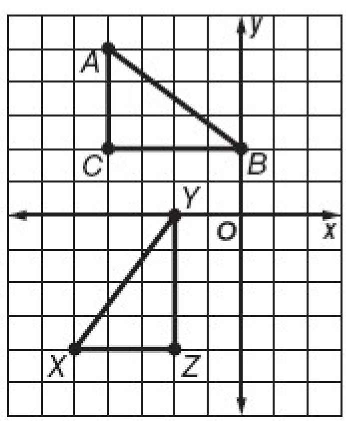
SOLUTION:



ΔXYZ is a rotation of ΔABC.

Here, $AC = 3$, $BC = 4$, by the Pythagorean Theorem, $AB = \sqrt{3^2 + 4^2}$ or 5. Similarly $YZ = 4$, $XZ = 3$, by the Pythagorean Theorem, $XY = \sqrt{3^2 + 4^2}$ or 5. Since $AB = XY$, $BC = YZ$, and $AC = XZ$, $\overline{AB} \cong \overline{XY}$, $\overline{BC} \cong \overline{YZ}$, and $\overline{AC} \cong \overline{XZ}$ by definition of congruence so $\Delta ABC \cong \Delta XYZ$ by SSS.

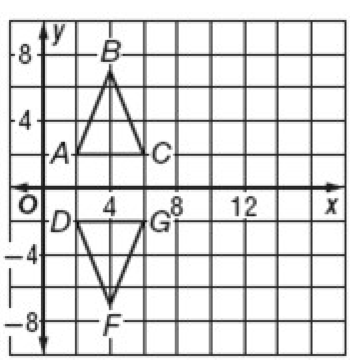
ANSWER:



ΔXYZ is a rotation of ΔABC. $AB = 5$, $BC = 4$, $AC = 3$, $XY = 5$, $YZ = 4$, $XZ = 3$. Since $AB = XY$, $BC = YZ$, and $AC = XZ$, $\overline{AB} \cong \overline{XY}$, $\overline{BC} \cong \overline{YZ}$, and $\overline{AC} \cong \overline{XZ}$, $\Delta ABC \cong \Delta XYZ$ by SSS.

20. $A(2, 2)$, $B(4, 7)$, $C(6, 2)$; $D(2, -2)$, $F(4, -7)$, $G(6, -2)$

SOLUTION:



Each vertex and its image are the same distance from the *y*-axis. This is a reflection. That is, ΔABC is a reflection of $\Delta DFG.$.

Use the distance formula. $\overline{AB}$ has end points *A*(2, 2) and *B*(4, 7).

$$\begin{aligned} AB &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} && \text{Distance Formula} \\ &= \sqrt{(4-2)^2 + (7-2)^2} && \text{Substitute.} \\ &= \sqrt{(2)^2 + (5)^2} && \text{Subtraction.} \\ &= \sqrt{4+25} && \text{Square terms.} \\ &= \sqrt{29} && \text{Addition.} \end{aligned}$$

$\overline{BC}$ has end points *B*(4, 7) and *C*(6, 2).

$$\begin{aligned} BC &= \sqrt{(6-4)^2 + (2-7)^2} && \text{Substitute.} \\ &= \sqrt{(2)^2 + (-5)^2} && \text{Simplify.} \\ &= \sqrt{4+25} \\ &= \sqrt{29} \end{aligned}$$

$\overline{CA}$ has end points *C*(6, 2) and *A*(2, 2).

$$\begin{aligned} CA &= \sqrt{(2-6)^2 + (2-2)^2} && \text{Substitute.} \\ &= \sqrt{(-4)^2 + (0)^2} && \text{Simplify.} \\ &= \sqrt{16+0} \\ &= 4 \end{aligned}$$

Similarly, find the lengths of $\overline{DF}$, $\overline{FG}$ and $\overline{GD}$. $\overline{DF}$ has end points *D*(2,–2) and *F*(4, –7).

$$\begin{aligned} DF &= \sqrt{(4-2)^2 + (-7-(-2))^2} && \text{Substitute.} \\ &= \sqrt{(2)^2 + (-5)^2} && \text{Simplify.} \\ &= \sqrt{4+25} \\ &= \sqrt{29} \end{aligned}$$

$\overline{FG}$ has end points *F*(4, –7) and *G*(6, –2).

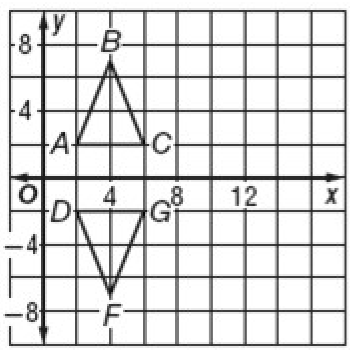
$$\begin{aligned} FG &= \sqrt{(6-4)^2 + (-2-(-7))^2} && \text{Substitute.} \\ &= \sqrt{(2)^2 + (5)^2} && \text{Simplify.} \\ &= \sqrt{4+25} \\ &= \sqrt{29} \end{aligned}$$

$\overline{GD}$ has end points $G(6, -2)$ and $D(2, -2)$.

$$\begin{aligned} GD &= \sqrt{(2-6)^2 + (-2-(-2))^2} && \text{Substitute.} \\ &= \sqrt{(-4)^2 + (0)^2} && \text{Simplify.} \\ &= \sqrt{16} \\ &= 4 \end{aligned}$$

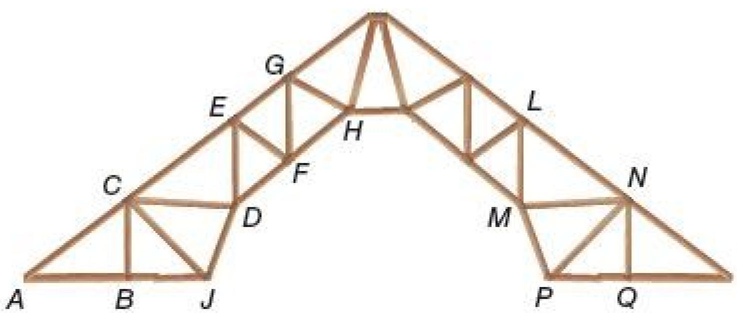
So, $\overline{AB} \cong \overline{DF}, \overline{BC} \cong \overline{FG}$ and $\overline{CA} \cong \overline{GD}$.
Each pair of corresponding sides has the same measure so they are congruent. $\Delta ABC \cong \Delta DFG$ by SSS.

ANSWER:



ΔABC is a reflection of ΔDFG. $AB = \sqrt{29}$, $BC = \sqrt{29}, CA = 4$, $DF = \sqrt{29}$, $FG = \sqrt{29}, GD = 4$.
$\Delta ABC \cong \Delta DFG$ by SSS.

CONSTRUCTION Identify the type of congruence transformation performed on each given triangle to generate the other triangle in the truss with matching left and right sides shown below.



21. ΔNMP to ΔCJD

SOLUTION:
Each point of ΔNMP and its image ΔCJD are the same distance from a center point. Therefore, this is a rotation.

ANSWER:
rotation

22. ΔEFD to ΔGHF

SOLUTION:
Each point of ΔEFD is moved the same distance and direction to map it onto ΔGHF. Therefore, this is a translation.

ANSWER:
translation

23. ΔCBJ to ΔNQP

SOLUTION:
If triangle CBJ is reflected in a vertical line that passes through the center of the truss, triangle NQP would be the resulting image. This is a reflection.

ANSWER:
reflection

AMUSEMENT RIDES Identify the type of congruence transformation shown in each picture as a reflection, translation, or rotation.

24. **Refer to the figure on page 300.**

SOLUTION:
Each person on the ride rotates about the center of the ride structure. This is a rotation.

ANSWER:
rotation

25. **Refer to the figure on page 300.**

SOLUTION:
This ride has a long bar that pivots at the center. The riders in the car rotate about the pivot point. This is a rotation.

ANSWER:
rotation

26. **Refer to the figure on page 300.**

SOLUTION:
The roller coaster riders move along the track so this is a translation.

ANSWER:
translation

27. **SCHOOL** Identify the transformations that are used to open a combination lock on a locker. If appropriate, identify the line of symmetry or center of rotation.

SOLUTION:
Rotation; the knob is the center of rotation. As you turn the dial on the lock, the edges of the dial are always the same distance from the center.

ANSWER:
Rotation; the knob is the center of rotation.

28. **CCSS STRUCTURE** Determine which capital letters of the alphabet have vertical and/or horizontal lines of reflection.

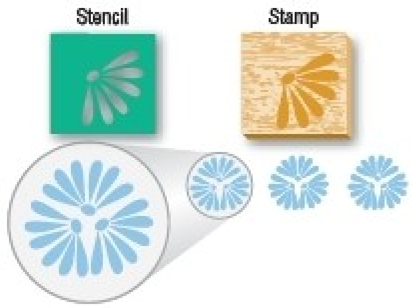
SOLUTION:

vertical: A, H, I, M, O, T, U, V, W, X, and Y; horizontal: B, C, D, E, H, I, K, O, and X

ANSWER:

vertical: A, H, I, M, O, T, U, V, W, X, and Y; horizontal: B, C, D, E, H, I, K, O, and X

29. **DECORATING** Tionne is redecorating her bedroom. She can use stencils or a stamp to create the design shown.
a. If Tionne used the stencil, what type of transformation was used to produce each flower in the design?
b. What type of transformation was used if she used the stamp to produce each flower in the design?



SOLUTION:

a. Tionne used the stencil on one side, then flipped it and used the other side, then flipped it again to create the third flower in the design. She could have also used the stencil, then turned it to create the second flower, and turned it again to create the third flower. So, she could have used reflections or rotations.
b. Tionne used the stamp, then turned it to create the second flower, and turned it again to create the third flower. So, she used rotations.

ANSWER:

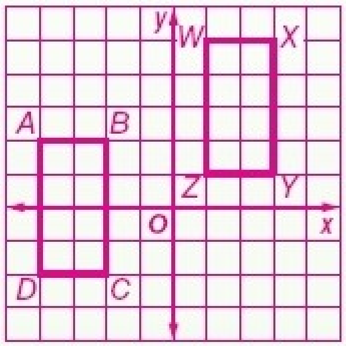
a. reflection or rotation
b. rotation

30. **MULTIPLE REPRESENTATIONS** In this problem, you will investigate the relationship between the ordered pairs of a figure and its translated image.
a. GEOMETRIC Draw congruent rectangles *ABCD* and *WXYZ* on a coordinate plane.
b. VERBAL How do you get from a vertex on *ABCD* to the corresponding vertex on *WXYZ* using only horizontal and vertical movement?
c. TABULAR Copy the table shown. Use your rectangles to fill in the *x*-coordinates, the *y*-coordinates, and the unknown value in the transformation column.
d. ALGEBRAIC Complete the following notation that represents the rule for the translation $ABCD \rightarrow WXYZ$: $(x, y) \rightarrow (x + ?, y + ?)$.

Rectangle *ABCD*	Transformation	Rectangle *WXYZ*
$A(?, ?)$	$(x_1 + ?, y_1 + ?)$	$W(?, ?)$
$B(?, ?)$	$(x_1 + ?, y_1 + ?)$	$X(?, ?)$
$C(?, ?)$	$(x_1 + ?, y_1 + ?)$	$Y(?, ?)$
$D(?, ?)$	$(x_1 + ?, y_1 + ?)$	$Z(?, ?)$

SOLUTION:

a. Sample answer:



b. Sample answer: You get from a vertex on *ABCD* to the corresponding vertex on *WXYZ* by moving 5 units to the right and 3 units up.

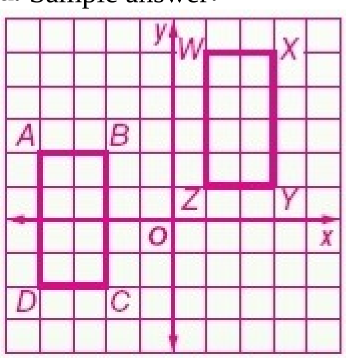
c. Sample answer:

Rectangle *ABCD*	Transformation	Rectangle *WXYZ*
$A(-4, 2)$	$(-4 + 5, 2 + 3)$	$W(1, 5)$
$B(-2, 2)$	$(-2 + 5, 2 + 3)$	$X(3, 5)$
$C(-2, -2)$	$(-2 + 5, -2 + 3)$	$Y(3, 1)$
$D(-4, -2)$	$(-4 + 5, -2 + 3)$	$Z(1, 1)$

d. Sample answer: $(x, y) \rightarrow (x + 5, y + 3)$

ANSWER:

a. Sample answer**:**



b. Sample answer: You get from a vertex on *ABCD* to the corresponding vertex on *WXYZ* by moving 5 units to the right and 3 units up.

c. Sample answer**:**

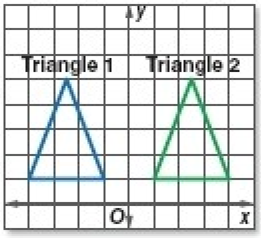
Rectangle *ABCD*	Transformation	Rectangle *WXYZ*
$A(-4, 2)$	$(-4 + 5, 2 + 3)$	$W(1, 5)$
$B(-2, 2)$	$(-2 + 5, 2 + 3)$	$X(3, 5)$
$C(-2, -2)$	$(-2 + 5, -2 + 3)$	$Y(3, 1)$
$D(-4, -2)$	$(-4 + 5, -2 + 3)$	$Z(1, 1)$

d. Sample answer: $(x, y) \rightarrow (x + 5, y + 3)$

31. **CHALLENGE** Use the diagram.

a. Identify two transformations of Triangle 1 that can result in Triangle 2.

b. What must be true of the triangles in order for more than one transformation on a preimage to result in the same image? Explain your reasoning.



SOLUTION:

a. Since each vertex and its image are the same difference from the y-axis, a reflection can map Triangle 2 to Triangle 1. Since each vertex and its image are in the same position, just 5 units to the right, a translation can map Triangle 2 to Triangle 1.

b. Sample answer: The triangles must be either isosceles or equilateral. When a triangle is isosceles or equilateral, a reflection and a translation result in the same image. They each have a line of symmetry so a reflection results in the same figure as a translation.

By counterexample, consider this scalene triangle that has been translated.

Compare the image to the image from a reflection. The two images are not the same.

ANSWER:

a. translation, reflection

b. Sample answer: The triangles must be either isosceles or equilateral. When triangles are isosceles or equilateral, they have a line of symmetry, so reflections result in the same figure.

32. **CCSS REASONING** A *dilation* is another type of transformation. In the diagram, a small paper clip has been dilated to produce a larger paper clip. Explain why dilations are not a congruence transformation.

Refer to the figure on page 301.

SOLUTION:

The images produced are not congruent to the original image.

ANSWER:

The images produced are not congruent to the original image.

OPEN ENDED Describe a real-world example of each of the following transformations, other than those given in this lesson.

33. reflection

SOLUTION:

Sample answer: A person looking in a mirror sees a reflection of himself or herself.

ANSWER:

Sample answer: A person looking in a mirror sees a reflection of himself or herself.

34. translation

SOLUTION:

Sample answer: A marching band moves across the field in a formation.

ANSWER:

Sample answer: A marching band moves across the field in a formation.

35. rotation

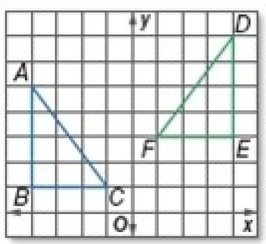
SOLUTION:

Sample answer: A faucet handle rotates when you turn the water on.

ANSWER:

Sample answer: A faucet handle rotates when you turn the water on.

36. **WRITING IN MATH** In the diagram ΔDEF is called a *glide reflection* of ΔABC. Based on the diagram, define a glide reflection. Is a glide reflection a congruence transformation? Explain your reasoning.



SOLUTION:

Sample answer: A glide reflection is a reflection over a line and then a translation in a direction that is parallel to the line of reflection. In a congruence transformation, the preimage and image are congruent.Yes; a glide reflection is a congruence transformation. In the diagram, *AB* = *DE*, *BC* = *EF*, and *AC* = *DF*, so $\overline{AB} \cong \overline{DE}$, $\overline{BC} \cong \overline{EF}$, and $\overline{AC} \cong \overline{DF}$, so $\Delta ABC \cong \Delta DEF$.

ANSWER:

Sample answer: A glide reflection is a reflection over a line and then a translation in a direction that is parallel to the line of reflection. In a congruence transformation, the preimage and image are congruent.Yes; a glide reflection is a congruence transformation. In the diagram, *AB* = *DE*, *BC* = *EF*, and *AC* = *DF*, so $\overline{AB} \cong \overline{DE}$, $\overline{BC} \cong \overline{EF}$, and $\overline{AC} \cong \overline{DF}$, so $\Delta ABC \cong \Delta DEF$.

37. **SHORT RESPONSE** Cindy is shopping for a new desk chair at a store where the desk chairs are 50% off. She also has a coupon for 50% off any one item. Cindy thinks that she can now get the desk chair for free. Is this true? If not, what will be the percent off she will receive with both the sale and the coupon?

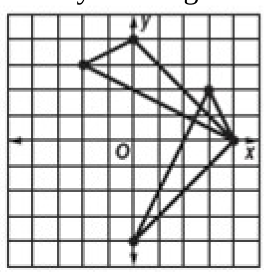
SOLUTION:

No; she will pay 50% of the 50% sale price which is 75% off the full price. If the desk chair she wants is $100 full price, the sale price is $50. With her coupon, she will pay $25 which is 75% off the full price.

ANSWER:

no; 75%

38. Identify the congruence transformation shown.



A dilation
B reflection
C rotation
D translation

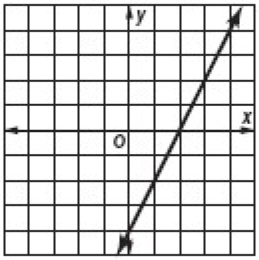
SOLUTION:

C

ANSWER:

C

39. Look at the graph below. What is the slope of the line shown?



F –2
G –1
H 1
J 2

SOLUTION:

Identify any two points on the line. (2, 0) and (4, 4).
Substitute the values in the slope formula.

$$\begin{aligned} m &= \frac{y_2 - y_1}{x_2 - x_1} && \text{Slope Formula} \\ &= \frac{4-0}{4-2} && \text{Substitute.} \\ &= \frac{4}{2} && \text{Simplify.} \\ &= 2 \end{aligned}$$

Therefore, the slope of the line is 2.
So, the correct choice is J.

ANSWER:

J

40. **SAT**/ACT What is the y-intercept of the line determined by the equation $3x - 4 = 12y - 3$?

A -12

B $-\frac{1}{12}$

C $\frac{1}{12}$

D $\frac{1}{4}$

E 12

SOLUTION:

Write the given equation in slope-intercept form.

$$\begin{aligned} 3x-4&=12y-3\\ 3x-4+3&=12y-3+3\\ 3x-1&=12y\\ \frac{3x}{12}-\frac{1}{12}&=y\\ \frac{x}{4}-\frac{1}{12}&=y \end{aligned}$$

That is, $y=\frac{1}{4}x-\frac{1}{12}$. Therefore, the y-intercept is $-\frac{1}{12}$.

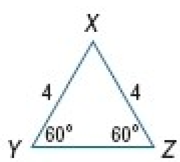
So, the correct choice is B.

ANSWER:

B

Find each measure.

41. *YZ*



SOLUTION:

The sum of the measures of the angles of a triangle is 180. Let x be the measure of unknown angle in the figure.

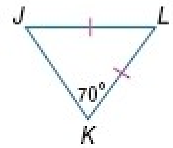
$$\begin{aligned} x+60+60&=180 && \text{Triangle Angle Sum Thm.}\\ x+120&=180 && \text{Simplify.}\\ x+120-120&=180-120 && -120 \text{ from each side.}\\ x&=60 && \text{Simplify.} \end{aligned}$$

Since all the angles are congruent, so it is an equilateral triangle. So, the length of *YZ* must be 4.

ANSWER:

4

42. $m\angle JLK$



SOLUTION:

Since ΔJLK is isosceles, $m\angle KJL = 70.$ The sum of the measures of the angles of a triangle is 180. So, $m\angle LKJ + m\angle KJL + m\angle JLK = 180$.

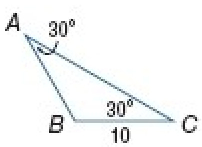
Substitute.

$$\begin{aligned} 70 + 70 + m\angle JLK &= 180 && \text{Triangle Angle Sum Thm.} \\ 140 + m\angle JLK &= 180 && \text{Simplify.} \\ 140 + m\angle JLK - 140 &= 180 - 140 && -140 \text{ from each side.} \\ m\angle JLK &= 40 && \text{Simplify.} \end{aligned}$$

ANSWER:

40

43. *AB*



SOLUTION:

This triangle has two congruent angles, so it is isosceles. So, $AB = BC = 10$.

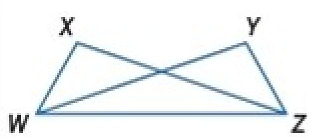
ANSWER:

10

PROOF Write a paragraph proof.

44. Given: $\angle YWZ \cong \angle XZW$ and $\angle YZW \cong \angle XWZ$

Prove: $\Delta WXZ \cong \Delta ZYW$



SOLUTION:

It is given that $\angle YWZ \cong \angle XZW$ and $\angle YZW \cong \angle XWZ$. By the Reflexive Property, $\overline{WZ} \cong \overline{WZ}$. Then $\Delta WXZ \cong \Delta ZYW$ by ASA.

ANSWER:

It is given that $\angle YWZ \cong \angle XZW$ and $\angle YZW \cong \angle XWZ$. By the Reflexive Property, $\overline{WZ} \cong \overline{WZ}$. Then $\Delta WXZ \cong \Delta ZYW$ by ASA.

45. **ROLLER COASTERS** The sign in front of the Electric Storm roller coaster states that all riders must be at least 54 inches tall to ride. If Andy is 5 feet 8 inches tall, can he ride the Electric Storm? Which law of logic leads you to this conclusion?

SOLUTION:

Yes, he can ride the Electric Storm; Law of Detachment

ANSWER:

yes; Law of Detachment

Find the coordinates of the midpoint of a segment with the given endpoints.

46. *A*(10, –12), *C*(5, –6)

SOLUTION:

Use the Midpoint Formula.

$$\left(\frac{x_1+x_2}{2},\frac{y_1+y_2}{2}\right)$$

Substitute.

$$\left(\frac{10+5}{2},\frac{-12-6}{2}\right)=(7.5,-9)$$

The midpoint of $\overline{AC}$ is (7.5, –9).

ANSWER:

(7.5, –9)

47. *A*(13, 14), *C*(3, 5)

SOLUTION:

Use the Midpoint Formula.

$$\left(\frac{x_1+x_2}{2},\frac{y_1+y_2}{2}\right)$$

Substitute.

$$\left(\frac{13+3}{2},\frac{14+5}{2}\right)=(8,9.5)$$

The midpoint of $\overline{AC}$ is (8, 9.5).

ANSWER:

(8, 9.5)

48. $A(-28, 8)$, $C(-10, 2)$

SOLUTION:

Use the Midpoint Formula.

$$\left(\frac{x_1+x_2}{2},\frac{y_1+y_2}{2}\right)$$

Substitute.

$$\left(\frac{-28-10}{2},\frac{8+2}{2}\right)=(-19,5)$$

The midpoint of $\overline{AC}$ is (–19, 5).

ANSWER:

(–19, 5)

49. $A(-12, 2)$, $C(-3, 5)$

SOLUTION:

Use the Midpoint Formula.

$$\left(\frac{x_1+x_2}{2},\frac{y_1+y_2}{2}\right)$$

Substitute.

$$\left(\frac{-12-3}{2},\frac{2+5}{2}\right)=(-7.5,3.5)$$

The midpoint of $\overline{AC}$ is (–7.5, 3.5).

ANSWER:

(–7.5, 3.5)

50. $A(0, 0)$, $C(3, -4)$

SOLUTION:

Use the Midpoint Formula.

$$\left(\frac{x_1+x_2}{2},\frac{y_1+y_2}{2}\right)$$

Substitute.

$$\left(\frac{0+3}{2},\frac{0-4}{2}\right)=(1.5,-2)$$

The midpoint of $\overline{AC}$ is (1.5, –2).

ANSWER:

(1.5, –2)

51. $A(2, 14)$, $C(0, 5)$

SOLUTION:

Use the Midpoint Formula.

$$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

Substitute.

$$\left(\frac{2+0}{2}, \frac{14+5}{2}\right) = (1, 9.5)$$

The midpoint of $\overline{AC}$ is (1, 9.5).

ANSWER:

(1, 9.5)