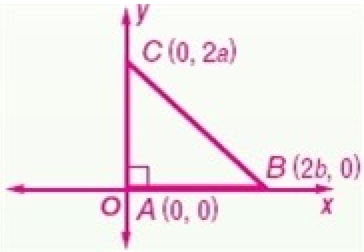
Position and label each triangle on the coordinate plane.

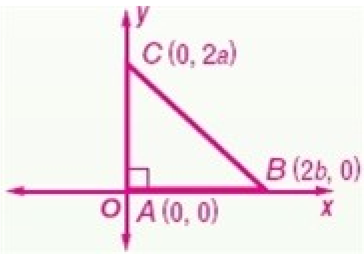
1. right $\triangle ABC$ with legs $\overline{AC}$ and $\overline{AB}$ so that $\overline{AC}$ is $2a$ units long and leg $\overline{AB}$ is $2b$ units long

SOLUTION:

Since this is a right triangle, two sides can be located on axis. Place the right angle of the triangle, $\angle A$, at the origin will allow the two legs to be along the x- and y-axes. Position the triangle in the first quadrant. Since C is on the y-axis, its x-coordinate is 0. Its y-coordinate is $2a$ because the leg is $2a$ units long. Since B is on the x-axis, its y-coordinate is 0. Its x-coordinate is $2b$ because the leg is $2b$ units long.



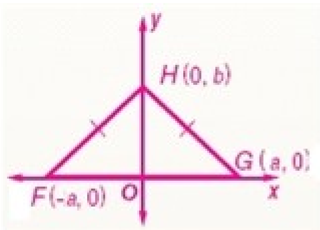
ANSWER:



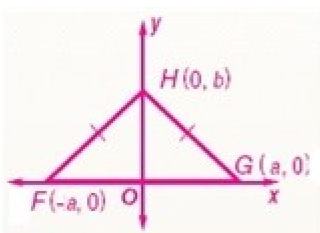
2. isosceles ΔFGH with base $\overline{FG}$ that is $2a$ units long

SOLUTION:

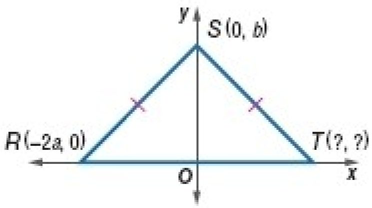
Since this is an isosceles triangle, two sides are congruent. Here, FG is $2a$ units long. Draw $\overline{FG}$ horizontally that passes through the origin, O. Since it is isosceles, draw $\overline{HO}$ on y axis which bisects $\overline{FG}$. So, the points F and G are equidistant from the origin. Since $FG = 2a$, $OG = a$ and $OF = a$. Since the point G is in the right side of the y-axis and on the x-axis, the coordinates of G are $(a, 0)$. Since the point F is in the left side of the y-axis and on the x-axis, the coordinates of F are $(–a, 0)$. Mark the point H such that $HG = HF$. Since the point H is on the y-axis, its x coordinate is 0. We can call the y-coordinate of H as b.



ANSWER:



Name the missing coordinate(s) of each triangle.

3. 

SOLUTION:

Since point *T* is on the *x*-axis, the *y*-coordinate of the point will be 0. On the triangle it is indicated that $\overline{RS} \cong \overline{TS}$.

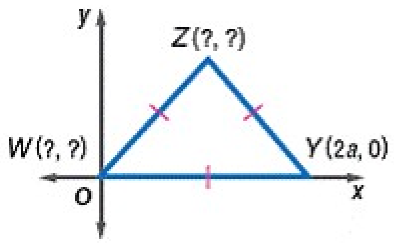
Let *n* be the *x*-coordinate of point *T*, and use the distance formula to find the value of *n* in terms of *a*.

$$\begin{aligned} RS &= ST && \text{Def. of congruent segments} \\ \sqrt{(0-(-2a))^2+(b-0)^2} &= \sqrt{(n-0)^2+(0-b)^2} && R(-2a, 0),\ S(0, b),\ T(n, 0) \\ \sqrt{4a^2+b^2} &= \sqrt{n^2+b^2} && \text{Simplify.} \\ 4a^2+b^2 &= n^2+b^2 && \text{Square each side.} \\ 4a^2 &= n^2 && \text{Subtract } b^2 \text{ from each side.} \\ 2a &= n && \text{Take the positive square root.} \end{aligned}$$

So, the coordinates of point *T* are (2*a*, 0).

ANSWER:

T(2*a*, 0)

4. 

SOLUTION:

W lies at the origin. So, W is (0, 0).
It is indicated on the diagram that $\overline{WZ} \cong \overline{YZ} \cong \overline{WY}$, so ΔWYZ is equilateral. The x-coordinate of Z is halfway between 0 and $2a$ or a.
Since point W and point Y lie on the x-axis at (0, 0) and $(2a, 0)$, $WY = 2a$.

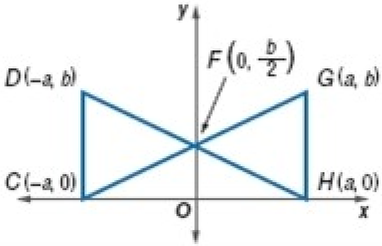
Let n represent the y-coordinate for point Z and use the distance formula to determine its value in terms of a.

$$\begin{aligned} WZ &= WY && \text{Def. of congruent segments} \\ \sqrt{(a-0)^2+(n-0)^2} &= 2a && W(0,0),\ Z(a,n),\ WY=2a \\ \sqrt{a^2+n^2} &= 2a && \text{Simplify.} \\ a^2+n^2 &= 4a^2 && \text{Square each side.} \\ n^2 &= 3a^2 && \text{Subtract } a^2 \text{ from each side.} \\ n &= \sqrt{3}a && \text{Take the positive square root.} \end{aligned}$$

Therefore, the missing coordinates are $W(0, 0)$, and $Z\left(a, \sqrt{3}a\right)$.

ANSWER:
$W(0, 0)$, $Z\left(a, \sqrt{3}a\right)$

5. **CCSS ARGUMENTS** Write a coordinate proof to show that $\Delta FGH \cong \Delta FDC$.



SOLUTION:

$DC = \sqrt{(-a-(-a)^2+(b-0)^2}$ or b
$GH = \sqrt{(a-a)^2+(b-0)^2}$ or b

Since $DC = GH$, $\overline{DC} \cong \overline{GH}$.

$DF = \sqrt{(0+a)^2+\left(\frac{b}{2}-b\right)^2}$ or $\sqrt{a^2+\frac{b^2}{4}}$
$GF = \sqrt{(a-0)^2+\left(b-\frac{b}{2}\right)^2}$ or $\sqrt{a^2+\frac{b^2}{4}}$
$CF = \sqrt{(0+a)^2+\left(\frac{b}{2}-0\right)^2}$ or $\sqrt{a^2+\frac{b^2}{4}}$
$HF = \sqrt{(a-0)^2+\left(0-\frac{b}{2}\right)^2}$ or $\sqrt{a^2+\frac{b^2}{4}}$

Since $DF = GF = CF = HF$, $\overline{DF} \cong \overline{GF} \cong \overline{CF} \cong \overline{HF}$. $\Delta FGH \cong \Delta FDC$ by SSS.

ANSWER:

$DC = \sqrt{(-a-(-a)^2+(b-0)^2}$ or b
$GH = \sqrt{(a-a)^2+(b-0)^2}$ or b
Since $DC = GH$, $\overline{DC} \cong \overline{GH}$.

$DF = \sqrt{(0+a)^2+\left(\frac{b}{2}-b\right)^2}$ or $\sqrt{a^2+\frac{b^2}{4}}$

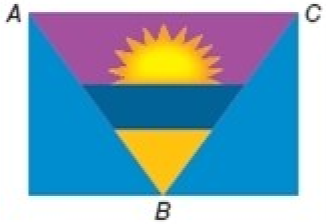
$GF = \sqrt{(a-0)^2+\left(b-\frac{b}{2}\right)^2}$ or $\sqrt{a^2+\frac{b^2}{4}}$

$CF = \sqrt{(0+a)^2+\left(\frac{b}{2}-0\right)^2}$ or $\sqrt{a^2+\frac{b^2}{4}}$

$HF = \sqrt{(a-0)^2+\left(0-\frac{b}{2}\right)^2}$ or $\sqrt{a^2+\frac{b^2}{4}}$

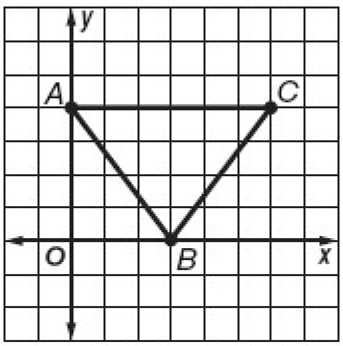
Since $DF = GF = CF = HF$, $\overline{DF} \cong \overline{GF} \cong \overline{CF} \cong \overline{HF}$. $\Delta FGH \cong \Delta FDC$ by SSS.

6. **FLAGS** Write a coordinate proof to prove that the large triangle in the center of the flag is isosceles. The dimensions of the flag are 4 feet by 6 feet and point *B* of the triangle bisects the bottom of the flag.



SOLUTION:

Given: $\triangle ABC$



Prove: $\triangle ABC$ is isosceles.

Proof: Use the Distance Formula to find *AB* and *BC*. $\triangle ABC$ with coordinates *A*(0, 4), *B*(3, 0), and *C*(6, 4).

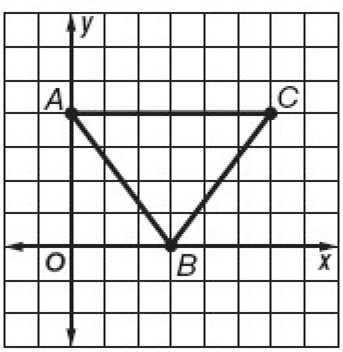
$AB = \sqrt{(3-0)^2 + (0-4)^2}$ or $\sqrt{25}$ or 5

$BC = \sqrt{(6-3)^2 + (4-0)^2}$ or $\sqrt{25}$ or 5

Since *AB* = *BC*, $\overline{AB} \cong \overline{BC}$. Since the legs are congruent, $\triangle ABC$ is isosceles.

ANSWER:

Given: $\triangle ABC$



Prove: $\triangle ABC$ is isosceles.

Proof: Use the Distance Formula to find *AB* and *BC*.

$AB = \sqrt{(4-0)^2 + (0-3)^2}$ *or* $\sqrt{25}$ or 5

$BC = \sqrt{(4-0)^2 + (6-3)^2}$ *or* $\sqrt{25}$ or 5

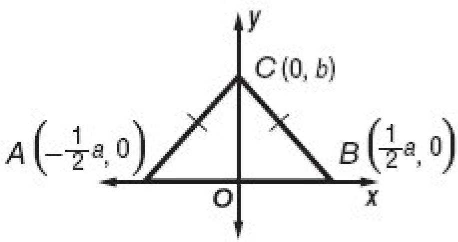
Since AB = BC, $\overline{AB} \cong \overline{BC}$. Since the legs are congruent, $\triangle ABC$ is isosceles.

Position and label each triangle on the coordinate plane.

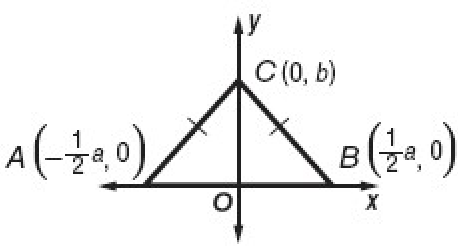
7. isosceles ΔABC with base $\overline{AB}$ that is *a* units long

SOLUTION:

Since this is an isosceles triangle, two sides are congruent. Here, *AB* is *a* units long. Draw $\overline{AB}$ horizontally that passes through the origin, *O*. Since it is isosceles, draw $\overline{CO}$ on *y* axis which bisects $\overline{AB}$. So, the points *A* and *B* are equidistance from the origin. Since *AB* = *a*, $OA = \frac{1}{2}a$ and $OB = \frac{1}{2}a$. Since the point *B* is in the right side of the *y*-axis and on the *x*-axis, the coordinates of *B* are $\left(\frac{1}{2}a, 0\right)$. Since the point *A* is in the left side of the *y*-axis and on the *x*-axis, the coordinates of *A* are $\left(-\frac{1}{2}a, 0\right)$. Mark the point *C* such that *AC* = *BC*. Since the point *C* is on the *y*-axis, its *x* coordinate is 0. Call the *y*-coordinate of *C* as *b*. The coordinates of point *C* are (0, *b*).



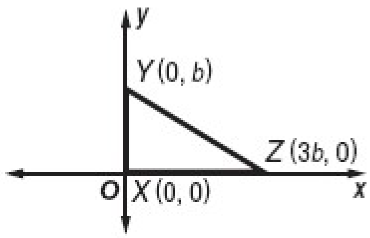
ANSWER:



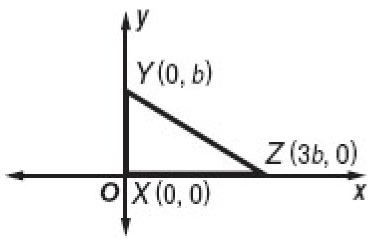
8. right $\triangle XYZ$ with hypotenuse $\overline{YZ}$, the length of $\overline{XY}$ is b units long, and the length of $\overline{XZ}$ is three times the length of $\overline{XY}$

SOLUTION:

Since this is a right triangle, two sides can be located on the axes. Place the right angle of the triangle, $\angle X$, at the origin will allow the two legs to be along the *x*- and *y*-axes. Position the triangle in the first quadrant. Since *Z* is on the *x*-axis, its *y*-coordinate is 0. Its *x*-coordinate is 3*b* because the leg is 3*b* units long. Since *Y* is on the *y*-axis, its *x*-coordinate is 0. Its *y*-coordinate is *b* because the leg is *b* units long.



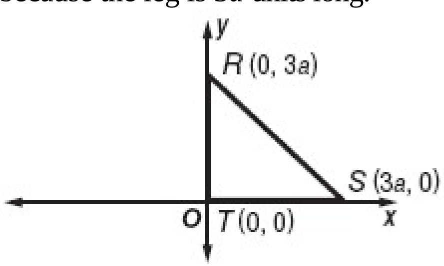
ANSWER:



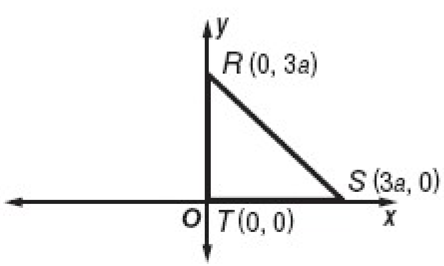
9. isosceles right $\triangle RST$ with hypotenuse $\overline{RS}$ and legs 3*a* units long

SOLUTION:

Since this is an isosceles right triangle, two sides can be located on axis. Placing the right angle of the triangle, $\angle T$, at the origin will allow the two legs to be along the *x*- and *y*-axes. Position the triangle in the first quadrant. Since *S* is on the *x*-axis, its *y*-coordinate is 0. Its *x*-coordinate is 3*a* because the leg is 3*a* units long. The given triangle is isosceles and $\overline{RS}$ is hypotenuse, so *TS* = *TR* = 3*a*.Since *R* is on the *y*-axis, its *x*-coordinate is 0. Its *y*-coordinate is 3*a* because the leg is 3*a* units long.



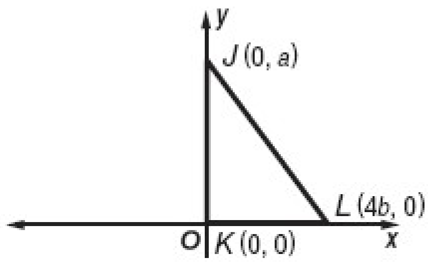
ANSWER:



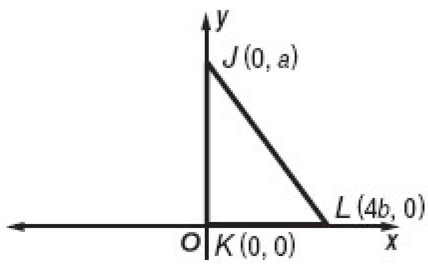
10. right $\triangle JKL$ with legs $\overline{JK}$ and $\overline{KL}$ so that $\overline{JK}$ is a units long and leg $\overline{KL}$ is $4b$ units long

SOLUTION:

Since this is a right triangle, two sides can be located on axis. Placing the right angle of the triangle, $\angle K$, at the origin will allow the two legs to be along the *x*- and *y*-axes. Position the triangle in the first quadrant. Since *L* is on the *x*-axis, its *y*-coordinate is 0. Its *x*-coordinate is 4*b* because the leg is 4*b* units long. Since *J* is on the *y*-axis, its *x*-coordinate is 0. Its *y*-coordinate is *a* because the leg is *a* units long.



ANSWER:



11. equilateral ΔGHJ with sides $\frac{1}{2}a$ units long

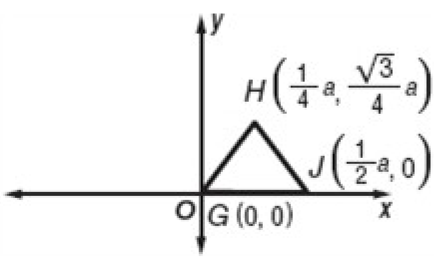
SOLUTION:

One side of the triangle can be located on the *x*-axis. Place point *G* at the origin and point *J* at $\left(\frac{1}{2}a, 0\right)$, so that $GJ = \frac{1}{2}a$. Place point *H* in the first quadrant so the triangle is equilateral and $\overline{GJ} \cong \overline{GH} \cong \overline{JH}$. The *x*-coordinate of *H* must be halfway between 0 and $\frac{1}{2}a$ or $\frac{1}{4}a$.

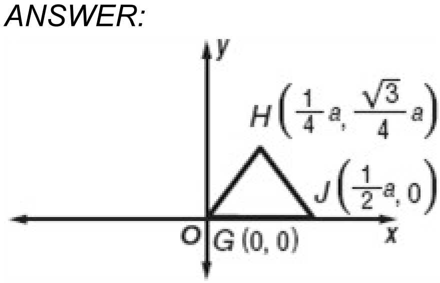
Let *n* represent the *y*-coordinate of point *H* and use the distance formula to determine its value in terms of *a*.

$$
\begin{aligned}
GH &= GJ && \text{Def. of congruent segments}\\
\sqrt{\left(\tfrac{1}{4}a-0\right)^2+(n-0)^2} &= \tfrac{1}{2}a && G(0,0),\ H\left(\tfrac{1}{4}a, n\right),\ GJ=\tfrac{1}{2}a\\
\sqrt{\tfrac{1}{16}a^2+n^2} &= \tfrac{1}{2}a && \text{Simplify.}\\
\tfrac{1}{16}a^2+n^2 &= \tfrac{1}{4}a^2 && \text{Square each side.}\\
n^2 &= \tfrac{3}{16}a^2 && \text{Subtract } \tfrac{1}{16}a^2 \text{ from each side.}\\
n &= \tfrac{\sqrt{3}}{4}a && \text{Take the positive square root.}
\end{aligned}
$$

Therefore, the coordinates of *H* should be $\left(\frac{1}{4}a, \frac{\sqrt{3}a}{4}\right)$.



ANSWER:



12. equilateral ΔDEF with sides 4b units long

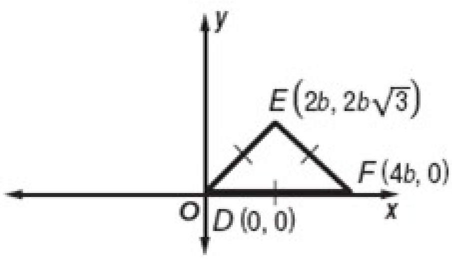
SOLUTION:

One side of the triangle can be located on the x-axis. Place point D at the origin and point F at (4b, 0), so that DF = 4b. Place point E in the first quadrant so the triangle is equilateral and $\overline{DF} \cong \overline{DE} \cong \overline{FE}$. The x-coordinate of E must be halfway between 0 and 4b or 2b.

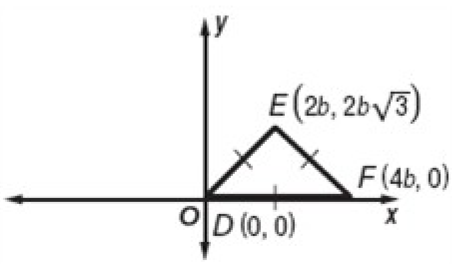
Let n represent the y-coordinate of point E and use the distance formula to determine its value in terms of a.

$$\begin{aligned} DE &= DF && \text{Def. of congruent segments} \\ \sqrt{(2b-0)^2+(n-0)^2} &= 4b && D(0,0),\ E(2b,n),\ DF=4b \\ \sqrt{4b^2+n^2} &= 4b && \text{Simplify.} \\ 4b^2+n^2 &= 16b^2 && \text{Square each side.} \\ n^2 &= 12b^2 && \text{Subtract } 4b^2 \text{ from each side.} \\ n &= 2b\sqrt{3} && \text{Take the positive square root.} \end{aligned}$$

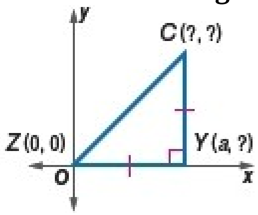
Therefore, the coordinates of E should be $\left(2b, 2b\sqrt{3}\right)$.



ANSWER:



Name the missing coordinate(s) of each triangle.

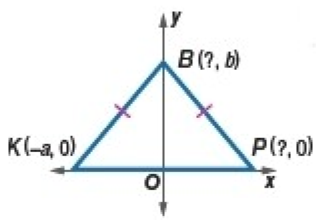
13. 

SOLUTION:

ΔZCY has two congruent sides, so it is isosceles. Here, $ZY = CY$. The point Y is on the x-axis, so its y-coordinate is 0. The coordinates of Y are $(a, 0)$. So, $ZY = CY = a$. The point C lies on the line $x = a$. So, its x-coordinate is a. It lies a units above the x-axis since $CY = a$. So, its y-coordinate is a. Hence C is (a, a).

ANSWER:

$C(a, a)$, $Y(a, 0)$

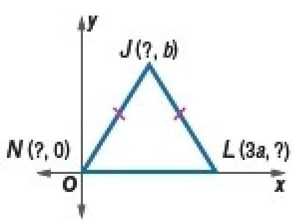
14. 

SOLUTION:

ΔKPB has two congruent sides, so it is isosceles. Here, $BP = BK$ and $\overline{OB}$ bisects $\overline{KP}$. So, $OK = OP = a$. Point P is on the right side of the y-axis and is on the x-axis, so the coordinates of P are $(a, 0)$. Point B is on the y axis, so its x-coordinate is zero. So, the coordinates of B are $(0, b)$.

ANSWER:

$P(a, 0)$, $B(0, b)$

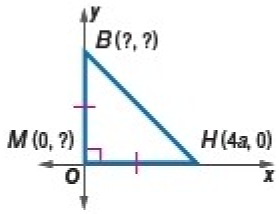
15. 

SOLUTION:

ΔNJL has two congruent sides, so it is isosceles. Here, $NJ = JL$. Point N is at origin, so its coordinates are (0, 0). Point L is on the x-axis, so its y-coordinate is 0. The coordinates of L are $(3a, 0)$. Find the coordinates of point J, that makes the triangle isosceles. This point is half way between N and L, so the x-coordinate is $1.5a$. Let b be the y-coordinate of J. So, the coordinates of J should be $(1.5a, b)$.

ANSWER:

$N(0, 0)$, $J(1.5a, b)$, $L(3a, 0)$

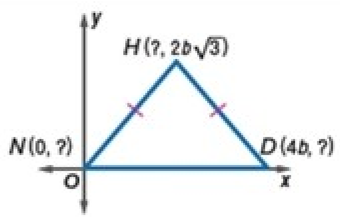
16. 

SOLUTION:

ΔMBH has two congruent sides, so it is isosceles. Point M is at origin, so its coordinates are (0, 0). So, $MB = MH = 4a$. Point B is on the y-axis, so its x-coordinate is 0. Also this point makes the triangle isosceles. So, its y-coordinate should be $4a$. The coordinates of B are $(0, 4a)$.

ANSWER:

$M(0, 0)$, $B(0, 4a)$

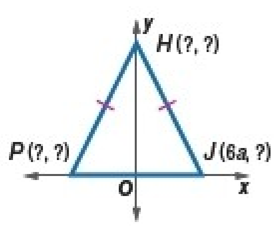
17. 

SOLUTION:

ΔNHD has two congruent sides, so it is isosceles. Here, $NH = HD$. Point N is at origin, so its coordinates are (0, 0). Point D is on the x-axis, so its y-coordinate is 0. The coordinates of D are $(4b, 0)$. Find the x-coordinate of point H, that makes the triangle isosceles. This point is half way between N and D, so the x-coordinate of H should be $\frac{4b}{2}$ or $2b$. Therefore, the coordinates of point H are $\left(2b, 2b\sqrt{3}\right)$.

ANSWER:

$H(2b, 2b\sqrt{3})$, $N(0, 0)$, $D(4b, 0)$

18. 

SOLUTION:

ΔPHJ has two congruent sides, so it is isosceles. $PH = HJ$ and $\overline{OH}$ bisects $\overline{PJ}$, so $OP = OJ = 6a$. Point P is on the left side of the y-axis and is on the x-axis, so the coordinates of point P are $(–6a, 0)$. Point J is on the x-axis, so its y-coordinate is zero. The coordinates of point J are $(6a, 0)$.Point H is on the y axis, so its x-coordinate is zero. We cannot write the y-coordinate of H in terms of a, so call it as b. So, the coordinates of H are $(0, b)$.

ANSWER:

$P(–6a, 0)$, $H(0, b)$, $J(6a, 0)$

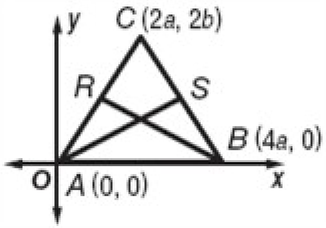
CCSS ARGUMENTS Write a coordinate proof for each statement.

19. The segments joining the base vertices to the midpoints of the legs of an isosceles triangle are congruent.

SOLUTION:

Draw isosceles triangle ABC on a coordinate plane, find the midpoints R and S of the two legs, and show that the segments connecting each midpoint with the opposite vertex have the same length. Start by placing a vertex at the origin and label it A. Use coordinates that are multiples of 2 because the Midpoint Formula involves dividing the sum of the coordinates by 2. So, place point B on the x-axis at $(4a, 0)$. Since the triangle is isosceles, the x-coordinate of point C is halfway between 0 and $4a$ or $2a$. We cannot write the y-coordinate in terms of a, so call it $2b$.

Given: Isosceles $\triangle ABC$ with $\overline{AC} \cong \overline{BC}$;
R and S are midpoints of legs $\overline{AC}$ and $\overline{BC}$.



Prove: $\overline{AS} \cong \overline{BR}$
Proof:

The coordinates of S are $\left(\frac{2a+4a}{2}, \frac{2b+0}{2}\right)$ or $(3a, b)$.

The coordinates of R are $\left(\frac{2a+0}{2}, \frac{2b+0}{2}\right)$ or (a, b).

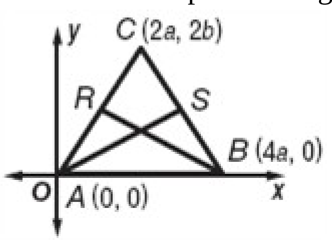
$AS = \sqrt{(3a-0)^2 + (b-0)^2}$ or $\sqrt{9a^2 + b^2}$

$BR = \sqrt{(4a-a)^2 + (0-b)^2}$ or $\sqrt{9a^2 + b^2}$

Since $AS = BR$, $\overline{AS} \cong \overline{BR}$.

ANSWER:

Given: Isosceles $\triangle ABC$ with $\overline{AC} \cong \overline{BC}$;
R and S are midpoints of legs $\overline{AC}$ and $\overline{BC}$.



Prove: $\overline{AS} \cong \overline{BR}$
Proof:

The coordinates of S are $\left(\frac{2a+4a}{2}, \frac{2b+0}{2}\right)$ or $(3a, b)$.

The coordinates of R are $\left(\frac{2a+0}{2}, \frac{2b+0}{2}\right)$ or (a, b).

$AS = \sqrt{(3a-0)^2 + (b-0)^2}$ or $\sqrt{9a^2 + b^2}$

$BR = \sqrt{(4a-a)^2 + (0-b)^2}$ or $\sqrt{9a^2 + b^2}$

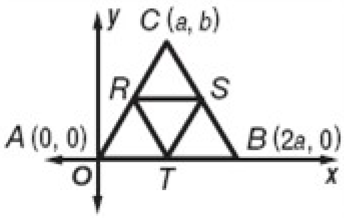
Since $AS = BR$, $\overline{AS} \cong \overline{BR}$.

20. The three segments joining the midpoints of the sides of an isosceles triangle form another isosceles triangle.

SOLUTION:

Draw isosceles triangle ABC on a coordinate plane, find the midpoints R, S, T of the three sides, and show that two of the three segments connecting the midpoints have the same length. Start by placing a vertex at the origin and label it A. Place point B on the x-axis at $(2a, 0)$. Since the triangle is isosceles, the x-coordinate of point C is halfway between 0 and $2a$ or a. We cannot write the y-coordinate in terms of a, so call it b.

Given: Isosceles triangle ABC; $\overline{BC} \cong \overline{AC}$;
R, S, and T are midpoints of their respective sides.



Prove: ΔRST is isosceles.
Proof:

Midpoint R is $\left(\frac{a+0}{2}, \frac{b+0}{2}\right)$ or $\left(\frac{a}{2}, \frac{b}{2}\right)$.

Midpoint S is $\left(\frac{a+2a}{2}, \frac{b+0}{2}\right)$ or $\left(\frac{3a}{2}, \frac{b}{2}\right)$.

Midpoint T is $\left(\frac{2a+0}{2}, \frac{0+0}{2}\right)$ or $(a, 0)$

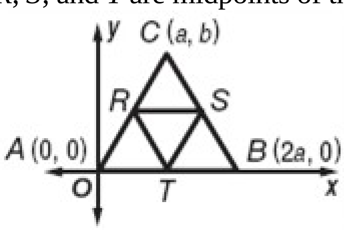
$RT = \sqrt{\left(\frac{a}{2} - a\right)^2 + \left(\frac{b}{2} - 0\right)^2}$ or $\sqrt{\left(\frac{a}{2}\right)^2 + \left(\frac{b}{2}\right)^2}$

$ST = \sqrt{\left(\frac{3a}{2} - a\right)^2 + \left(\frac{b}{2} - 0\right)^2}$ or $\sqrt{\left(\frac{a}{2}\right)^2 + \left(\frac{b}{2}\right)^2}$

So, $RT = ST$ and $\overline{RT} \cong \overline{ST}$. There is no need to find RS since these two sides already have equal measures. Therefore, by definition ΔRST is isosceles.

ANSWER:

Given: Isosceles triangle ABC; $\overline{BC} \cong \overline{AC}$;
R, S, and T are midpoints of their respective sides.



Prove: ΔRST is isosceles.
Proof:

Midpoint R is $\left(\frac{a+0}{2}, \frac{b+0}{2}\right)$ or $\left(\frac{a}{2}, \frac{b}{2}\right)$.

Midpoint S is $\left(\frac{a+2a}{2}, \frac{b+0}{2}\right)$ or $\left(\frac{3a}{2}, \frac{b}{2}\right)$.

Midpoint T is $\left(\frac{2a+0}{2}, \frac{0+0}{2}\right)$ or $(a, 0)$

$RT = \sqrt{\left(\frac{a}{2}-a\right)^2 + \left(\frac{b}{2}-0\right)^2}$ or $\sqrt{\left(\frac{a}{2}\right)^2 + \left(\frac{b}{2}\right)^2}$

$ST = \sqrt{\left(\frac{3a}{2}-a\right)^2 + \left(\frac{b}{2}-0\right)^2}$ or $\sqrt{\left(\frac{a}{2}\right)^2 + \left(\frac{b}{2}\right)^2}$

$RT = ST$ and $\overline{RT} \cong \overline{ST}$ and ΔRST is isosceles.

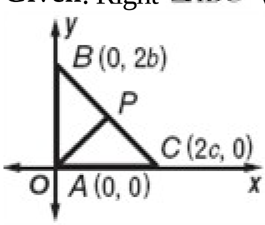
PROOF Write a coordinate proof for each statement.

21. The measure of the segment that joins the vertex of the right angle in a right triangle to the midpoint of the hypotenuse is one-half the measure of the hypotenuse.

SOLUTION:

Draw right triangle ABC on a coordinate plane, find the midpoint P of the hypotenuse, and show that the length of the segment joining the midpoint and the vertex of the right angle is one-half the length of the hypotenuse. Start by placing a vertex at the origin and label it A. Use coordinates that are multiples of 2 because the Midpoint Formula involves dividing the sum of the coordinates by 2. Place point C on the x-axis at $(2c, 0)$. Since the triangle is right, place point B on the y-axis at $(0, 2b)$.

Given: Right ΔABC with right $\angle BAC$; P is the midpoint of $\overline{BC}$.



Prove: $AP = \frac{1}{2}BC$

Proof:

Midpoint P is $\left(\frac{0+2c}{2}, \frac{2b+0}{2}\right)$ or (c, b).

$AP = \sqrt{(c-0)^2 + (b-0)^2}$ or $\sqrt{c^2 + b^2}$

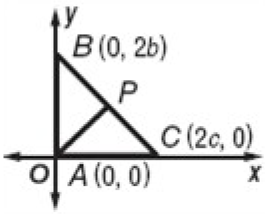
$BC = \sqrt{(2c-0)^2 + (0-2b)^2} = \sqrt{4c^2 + 4b^2}$ or $2\sqrt{c^2 + b^2}$

$\frac{1}{2}BC = \sqrt{c^2 + b^2}$

So, $AP = \frac{1}{2}BC.$

ANSWER:

Given: Right ΔABC with right $\angle BAC$; P is the midpoint of $\overline{BC}$.



Prove: $AP = \frac{1}{2}BC$

Proof:

Midpoint P is $\left(\frac{0+2c}{2}, \frac{2b+0}{2}\right)$ or (c, b).

$AP = \sqrt{(c-0)^2+(b-0)^2}$ or $\sqrt{c^2+b^2}$

$BC = \sqrt{(2c-0)^2+(0-2b)^2} = \sqrt{4c^2+4b^2}$ or $2\sqrt{c^2+b^2}$

$\frac{1}{2}BC = \sqrt{c^2+b^2}$

So, $AP = \frac{1}{2}BC.$

22. If a line segment joins the midpoints of two sides of a triangle, then its length is equal to one half the length of the third side.

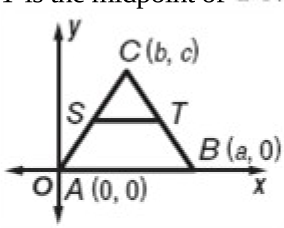
SOLUTION:

Draw general triangle ABC on a coordinate plane with one side on the x-axis, find the midpoints S and T of the two diagonal sides, and show that the segment connecting the midpoints has a length that is one-half the length of the side of the triangle on the x-axis. Start by placing a vertex at the origin and label it A. Place point B on the x-axis at $(a, 0)$. Place point C in quadrant I with coordinates (b, c).

Given: $\triangle ABC$

S is the midpoint of $\overline{AC}$.

T is the midpoint of $\overline{BC}$.



Prove: $ST = \frac{1}{2}AB$

Proof:

The coordinates of S are $\left(\frac{b+0}{2}, \frac{c+0}{2}\right)$ or $\left(\frac{b}{2}, \frac{c}{2}\right)$.

The coordinates of T are $\left(\frac{a+b}{2}, \frac{0+c}{2}\right)$ or $\left(\frac{a+b}{2}, \frac{c}{2}\right)$.

$ST = \sqrt{\left(\frac{a+b}{2}-\frac{b}{2}\right)^2+\left(\frac{c}{2}-\frac{c}{2}\right)^2}$ or $\frac{a}{2}$

$AB = \sqrt{(a-0)^2 + (0-0)^2}$ or a

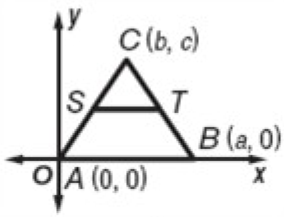
$ST = \frac{1}{2}AB$

ANSWER:

Given: ΔABC

S is the midpoint of $\overline{AC}$.

T is the midpoint of $\overline{BC}$.



Prove: $ST = \frac{1}{2}AB$

Proof:

The coordinates of S are $\left(\frac{b}{2}, \frac{c}{2}\right)$ and the coordinates of T are $\left(\frac{a+b}{2}, \frac{c}{2}\right)$

$ST = \sqrt{\left(\frac{a+b}{2} - \frac{b}{2}\right)^2 + \left(\frac{c}{2} - \frac{c}{2}\right)^2}$ or $\frac{a}{2}$

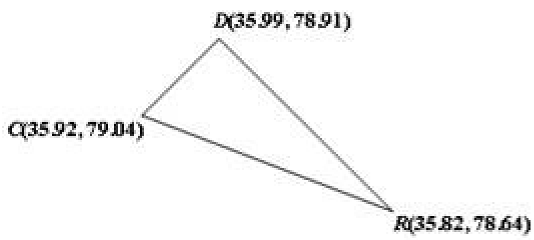
$AB = \sqrt{(a-0)^2 + (0-0)^2}$ or a

$ST = \frac{1}{2}AB$

23. **RESEARCH TRIANGLE** The cities of Raleigh, Durham, and Chapel Hill, North Carolina, form what is known as the Research Triangle. The approximate latitude and longitude of Raleigh are 35.82°N 78.64°W, of Durham are 35.99°N 78.91°W, and of Chapel Hill are 35.92°N 79.04°W. Show that the triangle formed by these three cities is scalene.

SOLUTION:

The first step is to label the coordinates of each location. Let *R* represent Raleigh, *D* represent Durham, and *C* represent Chapel Hill.



If no two sides of ΔCDR are congruent, then the triangle formed by these three cities is scalene.

$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$	Distance Formula
$DR = \sqrt{(35.99 - 35.82)^2 + (78.91 - 78.64)^2}$	$(x_1, y_1) = (35.82, 78.64)$ and $(x_2, y_2) = (35.99, 78.91)$
≈ 0.32	Simplify.
$CR = \sqrt{(35.92 - 35.82)^2 + (79.04 - 78.64)^2}$	$(x_1, y_1) = (35.82, 78.64)$ and $(x_2, y_2) = (35.92, 79.04)$
≈ 0.41	Simplify.
$CD = \sqrt{(35.99 - 35.92)^2 + (78.91 - 79.04)^2}$	$(x_1, y_1) = (35.92, 79.04)$ and $(x_2, y_2) = (35.99, 78.91)$
≈ 0.15	Simplify.

Since each side is a different length, ΔCDR is scalene. Therefore, the Research Triangle is scalene.

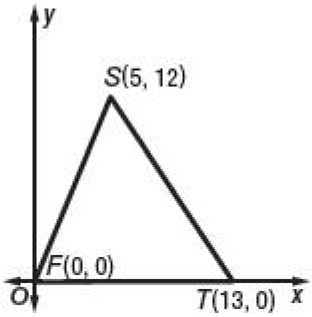
ANSWER:

The distance between Raleigh and Durham is about 0.32 units, between Raleigh and Chapel Hill is about 0.41 units, and between Durham and Chapel Hill is about 0.15 units. Since none of these distances are the same, the Research Triangle is scalene.

24. **PARTY PLANNING** Three friends live in houses with backyards adjacent to a neighborhood bike path. They decide to have a round-robin party using their three homes, inviting their friends to start at one house and then move to each of the other two. If one friend's house is centered at the origin, then the location of the other homes are (5, 12) and (13, 0). Write a coordinate proof to prove that the triangle formed by these three homes is isosceles.

SOLUTION:

Start by making a graph of the three homes on a coordinate plane. Let *F* represent the first friend's house , *S* represent the second friend's house, and *T* represent the third friend's house.

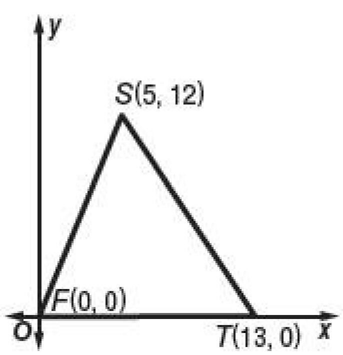


If any two sides of ΔFST are congruent, then the triangle is isosceles. Use the distance formula and a calculator to find the distance between each pair of houses.

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \quad \text{Distance Formula}$$
$$FS = \sqrt{(5-0)^2 + (12-0)^2} \quad (x_1, y_1) = (0,0) \text{ and } (x_2, y_2) = (5,12)$$
$$= 13 \quad \text{Simplify.}$$
$$FT = \sqrt{(13-0)^2 + (0-0)^2} \quad (x_1, y_1) = (0,0) \text{ and } (x_2, y_2) = (13,0)$$
$$= 13 \quad \text{Simplify.}$$
$$CD = \sqrt{(13-5)^2 + (0-12)^2} \quad (x_1, y_1) = (5,12) \text{ and } (x_2, y_2) = (13,0)$$
$$\approx 14.4 \quad \text{Simplify.}$$

Since the distance between the house at (0, 0) and the house at (13, 0) is the same as the distance between the house at (0, 0) and the house at (5, 12), the triangle formed by the three homes is isosceles.

ANSWER:



$FS = \sqrt{(5-0)^2 + (12-0)^2}$ or 13
$FT = \sqrt{(13-0)^2 + (0-0)^2}$ or 13

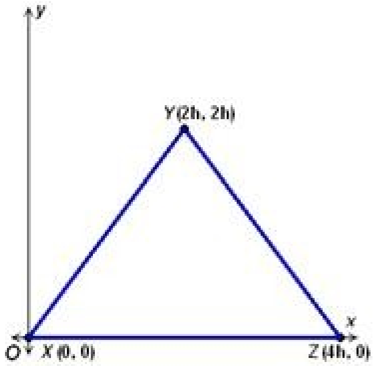
Since the distance between the first house at (0, 0) and the third house at (13, 0) is the same as the distance between the first house at (0, 0) and the second house at (5, 12), the triangle formed by the three homes is isosceles.

Draw ΔXYZ and find the slope of each side of the triangle. Determine whether the triangle is a right triangle. Explain

25. $X(0, 0)$, $Y(2h, 2h)$, $Z(4h, 0)$

SOLUTION:

First, place ΔXYZ on a coordinate plane. Then use the slope formula to find the slope of each side.



$$
\begin{aligned}
m &= \frac{y_2-y_1}{x_2-x_1} && \text{Slope formula}\\
m_{\overline{XY}} &= \frac{2h-0}{2h-0} \text{ or } 1 && (x_1, y_1) = (0,0); (x_2, y_2) = (2h, 2h)\\
m_{\overline{XZ}} &= \frac{0-0}{4h-0} \text{ or } 0 && (x_1, y_1) = (0,0); (x_2, y_2) = (4h, 0)\\
m_{\overline{YZ}} &= \frac{2h-0}{2h-4h} \text{ or } -1 && (x_1, y_1) = (4h,0); (x_2, y_2) = (2h, 2h)
\end{aligned}
$$

Since $1(-1) = -1$, $\overline{XY} \perp \overline{YZ}$. Therefore, ΔXYZ is a right triangle.

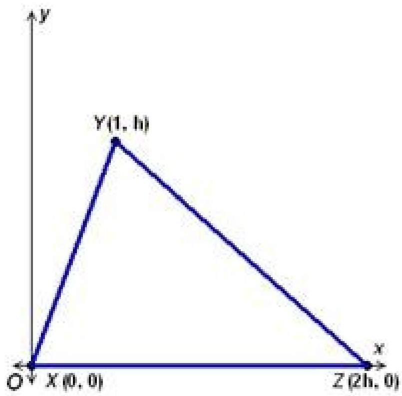
ANSWER:

slope of $\overline{XY}$ = 1, slope of $\overline{YZ}$ = –1, slope of $\overline{ZX}$ = 0; since 1(–1) = –1, $\overline{XY} \perp \overline{YZ}$. Therefore, ΔXYZ is a right triangle.

26. $X(0, 0)$, $Y(1, h)$, $Z(2h, 0)$

SOLUTION:

First, place ΔXYZ on a coordinate plane. Then use the slope formula to find the slope of each side.



m	$= \frac{y_2 - y_1}{x_2 - x_1}$	Slope formula
$m_{\overline{XY}}$	$= \frac{h - 0}{1 - 0}$ or h	$(x_1, y_1) = (0, 0); (x_2, y_2) = (1, h)$
$m_{\overline{XZ}}$	$= \frac{0 - 0}{2h - 0}$ or 0	$(x_1, y_1) = (0, 0); (x_2, y_2) = (2h, 0)$
$m_{\overline{YZ}}$	$= \frac{h - 0}{1 - 2h}$ or $\frac{h}{1 - 2h}$	$(x_1, y_1) = (2h, 0); (x_2, y_2) = (1, h)$

Since no two of the slopes have a product of –1, none of the sides are perpendicular. Therefore, ΔXYZ is not a right triangle.

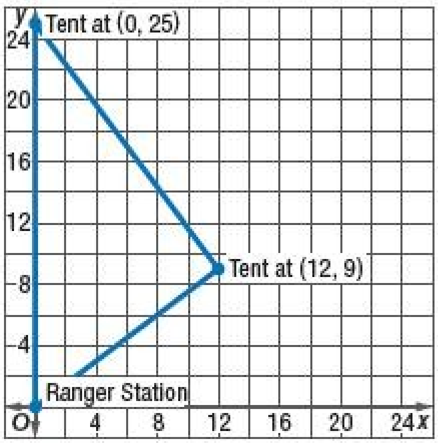
ANSWER:

Slope of $\overline{XY} = h$, slope of $\overline{YZ} = \frac{h}{1 - 2h}$, slope of $\overline{ZX} = 0$; no two slopes have a product of –1, so ΔXYZ is not a right triangle.

27. **CAMPING** Two families set up tents at a state park. If the ranger's station is located at (0, 0), and the locations of the tents are (0, 25) and (12, 9), write a coordinate proof to prove that the figure formed by the locations of the ranger's station and the two tents is a right triangle.

SOLUTION:

First graph the points.



To determine whether the triangle is a right triangle, find the slope of the line through (0, 0) to (12, 9) and compare it to the slope of the line through (12, 9), and (0, 25.

The slope between the tents is $-\frac{4}{3}$. The slope between the ranger's station and the tent located at (12, 9) is $\frac{3}{4}$. Since $-\frac{4}{3}\cdot\frac{3}{4}=-1$, the triangle formed by the tents and ranger's station is a right triangle.

ANSWER:

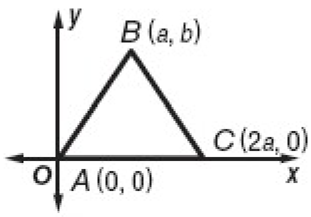
The slope between the tents is $-\frac{4}{3}$. The slope between the ranger's station and the tent located at (12, 9) is $\frac{3}{4}$. Since $-\frac{4}{3}\cdot\frac{3}{4}=-1$, the triangle formed by the tents and ranger's station is a right triangle.

28. **PROOF** Write a coordinate proof to prove that ΔABC is an isosceles triangle if the vertices are $A(0, 0)$, $B(a, b)$, and $C(2a, 0)$.

SOLUTION:

The first step is to place the triangle on a coordinate plane.Use the distance formula to find the lengths of the sides. If any two sides are congruent, then the triangle is isosceles.

Given: vertices $A(0, 0)$, $B(a, b)$, and $C(2a, 0)$



Prove: ΔABC is isosceles.

Proof:

$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ Distance Formula

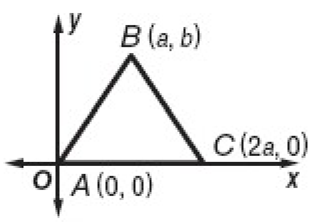
$AB = \sqrt{(a-0)^2 + (b-0)^2}$ or $\sqrt{a^2 + b^2}$

$BC = \sqrt{(2a-a)^2 + (0-b)^2}$ or $\sqrt{a^2 + b^2}$

Since, $AB = BC$, $\overline{AB} \cong \overline{BC}$. So, ΔABC is isosceles.

ANSWER:

Given: vertices $A(0, 0)$, $B(a, b)$, and $C(2a, 0)$



Prove: ΔABC is isosceles.

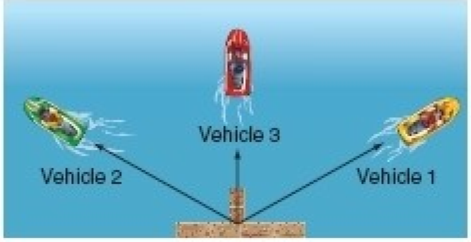
Proof:

$AB = \sqrt{(a-0)^2 + (b-0)^2}$ or $\sqrt{a^2 + b^2}$

$BC = \sqrt{(2a-a)^2 + (0-b)^2}$ or $\sqrt{a^2 + b^2}$

Since, $AB = BC$, $\overline{AB} \cong \overline{BC}$. So, ΔABC is isosceles.

29. WATER SPORTS Three personal watercraft vehicles launch from the same dock. The first vehicle leaves the dock traveling due northeast, while the second vehicle travels due northwest. Meanwhile, the third vehicle leaves the dock traveling due north.

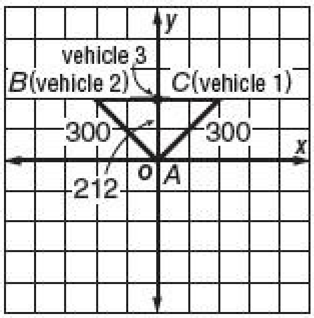


The first and second vehicles stop about 300 yards from the dock, while the third stops about 212 yards from the dock.

a. If the dock is located at (0, 0), sketch a graph to represent this situation. What is the equation of the line along which the first vehicle lies? What is the equation of the line along which the second vehicle lies? Explain your reasoning.
b. Write a coordinate proof to prove that the dock, the first vehicle, and the second vehicle form an isosceles right triangle.
c. Find the coordinates of the locations of all three watercrafts. Explain your reasoning.
d. Write a coordinate proof to prove that the positions of all three watercrafts are approximately collinear and that the third watercraft is at the midpoint between the other two.

SOLUTION:

a.



The equation of the line along which the first vehicle lies is $y = x$. The slope is 1 because the vehicle travels the same number of units north as it does east of the origin and the y-intercept is 0. The equation of the line along which the second vehicle lies is $y = -x$. The slope is –1 because the vehicle travels the same number of units north as it does west of the origin and the y-intercept is 0.

b. The paths taken by both the first and second vehicles are 300 yards long. Therefore the paths are congruent. If two sides of a triangle are congruent, then the triangle is isosceles. Since the slopes of the paths are negative reciprocals, they are perpendicular and the isosceles triangle is a right triangle.

c. The paths taken by the first two vehicles form the hypotenuse of a right triangle. From the graph, the paths of each boat form isosceles right triangles each with a hypotenuse of 300. Use the Pythagorean theorem to find the legs of these triangles.

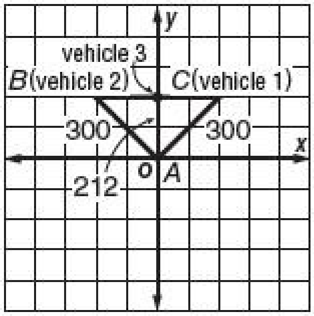
$$\begin{aligned} a^2 + b^2 &= c^2 && \text{Pythagorean Theorem} \\ h^2 + h^2 &= 300^2 && \text{Substitute.} \\ 2h^2 &= 90,000 && \text{Simplify.} \\ h^2 &= 45,000 && \div \text{ each side by 2.} \\ h &= 150\sqrt{2} && \text{Take the square root.} \end{aligned}$$

First vehicle, $(150\sqrt{2}, 150\sqrt{2})$; second vehicle, $(-150\sqrt{2}, 150\sqrt{2})$. The third vehicle travels due north and therefore, remains on the y-axis. The third vehicle is at (0, 212).

d. The y-coordinates of the first two vehicles are $150\sqrt{2} \approx 212.13$, while the y-coordinate of the third vehicle is 212. Since all three vehicles have approximately the same y-coordinate, they are approximately collinear. The midpoint of the first and second vehicle is $\left(\frac{150\sqrt{2} - 150\sqrt{2}}{2}, \frac{212 + 212}{2}\right)$ or (0, 212), the location of the third vehicle.

ANSWER:

a.



The equation of the line along which the first vehicle lies is $y = x$. The slope is 1 because the vehicle travels the same number of units north as it does east of the origin and the y-intercept is 0. The equation of the line along which the second vehicle lies is $y = -x$. The slope is –1 because the vehicle travels the same number of units north as it does west of the origin and the y-intercept is 0.

b. The paths taken by both the first and second vehicles are 300 yards long. Therefore the paths are congruent. If two sides of a triangle are congruent, then the triangle is isosceles.

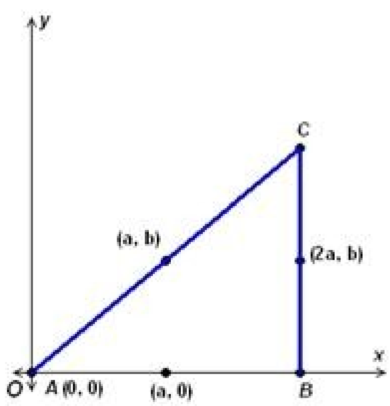
c. First vehicle, $(150\sqrt{2}, 150\sqrt{2})$; second vehicle, $(-150\sqrt{2}, 150\sqrt{2})$; third vehicle, (0, 212); the paths taken by the first two vehicles form the hypotenuse of a right triangle. Using the Pythagorean Theorem, the distance between the third vehicle and the first and second vehicle can be calculated. The third vehicle travels due north and therefore, remains on the y-axis.

d. The y-coordinates of the first two vehicles are $150\sqrt{2} \approx 212.13$, while the y-coordinate of the third vehicle is 212. Since all three vehicles have approximately the same y-coordinate, they are approximately collinear. The midpoint of the first and second vehicle is $\left(\frac{150\sqrt{2}-150\sqrt{2}}{2}, \frac{212+212}{2}\right)$ or (0, 212), the location of the third vehicle.

30. **REASONING** The midpoints of the sides of a triangle are located at $(a, 0)$, $(2a, b)$ and (a, b). If one vertex is located at the origin, what are the coordinates of the other vertices? Explain your reasoning.

SOLUTION:

Let A, B, and C represent the vertices of the triangle with A positioned at the origin. Let $(a, 0)$ be the midpoint of $\overline{AB}$, $(2a, b)$ be the midpoint of $\overline{BC}$, and (a, b) be the midpoint of $\overline{AC}$. Use the midpoint formula to determine the coordinates of B and C.



$$\begin{aligned} M &= \left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right) && \text{Midpoint Formula} \\ (a, 0) &= \left(\frac{0+x_2}{2}, \frac{0+y_2}{2}\right) && M_{\overline{AB}} = (a, 0), (x_1, y_1) = (0, 0) \\ 2a &= x_2 \qquad 0 = y_2 && \text{Solve for each coordinate.} \end{aligned}$$

So, the coordinates of point *B* are (2*a*, 0).

$$\begin{aligned} M &= \left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right) && \text{Midpoint Formula} \\ (a, b) &= \left(\frac{0+x_2}{2}, \frac{0+y_2}{2}\right) && M_{\overline{AC}} = (a, b), (x_1, y_1) = (0, 0) \\ 2a &= x_2 \qquad 2b = y_2 && \text{Solve for each coordinate.} \end{aligned}$$

So, the coordinates of point *C* are (2*a*, 2*b*).
Check by finding the midpoint of $\overline{BC}$.

$$\begin{aligned} M &= \left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right) && \text{Midpoint Formula} \\ M_{\overline{BC}} &= \left(\frac{2a+2a}{2}, \frac{0+2b}{2}\right) && (x_1, y_1) = (2a, 0), (x_2, y_2) = (2a, 2b) \\ M_{\overline{BC}} &= (2a, b) \checkmark && \text{Simplify.} \end{aligned}$$

Therefore, the coordinates of the other two vertices are (2*a*, 0) and (2*a*, *2b*).

ANSWER:

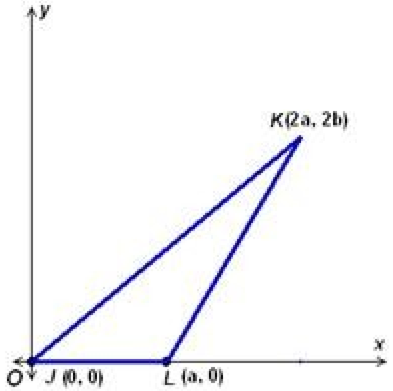
(2*a*, 0), (2*a*, 2*b*); Using the Midpoint Formula,
$(a, 0) = \left(\frac{0+x_2}{2}, \frac{0+y_2}{2}\right)$, so $x_2 = 2a$ and $y_2 = 0$.
$(a, b) = \left(\frac{0+x_2}{2}, \frac{0+y_2}{2}\right)$, so $x_2 = 2a$ and $y_2 = 2b$.

CHALLENGE Find the coordinates of point *L* so $\triangle JKL$ is the indicated type of triangle. Point *J* has coordinates (0, 0) and point *K* has coordinates (2*a*, 2*b*).

31. scalene triangle

SOLUTION:

Sample answer: Choose a point on the *x*- or *y*-axis with coordinates in terms of *a* or *b*, such that no two sides of the triangle will be congruent. Since point *J* has coordinates (0, 0) and point *K* has coordinates (2*a*, 2*b*), let point *L* have coordinates (*a*, 0). Use the distance formula to check that no two sides have the same length.



$$
\begin{aligned}
d &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} && \text{Distance Formula} \\
JK &= \sqrt{(2a - 0)^2 + (2b - 0)^2} && (x_1, y_1) = (0, 0) \text{ and } (x_2, y_2) = (2a, 2b) \\
&= \sqrt{4a^2 + 4b^2} \text{ or } 2\sqrt{a^2 + b^2} && \text{Simplify.} \\
JL &= \sqrt{(a - 0)^2 + (0 - 0)^2} && (x_1, y_1) = (0, 0) \text{ and } (x_2, y_2) = (a, 0) \\
&= \sqrt{a^2 + 0} \text{ or } a && \text{Simplify.} \\
KL &= \sqrt{(2a - a)^2 + (2b - 0)^2} && (x_1, y_1) = (a, 0) \text{ and } (x_2, y_2) = (2a, 2b) \\
&= \sqrt{a^2 + 4b^2} && \text{Simplify.}
\end{aligned}
$$

Since none of the sides have equal lengths, the triangle is scalene. Therefore, the coordinates of point *L* could be (*a*, 0).

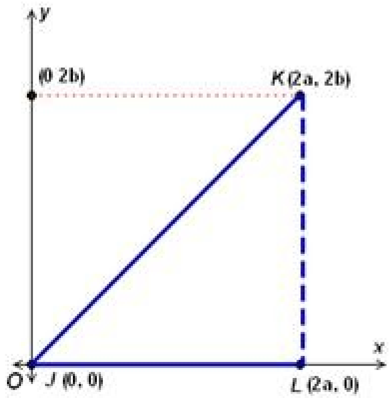
ANSWER:

Sample answer: (*a*, 0)

32. right triangle

SOLUTION:

Sample answer: Draw a vertical (or horizontal segment) from point K to the x-axis (or y-axis) to find a point L that would make ΔJKL a right triangle. The vertical segment from K would intersect the x-axis at $(2a, 0)$. Use the slopes of the sides to verify that ΔJKL is a right triangle.



$m = \frac{y_2 - y_1}{x_2 - x_1}$	Slope formula
$m_{\overline{JL}} = \frac{2a - 0}{0 - 0}$	$(x_1, y_1) = (0, 0); (x_2, y_2) = (2a, 0)$
$m_{\overline{JL}} = \frac{2a}{0}$ or undefined	Simplify.
$m_{\overline{KL}} = \frac{2a - 2a}{2b - 0}$	$(x_1, y_1) = (2a, 0); (x_2, y_2) = (2a, 2b)$
$m_{\overline{KL}} = \frac{0}{2b}$ or 0	Simplify.

Since the slopes are undefined and 0, $\overline{JL} \perp \overline{KL}$ and ΔJKL a right triangle. The horizontal segment would intersect the y-axis at $(0, 2b)$.
The slopes will again confirm that $\overline{JL} \perp \overline{KL}$, so this point would also make ΔJKL a right triangle.
Thus, the coordinates of L could be either $(2a, 0)$ or $(0, 2b)$.

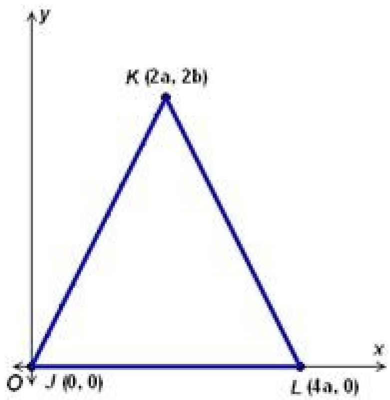
ANSWER:

Sample answer: $(2a, 0)$ or $(0, 2b)$

33. isosceles triangle

SOLUTION:

Sample answer: Since ΔJKL is to be isosceles, choose a point for L that is on the x-axis such that $JK = KL$. The y-coordinate of point L will be 0.The x-coordinate of K must be halfway between 0 and the x-coordinate for L because the triangle is isosceles. Since the x-coordinate for K is 2a, the x-coordinate for L must be $4a$. So, place L at $(4a, 0)$. Use the distance formula to verify that $\overline{JK} \cong \overline{KL}$.



$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \quad \text{Distance Formula}$$

$$JK = \sqrt{(2a - 0)^2 + (2b - 0)^2} \quad (x_1, y_1) = (0, 0) \text{ and } (x_2, y_2) = (2a, 2b)$$

$$= \sqrt{4a^2 + 4b^2} \text{ or } 2\sqrt{a^2 + b^2} \quad \text{Simplify.}$$

$$KL = \sqrt{(4a - 2a)^2 + (0 - 2b)^2} \quad (x_1, y_1) = (2a, 2b) \text{ and } (x_2, y_2) = (4a, 0)$$

$$= \sqrt{4a^2 + 4b^2} \text{ or } 2\sqrt{a^2 + b^2} \quad \text{Simplify.}$$

Since $JK = KL$, $\overline{JK} \cong \overline{KL}$ and ΔJKL is isosceles. Therefore, the coordinates of point L could be $(4a, 0)$.

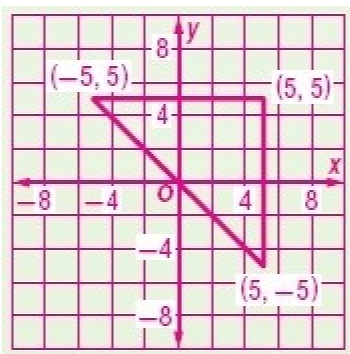
ANSWER:

Sample answer: $(4a, 0)$

34. **OPEN ENDED** Draw an isosceles right triangle on the coordinate plane so that the midpoint of its hypotenuse is the origin. Label the coordinates of each vertex.

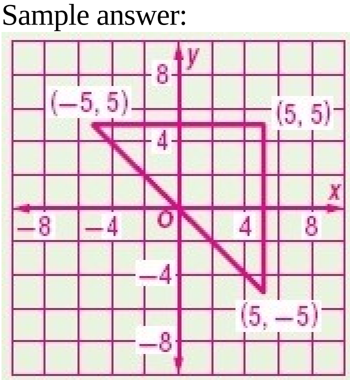
SOLUTION:

Sample answer: If (0, 0) is the midpoint and (a_1, b_1) and (a_2, b_2) are the endpoint of the hypotenuse, then (0, 0) = $\left(\frac{a_1+a_2}{2}, \frac{b_1+b_2}{2}\right)$. For this to be a true statement, $a_2 = -a_1$ and $b_2 = -b_1$. This means the *x*- and *y*-coordinates of the endpoints will have opposite signs. If the two legs of the right triangle are constructed so that one is vertical and the other is horizontal, the length of the horizontal segment will be *a* – (–a) or 2*a* and the length of the vertical segment will be *b* – (–*b*) or 2*b*. Since the triangle is isosceles, 2*a* = 2*b* or *a* = *b*. So, the *x*- and *y*-coordinates of the endpoints must have the same absolute values. Start by choosing endpoints for the hypotenuse that use the same number for the *x*- and *y*-values but have opposite signs in either endpoints, say (–5, 5) and (5, –5). The third point will be where the vertical and horizontal segments from the endpoints would intersect, say at (5, 5). (This could also be at (–5, –5) for this pair of endpoints.)



ANSWER:

Sample answer:



35. **CHALLENGE** Use a coordinate proof to show that if you add *n* units to each *x*-coordinate of the vertices of a triangle and *m* to each *y*-coordinate, the resulting figure is congruent to the original triangle.

SOLUTION:

Given: ΔABC with coordinates $A(0, 0)$, $B(a, b)$, and $C(c, d)$ and ΔDEF with coordinates $D(0 + n, 0 + m)$, $E(a + n, b + m)$, and $F(c + n, d + m)$.

Prove: $\Delta DEF \cong \Delta ABC$
Proof:

Use the Distance Formula, $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$.

$AB = \sqrt{(a-0)^2 + (b-0)^2}$ or $\sqrt{a^2 + b^2}$

$DE = \sqrt{(a+n-(0+n))^2 + (b+m-(0+m))^2}$ or $\sqrt{a^2 + b^2}$

Since $AB = DE$, $\overline{AB} \cong \overline{DE}$.

$BC = \sqrt{(c-a)^2 + (d-b)^2}$ or $\sqrt{c^2 - 2ac + a^2 + d^2 - 2bd + b^2}$

$EF = \sqrt{(c+n-(a+n))^2 + (d+m-(b+m))^2}$ or $\sqrt{c^2 - 2ac + a^2 + d^2 - 2bd + b^2}$

Since $BC = EF$, $\overline{BC} \cong \overline{EF}$.

$CA = \sqrt{(c-0)^2 + (d-0)^2}$ or $\sqrt{c^2 + d^2}$

$FD = \sqrt{(0+n-(c+n))^2 + (0+m-(d+m))^2}$ or $\sqrt{c^2 + d^2}$

Since $CA = FD$, $\overline{CA} \cong \overline{FD}$.
Therefore, $\Delta DEF \cong \Delta ABC$ by the SSS Postulate.

ANSWER:

Given: ΔABC with coordinates $A(0, 0)$, $B(a, b)$, and $C(c, d)$ and ΔDEF with coordinates $D(0 + n, 0 + m)$, $E(a + n, b + m)$, and $F(c + n, d + m)$.
Prove: $\Delta DEF \cong \Delta ABC$
Proof:

$AB = \sqrt{(a-0)^2 + (b-0)^2}$ or $\sqrt{a^2 + b^2}$

$DE = \sqrt{(a+n-(0+n))^2 + (b+m-(0+m))^2}$ or $\sqrt{a^2 + b^2}$

Since $AB = DE$, $\overline{AB} \cong \overline{DE}$.

$BC = \sqrt{(c-a)^2 + (d-b)^2}$ or $\sqrt{c^2 - 2ac + a^2 + d^2 - 2bd + b^2}$

$EF = \sqrt{(c+n-(a+n))^2 + (d+m-(b+m))^2}$ or $\sqrt{c^2 - 2ac + a^2 + d^2 - 2bd + b^2}$

Since $BC = EF$, $\overline{BC} \cong \overline{EF}$.

$CA = \sqrt{(c-0)^2 + (d-0)^2}$ or $\sqrt{c^2 + d^2}$

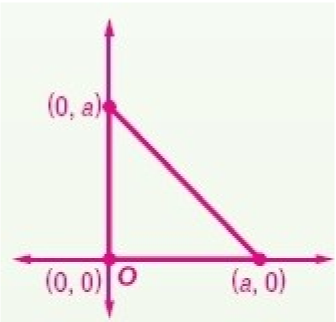
$FD = \sqrt{(0+n-(c+n))^2 + (0+m-(d+m))^2}$ or $\sqrt{c^2 + d^2}$

Since $CA = FD$, $\overline{CA} \cong \overline{FD}$.
Therefore, $\Delta DEF \cong \Delta ABC$ by the SSS Postulate.

36. **CCSS REASONING** A triangle has vertex coordinates (0, 0) and (a, 0). If the coordinates of the third vertex are in terms of a, and the triangle is isosceles, identify the coordinates and position the triangle on the coordinate plane.

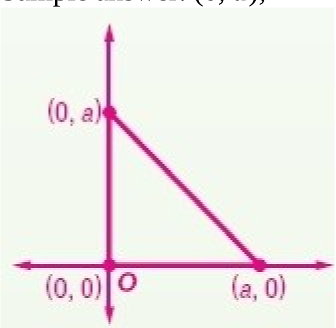
SOLUTION:

Sample answer: Since two vertices of the triangle are (0, 0) and (a, 0), one side of the triangle is on the x-axis and has a length of a. To create an isosceles triangle, choose the point (0, a) on the y-axis such that a second side of the triangle will also have a length of a. Connect (a, 0) and (0, a) to form the third side an isosceles right triangle. Therefore, a third vertex in terms of a that would make the triangle isosceles could be (0, a).



ANSWER:

Sample answer: (0, a);



37. **WRITING IN MATH** Explain why following each guideline below for placing a triangle on the coordinate plane is helpful in proving coordinate proofs.

a. Use the origin as a vertex of the triangle.
b. Place at least one side of the triangle on the x- or y-axis.
c. Keep the triangle within the first quadrant if possible.

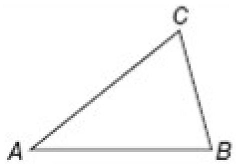
SOLUTION:

a. Using the origin as a vertex of the triangle makes calculations easier because the coordinates are (0, 0).
b. Placing at least one side of the triangle on the x- or y-axis makes it easier to calculate the length of the side since one of the coordinates will be 0.
c. Keeping a triangle within the first quadrant makes all of the coordinates positive, and makes the calculations easier.

ANSWER:

a. Using the origin as a vertex of the triangle makes calculations easier because the coordinates are (0, 0).
b. Placing at least one side of the triangle on the x- or y-axis makes it easier to calculate the length of the side since one of the coordinates will be 0.
c. Keeping a triangle within the first quadrant makes all of the coordinates positive, and makes the calculations easier.

38. **GRIDDED RESPONSE** In the figure below, $m\angle B = 76$. The measure of $\angle A$ is half the measure of $\angle B$. What is $m\angle C$?



SOLUTION:

Here, $m\angle B = 76$ and $m\angle A = \frac{1}{2}m\angle B$.

So, $m\angle A = 38$.

By the Triangle Angle-Sum Theorem, $m\angle A + m\angle B + m\angle C = 180.$

Substitute.

$$
\begin{aligned}
38 + 76 + m\angle C &= 180 && \text{Substitute.}\\
114 + m\angle C &= 180 && \text{Simplify.}\\
114 + m\angle C - 114 &= 180 - 114 && -114 \text{ from each side.}\\
m\angle C &= 66 && \text{Simplify.}
\end{aligned}
$$

ANSWER:

66

39. **ALGEBRA** What is the x-coordinate of the solution to the system of equations shown below?

$$\begin{cases} 2x - 3y = 3 \\ -4x + 2y = -18 \end{cases}$$

A –6
B –3
C 3
D 6

SOLUTION:

Multiply the first equation by 2 and the second equation by 3 and add the resulting equations.

$$
\begin{array}{r}
4x - 6y = 6 \\
\underline{-12x + 6y = -54} \\
-8x \qquad = -48
\end{array}
$$

Divide each side by –8.

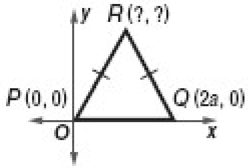
$x = 6$

So, the correct choice is D.

ANSWER:

D

40. What are the coordinates of point *R* in the triangle?



F $\left(\frac{a}{2}, a\right)$

G (a, b)

H $\left(\frac{b}{2}, a\right)$

J $\left(\frac{b}{2}, \frac{a}{2}\right)$

SOLUTION:

The given triangle has two congruent sides, *RQ* and *RP*, so it is isosceles. $\triangle RPQ$ is isosceles, so the *x*-coordinate of *R* is located half way between 0 and $2a$ or a. We cannot write the y-coordinate in terms of a, so call it as b. The coordinates of point *R* are (a, b). The correct choice is G.

ANSWER:

G

41. **SAT/ACT** For all *x*, $17x^5 + 3x^2 + 2 - (-4x^5 + 3x^3 - 2) =$

A $13x^5 + 3x^3 + 3x^2$

B $13x^5 + 6x^2 + 4$

C $21x^5 - 3x^3 + 3x^2 + 4$

D $21x^5 + 3x^2 + 3x^3$

E $21x^5 + 3x^2 + 3x^2 + 4$

SOLUTION:

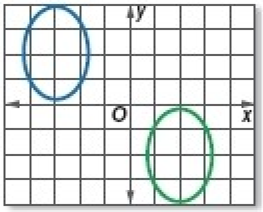
$$17x^5 + 3x^2 + 2 - (-4x^5 + 3x^3 - 2)$$
$$= 17x^5 + 3x^2 + 2 + 4x^5 - 3x^3 + 2 \quad \text{Distributive Property}$$
$$= 21x^5 - 3x^3 + 3x^2 + 4 \quad \text{Combine like terms.}$$

The correct choice is C.

ANSWER:

C

Identify the type of congruence transformation shown as a *reflection, translation,* or *rotation*.

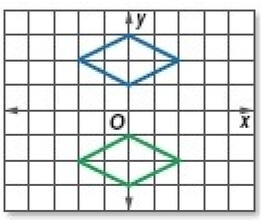
42. 

SOLUTION:

The image in green can be found by translating the blue figure to the right and down or by rotating the figure 180 degrees. Each point of the figure and its image are in the same position, just 5 units right and about 4 units down. This is a translation. Each point of the figure and its image are the same distance from (–0.5, 0). The angles formed by each pair of corresponding points and this point are congruent. This is a rotation.

ANSWER:

translation or rotation

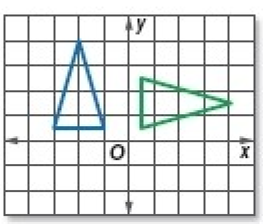
43. 

SOLUTION:

The image in green can be found by translating the blue figure down, by reflecting the figure in the *x*-axis, or by rotating the figure 180 degrees in either direction. Since each vertex and its image are the same distance from the *x*-axis, this is a reflection. Since each vertex and its image are in the same position, just 5 units down this is a translation. This is a rotation because each vertex and its image are the same distance from the origin and the angles formed by each pair of corresponding points and the origin are congruent.

ANSWER:

reflection, translation, or rotation

44. 

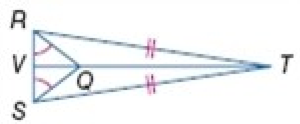
SOLUTION:

The image in green can be found by rotating the figure 90 degrees clockwise. This is a rotation because each vertex and its image are the same distance from the origin and the angles formed by each pair of corresponding points and the origin are congruent.

ANSWER:

rotation

Refer to the figure.



45. Name two congruent angles.

SOLUTION:

$\angle TSR \cong \angle TRS$
$\angle QRS \cong \angle QSR$

ANSWER:

Sample answer: $\angle TSR \cong \angle TRS$

46. Name two congruent segments.

SOLUTION:

$\overline{RQ} \cong \overline{QS}$
$\overline{RT} \cong \overline{ST}$

ANSWER:

Sample answer: $\overline{RQ} \cong \overline{QS}$

47. Name a pair of congruent triangles.

SOLUTION:

$\Delta RQV \cong \Delta SQV$

ANSWER:

Sample answer: $\Delta RQV \cong \Delta SQV$

48. **RAMPS** The Americans with Disabilities Act requires that wheelchair ramps have at least a 12-inch run for each rise of 1 inch.

a. Determine the slope represented by this requirement.

b. The maximum length that the law allows for a ramp is 30 feet. How many inches tall is the highest point of this ramp?

SOLUTION:

a. We know that

$\text{slope} = \frac{\text{rise}}{\text{run}}$.

Substitute.

$\text{slope} = \frac{1}{12}$

b. We know that 1 ft = 12 in.
Here, the length of 12 inch ramp has 1 inch height, that is, the length of 1 feet ramp has 1 inch height. Therefore, the maximum height of 30 feet ramp is 30 inches.

ANSWER:

a. $\frac{1}{12}$

b. 30 in.

Find the distance between each pair of points. Round to the nearest tenth.

49. $X(5, 4)$ and $Y(2, 1)$

SOLUTION:

Use the distance formula to find XY.

$$\begin{aligned} XY &= \sqrt{(2-5)^2 + (1-4)^2} && \text{Distance Formula} \\ &= \sqrt{(-3)^2 + (-3)^2} && \text{Simplify.} \\ &= \sqrt{9+9} && \text{Square each term.} \\ &= \sqrt{18} && \text{Add.} \\ &\approx 4.2 && \text{Take the square root.} \end{aligned}$$

ANSWER:

4.2

50. $A(1, 5)$ and $B(-2, -3)$

SOLUTION:

Use the distance formula to find *AB*.

$$\begin{aligned} AB &= \sqrt{(-2-1)^2 + (-3-5)^2} && \text{Distance Formula} \\ &= \sqrt{(-3)^2 + (-8)^2} && \text{Simplify.} \\ &= \sqrt{9+64} && \text{Square each term.} \\ &= \sqrt{73} && \text{Add.} \\ &\approx 8.5 && \text{Take the square root.} \end{aligned}$$

ANSWER:

8.5

51. $J(-2, 6)$ and $K(1, 4)$

SOLUTION:

Use the distance formula to find *JK*.

$$\begin{aligned} JK &= \sqrt{(1-(-2))^2 + (4-6)^2} && \text{Distance Formula} \\ &= \sqrt{(3)^2 + (-2)^2} && \text{Simplify.} \\ &= \sqrt{9+4} && \text{Square each term.} \\ &= \sqrt{13} && \text{Add.} \\ &\approx 3.6 && \text{Take the square root.} \end{aligned}$$

ANSWER:

3.6