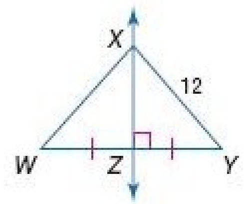
5-1 Bisectors of Triangles

Find each measure.

1. *XW*



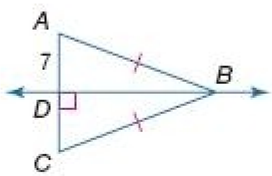
SOLUTION:

Given that $WZ = ZY$ and $\overleftrightarrow{XZ} \perp \overline{WY}$.
By the Perpendicular Bisector Theorem, $XW = XY$.
Therefore, $XW = 12$.

ANSWER:

12

2. *AC*



SOLUTION:

In the figure, $AB = BC$. By the converse of the Perpendicular Bisector Theorem, $\overleftrightarrow{BD}$ is a perpendicular bisector of $\overline{AC}$.
Therefore, $AC = DC$.
Since $AD = 7$, $DC = 7$.

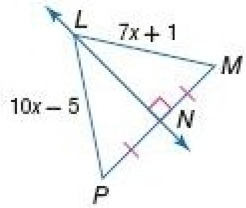
By the Segment Addition Postulate,

$$\begin{aligned} AC &= AD + DC \\ &= 7 + 7 \\ &= 14 \end{aligned}$$

ANSWER:

14

3. *LP*



SOLUTION:

Given that $PN = MN$ and $\overleftrightarrow{LN} \perp \overline{PM}$.
So *LM* = *LP*, by the Perpendicular Bisector Theorem.
Therefore, $10x - 5 = 7x + 1$.
Solve for *x*.

$$\begin{aligned} LP &= LM \\ 10x - 5 &= 7x + 1 \\ 10x - 7x - 5 &= 7x - 7x + 1 \\ 3x - 5 + 5 &= 1 = 5 \\ 3x &= 6 \\ x &= 2 \end{aligned}$$

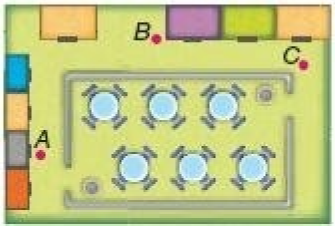
Substitute *x* = 2 in the expression for *LP*.

$$\begin{aligned} LP &= 10x - 5 \\ &= 10(2) - 5 \\ &= 15 \end{aligned}$$

ANSWER:

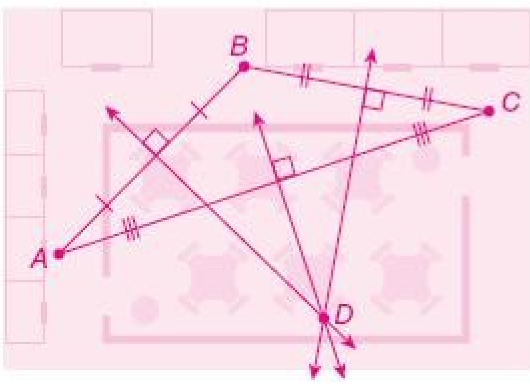
15

4. **ADVERTISING** Four friends are passing out flyers at a mall food court. Three of them take as many flyers as they can and position themselves as shown. The fourth one keeps the supply of additional flyers. Copy the positions of points *A*, *B*, and *C*. Then position the fourth friend at *D* so that she is the same distance from each of the other three friends.

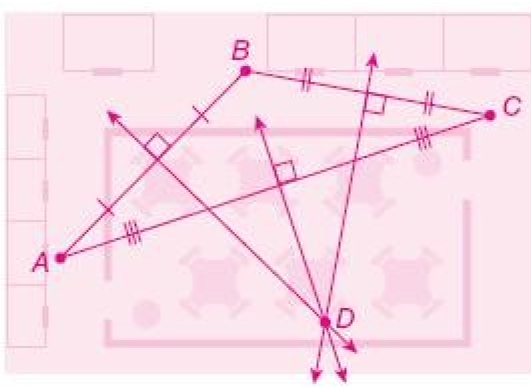


SOLUTION:

You will need to find the circumcenter of the triangle formed by points A, B, and C. This can be done by constructing the perpendicular bisectors of each side of the triangle and finding their point of concurrency. Point D, as shown in the diagram, is where the 4th friend should position herself so that she is equidistant to the others.

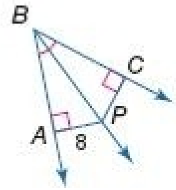


ANSWER:



Find each measure.

5. CP



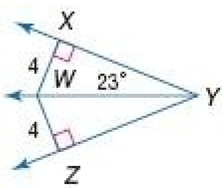
SOLUTION:

Given: $\overrightarrow{BP}$ is the angle bisector of $\angle ABC$, $\overrightarrow{BA} \perp \overline{PA}$, and $\overrightarrow{BC} \perp \overline{PC}$.
By the Angle Bisector Theorem, $AP = CP$.
So, $CP = 8$.

ANSWER:

8

6. $m\angle WYZ$



SOLUTION:

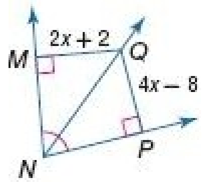
Given: $WX = WZ$; $\overrightarrow{YX} \perp \overline{XW}$ and $\overrightarrow{YZ} \perp \overline{ZW}$.
$\overrightarrow{YW}$ bisects $\angle XYZ$ by the converse of the Angle Bisector Theorem.
Therefore, $m\angle WYZ = m\angle WYX = 23$.

ANSWER:

23°

5-1 Bisectors of Triangles

7. QM



SOLUTION:

From the figure, $\overrightarrow{NQ}$ is the angle bisector of $\angle MNP$, $\overrightarrow{NP} \perp \overline{PQ}$, and $\overrightarrow{NM} \perp \overline{MQ}$.
By the Angle Bisector Theorem, $MQ = PQ$.

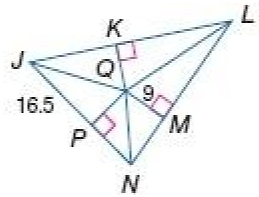
$$
\begin{aligned}
PQ &= MQ \\
4x - 8 &= 2x + 2 \\
4x - 2x - 8 &= 2x - 2x + 2 \\
2x - 8 &= 2 \\
2x - 8 + 8 &= 2 + 8 \\
2x &= 10 \\
x &= 5
\end{aligned}
$$

$$
\begin{aligned}
MQ &= 2x + 2 \\
&= 2(5) + 2 \\
&= 10
\end{aligned}
$$

ANSWER:

12

8. **CCSS SENSE-MAKING** Find JQ if Q is the incenter of ΔJLN.



SOLUTION:

Since Q is the incenter of ΔJLN, $PQ = QM = 9$.
Use the Pythagorean Theorem in triangle JPQ.

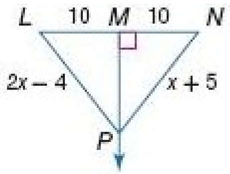
$$
\begin{aligned}
JQ^2 &= \sqrt{16.5^2 + 9^2} \\
&\approx 18.8
\end{aligned}
$$

ANSWER:

18.8

Find each measure.

9. *NP*



SOLUTION:

From the figure, $LM = MN$ and $\overrightarrow{MP} \perp \overline{LN}$.

By the Perpendicular Bisector Theorem, $LP = NP$.

$$2x - 4 = x + 5$$
$$x = 9$$

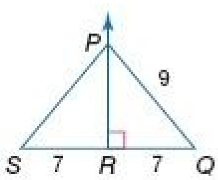
Substitute $x = 9$ in the expression for NP.

$$NP = x + 5$$
$$= 9 + 5$$
$$= 14$$

ANSWER:

14

10. *PS*



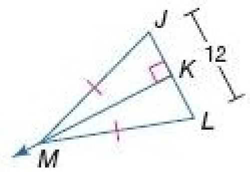
SOLUTION:

Given that $SR = RQ$ and $\overrightarrow{PR} \perp \overline{SQ}$.
By the Perpendicular Bisector Theorem, $PS = PQ$.
Therefore, $PS = 9$.

ANSWER:

9

11. *KL*



SOLUTION:

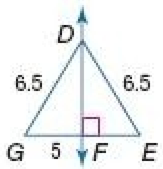
Here $JM = LM$. By the converse of the Perpendicular Bisector Theorem, $\overleftrightarrow{KL}$ is a perpendicular bisector of $\overline{JL}$. Therefore, $JK = KL$.

$$JL = JK + KL$$
$$12 = KL + KL$$
$$12 = 2KL$$
$$6 = KL$$

ANSWER:

6

12. *EG*



SOLUTION:

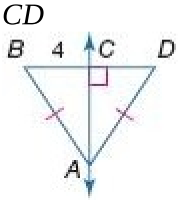
Given: $GD = ED$. By the converse of the Perpendicular Bisector Theorem, $\overleftrightarrow{DF}$ is a perpendicular bisector of $\overline{GE}$. Therefore, $GF = FE = 5$.

$$\begin{aligned} EG &= GF + FE \\ &= 5 + 5 \\ &= 10 \end{aligned}$$

ANSWER:

10

13. *CD*



SOLUTION:

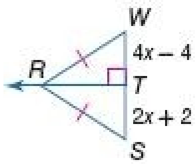
Given: $AB = AD$. By the converse of the Perpendicular Bisector Theorem, $\overleftrightarrow{AC}$ is a perpendicular bisector of $\overline{BD}$. Therefore, $BC = CD$

$CD = 4$.

ANSWER:

4

14. *SW*



SOLUTION:

$WT = TS$ by the converse of the Perpendicular Bisector Theorem.
That is, $4x - 4 = 2x + 2$.

Solve for *x*.

$$\begin{aligned} 4x - 4 &= 2x + 2 \\ 4x - 2x - 4 &= 2x - 2x + 2 \\ 2x - 4 + 4 &= 2 + 4 \\ 2x &= 6 \\ x &= 3 \end{aligned}$$

Substitute $x = 3$ in the expression for *WT*.

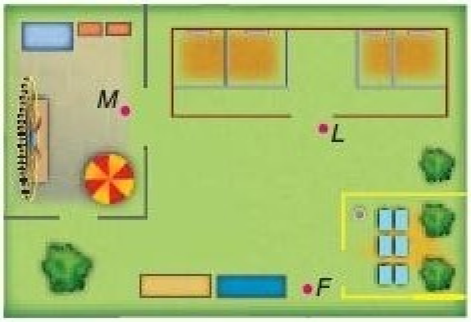
$WT = 4x - 4 = 4(3) - 4 = 8,$

$TS = 8$

ANSWER:

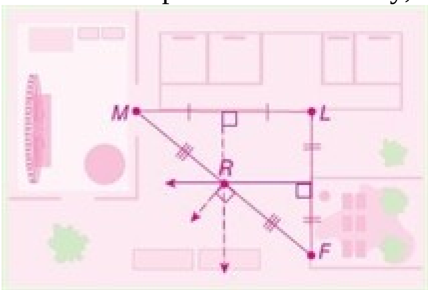
16

15. **STATE FAIR** The state fair has set up the location of the midway, livestock competition, and food vendors. The fair planners decide that they want to locate the portable restrooms the same distance from each location. Copy the positions of points *M*, *L*, and *F*. Then find the location for the restrooms and label it *R*.

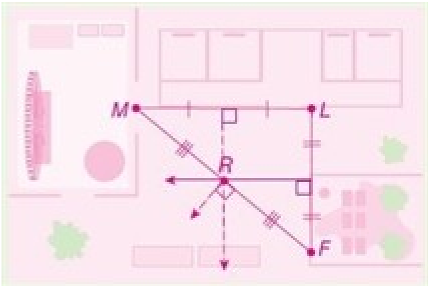


SOLUTION:

You can find point *R* by constructing the perpendicular bisectors of each side of the triangle formed by points *F*, *L*, and *M*. Their point of concurrency, *R*, is equidistant from each vertex of the triangle.



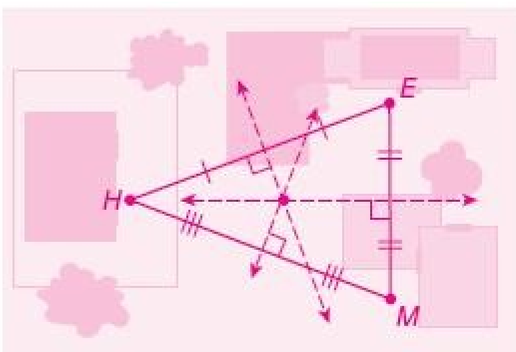
ANSWER:



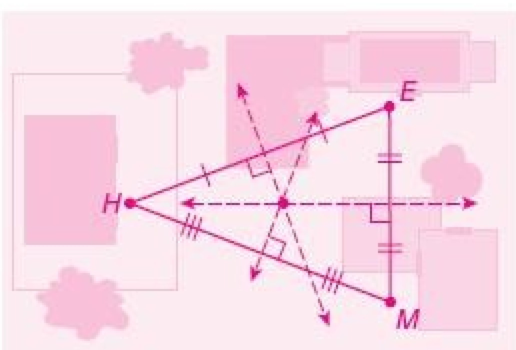
16. **SCHOOL** A school system has built an elementary, middle, and high school at the locations shown in the diagram. Copy the positions of points *E*, *M*, and *H*. Then find the location for the bus yard *B* that will service these schools so that it is the same distance from each school.

SOLUTION:

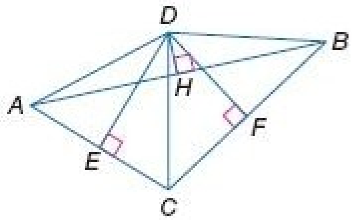
To find the best place for the bus yard, you will need to construct the perpendicular bisector of each side of the triangle. Their point of concurrency is point *B*, which is the same distance from each point *H*, *E*, and *M*.



ANSWER:



Point D is the circumcenter of $\triangle ABC$. List any segment(s) congruent to each segment.



17. $\overline{AD}$

SOLUTION:

Circumcenter D is equidistant from the vertices of the triangle ABC.
$\overline{AD} \cong \overline{CD} \cong \overline{BD}$.

ANSWER:

$\overline{CD}, \overline{BD}$

18. $\overline{BF}$

SOLUTION:

$\overline{DF}$ is the perpendicular bisector of $\overline{BC}$. So, $\overline{BF} \cong \overline{CF}$.

ANSWER:

$\overline{CF}$

19. $\overline{AH}$

SOLUTION:

$\overline{DH}$ is the perpendicular bisector of $\overline{AB}$. So, $\overline{AH} \cong \overline{BH}$.

ANSWER:

$\overline{BH}$

20. $\overline{DC}$

SOLUTION:

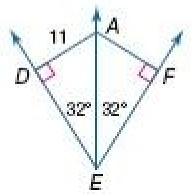
Circumcenter D is equidistant from the vertices of the triangle ABC.
Therefore, $\overline{DC} \cong \overline{DA} \cong \overline{DB}$.

ANSWER:

$\overline{DA}, \overline{DB}$

5-1 Bisectors of Triangles

Find each measure.



21. *AF*

SOLUTION:

By the Angle Bisector Theorem, *AF* = *AD* = 11.

ANSWER:

11

22. $m\angle DBA$

SOLUTION:

$m\angle DBA = m\angle DBC = 17$ by the converse of the Angle Bisector Theorem.

ANSWER:

$17°$

23. $m\angle PNM$

SOLUTION:

The converse of the Angle Bisector Theorem says $\angle MNQ \cong \angle PNQ$.
That is, $(3x + 5)° = (4x - 8)°$.
Solve the equation for *x*.

$$\begin{aligned} 3x + 5 &= 4x - 8 \\ 3x - 3x + 5 &= 4x - 3x - 8 \\ 5 &= x - 8 \\ 5 + 8 &= x - 8 + 8 \\ 13 &= x \end{aligned}$$

Now,

$$\begin{aligned} m\angle PNM &= m\angle MNQ + m\angle PNQ \\ &= [3x + 5] + [4x - 8] \\ &= [3(13) + 5] + [4(13) - 8] \\ &= 39 + 5 + 52 - 8 \\ &= 88 \end{aligned}$$

ANSWER:

$88°$

5-1 Bisectors of Triangles

24. *XA*

SOLUTION:
By the Angle Bisector Theorem, $XA = ZA = 4$.

ANSWER:
4

25. $m\angle PQS$

SOLUTION:
In triangle *QRS*, $m\angle QRS + m\angle RSQ + m\angle SQR = 180$.
Substitute the known values.

$$\begin{aligned} 90 + 48 + m\angle SQR &= 180 \\ 138 + m\angle SQR &= 180 \\ m\angle SQR &= 180 - 138 \\ &= 42 \end{aligned}$$

By the converse of the Angle Bisector Theorem, $\angle PQS \cong \angle SQR$. Therefore $m\angle PQS = 42$.

ANSWER:
$42°$

26. *PN*

SOLUTION:
Here $PN = LP$, by the Angle Bisector Theorem.

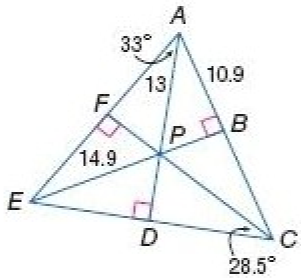
$$\begin{aligned} 3x + 6 &= 4x - 2 \\ 3x - 3x + 6 &= 4x - 3x - 2 \\ 6 &= x - 2 \\ 6 + 2 &= x - 2 + 2 \\ 8 &= x \end{aligned}$$

Substitute $x = 8$ in the expression for *PN*.

$$\begin{aligned} PN &= 4x - 2 \\ &= 4(8) - 2 \\ &= 30 \end{aligned}$$

ANSWER:
30

CCSS SENSE-MAKING Point *P* is the incenter of $\triangle AEC$. Find each measure below.



27. *PB*

SOLUTION:

Use the Pythagorean Theorem in the right triangle *ABP*.

$$\begin{aligned} PB^2 &= 13^2 - 10.9^2 \\ &= 169 - 118.81 \\ &= 50.19 \\ PB &\approx 7.1 \end{aligned}$$

ANSWER:

7.1

28. *DE*

SOLUTION:

$\overline{PC}$ is the angle bisector of $\angle BCD$. By the Angle Bisector Theorem, *PD* = *PB*. Or *PD* = 7.1.
In triangle *PDE*,

$$\begin{aligned} ED^2 &= 14.9^2 - 7.1^2 \\ &= 222.01 - 50.41 \\ &= 171.69 \\ ED &\approx 13.1. \end{aligned}$$

ANSWER:

13.1

29. $m\angle DAC$

SOLUTION:

By the Converse of the Angle Bisector Theorem, $\overline{AD}$ is the angle bisector of $\angle EAC$. Therefore, $m\angle DAC = m\angle DAE = 33°$.

ANSWER:

33°

30. $\angle DEP$

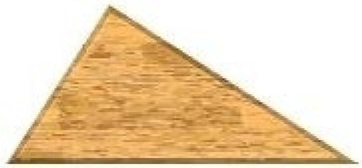
SOLUTION:

By SAS postulate, $\Delta DEP \cong \Delta DCP$.
So, $\angle DEP = \angle DCP = 28.5°$.

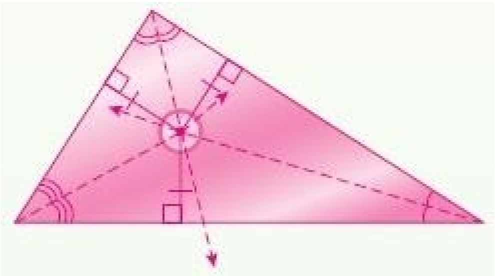
ANSWER:

$28.5°$

31. **INTERIOR DESIGN** You want to place a centerpiece on a corner table so that it is located the same distance from each edge of the table. Make a sketch to show where you should place the centerpiece. Explain your reasoning.

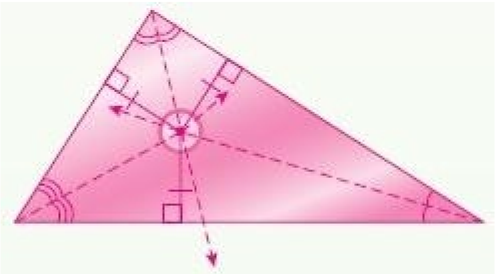


SOLUTION:



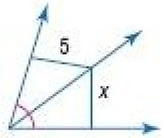
Find the point of concurrency of the angle bisectors of the triangle, the incenter. This point is equidistant from each side of the triangle.

ANSWER:



Find the point of concurrency of the angle bisectors of the triangle, the incenter. This point is equidistant from each side of the triangle.

Determine whether there is enough information given in each diagram to find the value of *x*. Explain your reasoning.

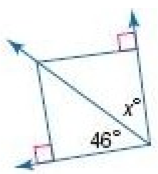
32. 

SOLUTION:

There is not enough information to find the value of x. In order to determine the value of x, we would need to see markings indicating whether the segments drawn from the bisecting ray are perpendicular to each side of the angle being bisected. Since there are no markings indicating perpendicularity, no conclusion can be made.

ANSWER:

No; we need to know if the segments are perpendicular to the rays.

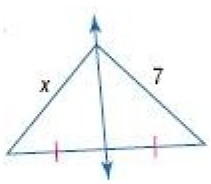
33. 

SOLUTION:

There is not enough information to find the value of x. In order to determine the value of x, we would need to see markings indicating whether the segments drawn from the ray are congruent to each other. Since there are no markings indicating congruence, no conclusion can be made.

ANSWER:

No; we need to know whether the perpendicular segments are congruent to each other.

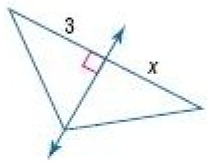
34. 

SOLUTION:

There is not enough information to find the value of x. In order to determine the value of x, we would need to see markings indicating whether the segment bisector is perpendicular to the side of the triangle that is bisected. Since there are no markings indicating perpendicularity, no conclusion can be made.

ANSWER:

No; we need to know if the segment bisector is a perpendicular bisector.

35. 

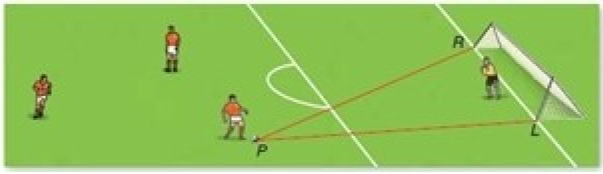
SOLUTION:

There is not enough information to find the value of *x*. According to the Converse of the Perpendicular Bisector Theorem, we would need to know if the point of the triangle is equidistant to the endpoints of the segment. Since we don't know if the hypotenuses of the two smaller triangles are congruent, we can't assume that the other side of the big triangle is bisected.

ANSWER:

No; we need to know whether the hypotenuses of the triangles are congruent.

36. **SOCCER** A soccer player *P* is approaching the opposing team's goal as shown in the diagram. To make the goal, the player must kick the ball between the goal posts at *L* and *R*. The goalkeeper faces the kicker. He then tries to stand so that if he needs to dive to stop a shot, he is as far from the left-hand side of the shot angle as the right-hand side.



a. Describe where the goalkeeper should stand. Explain your reasoning.

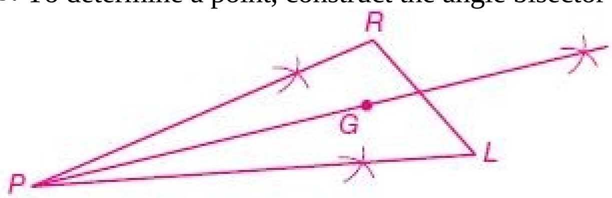
b. Copy ΔPRL . Use a compass and a straightedge to locate point *G*, the desired place for the goalkeeper to stand.

c. If the ball is kicked so it follows the path from *P* to *R*, construct the shortest path the goalkeeper should take to block the shot. Explain your reasoning.

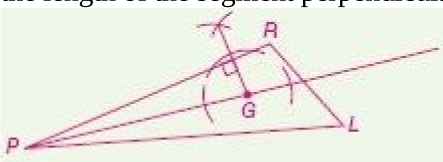
SOLUTION:

a. The goalkeeper should stand along the angle bisector of the opponent's shot angle, since the distance to either side of the angle is the same along this line.

b. To determine a point, construct the angle bisector of $\angle P$ and then mark point *G* anywhere on $\overrightarrow{PG}$.



c. We can find the shortest distance from point *G* to either side of $\angle RPL$ by constructing a line that is perpendicular to each side of the angle that passes through point *G*. The shortest distance to a line from a point not on the line is the length of the segment perpendicular to the line from the point



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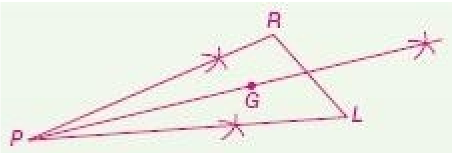
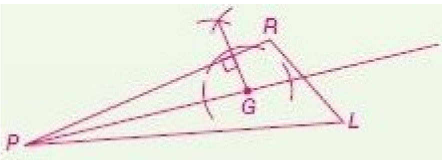
ANSWER:

a. The goalkeeper should stand along the angle bisector of the opponent's shot angle, since the distance to either side of the angle is the same along this line.

b.



c.



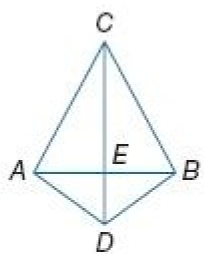
The shortest distance to a line from a point not on the line is the length of the segment perpendicular to the line from the point.

PROOF Write a two-column proof.

37. Theorem 5.2

Given: $\overline{CA} \cong \overline{CB}, \overline{AD} \cong \overline{BD}$

Prove: *C* and *D* are on the perpendicular bisector of $\overline{AB}$



SOLUTION:

As in all proofs, you need to think backwards. What would you need to do to prove that a segment in a perpendicular bisector? You can prove it in two parts - first, that $\overline{CD}$ and $\overline{AB}$ are perpendicular to each other and then, that $\overline{AB}$ is bisected. This will involve proving two triangles are congruent so that you can get congruent corresponding parts (CPCTC). Start by considering which triangles you can make congruent to each other using the given information. There are three pairs of triangles in the diagram, so mark the given information to lead you to the correct pair. Once you have determined that $\Delta ACD \cong \Delta BCD$, then think about what congruent corresponding parts you need to make $\Delta CEA \cong \Delta CEB$. To prove that two segments are perpendicular, consider what CPCTC you need to choose that would eventually prove that two segments form a right angle. In addition, which CPCTC would you choose to prove that a segment is bisected?

Proof:

Statement(Reasons)

1. $\overline{CA} \cong \overline{CB}, \overline{AD} \cong \overline{BD}$ (Given)
2. $\overline{CD} \cong \overline{CD}$ (Congruence of segments is reflexive.)
3. $\Delta ACD \cong \Delta BCD$ (SSS)
4. $\angle ACD \cong \angle BCD$ (CPCTC)
5. $\overline{CE} \cong \overline{CE}$ (Congruence of segments is reflexive.)
6. $\Delta CEA \cong \Delta CEB$ (SAS)
7. $\overline{AE} \cong \overline{BE}$ (CPCTC)
8. *E* is the midpoint of $\overline{AB}$. (Definition of midpoint)

9. $\angle CEA \cong \angle CEB$ (CPCTC)
10. $\angle CEA$ and $\angle CEB$ form a linear pair.(Definition of linear pair)
11. $\angle CEA$ and $\angle CEB$ are supplementary. (Supplementary Theorem)
12. $m\angle CEA + m\angle CEB = 180$ (Definition of supplementary)
13. $m\angle CEA + m\angle CEA = 180$ (Substitution Property)
14. $2m\angle CEA = 180$(Substitution Property)
15. $m\angle CEA = 90$(Division Property)
16. $\angle CEA$ and $\angle CEB$ are right angles.(Definition of right angle)
17. $\overline{CD} \perp \overline{AB}$ (Definition of Perpendicular lines)
18. $\overline{CD}$ is the perpendicular bisector of $\overline{AB}$. (Definition of perpendicular bisector)
19. *C* and *D* are on the perpendicular bisector of $\overline{AB}$. (Definition of point on a line)

ANSWER:

Statement(Reasons)

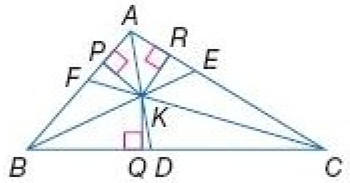
1. $\overline{CA} \cong \overline{CB}$, $\overline{AD} \cong \overline{BD}$ (Given)
2. $\overline{CD} \cong \overline{CD}$ (Congruence of segments is reflexive.)
3. $\Delta ACD \cong \Delta BCD$ (SSS)
4. $\angle ACD \cong \angle BCD$ (CPCTC)
5. $\overline{CE} \cong \overline{CE}$ (Congruence of segments is reflexive.)
6. $\Delta CEA \cong \Delta CEB$ (SAS)
7. $\overline{AE} \cong \overline{BE}$ (CPCTC)
8. *E* is the midpoint of $\overline{AB}$. (Definition of midpoint)
9. $\angle CEA \cong \angle CEB$ (CPCTC)
10. $\angle CEA$ *and* $\angle CEB$ form a linear pair.(Definition of linear pair)
11. $\angle CEA$ and $\angle CEB$ are supplementary. (Supplementary Theorem)
12. $m\angle CEA + m\angle CEB = 180$(Definition of supplementary)
13. $m\angle CEA + m\angle CEA = 180$ (Substitution Property)
14. $2m\angle CEA = 180$(Substitution Property)
15. $m\angle CEA = 90$ (Division Property)
16. $\angle CEA$ and $\angle CEB$ are right angles.(Definition of right angle)
17. $\overline{CD} \perp \overline{AB}$ (Definition of Perpendicular lines)
18. $\overline{CD}$ is the perpendicular bisector of $\overline{AB}$. (Definition of perpendicular bisector)
19. *C* and *D* are on the perpendicular bisector of $\overline{AB}$. (Definition of point on a line)

38. Theorem 5.6

Given: ΔABC, angle bisectors $\overline{AD}, \overline{BE},$ and $\overline{CF}$

$\overline{KP} \perp \overline{AB}, \overline{KQ} \perp \overline{BC}, \overline{KR} \perp \overline{AC}$

Prove: $KP = KQ = KR$



SOLUTION:

Consider what it means if $\overline{AD}, \overline{BE},$ and $\overline{CF}$ are all angle bisectors and meet at point K. The key to this proof is the Angle Bisector Theorem.

Proof:

Statement(Reasons)

1. $\Delta ABC,$ angle bisectors $\overline{AD}, \overline{BE},$ and $\overline{CF}$
$\overline{KP} \perp \overline{AB}, \overline{KQ} \perp \overline{BC}, \overline{KR} \perp \overline{AC}$ (Given)
2. $KP = KQ$, $KQ = KR$, $KP = KR$ (Any point on the angle bisector is equidistant from the sides of the angle.)
3. $KP = KQ = KR$(Transitive Property)

ANSWER:

Proof:

Statement(Reasons)

1. $\Delta ABC,$ angle bisectors $\overline{AD}, \overline{BE},$ and $\overline{CF}$
$\overline{KP} \perp \overline{AB}, \overline{KQ} \perp \overline{BC}, \overline{KR} \perp \overline{AC}$ (Given)
2. $KP = KQ$, $KQ = KR$, $KP = KR$ (Any point on the angle bisector is equidistant from the sides of the angle.)
3. $KP = KQ = KR$(Transitive Property)

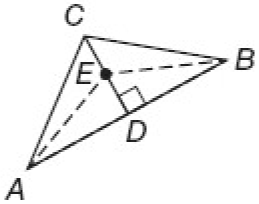
CCSS ARGUMENTS Write a paragraph proof of each theorem.

39. Theorem 5.1

SOLUTION:

If we know that $\overline{CD}$ is the perpendicular bisector to $\overline{AB}$, then what two congruent triangle pairs can we get from this given information? Think about what triangles we can make congruent to prove that $\overline{EA} \cong \overline{EB}$?

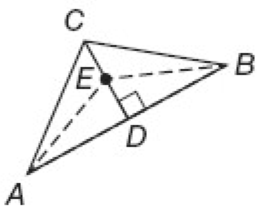
Given: $\overline{CD}$ is the $\perp$ bisector of $\overline{AB}$.
E is a point on *CD*.
Prove: *EA* = *EB*



Proof: $\overline{CD}$ is the $\perp$ bisector of $\overline{AB}$. By definition of bisector, *D* is the midpoint of $\overline{AB}$. Thus, $\overline{AD} \cong \overline{BD}$ by the Midpoint Theorem. $\angle CDA$ and $\angle CDB$ are right angles by the definition of perpendicular. Since all right angles are congruent, $\angle CDA \cong \angle CDB$. Since *E* is a point on $\overline{CD}$, $\angle EDA$ and $\angle EDB$ are right angles and are congruent. By the Reflexive Property, $\overline{ED} \cong \overline{ED}$. Thus $\Delta EDA \cong \Delta EDB$ by SAS. $\overline{EA} \cong \overline{EB}$ because CPCTC, and by definition of congruence, *EA* = *EB*.

ANSWER:

Given: $\overline{CD}$ is the $\perp$ bisector of $\overline{AB}$.
E is a point on *CD*.
Prove: *EA* = *EB*



Proof: $\overline{CD}$ is the $\perp$ bisector of $\overline{AB}$. By definition of bisector, *D* is the midpoint of $\overline{AB}$. Thus, $\overline{AD} \cong \overline{BD}$ by the Midpoint Theorem. $\angle CDA$ and $\angle CDB$ are right angles by the definition of perpendicular. Since all right angles are congruent, $\angle CDA \cong \angle CDB$. Since *E* is a point on $\overline{CD}$, $\angle EDA$ and $\angle EDB$ are right angles and are congruent. By the Reflexive Property, $\overline{ED} \cong \overline{ED}$. Thus $\Delta EDA \cong \Delta EDB$ by SAS. $\overline{EA} \cong \overline{EB}$ because CPCTC, and by definition of congruence, *EA* = *EB*.

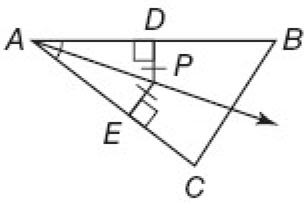
5-1 Bisectors of Triangles

40. Theorem 5.5

SOLUTION:

If PD and PE are equidistant to the sides of $\angle BAC$, then they are also perpendicular to those sides.If you can prove $\Delta DAP \cong \Delta EAP$, then consider what CPCTC would need to be chosen to prove that $\overline{AP}$ is an angle bisector of $\angle BAC$.

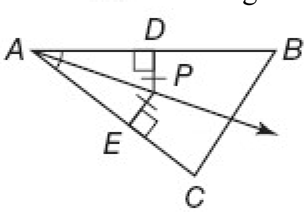
Given: $\angle BAC$
P is in the interior of $\angle BAC$
PD = *PE*
Prove: $\overline{AP}$ is the angle bisector of $\angle BAC$



Proof: Point *P* is on the interior of $\angle BAC$ of ΔBAC and *PD* = *PE*. By definition of congruence, $\overline{PD} \cong \overline{PE}$. $\overline{PD} \perp \overline{AB}$ and $\overline{PE} \perp \overline{AC}$ since the distance from a point to a line is measured along the perpendicular segment from the point to the line. $\angle ADP$ and $\angle AEP$ are right angles by the definition of perpendicular lines and ΔADP and ΔAEP are right triangles by the definition of right triangles. By the Reflexive Property, $\overline{AP} \cong \overline{AP}$. Thus, $\Delta ADP \cong \Delta AEP$ by HL. $\angle DAP \cong \angle EAP$ because CPCTC, and $\overline{AP}$ is the angle bisector of $\angle BAC$ by the definition of angle bisector.

ANSWER:

Given: $\angle BAC$
P is in the interior of $\angle BAC$
PD = *PE*
Prove: $\overline{AP}$ is the angle bisector of $\angle BAC$



Proof: Point *P* is on the interior of $\angle BAC$ of ΔBAC and *PD* = *PE*. By definition of congruence, $\overline{PD} \cong \overline{PE}$. $\overline{PD} \perp \overline{AB}$ and $\overline{PE} \perp \overline{AC}$ since the distance from a point to a line is measured along the perpendicular segment from the point to the line. $\angle ADP$ and $\angle AEP$ are right angles by the definition of perpendicular lines and ΔADP and ΔAEP are right triangles by the definition of right triangles. By the Reflexive Property, $\overline{AP} \cong \overline{AP}$. Thus, $\Delta ADP \cong \Delta AEP$ by HL. $\angle DAP \cong \angle EAP$ because CPCTC, and $\overline{AP}$ is the angle bisector of $\angle BAC$ by the definition of angle bisector.

COORDINATE GEOMETRY Write an equation in slope-intercept form for the perpendicular bisector of the segment with the given endpoints. Justify your answer.

41. $A(-3, 1)$ and $B(4, 3)$

SOLUTION:

The slope of the segment AB is $\frac{3-1}{4+3}$ or $\frac{2}{7}$. So, the slope of the perpendicular bisector is $-\frac{7}{2}$.

The perpendicular bisector passes through the midpoint of the segment AB. The midpoint of AB is $\left(\frac{-3+4}{2}, \frac{3-1}{2}\right)$ or $\left(\frac{1}{2}, 2\right)$.

The slope-intercept form for the equation of the perpendicular bisector of AB is:

$$\begin{aligned}(y-2) &= -\frac{7}{2}\left(x-\frac{1}{2}\right)\\ y-2 &= -\frac{7}{2}x+\frac{7}{4}\\ y &= -\frac{7}{2}x+\frac{15}{4}\end{aligned}$$

ANSWER:

$y=-\frac{7}{2}x+\frac{15}{4}$; The perpendicular bisector bisects the segment at the midpoint of the segment. The midpoint is $\left(\frac{1}{2}, 2\right)$. The slope of the given segment is $\frac{2}{7}$, so the slope of the perpendicular bisector is $-\frac{7}{2}$.

42. $C(–4, 5)$ and $D(2, –2)$

SOLUTION:

The slope of the segment *CD* is $\frac{-2-5}{2+4}$ or $-\frac{7}{6}$. So, the slope of the perpendicular bisector is $\frac{6}{7}$.
The perpendicular bisector passes through the midpoint of the segment *CD*. The midpoint of *CD* is $\left(\frac{-4+2}{2}, \frac{5-2}{2}\right)$ or $\left(-1, \frac{3}{2}\right)$.

The slope-intercept form for the equation of the perpendicular bisector of *CD* is:

$$\left(y-\frac{3}{2}\right)=\frac{6}{7}(x+1)$$
$$y-\frac{3}{2}=\frac{6}{7}x+\frac{6}{7}$$
$$y=\frac{6}{7}x+\frac{33}{14}$$

ANSWER:

$y=\frac{6}{7}x+\frac{33}{14}$; The perpendicular bisector bisects the segment at the midpoint of the segment. The midpoint is $\left(-1,\frac{3}{2}\right)$. The slope of the given segment is $-\frac{7}{6}$, so the slope of the perpendicular bisector is $\frac{6}{7}$.

43. **PROOF** Write a two-column proof of Theorem 5.4.

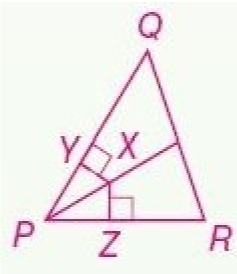
SOLUTION:

The key to this proof is to think about how to get $\triangle PYX \cong \triangle PZX$. Use your given information to get congruent corresponding parts. Remember that an angle bisector makes two *angles* congruent.

Given: $\overline{PX}$ bisects $\angle QPR$. $\overline{XY} \perp \overline{PQ}$ and $\overline{XZ} \perp \overline{PR}$

Prove: $\overline{XY} \cong \overline{XZ}$

Proof:



Statements (Reasons)

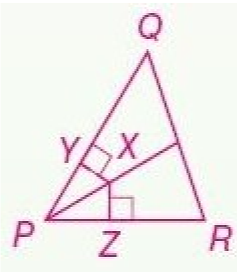
1. $\overline{PX}$ bisects $\angle QPR$, $\overline{XY} \perp \overline{PQ}$ and $\overline{XZ} \perp \overline{PR}$. (Given)
2. $\angle YPX \cong \angle ZPX$ (Definition of angle bisector)
3. $\angle PYX$ and $\angle PZX$ are right angles. (Definition of perpendicular)
4. $\angle PYX \cong \angle PZX$ (Right angles are congruent.)
5. $\overline{PX} \cong \overline{PX}$ (Reflexive Property)
6. $\Delta PYX \cong \Delta PZX (AAS)$
7. $\overline{XY} \cong \overline{XZ}$ (CPCTC)

ANSWER:

Given: $\overline{PX}$ bisects $\angle QPR$. $\overline{XY} \perp \overline{PQ}$ and $\overline{XZ} \perp \overline{PR}$.

Prove: $\overline{XY} \cong \overline{XZ}$

Proof:



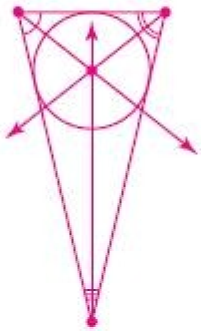
Statements (Reasons)

1. $\overline{PX}$ bisects $\angle QPR$, $\overline{XY} \perp \overline{PQ}$ and $\overline{XZ} \perp \overline{PR}$. (Given)
2. $\angle YPX \cong \angle ZPX$ (Definition of angle bisector)
3. $\angle PYX$ and $\angle PZX$ are right angles. (Definition of perpendicular)
4. $\angle PYX \cong \angle PZX$ (Right angles are congruent.)
5. $\overline{PX} \cong \overline{PX}$ (Reflexive Property)
6. $\Delta PYX \cong \Delta PZX (\mathrm{AAS})$
7. $\overline{XY} \cong \overline{XZ}$ (CPCTC)

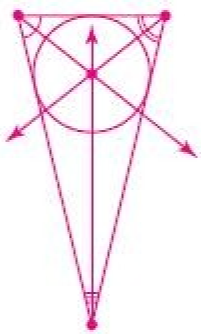
44. **GRAPHIC DESIGN** Mykia is designing a pennant for her school. She wants to put a picture of the school mascot inside a circle on the pennant. Copy the outline of the pennant and locate the point where the center of the circle should be to create the largest circle possible. Justify your drawing.

SOLUTION:

We need to find the incenter of the triangle by finding the intersection point of the angle bisectors. To make the circle, place your compass on the incenter, mark a radius that is perpendicular to each side (the shortest length) and make a circle.When the circle is as large as possible, it will touch all three sides of the pennant.



ANSWER:



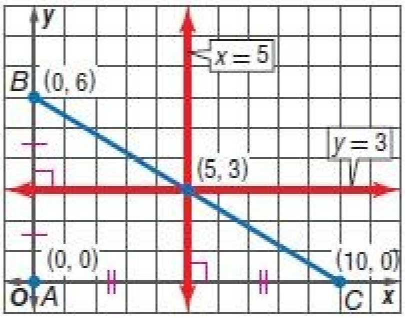
When the circle is as large as possible, it will touch all three sides of the pennant. We need to find the incenter of the triangle by finding the intersection point of the angle bisectors.

COORDINATE GEOMETRY Find the coordinates of the circumcenter of the triangle with the given vertices. Explain.

45. $A(0, 0)$, $B(0, 6)$, $C(10, 0)$

SOLUTION:

Graph the points.



Since the circumcenter is formed by the perpendicular bisectors of each side of the triangle, we need to draw the perpendicular bisectors of the two legs of the triangle and see where they intersect.

The equation of a line of one of the perpendicular bisectors is $y = 3$ because it is a horizontal line through point (0,3), the midpoint of the vertical leg.

The equation of a line of another perpendicular bisector is $x = 5$, because it is a vertical line through the point (5,0), the midpoint of the horizontal leg.

These lines intersect at (5, 3), because it will have the x-value of the line $x = 5$ and the y-value of the line $y = 3$.

The circumcenter is located at (5, 3).

ANSWER:

The equation of a line of one of the perpendicular bisectors is $y = 3$. The equation of a line of another perpendicular bisector is $x = 5$. These lines intersect at (5, 3). The circumcenter is located at (5, 3).

46. $J(5, 0)$, $K(5, –8)$, $L(0, 0)$

SOLUTION:

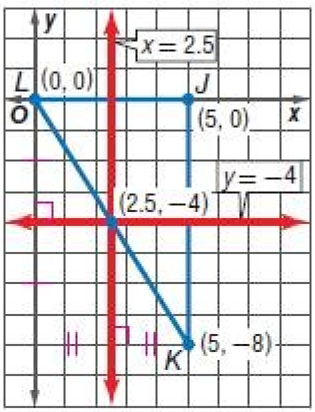
Since the circumcenter is formed by the perpendicular bisectors of each side of the triangle, we need to draw the perpendicular bisectors of the two legs of the triangle and see where they intersect.

The equation of a line of one of the perpendicular bisectors is $y = –4$ because it is a horizontal line through point (5, –4), the midpoint of the vertical leg.

The equation of a line of another perpendicular bisector is $x = 2.5$, because it is a vertical line through the point (2.5, 0), the midpoint of the horizontal leg.

These lines intersect at (2.5, –4), because it will have the x-value of the line $x = 2.5$ and the y-value of the line $y = –4$.

The circumcenter is located at (2.5, –4).



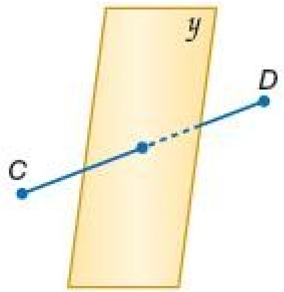
ANSWER:

The equation of a line of one perpendicular bisector is $y = –4$. The equation of a line of another perpendicular bisector is $x = 2.5$. These lines intersect at (2.5, –4). The circumcenter is located at (2.5, –4).

47. **LOCUS** Consider $\overline{CD}$. Describe the set of all points in space that are equidistant from *C* and *D*.

SOLUTION:

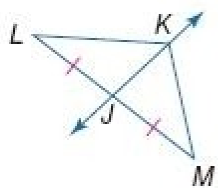
Since we are considering all points *in space*, we need to consider more than just the perpendicular bisector of CD. The solution is a plane that is perpendicular to the plane in which $\overline{CD}$ lies and bisects $\overline{CD}$.



ANSWER:

a plane perpendicular to the plane in which $\overline{CD}$ lies and bisecting $\overline{CD}$.

48. **ERROR ANALYSIS** Claudio says that from the information supplied in the diagram, he can conclude that *K* is on the perpendicular bisector of $\overline{LM}$. Caitlyn disagrees. Is either of them correct? Explain your reasoning.



SOLUTION:

Caitlyn is correct; Based on the markings, we only know that *J* is the midpoint of $\overline{LM}$. We don't know if $\overline{KJ}$ is perpendicular to $\overline{LM}$.

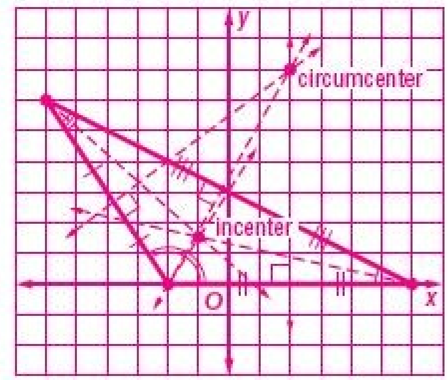
ANSWER:

Caitlyn; *K* is only on the perpendicular bisector of $\overline{LM}$ if $\overline{LK} \cong \overline{MK}$, but we are not given this information in the diagram.

49. **OPEN ENDED** Draw a triangle with an incenter located inside the triangle but a circumcenter located outside. Justify your drawing by using a straightedge and a compass to find both points of concurrency.

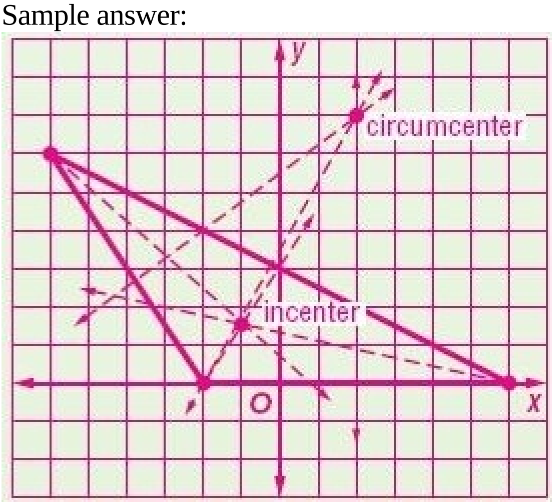
SOLUTION:

Knowing that the incenter of a triangle is *always* found inside any triangle, the key to this problem is to think about for which type of triangle the circumcenter would fall on the outside. Experiment with acute, right and obtuse triangles to find the one that will work.



ANSWER:

Sample answer:

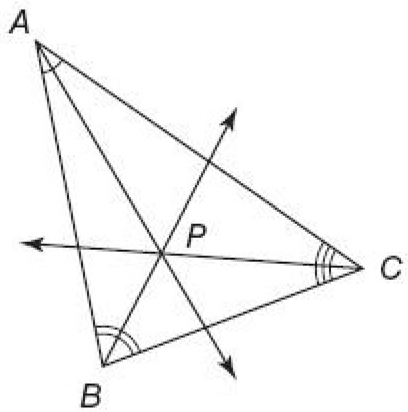


CCSS ARGUMENTS Determine whether each statement is *sometimes, always,* or *never* true. Justify your reasoning using a counterexample or proof.

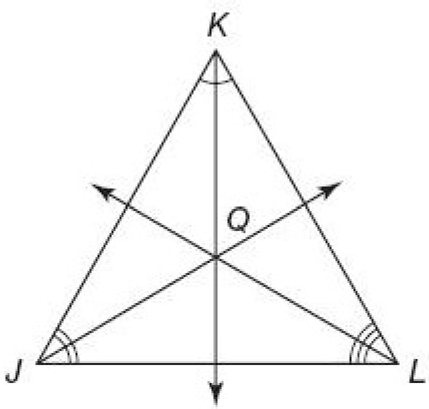
50. The angle bisectors of a triangle intersect at a point that is equidistant from the vertices of the triangle.

SOLUTION:

It is the circumcenter, formed by the perpendicular bisectors, that is equidistant to the vertices of a triangle. Consider for what type of triangle the incenter, formed by the angle bisectors, would be the same location as the circumcenter. Sometimes; if the triangle is equilateral, then this is true, but if the triangle is isosceles or scalene, the statement is false.



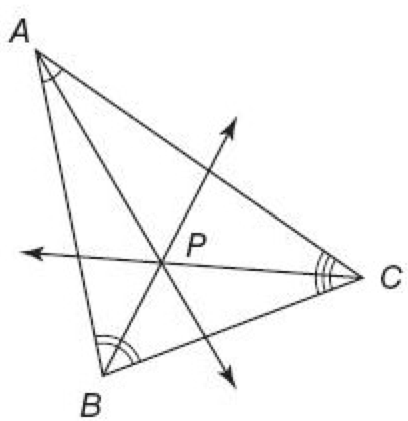
$AP \neq BP \neq CP$.



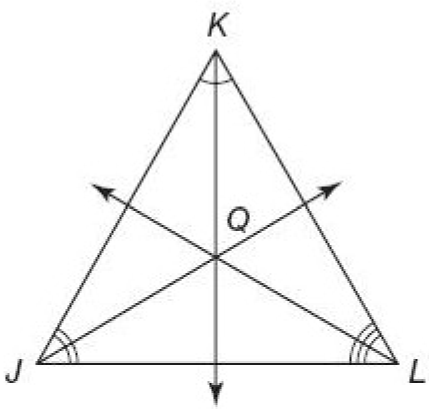
$JQ = KQ = LQ$

ANSWER:

Sometimes; if the triangle is equilateral, then this is true, but if the triangle is isosceles or scalene, the statement is false.



$AP \neq BP \neq CP$.



$JQ = KQ = LQ$

51. In an isosceles triangle, the perpendicular bisector of the base is also the angle bisector of the opposite vertex.

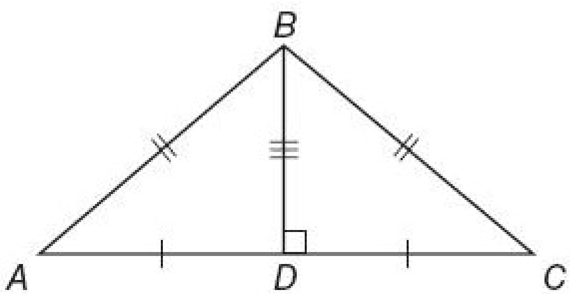
SOLUTION:

Always. It is best to show this through a proof that would show that, given an isosceles triangle divided by a perpendicular bisector, that it creates two congruent triangles. Therefore, by CPCTC, the two angles formed by the perpendicular bisector are congruent and form an angle bisector.

Given: ΔABC is isosceles with legs $\overline{AB}$ and $\overline{BC}$; $\overline{BD}$ is the $\perp$ bisector of $\overline{AC}$.

Prove: $\overline{BD}$ is the angle bisector of $\angle ABC$.

Proof:



Statements (Reasons)

1. ΔABC is isosceles with legs $\overline{AB}$ and $\overline{BC}$. (Given)
2. $\overline{AB} \cong \overline{BC}$ (Definition of isosceles triangle)
3. $\overline{BD}$ is the $\perp$ bisector of $\overline{AC}$.(Given)
4. D is the midpoint of $\overline{AC}$. (Definition of segment bisector)
5. $\overline{AD} \cong \overline{DC}$ (Definition of midpoint)
6. $\overline{BD} \cong \overline{BD}$ (Reflexive Property)
7. $\Delta ABC \cong \Delta CBD$(SSS)
8. $\angle ABD \cong \angle CBD$(CPCTC)
9. $\overline{BD}$ is the angle bisector of $\angle ABC$. (Definition of angle bisector)

ANSWER:

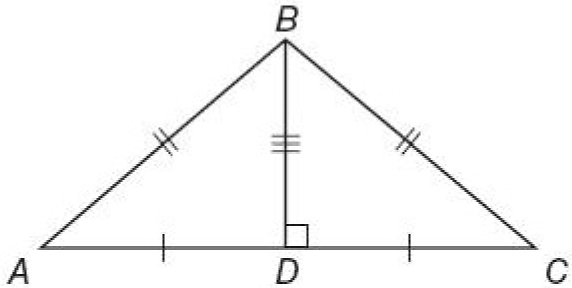
Always.

Given: ΔABC is isosceles with legs $\overline{AB}$ and $\overline{BC}$; $\overline{BD}$ is the $\perp$ bisector of $\overline{AC}$.

Prove: $\overline{BD}$ is the angle bisector of $\angle ABC$.

Proof:

5-1 Bisectors of Triangles

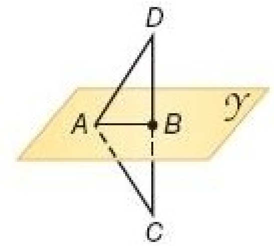


Statements (Reasons)

1. ΔABC is isosceles with legs $\overline{AB}$ and $\overline{BC}$. (Given)
2. $\overline{AB} \cong \overline{BC}$ (Def. of isosceles Δ)
3. $\overline{BD}$ is the $\perp$ bisector of $\overline{AC}$.(Given)
4. D is the midpoint of $\overline{AC}$. (Def. of segment bisector)
5. $\overline{AD} \cong \overline{DC}$ (Def. of midpoint)
6. $\overline{BD} \cong \overline{BD}$ (Reflexive Property)
7. $\Delta ABC \cong \Delta CBD$**(SSS)**
8. $\angle ABD \cong \angle CBD$**(CPCTC)**
9. $\overline{BD}$ is the angle bisector of $\angle ABC$. (Def. $\angle$ bisector)

CHALLENGE Write a two-column proof for each of the following.

52. **Given:** Plane Y is a perpendicular bisector of $\overline{DC}$.
Prove: $\angle ADB \cong \angle ACB$



SOLUTION:

A plane that is a perpendicular bisector of a segment behaves the same way as a segment that is a perpendicular bisector. It creates two right angles and two congruent corresponding segments. Use them to prove two triangles congruent in this diagram.

Proof:
Statements (Reasons)

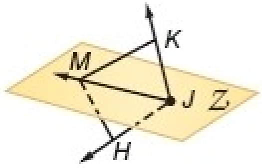
1. Plane Y is a perpendicular bisector of $\overline{DC}$. (Given)
2. $\angle DBA$ and $\angle CBA$ are right angles, $\overline{DB} \cong \overline{CB}$ (Definition of $\perp$ bisector)
3. $\angle DBA \cong \angle CBA$ (Right angles are congruent.)
4. $\overline{AB} \cong \overline{AB}$ (Reflexive Property)
5. $\triangle DBA \cong \triangle CBA$ (SAS)
6. $\angle ADB \cong \angle ACB$ (CPCTC)

ANSWER:

Proof:
Statements (Reasons)

1. Plane Y is a perpendicular bisector of $\overline{DC}$. (Given)
2. $\angle DBA$ and $\angle CBA$ are rt. $\angle s$, $\overline{DB} \cong \overline{CB}$ (Def. of $\perp$ bisector)
3. $\angle DBA \cong \angle CBA$ (Right angles are congruent.)
4. $\overline{AB} \cong \overline{AB}$ (Reflexive Property)
5. $\triangle DBA \cong \triangle CBA$ (SAS)
6. $\angle ADB \cong \angle ACB$ (CPCTC)

53. **Given:** Plane Z is an angle bisector of $\angle KJH$. $\overline{KJ} \cong \overline{HJ}$
Prove: $\overline{MH} \cong \overline{MK}$



SOLUTION:

A plane that is an angle bisector of a segment behaves the same way as a segment that is an angle bisector. It creates two congruent angles. Use these angles, along with the other given and the diagram, to prove two triangles congruent.

Proof:
Statements (Reasons)

1. Plane Z is an angle bisector of $\angle KJH$; $\overline{KJ} \cong \overline{HJ}$ (Given)
2. $\angle KJM \cong \angle HJM$ (Definition of angle bisector)
3. $\overline{JM} \cong \overline{JM}$ (Reflexive Property)
4. $\Delta KJM \cong \Delta HJM$ (SAS)
5. $\overline{MH} \cong \overline{MK}$ (CPCTC)

ANSWER:

Proof:
Statements (Reasons)

1. Plane Z is an angle bisector of $\angle KJH$; $\overline{KJ} \cong \overline{HJ}$ (Given)
2. $\angle KJM \cong \angle HJM$ (Definition of angle bisector)
3. $\overline{JM} \cong \overline{JM}$ (Reflexive Property)
4. $\Delta KJM \cong \Delta HJM$ (SAS)
5. $\overline{MH} \cong \overline{MK}$ (CPCTC)

54. **WRITING IN MATH** Compare and contrast the perpendicular bisectors and angle bisectors of a triangle. How are they alike? How are they different? Be sure to compare their points of concurrency.

SOLUTION:

Start by making a list of what the different bisectors do. Consider *what* they bisect, as well as the point of concurrency they form. Discuss the behaviors of these points of concurrency in different types of triangles. Are they ever found outside the triangle? Inside the triangle? On the triangle?

The bisectors each bisect something, but the perpendicular bisectors bisect segments while angle bisectors bisect angles. They each will intersect at a point of concurrency. The point of concurrency for perpendicular bisectors is the circumcenter. The point of concurrency for angle bisectors is the incenter. The incenter always lies in the triangle, while the circumcenter can be inside, outside, or on the triangle.

ANSWER:

The bisectors each bisect something, but the perpendicular bisectors bisect segments while angle bisectors bisect angles. They each will intersect at a point of concurrency. The point of concurrency for perpendicular bisectors is the circumcenter. The point of concurrency for angle bisectors is the incenter. The incenter always lies in the triangle, while the circumcenter can be inside, outside, or on the triangle.

55. **ALGEBRA** An object is projected straight upward with initial velocity *v* meters per second from an initial height of *s* meters. The distance *d* in meters the object travels after *t* seconds is given by $d = -10t^2 + vt + s$. Sherise is standing at the edge of a balcony 54 meters above the ground and throws a ball straight up with an initial velocity of 12 meters per second. After how many seconds will it hit the ground?

A 3 seconds
B 4 seconds
C 6 seconds
D 9 seconds

SOLUTION:

Substitute the known values in the distance expression.

$$0 = -10t^2 + (12)t + 54$$
$$-10t^2 + 12t + 54 = 0$$

Solve the equation for *t*.

$$\begin{aligned} t &= \frac{-12 \pm \sqrt{144 - 4(-10)(54)}}{2(-10)} \\ &= \frac{-12 \pm \sqrt{2304}}{-20} \\ &= \frac{-12 \pm 48}{-20} \\ &= -1.8 \text{ or } 3 \end{aligned}$$

Discard the negative value, as it has no real world meaning in this case.
The correct choice is A.

ANSWER:

A

56. **SAT/ACT** For $x \neq -3, \frac{3x+9}{x+3} =$

F $x + 12$
G $x + 9$
H $x + 3$
J x
K 3

SOLUTION:

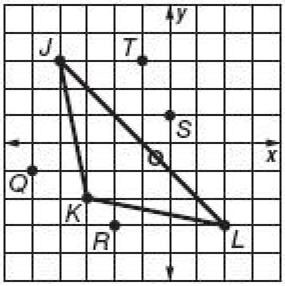
$$\begin{aligned} \frac{3x+9}{x+3} &= \frac{3(x+3)}{x+3} \\ &= 3 \end{aligned}$$

The correct choice is K.

ANSWER:

K

57. A line drawn through which of the following points would be a perpendicular bisector of ΔJKL ?



A *T* and *K*
B *L* and *Q*
C *J* and *R*
D *S* and *K*

SOLUTION:

The line passes through the points *K* and *S* will be a perpendicular bisector of ΔJKL.
The correct choice is D.

ANSWER:

D

58. **SAT/ACT** For $x \neq -3$, $\frac{3x+9}{x+3} =$

F $x + 12$
G $x + 9$
H $x + 3$
J x
K 3

SOLUTION:

Simplify the expression by first removing the GCF.

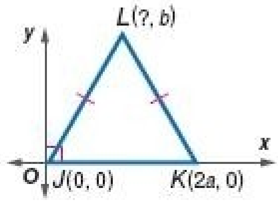
$$\begin{aligned} \frac{3x+9}{x+3} &= \frac{3(x+3)}{x+3} && \text{Remove the GCF of 3.} \\ &= 3 && \text{Cancel the common factor.} \end{aligned}$$

Therefore, the correct choice is K.

ANSWER:

K

Name the missing coordinate(s) of each triangle.

59. 

SOLUTION:

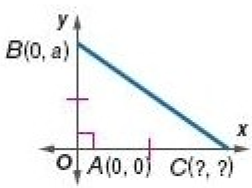
The line drawn through the point L is the perpendicular bisector of $\overleftrightarrow{JK}$. Therefore, x-coordinate of L is x-coordinate of the midpoint of JK.

The x-coordinate of L is $\frac{0+2a}{2}$ or a.

Thus, the coordinates of L is (a, b).

ANSWER:

$L(a, b)$

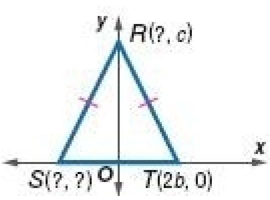
60. 

SOLUTION:

Since $AC = AB$, the distance from the origin to point B and C is the same. They are both a length away from the origin. The coordinate of C is $(a, 0)$ because it is on the x-axis and is a units away from the origin.

ANSWER:

$C(a, 0)$

61. 

SOLUTION:

Since the triangle is an isosceles triangle, S and T are equidistant from O. So, the coordinates of S are $(–2b, 0)$. R lies on the y-axis. So, its x-coordinate is 0. The coordinates of R are $(0, c)$

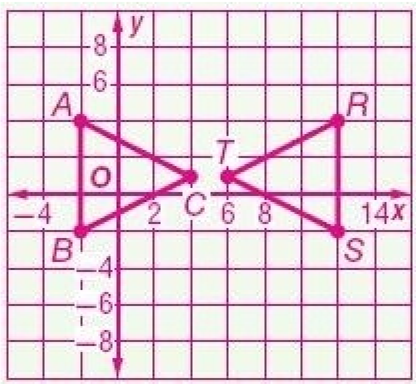
ANSWER:

$S(–2b, 0)$ and $R(0, c)$

COORDINATE GEOMETRY Graph each pair of triangles with the given vertices. Then identify the transformation and verify that it is a congruence transformation.

62. $A(-2, 4)$, $B(-2, -2)$, $C(4, 1)$; $R(12, 4)$, $S(12, -2)$, $T(6, 1)$

SOLUTION:



ΔRST is a reflection of ΔABC.

Find the length of each side.

$$AB = \sqrt{(-2+2)^2 + (-2-4)^2} = 6;$$

$$BC = \sqrt{(4+2)^2 + (1+2)^2} = \sqrt{45};$$

$$AC = \sqrt{(4+2)^2 + (1-4)^2} = \sqrt{45};$$

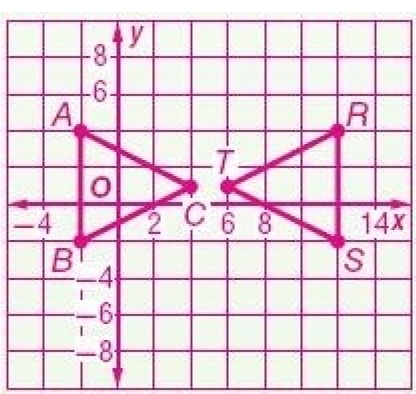
$$RS = \sqrt{(12-12)^2 + (4+2)^2} = 6;$$

$$TS = \sqrt{(6-12)^2 + (1+2)^2} = \sqrt{45};$$

$$TR = \sqrt{(12-6)^2 + (1-4)^2} = \sqrt{45}$$

$\Delta ABC \cong \Delta RST$ by SSS.

ANSWER:



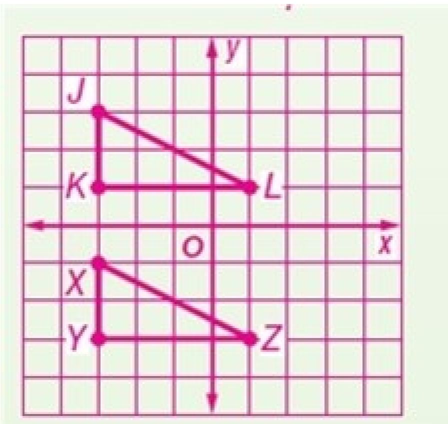
ΔRST is a reflection of ΔABC; $AB = 6$, $BC = \sqrt{45}$, $AC = \sqrt{45}$, $TR = \sqrt{45}$, $RS = 6$, $TS = \sqrt{45}$.

$\Delta ABC \cong \Delta RST$ by SSS.

5-1 Bisectors of Triangles

63. $J(-3, 3)$, $K(-3, 1)$, $L(1, 1)$; $X(-3, -1)$, $Y(-3, -3)$, $Z(1, -3)$

SOLUTION:



ΔJKL is a translation of ΔXYZ.

Find the length of each side.

$JK = \sqrt{(-3+3)^2 + (1-3)^2} = 2;$

$KL = \sqrt{(1+3)^2 + (1-1)^2} = 4;$

$JL = \sqrt{(1+3)^2 + (1-3)^2} = \sqrt{20};$

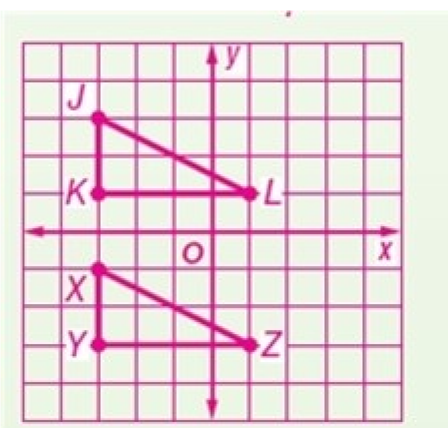
$XY = \sqrt{(-3+3)^2 + (-3+1)^2} = 2;$

$YZ = \sqrt{(1+3)^2 + (-3+3)^2} = 4;$

$ZX = \sqrt{(1+3)^2 + (-3+1)^2} = \sqrt{20}$

$\Delta JKL \cong \Delta XYZ$ by SSS.

ANSWER:



ΔJKL is a translation of ΔXYZ; $JK = 2$, $KL = 4$, $JL = \sqrt{20}$, $XY = 2$, $YZ = 4$, $XZ = \sqrt{20}$. $\Delta JKL \cong \Delta XYZ$ by SSS.

Find the distance from the line to the given point.

64. $y = 5, (-2, 4)$

SOLUTION:

The slope of an equation perpendicular to $y = 5$ will be undefined, or the line will be a vertical line. The equation of a vertical line through (–2, 4) is $x = -2$.
The point of intersection of the two lines is (–2, 5).
Use the Distance Formula to find the distance between the points (–2, 4) and (–2, 5).

$$\begin{aligned} d &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\ &= \sqrt{(-2+2)^2 + (5-4)^2} \\ &= \sqrt{0+1} \\ &= 1 \end{aligned}$$

Therefore, the distance between the line and the point is 1 unit.

ANSWER:

1

65. $y = 2x + 2, (-1, -5)$

SOLUTION:

The slope of an equation perpendicular to $y = 2x + 2$ will be $-\frac{1}{2}$. A line with a slope $-\frac{1}{2}$ and that passes through the point (–1, –5) will have the equation,

$$\begin{aligned} y + 5 &= -\frac{1}{2}(x + 1). \\ 2y + 10 &= -x - 1 \\ y &= -\frac{1}{2}x - \frac{11}{2} \end{aligned}$$

Solve the two equations to find the point of intersection.
The left sides of the equations are the same. So, equate the right sides and solve for *x*.

$$\begin{aligned} -\frac{1}{2}x - \frac{11}{2} &= 2x + 2 \\ -\frac{1}{2}x - 2x - \frac{11}{2} &= 2x - 2x + 2 \\ -\frac{1}{2}x - \frac{4}{2}x - \frac{11}{2} &= 2 \\ -\frac{5}{2}x - \frac{11}{2} + \frac{11}{2} &= \frac{4}{2} + \frac{11}{2} \\ -\frac{5}{2}x &= \frac{15}{2} \\ x &= -3 \end{aligned}$$

Use the value of *x* to find the value of *y*.

$$\begin{aligned} y &= 2x + 2 \\ &= 2(-3) + 2 \\ &= -4 \end{aligned}$$

The point of intersection of the two lines is (–3, –4).

Use the Distance Formula to find the distance between the points (–3, –4) and (–1, –5).

$$\begin{aligned} d &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\ &= \sqrt{(-1 + 3)^2 + (-5 + 4)^2} \\ &= \sqrt{4 + 1} \\ &= \sqrt{5} \end{aligned}$$

Therefore, the distance between the line and the point is $\sqrt{5}$ units.

ANSWER:

$\sqrt{5}$

66. $2x-3y=-9,(2,0)$

SOLUTION:

The slope of an equation perpendicular to $2x-3y=-9$ will be $-\frac{3}{2}$. A line with a slope $-\frac{3}{2}$ and that passes through the point (2, 0) will have the equation,

$$\begin{aligned} y+0&=-\frac{3}{2}(x-2).\\ 2y&=-3x+6\\ y&=-\frac{3}{2}x+3 \end{aligned}$$

Solve the two equations to find the point of intersection.
The left sides of the equations are the same. So, equate the right sides and solve for x.

$$\begin{aligned} -\frac{3}{2}x+3&=\frac{2}{3}x+3\\ -\frac{3}{2}x-\frac{2}{3}x+3&=\frac{2}{3}x-\frac{2}{3}x+3\\ -\frac{9}{6}x-\frac{4}{6}x+3&=3\\ -\frac{1}{6}x+3-3&=3-3\\ -\frac{11}{6}x&=0\\ x&=0 \end{aligned}$$

Use the value of x to find the value of y.

$$\begin{aligned} y&=-\frac{3}{2}x+3\\ &=-\frac{3}{2}(0)+3\\ &=3 \end{aligned}$$

The point of intersection of the two lines is (0, 3).
Use the Distance Formula to find the distance between the points (0, 3) and (2, 0).

$$\begin{aligned} d&=\sqrt{(2-0)^2+(0-3)^2}\\ &=\sqrt{4+9}\\ &=\sqrt{13} \end{aligned}$$

Therefore, the distance between the line and the point is $\sqrt{13}$ units.

ANSWER:

$\sqrt{\mathbf{13}}$

67. **AUDIO ENGINEERING** A studio engineer charges a flat fee of $450 for equipment rental and $42 an hour for recording and mixing time. Write the equation that shows the cost to hire the studio engineer as a function of time. How much would it cost to hire the studio engineer for 17 hours?

SOLUTION:

Let t be the time taken for recording and mixing in hours.
The equation that represents the cost for hiring a studio engineer is $m = 42t + 450$.
Substitute t = 17 in the expression to find the cost to hire the studio engineer for 17 hours.

$$\begin{aligned} m &= 42(17) + 450 \\ &= 1164 \end{aligned}$$

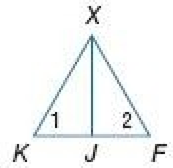
ANSWER:

$m = 42t + 450$; $1164

PROOF Write a two-column proof for each of the following.

68. **Given:** ΔXKF is equilateral. $\overline{XJ}$ bisects $\angle X$.

Prove: J is the midpoint of $\overline{KF}$.



SOLUTION:

Consider what the given information that ΔXKF is an equilateral triangle is telling you - not only are all the sides congruent but all the angles are congruent as well. You will need to use both of these ideas to get $\Delta KXJ \cong \Delta FXJ$. Then, consider what CPCTC you will need to prove that J is a midpoint of $\overline{KF}$.

Proof:

Statements (Reasons)

1. ΔXKF is equilateral. (Given)
2. $\angle 1 \cong \angle 2$ (Equilateral triangles are equiangular.)
3. $\overline{KX} \cong \overline{FX}$ (Definition of equilateral triangle).
4. $\overline{XJ}$ bisects $\angle X$. (Given)
5. $\angle KXJ \cong \angle FXJ$ (Definition of angle bisector)
6. $\Delta KXJ \cong \Delta FXJ$ (ASA)
7. $\overline{KJ} \cong \overline{FJ}$ (CPCTC)
8. J is the midpoint of $\overline{KF}$. (Definition of midpoint)

ANSWER:

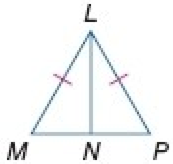
Proof:

Statements (Reasons)

1. ΔXKF is equilateral. (Given)
2. $\angle 1 \cong \angle 2$ (Equilateral Δs are equiangular.)
3. $\overline{KX} \cong \overline{FX}$ (Def. of equilateral Δ)
4. $\overline{XJ}$ bisects $\angle X$. (Given)
5. $\angle KXJ \cong \angle FXJ$ (Def. of $\angle$ bisector)
6. $\Delta KXJ \cong \Delta FXJ$ (ASA)
7. $\overline{KJ} \cong \overline{FJ}$ (CPCTC)
8. J is the midpoint of $\overline{KF}$. (Def. of midpoint)

69. **Given:** ΔMLP is isosceles. N is the midpoint of $\overline{MP}$.

Prove: $\overline{LN} \perp \overline{MP}$.



SOLUTION:

The trickier part of this proof is how to prove two lines are perpendicular to each other once you get $\angle LNM \cong \angle LNP$. You will need to think about how to progress from making them congruent to making them add up to 180 degrees.Then, once you have them adding up to 180 degrees, how can you prove that just one of them equals 90 degrees?

Proof:
Statements (Reasons)
1. ΔMLP is isosceles. (Given)
2. $\overline{ML} \cong \overline{PL}$ (Definition of isosceles triangle)
3. $\angle M \cong \angle P$ (Isosceles Triangle Theorem)
4. N is the midpoint of $\overline{MP}$. (Given)
5. $\overline{MN} \cong \overline{PN}$ (Definition of midpoint)
6. $\Delta MNL \cong \Delta PNL$ (SAS)
7. $\angle LNM \cong \angle LNP$ (CPCTC)
8. $m\angle LNM = m\angle LNP$ (Definition of Congruent Angles)
9. $\angle LNM$ and $\angle LNP$ are a linear pair. (Definition of a linear pair)
10. $m\angle LNM + m\angle LNP = 180$ (Sum of measures of linear pair of angles is equal to 180.)
11. $2m\angle LNM = 180$ (Substitution Property)
12. $m\angle LNM = 90$ (Division Property)
13. $\angle LNM$ is a right angle. (Definition of right angles)
14. $\overline{LN} \perp \overline{MP}$ (Definition of perpendicular lines)

ANSWER:
Proof:
Statements (Reasons)
1. ΔMLP is isosceles. (Given)
2. $\overline{ML} \cong \overline{PL}$ (Definition of isosceles Δ)
3. $\angle M \cong \angle P$ (Isosceles Δ Th.)
4. N is the midpoint of $\overline{MP}$. (Given)
5. $\overline{MN} \cong \overline{PN}$ (Def. of midpoint)
6. $\Delta MNL \cong \Delta PNL$ (SAS)
7. $\angle LNM \cong \angle LNP$ (CPCTC)
8. $m\angle LNM = m\angle LNP$ (Def. of $\cong \angle s$)
9. $\angle LNM$ and $\angle LNP$ are a linear pair. (Def. of a linear pair)
10. $m\angle LNM + m\angle LNP = 180$ (Sum of measures of linear pair of $\angle s = 180$)
11. $2m\angle LNM = 180$ (Substitution)
12. $m\angle LNM = 90$ (Division)
13. $\angle LNM$ is a right angle. (Def. of rt. $\angle$)
14. $\overline{LN} \perp \overline{MP}$ (Def. of $\perp$)