In $\triangle ACE$, P is the centroid, $PF = 6$, and $AD = 15$. Find each measure.
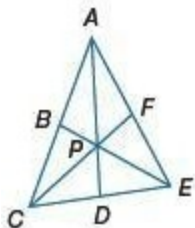
1. *PC*

   SOLUTION:

   Since P is the centroid of the triangle ACE, $PC = \frac{2}{3}CF$ by the Centroid Theorem.

   We know that $CF = PC + PF$.

   $$PC = \frac{2}{3}(PC + PF)$$
   $$PC = \frac{2}{3}(PC + 6)$$
   $$\frac{PC}{3} = 4$$
   $$PC = 12$$

   ANSWER:

   12

2. *AP*

   SOLUTION:

   Since P is the centroid of the triangle ACE, $AP = \frac{2}{3}AD$ by the Centroid Theorem.

   $$AP = \frac{2}{3}(15)$$
   $$= 10$$

   ANSWER:

   10

3. **INTERIOR DESIGN** An interior designer is creating a custom coffee table for a client. The top of the table is a glass triangle that needs to balance on a single support. If the coordinates of the vertices of the triangle are at (3, 6), (5, 2), and (7, 10), at what point should the support be placed?

SOLUTION:
The triangle has a balance point at centroid. Find the centroid of the triangular coffee table.

Use the centroid formula $\left(\dfrac{x_1 + x_2 + x_3}{3}, \dfrac{y_1 + y_2 + y_3}{3} \right)$.

The centroid of the triangle is $\left(\dfrac{3+5+7}{3}, \dfrac{6+2+10}{3} \right)$ or $(5,6)$.

Let the points be $A(3, 6)$, $B(5, 2)$, and $C(7, 10)$.

The midpoint D of $\overline{BC}$ is $\left(\dfrac{5+7}{2}, \dfrac{2+10}{2} \right)$ or $(6,6)$. Note that $\overline{AD}$ is a line that connects the vertex A and D, the midpoint of $\overline{BC}$.

The distance from $D(6, 6)$ to $A(3, 6)$ is $6 - 3$ or 3 units.

If P is the centroid of the triangle ABC, then $AC = \dfrac{2}{3}AD$. So, the centroid is $\dfrac{2}{3}(3)$ or 2 units to the right of A. The coordinates of the centroid (P) are $(3 + 2, 6)$ or $(5, 6)$.

ANSWER:
(5, 6)

4. **COORDINATE GEOMETRY** Find the coordinates of the orthocenter of triangle ABC with vertices $A(-3, 3)$, B $(-1, 7)$, and $C(3, 3)$.

SOLUTION:

The slope of $\overline{AB}$ is $\dfrac{7-3}{-1+3} = \dfrac{4}{2} = \dfrac{2}{1}$. So, the slope of the altitude, which is perpendicular to $\overline{AB}$ is $-\dfrac{1}{2}$. Now, the equation of the altitude from C to $\overline{AB}$ is:

$$y - 3 = -\frac{1}{2}(x - 3)$$

$$y - 3 = -\frac{1}{2}x + \frac{3}{2}$$

$$y - 3 + 3 = -\frac{1}{2}x + \frac{3}{2} + 3$$

$$y = -\frac{1}{2}x + \frac{9}{2}$$

Use the same method to find the equation of the altitude from A to $\overline{BC}$. That is, $y - 3 = 1(x + 3)$ or $y = x + 6$.

Solve the equations to find the intersection point of the altitudes.

$$x + 6 = -\frac{1}{2}x + \frac{9}{2}$$

$$2(x + 6) = 2\left(-\frac{1}{2}x + \frac{9}{2}\right)$$

$$2x + 12 = -x + 9$$

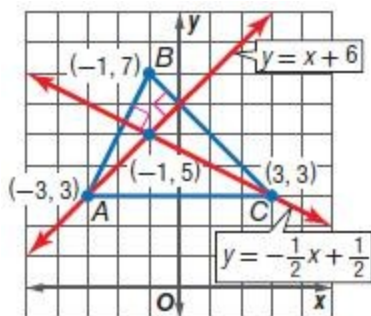$$2x - x + 12 = -x - x + 9$$

$$x + 12 = 9$$

$$x + 12 - 12 = 9 - 12$$

$$3x = -3$$

$$x = -1$$

$$y = (-1) + 6 = 5$$

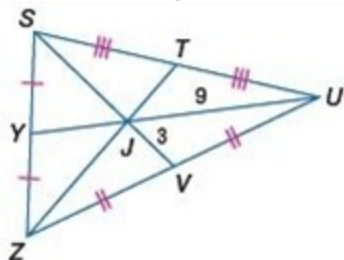So, the coordinates of the orthocenter of $\triangle ABC$ is $(-1, 5)$.

ANSWER:

$(-1, 5)$

In △SZU , UJ = 9, VJ = 3, and ZT = 18.
Find each length.

5. *YJ*

SOLUTION:

Since $\overline{SY} \cong \overline{YZ}$, Y is the midpoint of $\overline{SZ}$ and $\overline{UY}$ is a median of $\triangle ZSU$. Similarly, points *T* and *V* are also midpoints of $\overline{SU}$ and $\overline{ZU}$, respectively, so $\overline{TZ}$ and $\overline{SV}$ are also medians. Therefore, point *J* is the centroid of $\triangle ZSU$ and, according to the Centroid Theorem, *UJ* is $\frac{2}{3}$ of *UY*.

$$UJ = \frac{2}{3}(UY)$$

$$UJ = \frac{2}{3}(YJ + UJ)$$

$$\frac{3UJ}{2} - UJ = YJ$$

$$\frac{UJ}{2} = YJ$$

$$\frac{9}{2} = YJ$$

$$YJ = 4.5$$

ANSWER:

4.5

6. *SJ*

   SOLUTION:

   Since $\overline{ZV} \cong \overline{VU}$, *V* is the midpoint of $\overline{UZ}$ and $\overline{SV}$ is a median of $\triangle ZSU$. Similarly, points *Y* and *T* are also midpoints of $\overline{SZ}$ and $\overline{SU}$, respectively, so $\overline{YU}$ and $\overline{ZT}$ are also medians. Therefore, point *J* is the centroid of $\triangle ZSU$ and, according to the Centroid Theorem, *SJ* is $\frac{2}{3}$ of *SV*.

   $$SJ = \frac{2}{3}SV$$

   $$= \frac{2}{3}(SJ + VJ)$$

   $$\frac{3SJ}{2} - SJ = VJ$$

   $$\frac{SJ}{2} = 3$$

   $$SJ = 6$$

   ANSWER:

   6

7. *YU*

   SOLUTION:

   Since $\overline{SY} \cong \overline{YZ}$, *Y* is the midpoint of $\overline{SZ}$ and $\overline{UY}$ is a median of $\triangle ZSU$. Similarly, points *T* and *V* are also midpoints of $\overline{SU}$ and $\overline{ZU}$, respectively, so $\overline{TZ}$ and $\overline{SV}$ are also medians. Therefore, point *J* is the centroid of $\triangle ZSU$ and, according to the Centroid Theorem, *UJ* is $\frac{2}{3}$ of *UY*.

   $$UJ = \frac{2}{3}(UY)$$

   $$UJ = \frac{2}{3}(YJ + UJ)$$

   $$\frac{3UJ}{2} - UJ = YJ$$

   $$\frac{UJ}{2} = YJ$$

   $$\frac{9}{2} = YJ$$

   $$YJ = 4.5$$

   Now, since *YU* is the sum of *YJ* and *UJ,* we can add them to find *YU*.

   $$YU = YJ + UJ$$

   $$= 4.5 + 9$$

   $$= 13.5$$

   ANSWER:

   13.5

8. *SV*

SOLUTION:

Since $\overline{ZV} \cong \overline{VU}$, *V* is the midpoint of $\overline{UZ}$ and $\overline{SV}$ is a median of $\triangle ZSU$. Similarly, points *Y* and *T* are also midpoints of $\overline{SZ}$ and $\overline{SU}$, respectively, so $\overline{YU}$ and $\overline{ZT}$ are also medians. Therefore, point *J* is the centroid of $\triangle ZSU$ and, according to the Centroid Theorem, *SJ* is $\frac{2}{3}$ of *SV*.

$$SJ = \frac{2}{3}SV$$
$$= \frac{2}{3}(SJ + VJ)$$
$$\frac{3SJ}{2} - SJ = VJ$$
$$\frac{SJ}{2} = 3$$
$$SJ = 6$$

Therefore, to find *SV*, we can now add *SJ* and *VJ*.
$$SV = SJ + VJ$$
$$= 6 + 3$$
$$= 9$$

ANSWER:

9

9. *JT*

SOLUTION:

Since $\overline{ST} \cong \overline{TU}$, *T* is the midpoint of $\overline{US}$ and $\overline{ZT}$ is a median of $\triangle ZSU$. Similarly, points *Y* and *V* are also midpoints of $\overline{SZ}$ and $\overline{ZU}$, respectively, so $\overline{YU}$ and $\overline{SV}$ are also medians. Therefore, point J is the centroid of $\triangle ZSU$ and, according to the Centroid Theorem, *ZJ* is $\frac{2}{3}$ of *ZT*, which we know *ZT* equals 18 from the given information. To find *JT*, subtract *ZJ* from *ZT*.

$$ZJ = \frac{2}{3}ZT$$
$$= \frac{2}{3}(18)$$
$$= 12$$
$$JT = ZT - ZJ$$
$$= 18 - 12$$
$$= 6$$

ANSWER:

6

10. *ZJ*

SOLUTION:

Since $\overline{ST} \cong \overline{TU}$, *T* is the midpoint of $\overline{US}$ and $\overline{ZT}$ is a median of $\triangle ZSU$. Similarly, points *Y* and *V* are also midpoints of $\overline{SZ}$ and $\overline{ZU}$, respectively, so $\overline{YU}$ and $\overline{SV}$ are also medians. Therefore, point *J* is the centroid of $\triangle ZSU$ and, according to the Centroid Theorem, *ZJ* is $\frac{2}{3}$ of *ZT*, which we know *ZT* equals 18 from the given information.

$$ZJ = \frac{2}{3}ZT$$

$$= \frac{2}{3}(18)$$

$$= 12$$

ANSWER:

12

COORDINATE GEOMETRY Find the coordinates of the centroid of each triangle with the given vertices.
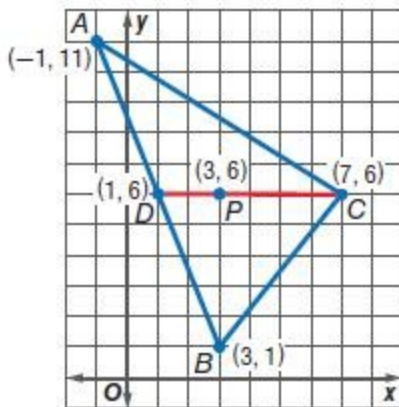
11. *A*(−1, 11), *B*(3, 1), *C*(7, 6)

SOLUTION:

The midpoint *D* of $\overline{AB}$ is $\left(\frac{-1+3}{2}, \frac{11+1}{2}\right)$ or (1, 6). Note that $\overline{DC}$ is a line that connects the vertex *C* and *D*, the midpoint of $\overline{AB}$.

The distance from *D*(1, 6) to *C*(7, 6) is 7 − 1 or 6 units.

If *P* is the centroid of the triangle *ABC*, then $PC = \frac{2}{3}DC$. So, the centroid is $\frac{2}{3}(6)$ or 4 units to the left of *C*. The coordinates of the centroid (*P*) are (7 − 4, 6) or (3, 6).
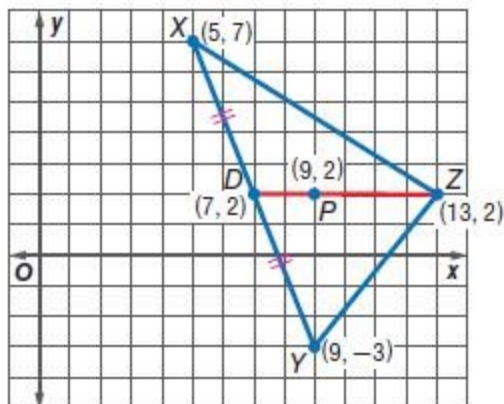

ANSWER:

(3, 6)

12. $X(5,\ 7),\ Y(9,\ -3),\ Z(13,\ 2)$

   SOLUTION:

   The midpoint D of $\overline{XY}$ is $\left(\dfrac{5+9}{2}, \dfrac{7-3}{2}\right)$ or $(7, 2)$. Note that $\overline{DZ}$ is a line that connects the vertex Z and D, the midpoint of $\overline{XY}$.

   The distance from $D(7, 2)$ to $Z(13, 2)$ is $13 - 7$ or 6 units.

   If P is the centroid of the triangle XYZ, then $PZ = \dfrac{2}{3}DZ$. So, the centroid is $\dfrac{2}{3}(6)$ or 4 units to the left of Z. The coordinates of the centroid(P) are $(13-4, 2)$ or $(9, 2)$.

   ANSWER:
   $(9, 2)$

13. **INTERIOR DESIGN** Emilia made a collage with pictures of her friends. She wants to hang the collage from the ceiling in her room so that it is parallel to the ceiling. A diagram of the collage is shown in the graph at the right. At what point should she place the string?
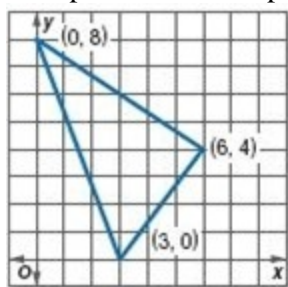
   SOLUTION:
   The triangle has a balance point at centroid. Find the centroid of the picture.

   Use the centroid formula $\left(\dfrac{x_1+x_2+x_3}{3}, \dfrac{y_1+y_2+y_3}{3}\right)$.

   The centroid of the triangle is $\left(\dfrac{0+3+6}{3}, \dfrac{8+0+4}{3}\right)$ or $(3, 4)$.

   ANSWER:
   $(3, 4)$

COORDINATE GEOMETRY Find the coordinates of the orthocenter of each triangle with the given vertices.

14. $J(3, -2), K(5, 6), L(9, -2)$

SOLUTION:

The slope of $\overline{JK}$ is $\dfrac{6+2}{5-3}$ or 4. So, the slope of the altitude, which is perpendicular to $\overline{JK}$ is $-\dfrac{1}{4}$. Now, the equation of the altitude from L to $\overline{JK}$ is:
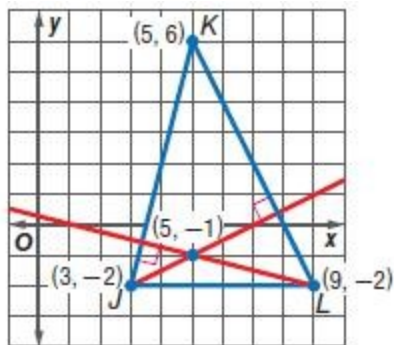
$$y+2 = -\frac{1}{4}(x-9)$$
$$y = -\frac{1}{4}x + \frac{1}{4}$$

Use the same method to find the equation of the altitude from J to $\overline{KL}$. That is, $y+2 = \dfrac{1}{2}(x-3)$ or $y = \dfrac{1}{2}x - \dfrac{7}{2}$.

Solve the equations to find the intersection point of the altitudes.

$$-\frac{1}{4}x + \frac{1}{4} = \frac{1}{2}x - \frac{7}{2}$$
$$-\frac{1}{4}x - \frac{1}{2}x = -\frac{7}{2} - \frac{1}{4}$$
$$-\frac{3}{4}x = -\frac{15}{4}$$
$$x = 5$$
$$y = -\frac{1}{4}(5) + \frac{1}{4}$$
$$= \frac{-4}{4}$$
$$y = -1$$

So, the coordinates of the orthocenter of $\triangle JKL$ is $(5, -1)$.

ANSWER:

$(5, -1)$

15. *R*(–4, 8), *S*(–1, 5), *T*(5, 5)

   SOLUTION:

   The slope of $\overline{RS}$ is $\dfrac{5-8}{-1+4}$ or –1. So, the slope of the altitude, which is perpendicular to $\overline{RS}$ is 1. Now, the

   equation of the altitude from *T* to $\overline{RS}$ is:
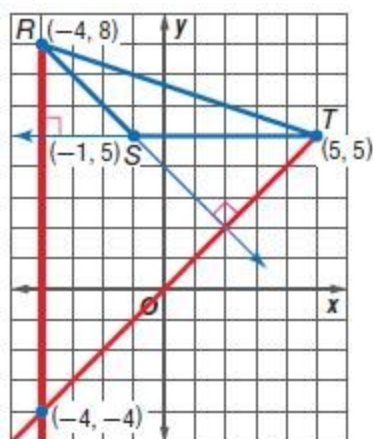
   $$y - 5 = 1(x - 5)$$
   $$y = x$$

   Use the same way to find the equation of the altitude from *R* to $\overline{ST}$. That is, $y - 8 = -\dfrac{1}{0}(x + 4)$ or $x = -4$

   Solve the equations to find the intersection point of the altitudes.

   $$x = -4; y = -4;$$

   So, the coordinates of the orthocenter of $\triangle RST$ is (–4, –4).

   ANSWER:

   (–4, –4)

Identify each segment $\overline{BD}$ as a(n) altitude, median, or perpendicular bisector.
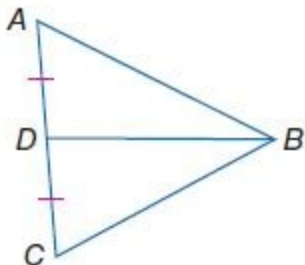
16. Refer to the image on page 340.

SOLUTION:

$\overline{BD}$ is an altitude here because is it perpendicular to $\overline{AC}$.

ANSWER:

altitude

17. Refer to the image on page 340.

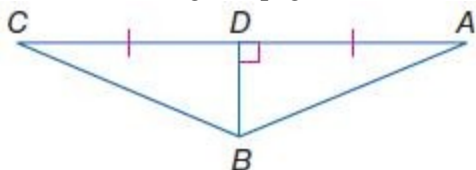

SOLUTION:

$\overline{BD}$ is a median because D is the midpoint of the side opposite vertex B.

ANSWER:

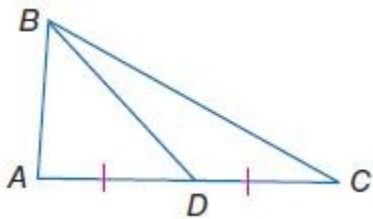median

18. Refer to the image on page 340.

SOLUTION:

$\overline{BD}$ is perpendicular to $\overline{AC}$, as well as meets at the midpoint D. Therefore, $\overline{BD}$ is an altitude, median and perpendicular bisector.

ANSWER:

perpendicular bisector, altitude, median

19. Refer to the image on page 340.

SOLUTION:

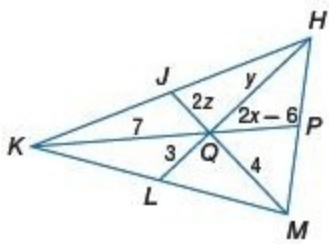$\overline{BD}$ is a median because it connects point B to point D, which is the midpoint of $\overline{AC}$.

ANSWER:

median

20. **CCSS SENSE-MAKING** In the figure, if *J, P,* and *L* are the midpoints of $\overline{KH}$, $\overline{HM}$, and $\overline{MK}$, respectively, find *x, y,* and *z*.

SOLUTION:

By the Centroid Theorem, $HQ = \frac{2}{3}(LH); KQ = \frac{2}{3}(KP); QM = \frac{2}{3}(JM).$

$$HQ = \frac{2}{3}(LH)$$
$$y = \frac{2}{3}(3+y)$$
$$3y = 6+2y$$
$$y = 6$$

$$KQ = \frac{2}{3}(KP);$$
$$7 = \frac{2}{3}(7 + 2x - 6);$$
$$21 = 2 + 4x$$
$$19 = 4x$$
$$x = 4.75$$

$$QM = \frac{2}{3}(JM);$$
$$4 = \frac{2}{3}(2z + 4)$$
$$12 = 4z + 8$$
$$4 = 4z$$
$$z = 1$$

ANSWER:

$x = 4.75, y = 6, z = 1$

Copy and complete each statement for $\triangle RST$ **for medians** $\overline{RM}$, $\overline{SL}$ **and** $\overline{TK}$ **, and centroid** J**.**

21. $SL = x(JL)$

SOLUTION:
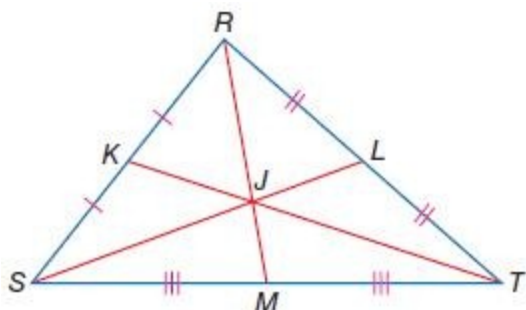
$$SL = JL + SJ$$
$$= JL + \frac{2}{3}SL$$
$$SL - \frac{2}{3}SL = JL$$
$$\frac{SL}{3} = JL$$
$$SL = 3JL$$

ANSWER:

3

22. $JT = x(TK)$

SOLUTION:

By the Centroid Theorem, $JT = \frac{2}{3}TK$.

ANSWER:

$$\frac{2}{3}$$

23. $JM = x(RJ)$
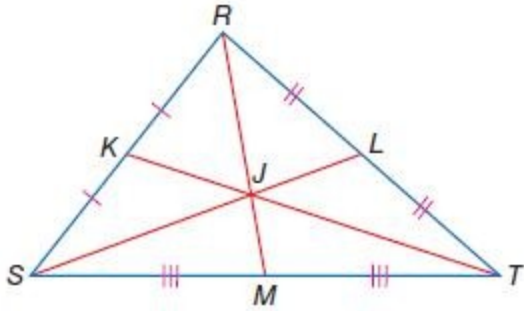
   SOLUTION:

   $$RJ = \frac{2}{3}(RM)$$
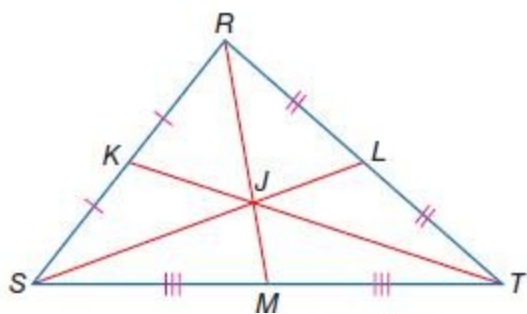 $$= \frac{2}{3}(RJ + JM)$$
   $$3RJ - 2RJ = 2JM$$
   $$RJ = 2JM$$
   $$JM = \frac{1}{2}RJ$$

   ANSWER:

   $\dfrac{1}{2}$

ALGEBRA Use the figure.
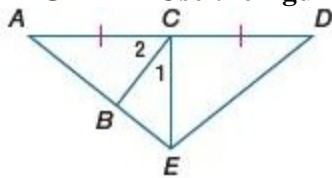
24. If $\overline{EC}$ is an altitude of $\triangle AED$, $m\angle 1 = 2x + 7$, and $m\angle 2 = 3x + 13$, find $m\angle 1$ and $m\angle 2$.

SOLUTION:

By the definition of altitude, $m\angle 1 + m\angle 2 = 90$.

$$2x + 7 + 3x + 13 = 90$$
$$5x + 20 = 90$$
$$5x = 70$$
$$x = 14$$

Substitute 14 for x in $m\angle 1$ and $m\angle 2$.

$$m\angle 1 = 2x + 7$$
$$= 2(14) + 7$$
$$= 35$$
$$m\angle 2 = 3x + 13$$
$$= 3(14) + 13$$
$$= 55$$

ANSWER:

$m\angle 1 = 35, m\angle 2 = 55$

25. Find the value of x if $AC = 4x - 3$, $DC = 2x + 9$, $m \angle ECA = 15x + 2$, and $\overline{EC}$ is a median of $\triangle AED$. Is $\overline{EC}$ also an altitude of $\triangle AED$? Explain.

    SOLUTION:
    Given: $AC = DC$.

    $$4x - 3 = 2x + 9$$
    $$4x - 2x - 3 = 2x - 2x + 9$$
    $$2x - 3 = 9$$
    $$2x - 3 + 3 = 9 + 3$$
    $$2x = 12$$
    $$x = 6$$

    Substitute 6 for x in $m \angle ECA$.

    $$m \angle ECA = 15x + 2$$
    $$= 15(6) + 2$$
    $$= 92$$

    $\overline{EC}$ is not an altitude of $\triangle AED$ because $m \angle ECA = 92$.

    ANSWER:
    6; no; because $m \angle ECA = 92$

26. **GAMES** The game board shown is shaped like an equilateral triangle and has indentations for game pieces. The game's objective is to remove pegs by jumping over them until there is only one peg left. Copy the game board's outline and determine which of the points of concurrency the blue peg represents: circumcenter, incenter, centroid, or orthocenter. Explain your reasoning.
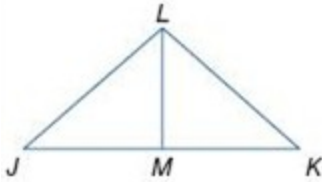

    SOLUTION:
    To determine each point of concurrency, you must perform their corresponding constructions. You may want to make a different tracing for each center so your lines and arcs won't get confusing.To determine if the blue peg is a circumcenter, construct the perpendicular bisectors of each side. To determine if the blue peg is the incenter, you need to construct the angle bisectors of each side. The centroid can be determined by constructing the midpoints of each side and connecting them to the opposite vertices. Finally, the orthocenter can be constructed by finding the altitudes from each vertex to the opposite side. The blue peg represents all of the centers, including the circumcenter, incenter, centroid, and orthocenter.

    ANSWER:
    Circumcenter, incenter, centroid, orthocenter; Sample answer: The angle bisector of each angle also bisects the opposite side and is perpendicular to the opposite side of the triangle, so it also represents the perpendicular bisector, the median, and the altitude. That means that the blue peg represents all of the centers, including the circumcenter, incenter, centroid, and orthocenter.

5-2 Medians and Altitudes of Triangles

CCSS ARGUMENTS Use the given information to determine whether
$\overline{LM}$ **is a** *perpendicular bisector, median,* **and/or an** *altitude* **of** $\triangle JKL$ **.**

27. $\overline{LM} \perp \overline{JK}$

SOLUTION:

Since $\overline{LM} \perp \overline{JK}$, $\overline{LM}$ is an altitude by the definition of altitude. We don't know if it is a perpendicular bisector because it is not evident that M is the midpoint of $\overline{JK}$.

ANSWER:

altitude

28. $\triangle JLM \cong \triangle KLM$

SOLUTION:

Since $\triangle JLM \cong \triangle KLM$, we know that $\overline{JM} \cong \overline{MK}$ and $\angle LMJ \cong \angle LMK$ by CPCTC. Since $\angle LMJ \cong \angle LMK$ and they are a linear pair, then we know they are right angles and $\overline{LM} \perp \overline{JK}$. Therefore, $\overline{LM}$ is the perpendicular bisector, median, and altitude of $\triangle JKL$.

ANSWER:

perpendicular bisector, median, altitude

29. $\overline{JM} \cong \overline{KM}$

SOLUTION:

Since we know that $\overline{JM} \cong \overline{MK}$ then M is the midpoint of $\overline{JK}$. Therefore, $\overline{LM}$ is the median of $\triangle JKL$.

ANSWER:

median

30. $\overline{LM} \perp \overline{JK}$ and $\overline{JL} \cong \overline{KL}$

SOLUTION:

Since $\overline{LM} \perp \overline{JK}$ and $\overline{JL} \cong \overline{KL}$, we can prove $\triangle JLM \cong \triangle KLM$ by HL. Therefore, we know that $\overline{JM} \cong \overline{MK}$ by CPCTC, making M the midpoint of $\overline{JK}$. Therefore, $\overline{LM}$ is the perpendicular bisector, median, and altitude of $\triangle JKL$.
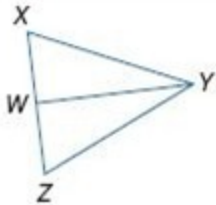
ANSWER:

perpendicular bisector, median, altitude

PROOF Write a paragraph proof.

31. **Given:** $\triangle XYZ$ is isosceles. $\overline{WY}$ bisects $\angle Y$.

    Prove: $\overline{WY}$ is a median.

SOLUTION:

When solving this proofs like these it helps to think backwards. What do you need to do to prove that $\overline{WY}$ is a median of the triangle? To be considered a median, then it must be formed by a segment with one endpoint on a vertex of the triangle and the other endpoint at the midpoint of the opposite side. To prove that $\overline{WY}$ is a median, you must prove that W is the midpoint of $\overline{XZ}$ or that $\overline{XW} \cong \overline{WZ}$. Use the given information to prove that the two triangles in the diagram are congruent and then use $\overline{XW} \cong \overline{WZ}$ as your CPCTC statement.

Given: $\triangle XYZ$ is isosceles. $\overline{WY}$ bisects $\angle Y$.

Prove: $\overline{WY}$ is a median.

Proof: Since $\triangle XYZ$ is isosceles, $\overline{XY} \cong \overline{YZ}$.

By the definition of angle bisector, $\angle XYW \cong \angle ZYW$. $\overline{YW} \cong \overline{YW}$ by the Reflexive Property. So, by SAS, $\triangle XYW \cong \triangle ZYW$.

By CPCTC, $\overline{XW} \cong \overline{ZW}$. By the definition of a midpoint, W is the midpoint of $\overline{XZ}$. By the definition of a median, $\overline{WY}$ is a median.

ANSWER:

Given: $\triangle XYZ$ is isosceles. $\overline{WY}$ bisects $\angle Y$.

Prove: $\overline{WY}$ is a median.

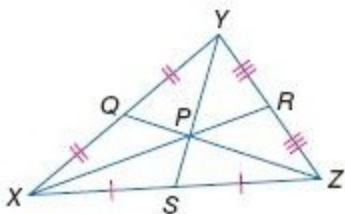Proof: Since $\triangle XYZ$ is isosceles, $\overline{XY} \cong \overline{YZ}$. By the definition of angle bisector,

$\angle XYW \cong \angle ZYW$. $\overline{YW} \cong \overline{YW}$ by the Reflexive Property. So, by SAS, $\triangle XYW \cong \triangle ZYW$. By CPCTC,

$\overline{XW} \cong \overline{ZW}$. By the definition of a midpoint, W is the midpoint of $\overline{XZ}$. By the definition of a median, $\overline{WY}$ is a median.

PROOF Write an algebraic proof.

32. **Given:** $\triangle XYZ$ with medians $\overline{XR}$, $\overline{YS}$, $\overline{ZQ}$.

    Prove: $\dfrac{XP}{PR} = 2$

SOLUTION:

Given: $\triangle XYZ$ with medians $\overline{XR}$, $\overline{YS}$, and $\overline{ZQ}$.

Prove: $\dfrac{XP}{PR} = 2$

Consider what algebraic relationships exist for the medians of a triangle. You know that, according to the Centroid Theorem, the centroid of a triangle lies $\frac{2}{3}$ the distance from one vertex to the midpoint of the opposite side. Using this, you can set up an equation relating XP and XR You also can write a statement relating XR to XP and PR. Notice how you have XR in your first two statements. Think about how you can substitute one statement into the other to just leave XP and XR in your equation. Keep simplifying until you reach what you are trying to prove.

Proof:

Statements (Reasons)

1. $\triangle XYZ$ with medians $\overline{XR}, \overline{YS}$, and $\overline{ZQ}$.
(Given)

2. $XP = \dfrac{2}{3} XR$ (Centroid Theorem)

3. $XR = XP + PR$ (Segment Addition Postulate)

4. $XP = \dfrac{2}{3}(XR + PR)$ (Substitution Property)

5. $XP = \dfrac{2}{3} XP + \dfrac{2}{3} PR$ (Distributive Property)

6. $\dfrac{1}{3} XP = \dfrac{2}{3} PR$ (Subtraction Property)

7. $XP = 2PR$ (Multiplication Property)

8. $\dfrac{XP}{PR} = 2$ (Division Property)

ANSWER:

Given: $\triangle XYZ$ with medians $\overline{XR}, \overline{YS}$, and $\overline{ZQ}$.

Prove: $\dfrac{XP}{PR} = 2$

Proof:

Statements (Reasons)

1. $\triangle XYZ$ with medians $\overline{XR}, \overline{YS}$, and $\overline{ZQ}$.
(Given)

2. $XP = \dfrac{2}{3} XR$ (Centroid Theorem.)

3. $XR = XP + PR$ (Segment Addition Post.)

4. $XP = \dfrac{2}{3}(XR + PR)$ (Substitution Property)

5. $XP = \dfrac{2}{3} XP + \dfrac{2}{3} PR$ (Distributive. Property)

6. $\dfrac{1}{3} XP = \dfrac{2}{3} PR$ (Subtraction Property.)

7. $XP = 2PR$(Multiplication Property)

8. $\dfrac{XP}{PR} = 2$ (Division. Property)

33. **MULTIPLE REPRESENTATIONS** In this problem, you will investigate the location of the points of concurrency for any equilateral triangle.
   **a. CONCRETE** Construct three different equilateral triangles on patty paper and cut them out. Fold each triangle to locate the circumcenter, incenter, centroid, and orthocenter.
    b. VERBAL Make a conjecture about the relationships among the four points of concurrency of any equilateral triangle.
    c. GRAPHICAL Position an equilateral triangle and its circumcenter, incenter, centroid, and orthocenter on the coordinate plane using variable coordinates. Determine the coordinates of each point of concurrency.

    SOLUTION:
    a. Using a different piece of patty paper for each equilateral triangle, fold the triangles to find their circumcenter (by making the perpendicular bisectors), the incenter (by making the angle bisectors), centroid (by constructing the medians) and the orthocenter (by making the altitudes of each side).
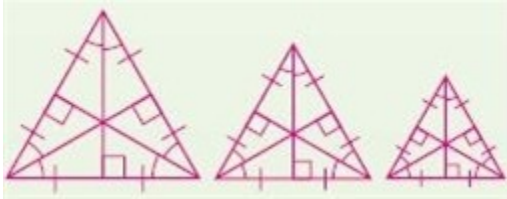
    b. Sample answer: The four points of concurrency of an equilateral triangle are all the same point.

    c. This problem can be made easier by placing the triangle on the coordinate plane in such a way that one vertex is placed at the origin and one side of the triangle lies along the *x*-axis. Additionally, by choosing a coordinate for the vertex on the *x*-axis that is divisible by two, you can easily compute the coordinate of the third vertex.

    Using the coordinates $(0, 0)$ and $(4a, 0)$ for the two vertices on the *x*-axis, we need to find the coordinates of the third vertex. Since it is located above the midpoint of the side across from it, we know the *x*-value is $2a$.

    Using the Converse of the Pythagorean Theorem, we can set up an equation to find the *y*-value, which is also the height of the triangle. The height divides the equilateral triangle into two parts, with a height (h) and half of the base of the equilateral triangle ($2a$), as its legs, and $4a$ as its hypotenuse.

    Using the equation below, we can find the height of the triangle:
    $$(4a)^2 = (2a)^2 + h^2$$
    $$16a^2 = 4a^2 + h^2$$
    $$16a^2 - 4a^2 = h^2$$
    $$12a^2 = h^2$$
 $$\sqrt{12a^2} = \sqrt{h^2}$$
    $$2a\sqrt{3} = h$$

    Therefore, the height of the triangle, and the *y*-value of the third vertex is $2a\sqrt{3}$.

To locate the coordinate of the points of concurrency, which are all concurrent in an equilateral triangle, we need to use the Centroid Theorem to find the location that is $\frac{2}{3}$ from the vertex to the midpoint of the opposite side. This would be the point that is $\frac{2}{3}$ of the distance from $\left(2a,\ 2a\sqrt{3}\right)$ along the height, or altitude of the triangle. This can be found as shown below:
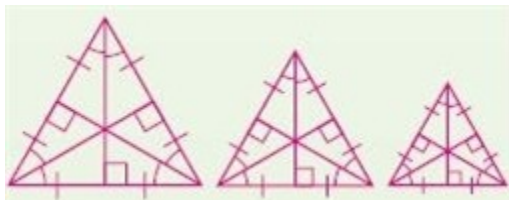
$$\tfrac{2}{3}\left(2a\sqrt{3}\right) = \frac{4a\sqrt{3}}{3}$$

The y-value of the centroid is $2a\sqrt{3} - \dfrac{4a\sqrt{3}}{3} = \dfrac{2a\sqrt{3}}{3}$. Therefore the coordinate of the points of concurrency of the equilateral triangle would be $\left(2a,\ \dfrac{2a\sqrt{3}}{3}\right)$.
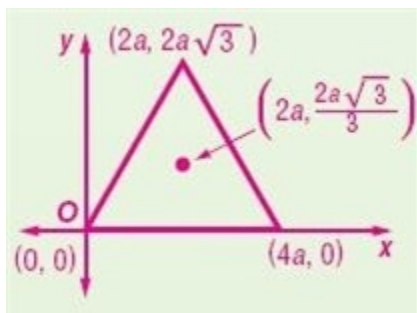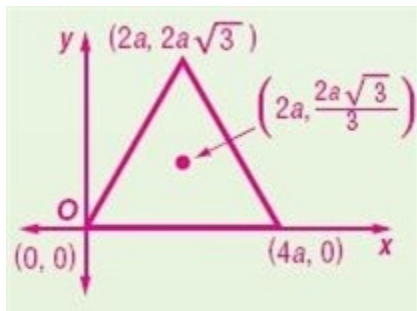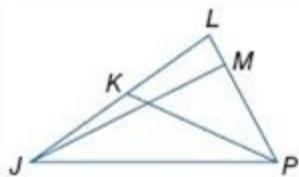

ANSWER:

a.

b. Sample answer: The four points of concurrency of an equilateral triangle are all the same point.

c.

ALGEBRA In $\triangle JLP$, $m \angle JMP = 3x - 6$, $JK = 3y - 2$, **and** $LK = 5y - 8$.

34. If $\overline{JM}$ is an altitude of $\triangle JLP$, find x.

SOLUTION:

Since $\overline{JM}$ is an altitude of $\triangle JLP$, $m\angle JMP = 90$.

$3x - 6 = 90$

$\quad 3x = 96$

$\quad\ x = 32$

ANSWER:

32

35. Find LK if $\overline{PK}$ is a median.

SOLUTION:

Since $\overline{PK}$ is a median, $JK = KL$.

$3y - 2 = 5y - 8$

$3y - 5y - 2 = 5y - 5y - 8$

$-2y - 2 = -8$

$-2y - 2 + 2 = -8 + 2$

$\quad\quad -2y = -6$

$\quad\quad\quad y = 3$

Substitute 3 for y in LK.

$LK = 5y - 8$

$\quad = 5(3) - 8$

$\quad = 15 - 8$

$\quad = 7$

ANSWER:

7

PROOF Write a coordinate proof to prove the Centroid Theorem.
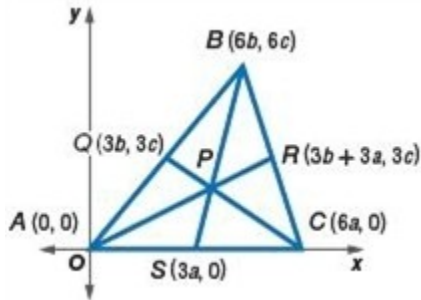
36. **Given:** $\triangle ABC$, medians $\overline{AR}$, $\overline{BS}$, and $\overline{CQ}$

    Prove: The medians intersect at point P and P is two thirds of the distance from each vertex to the midpoint of the opposite side.

    (*Hint*: First, find the equations of the lines containing the medians. Then find the coordinates of point P and show that all three medians intersect at point P. Next, use the Distance Formula and multiplication to show $AP = \dfrac{2}{3}AR$,

    $BP = \dfrac{2}{3}BS$, and $CP = \dfrac{2}{3}CQ$.)

    SOLUTION:

    To begin this proof, you will need to compute the slopes of the three medians. Using these slopes, determine the equation of each line containing these medians. Set the equations of two medians equal to each other to determine the coordinates of point P, the centroid. You can then choose a different pair of medians to check if you get the same point P or you can plug point P into the third median to confirm that it does lie on that line, as well. Once you have proven that P is the median of the triangle, you can use its coordinates to find the lengths of the parts of each median, using the Distance formula. Once this is done, you can substitute in the lengths of the medians to verify that

    $AP = \dfrac{2}{3}AR, BP = \dfrac{2}{3}BS$, and $CP = \dfrac{2}{3}CQ$.

    Proof:

    Slope of $\overline{AR} = \dfrac{3c}{3b+3a} = \dfrac{c}{b+a}$;

    Slope of $\overline{BS} = \dfrac{6c}{6b-3a} = \dfrac{2c}{2b-a}$;

    Slope of $\overline{CQ} = \dfrac{3c}{3b-6a} = \dfrac{c}{b-2a}$;

    $\overline{AR}$ contained in the line $y = \left(\dfrac{c}{b+a}\right)x$, $\overline{BS}$ contained in the line $y = \left(\dfrac{2c}{2b+a}\right)(x-3a)$, $\overline{CQ}$ contained in the

    line $y = \left(\dfrac{c}{b-2a}\right)(x-6a)$.

    To find the coordinates of P, find the intersection point of two medians, $\overline{BS}$ and $\overline{CQ}$.

    $y = \left(\dfrac{2c}{2b-a}\right)(x-3a)$ and $y = \left(\dfrac{c}{b-2a}\right)(x-6a)$

$$\frac{2c}{2b-a}(x-3a)=\frac{c}{b-2a}(x-6a)$$

$$2c(x-3a)(b-2a)=c(x-6a)(2b-a)$$

$$2c\left(bx-2ax-3ab+6a^2\right)=c\left(2bx-ax-12ab+6a^2\right)$$

$$2bcx-4acx-6abc+12a^2c=2bcx-acx-12abc+6a^2c$$

$$-3acx=-6abc-6a^2c$$

$$x=2b+2a$$

Find y.

$$y=\frac{2c}{2b-a}(x-3a)=\frac{2c}{2b-a}(2b+2a-3a)=\frac{2c(2b-a)}{2b-a}=2c$$

So the coordinates of P are $(2b + 2a, 2c)$. Now, show that P is on $\overline{AR}$.

$$y=\left(\frac{c}{b+a}\right)(2b+2a)=\frac{2c(b+a)}{b+a}=2c.$$

Thus, the three medians intersect at the same point.

Find the lengths of $\overline{AR}$, $\overline{AP}$, $\overline{BS}$, $\overline{BP}$, $\overline{CQ}$, and $\overline{CP}$ using the Distance Formula.

$$AR=\sqrt{\left((3b+3a)-0^2\right)+(3c-0)^2}$$

$$=\sqrt{(3(b+a))^2+(3c)^2}$$

$$=\sqrt{9\left((b+a)^2+c^2\right)}$$

$$=3\sqrt{(b+a)^2+c^2}$$

$$AP=\sqrt{\left((2b+2a)-0^2\right)+(2c-0)^2}$$

$$=\sqrt{(2(b+a))^2+\left(2c^2\right)}$$

$$=\sqrt{4\left((b+a)^2+c^2\right)}$$

$$=2\sqrt{(b+a)^2+c^2}$$

$$BS=\sqrt{(6b-3a)^2+(6c-0)^2}$$

$$=\sqrt{(3(2b-a))^2+(3(2c))^2}$$

$$=\sqrt{9(2b-a)^2+9(2c)^2}$$

$$=3\sqrt{(2b-a)^2+4c^2}$$

$$BP = \sqrt{\left(6b - (2b + 2a)\right)^2 + (6c - 2c)^2}$$
$$= \sqrt{(4b - 2a)^2 + (4c)^2}$$
$$= \sqrt{(2(2b - a))^2 + (2(2c))^2}$$
$$= 2\sqrt{(2b - a)^2 + 4c^2}$$

$$CQ = \sqrt{(6a - 3b)^2 + (0 - 3c)^2}$$
$$= \sqrt{(3(2a - b))^2 + (-3c)^2}$$
$$= \sqrt{9\left((2a - b) + c^2\right)}$$
$$= 3\sqrt{(2a - b)^2 + c^2}$$

$$CP = \sqrt{\left(6a - (2b + 2a)\right) + (0 - 2c)^2}$$
$$= \sqrt{(4a - 2b)^2 + (-2c)^2}$$
$$= \sqrt{(2(2a - b))^2 + 4c^2}$$
$$= \sqrt{4(2a - b)^2 + 4c^2}$$
$$= 2\sqrt{(2a - b)^2 + c^2}$$

Show that the *P* is two thirds of the distance from the vertices to the midpoints.

$$\frac{2}{3}AR = \frac{2}{3}\left(3\sqrt{(b + a)^2 + c^2}\right)$$
$$= 2\sqrt{(b + a)^2 + c^2}$$
$$= AP$$

$$\frac{2}{3}BS = \frac{2}{3}\left(3\sqrt{(2b - a)^2 + 4c^2}\right)$$
$$= 2\sqrt{(2b - a)^2 + 4c^2}$$
$$= BP$$

$$\frac{2}{3}CQ = \frac{2}{3}\left(3\sqrt{(2a - b)^2 + c^2}\right)$$
$$= 2\sqrt{(2a - b)^2 + c^2}$$
$$= CP$$

Thus, $AP = \frac{2}{3}AR$, $BP = \frac{2}{3}BS$,and $CP = \frac{2}{3}CQ$.

ANSWER:

Proof:

Slope of $\overline{AR} = \dfrac{3c}{3b+3a} = \dfrac{c}{b+a}$;

Slope of $\overline{BS} = \dfrac{6c}{6b-3a} = \dfrac{2c}{2b-a}$;

Slope of $\overline{CQ} = \dfrac{3c}{3b-6a} = \dfrac{c}{b-2a}$;

$\overline{AR}$ contained in the line $y = \left(\dfrac{c}{b+a}\right)x,$ $\overline{BS}$ contained in the line $y = \left(\dfrac{2c}{2b+a}\right)(x-3a),$ $\overline{CQ}$ contained in the

line $y = \left(\dfrac{c}{b-2a}\right)(x-6a).$

To find the coordinates of P, find the intersection point of two medians, $\overline{BS}$ and $\overline{CQ}$.

$y = \left(\dfrac{2c}{2b-a}\right)(x-3a)$ and $y = \left(\dfrac{c}{b-2a}\right)(x-6a)$

$$\dfrac{2c}{2b-a}(x-3a) = \dfrac{c}{b-2a}(x-6a)$$

$$2c(x-3a)(b-2a) = c(x-6a)(2b-a)$$

$$2c\left(bx-2ax-3ab+6a^2\right) = c\left(2bx-ax-12ab+6a^2\right)$$

$$2bcx-4acx-6abc+12a^2c = 2bcx-acx-12abc+6a^2c$$

$$-3acx = -6abc-6a^2c$$

$$x = 2b+2a$$

Find y.

$$y = \dfrac{2c}{2b-a}(x-3a) = \dfrac{2c}{2b-a}(2b+2a-3a) = \dfrac{2c(2b-a)}{2b-a} = 2c$$

So the coordinates of P are $(2b + 2a, 2c)$.

Now, show that P is on $\overline{AR}$.

$$y = \left(\dfrac{c}{b+a}\right)(2b+2a) = \dfrac{2c(b+a)}{b+a} = 2c.$$

Thus, the three medians intersect at the same point.

Find the lengths of $\overline{AR}, \overline{AP}, \overline{BS}, \overline{BP}, \overline{CQ}$, and $\overline{CP}$ using the Distance Formula.

$$AR = \sqrt{\left((3b+3a)-0^2\right)+(3c-0)^2}$$

$$= \sqrt{\left(3(b+a)\right)^2+(3c)^2}$$

$$= \sqrt{9\left((b+a)^2+c^2\right)}$$

$$= 3\sqrt{(b+a)^2+c^2}$$

$$AP = \sqrt{\left((2b+2a)-0^2\right)+(2c-0)^2}$$
$$= \sqrt{\left(2(b+a)\right)^2+\left(2c^2\right)}$$
$$= \sqrt{4\left((b+a)^2+c^2\right)}$$
$$= 2\sqrt{(b+a)^2+c^2}$$

$$BS = \sqrt{(6b-3a)^2+(6c-0)^2}$$
$$= \sqrt{\left(3(2b-a)\right)^2+\left(3(2c)\right)^2}$$
$$= \sqrt{9(2b-a)^2+9(2c)^2}$$
$$= 3\sqrt{(2b-a)^2+4c^2}$$

$$BP = \sqrt{\left(6b-(2b+2a)\right)^2+(6c-2c)^2}$$
$$= \sqrt{(4b-2a)^2+(4c)^2}$$
$$= \sqrt{\left(2(2b-a)\right)^2+\left(2(2c)\right)^2}$$
$$= 2\sqrt{(2b-a)^2+4c^2}$$

$$CQ = \sqrt{(6a-3b)^2+(0-3c)^2}$$
$$= \sqrt{\left(3(2a-b)\right)^2+(-3c)^2}$$
$$= \sqrt{9\left((2a-b)+c^2\right)}$$
$$= 3\sqrt{(2a-b)^2+c^2}$$

$$CP = \sqrt{\left(6a-(2b+2a)\right)+(0-2c)^2}$$
$$= \sqrt{(4a-2b)^2+(-2c)^2}$$
$$= \sqrt{\left(2(2a-b)\right)^2+4c^2}$$
$$= \sqrt{4(2a-b)^2+4c^2}$$
$$= 2\sqrt{(2a-b)^2+c^2}$$

Show that the *P* is two thirds of the distance from the vertices to the midpoints.

$$\frac{2}{3}AR = \frac{2}{3}\left(3\sqrt{(b+a)^2+c^2}\right)$$
$$= 2\sqrt{(b+a)^2+c^2}$$
$$= AP$$
$$\frac{2}{3}BS = \frac{2}{3}\left(3\sqrt{(2b-a)^2+4c^2}\right)$$
$$= 2\sqrt{(2b-a)^2+4c^2}$$
$$= BP$$
$$\frac{2}{3}CQ = \frac{2}{3}\left(3\sqrt{(2a-b)^2+c^2}\right)$$
$$= 2\sqrt{(2a-b)^2+c^2}$$
$$= CP$$

Thus, $AP = \frac{2}{3}AR$, $BP = \frac{2}{3}BS$, and $CP = \frac{2}{3}CQ$.

37. **ERROR ANALYSIS** Based on the figure at the right, Luke says that $\frac{2}{3}AP = AD$. Kareem disagrees. Is either of them correct? Explain your reasoning.
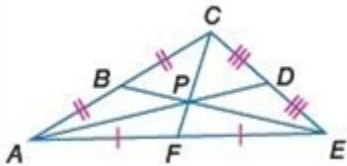
SOLUTION:

Sample answer: Kareem is correct. Luke has the segment lengths transposed. The shorter segment is 2/3 of the entire median, not the other way around. According to the Centroid Theorem, $AP = \frac{2}{3}AD$.

ANSWER:

Sample answer: Kareem is correct. According to the Centroid Theorem, $AP = \frac{2}{3}AD$. The segment lengths are transposed.

38. **CCSS ARGUMENTS** Determine whether the following statement is *true* or *false*. If true, explain your reasoning. If false, provide a counterexample.
The orthocenter of a right triangle is always located at the vertex of the right angle.

SOLUTION:

It is true that "*The orthocenter of a right triangle is always located at the vertex of the right angle*".

Sample answer: As shown in the right triangle below, the altitudes from the two nonright vertices (*A* and *B*) will always be the legs of the triangle, which intersect at the vertex that contains the right angle (*C*). The altitude to the hypotenuse of the triangle originates at the vertex, so the three altitudes (the red rays) intersect there. Therefore, the vertex of a right triangle will always be the orthocenter.
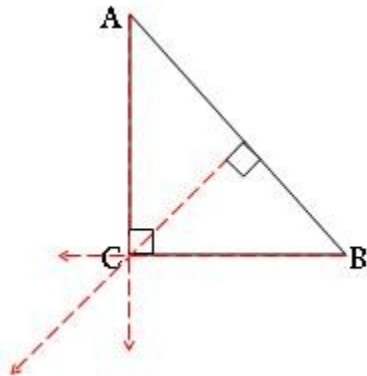

ANSWER:

True; sample answer: In a right triangle, the altitudes from the two nonright vertices will always be the legs of the triangle, which intersect at the vertex that contains the right angle. The altitude to the hypotenuse of the triangle originates at the vertex, so the three altitudes intersect there. Therefore, the vertex of a right triangle will always be the orthocenter.

39. **CHALLENGE** $\triangle ABC$ has vertices $A(-3, 3)$, $B(2, 5)$, and $C(4, -3)$. What are the coordinates of the centroid of $\triangle ABC$? Explain the process you used to reach your conclusion.

SOLUTION:
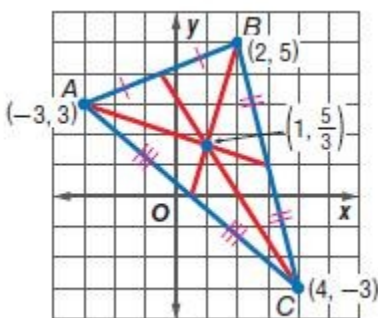
The midpoint D of $\overline{AC}$ is $\left(\dfrac{-3+4}{2}, \dfrac{3-3}{2}\right)$ or $\left(\dfrac{1}{2}, 0\right)$. Note that $\overline{DB}$ is a line that connects the vertex B and D, the midpoint of $\overline{AC}$.

The slope of the line $\overline{DB}$ is $\left(\dfrac{5-0}{2-\frac{1}{2}}\right)$ or $\left(\dfrac{10}{3}\right)$.

The equation of $\overline{DB}$ is:

$$y - 5 = \frac{10}{3}(x - 2)$$

$$y - 5 = \frac{10}{3}x - \frac{20}{3}$$

$$y - 5 + 5 = \frac{10}{3}x - \frac{20}{3} + 5$$

$$y = \frac{10}{3}x - \frac{5}{3}$$

Use the same method to find the equation of the line between point A and the midpoint of $\overline{BC}$.

That is, $y = -\frac{1}{3}x + 2$.

Solving the system of equations, we get the intersection point $\left(1, \frac{5}{3}\right)$. So, the centroid of the triangle is $\left(1, \frac{5}{3}\right)$.

ANSWER:

$\left(1, \frac{5}{3}\right)$; Sample answer: I found the midpoint of $\overline{AC}$ and used it to find the equation for the line that contains point B

and the midpoint of $\overline{AC}$, $y = \frac{10}{3}x - \frac{5}{3}$. I also found the midpoint of $\overline{BC}$ and the equation for the line between point A

and the midpoint of $\overline{BC}$, $y = -\frac{1}{3}x + 2$. I solved the system of two equations for x and y to get the coordinates of the

centroid, $\left(1, \frac{5}{3}\right)$.
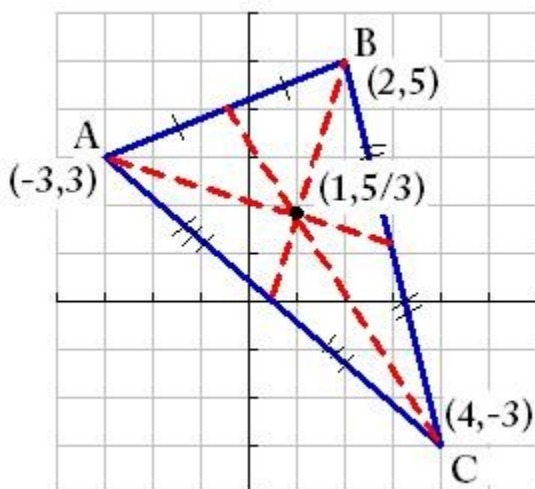
40. **WRITING IN MATH** Compare and contrast the perpendicular bisectors, medians, and altitudes of a triangle.
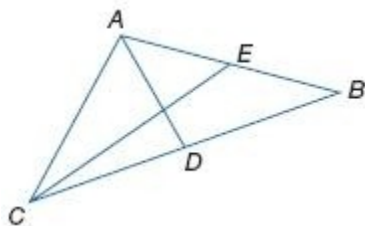
    SOLUTION:

    To consider this answer, start by making a list of the properties of each item. Consider which ones must pass through the vertex and which must meet at the midpoint of the opposite side, as well as which ones must be perpendicular to the opposite side. Try comparing and contrasting two items at a time until all three have been compared. Sample answer: The perpendicular bisector and the median pass through the midpoint on the side of the triangle, but only the median always passes through the vertex opposite the side. The perpendicular bisector and the altitude are both perpendicular to the side, but do not necessarily pass through a common point on the side of the triangle. The median and the altitude both pass through the vertex, but do not necessarily pass through a common point on the side of the triangle.

    ANSWER:

    Sample answer: The perpendicular bisector and the median pass through a common point on the side of the triangle, but only the median always passes through the vertex opposite the side. The perpendicular bisector and the altitude are both perpendicular to the side, but do not necessarily pass through a common point on the side of the triangle. The median and the altitude both pass through the vertex, but do not necessarily pass through a common point on the side of the triangle.

41. **CHALLENGE** In the figure, segments $\overline{AD}$ and $\overline{CE}$ are medians of $\triangle ACB$, $\overline{AD} \perp \overline{CE}$, $AB = 10$, and $CE = 9$. Find CA.

    SOLUTION:

    Since $\overline{CE}$ and $\overline{AD}$ are medians, the intersect at the centroid of the triangle. Let P be the centroid, the intersection point of the medians. By the Centroid Theorem, $CP = \frac{2}{3}CE.$ That is, $CP = 6$. And $PE = 9 - 6 = 3$. Because $\overline{AD} \perp \overline{CE}$, then two right triangles are formed, $\triangle APE$ and $\triangle APC$. Using the converse of the Pythagorean Theorem, we can find the lengths of $\overline{AP}$ and then $\overline{CA}$, as shown below:

    In the right triangle APE,
    $$AP = \sqrt{5^2 - 3^2}$$
    $$= \sqrt{25 - 9}$$
    $$= 4$$

    In the right triangle APC,
    $$CA = \sqrt{6^2 + 4^2}$$
    $$= \sqrt{36 + 16}$$
    $$= 2\sqrt{13}$$

    ANSWER:

    $2\sqrt{13}$

42. **OPEN ENDED** In this problem, you will investigate the relationships among three
points of concurrency in a triangle.
a. Draw an acute triangle and find the circumcenter, centroid, and orthocenter.
b. Draw an obtuse triangle and find the circumcenter, centroid, and orthocenter.
c. Draw a right triangle and find the circumcenter, centroid, and orthocenter.
d. Make a conjecture about the relationships among the circumcenter, centroid, and orthocenter.

SOLUTION:
a. Make an acute triangle and construct the centroid (medians), circumcenter (perpendicular bisectors), and
orthocenter (altitudes).Pay attention to how these three centers relate to each other.
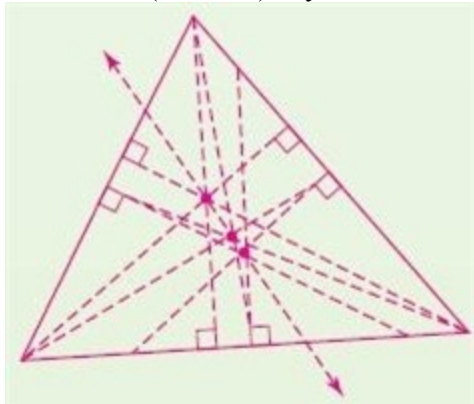
b. Make an obtuse triangle and construct the centroid (medians), circumcenter (perpendicular bisectors), and
orthocenter (altitudes). Pay attention to how these three centers relate to each other.
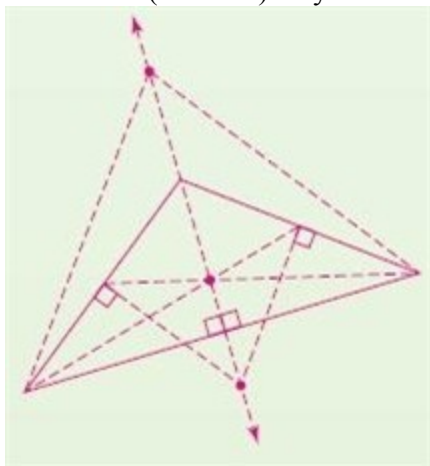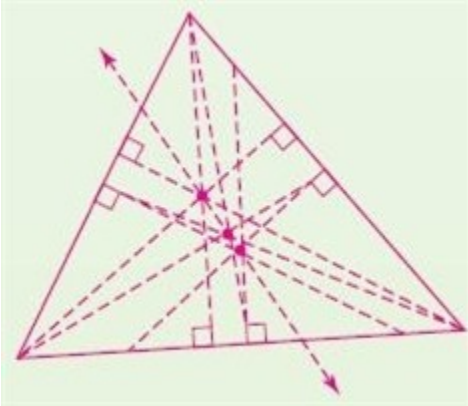

c. Make a right triangle and construct the centroid (medians), circumcenter (perpendicular bisectors), and
orthocenter (altitudes). Pay attention to how these three centers relate to each other.
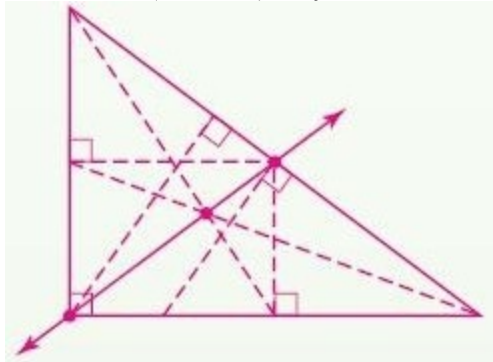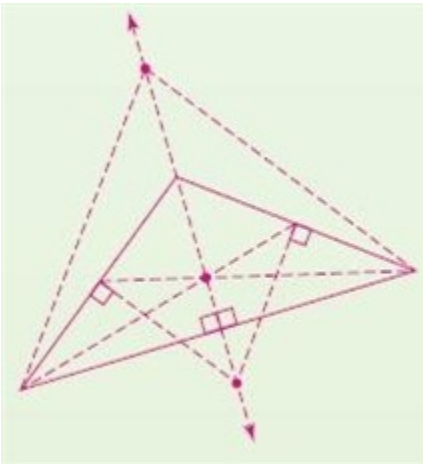
d. Sample answer: The circumcenter, centroid, and orthocenter are all collinear.

ANSWER:

a.

b.

c.
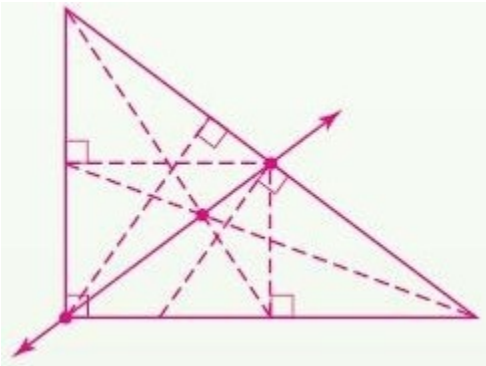
d. Sample answer: The circumcenter, centroid, and orthocenter are all collinear.

43. **WRITING IN MATH** Use area to explain why the centroid of a triangle is its center of gravity. Then use this explanation to describe the location for the balancing point for a rectangle.
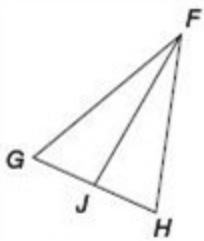
SOLUTION:

When considering this answer, think about what the medians of a triangle do to the triangle. Since each median divides the triangle into two smaller triangles of equal area, the triangle can be balanced along any one of those lines. To balance the triangle on one point, you need to find the point where these three balance lines intersect. The balancing point for a rectangle is the intersection of the segments connecting the midpoints of the opposite sides, since each segment connecting these midpoints of a pair of opposite sides divides the rectangle into two parts with equal area.

ANSWER:

Sample answer: Each median divides the triangle into two smaller triangles of equal area, so the triangle can be balanced along any one of those lines. To balance the triangle on one point, you need to find the point where these three balance lines intersect. The balancing point for a rectangle is the intersection of the segments connecting the midpoints of the opposite sides, since each segment connecting these midpoints of a pair of opposite sides divides the rectangle into two parts with equal area.

44. In the figure below, $\overline{GJ} \cong \overline{HJ}$. Which must be true?

   A $\overline{FJ}$ is an altitude of $\triangle FGH$.

   B $\overline{FJ}$ is an angle bisector of $\triangle FGH$.

   C $\overline{FJ}$ is a median of $\triangle FGH$.

   D $\overline{FJ}$ is a perpendicular bisector of $\triangle FGH$.

SOLUTION:

We are given that $\overline{GJ} \cong \overline{HJ}$, therefore J is the midpoint of $\overline{GH}$. We don't know if $\overline{FJ} \perp \overline{GH}$, so all we can conclude is that $\overline{FJ}$ is a median of $\triangle FGH$. The correct choice is C.

ANSWER:

C

45. **GRIDDED RESPONSE** What is the *x*-intercept of the graph of $4x - 6y = 12$?

SOLUTION:

Substitute $y = 0$ in the equation to find the *x*-intercept.

$$4x - 6(0) = 12$$
$$4x = 12$$
$$x = 3$$

ANSWER:

3

46. **ALGEBRA** Four students have volunteered to fold pamphlets for a local community action group. Which student is the fastest?

Student	Folding Speed
Neiva	1 page every 3 seconds
Sarah	2 pages every 10 seconds
Quinn	30 pages per minute
Deron	45 pages in 2 minutes

F Deron

G Neiva

H Quinn

J Sarah

SOLUTION:

Find the folding speed of the students per second.

Neiva: $\dfrac{1}{3}$ pages per second.

Sarah: $\dfrac{2}{10}$ or $\dfrac{1}{5}$ pages per second.

Quinn: $\dfrac{30}{60}$ or $\dfrac{1}{2}$ pages per second.

Deron: $\dfrac{45}{120}$ or $\dfrac{3}{8}$ pages per second.

Among these students, Quinn is the fastest student in folding pamphlets.
The correct choice is H.

ANSWER:

H

47. **SAT/ACT** 80 percent of 42 is what percent of 16?

   A 240

   B 210

   C 150

   D 50

   E 30

   SOLUTION:

   Let x be the percent of 16, which is equal to the 80 percent of 42.

   $$80\% \text{ of } 42 = x\% \text{ of } 16$$

   $$\frac{80}{100} \cdot 42 = \frac{x}{100} \cdot 16$$

   $$\frac{80(42)}{100} = x\left(\frac{16}{100}\right)$$

   $$\frac{80(42)}{100}\left(\frac{100}{16}\right) = x\left(\frac{16}{100}\right)\left(\frac{100}{16}\right)$$

   $$\frac{80(42)}{100}\left(\frac{100}{16}\right) = x$$

   $$\frac{80(42)}{16} = x$$

   $$210 = x$$
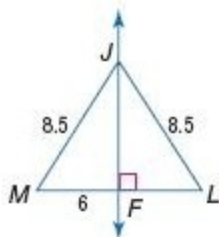
   The correct choice is B.

   ANSWER:

   B

   Find each measure.

48. *LM*

   SOLUTION:

   From the figure, $LM = MF + FL$.

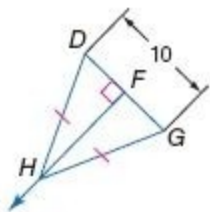 $FL = MF = 6$ because $\overline{FJ}$ is the perpendicular bisector of $\overline{ML}$.

   Therefore, $LM = 6 + 6 = 12$.

   ANSWER:

   12

49. *DF*

SOLUTION:

By the Converse of Perpendicular Bisector Theorem, $FG = DF$.
By the Segment Addition Postulate, $DG = DF + FG$.
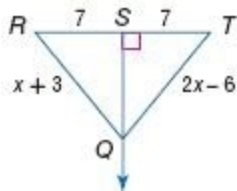Therefore, $DG = 2DF$.
$$DG = 2DF$$
$$10 = 2DF$$
$$DF = 5$$

ANSWER:

5

50. *TQ*

SOLUTION:

By the Perpendicular Bisector Theorem, $RQ = TQ$.
$$2x - 6 = x + 3$$
$$x = 9$$
$$TQ = 2(9) - 6$$
$$= 18 - 6$$
$$= 12$$

ANSWER:

12

**Position and label each triangle on the coordinate plane.**

51. right $\triangle XYZ$ with hypotenuse $\overline{XZ}$, *ZY* is twice *XY,* and $\overline{XY}$ is *b* units long

SOLUTION:

By placing Y at the origin, the legs of the right triangle are positioned along the *x*-axis and *y*-axis. Since $\overline{YX}$ is *b* units long, you can place point X along the *y*-axis and make the coordinates (0,*b*). Since $\overline{ZY}$ is twice the length of $\overline{XY}$ and it lies along the *x*-axis, its coordinates would be (2*b*,0).
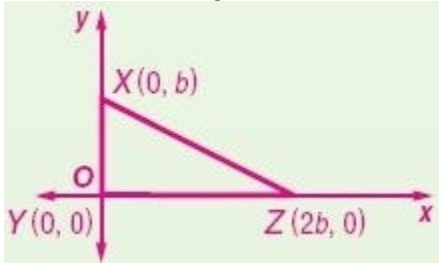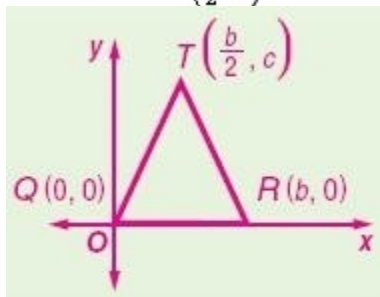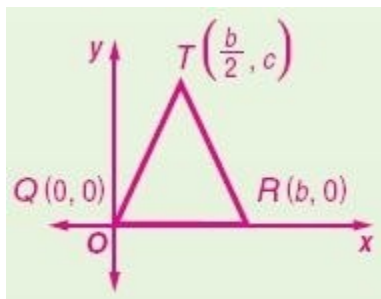

ANSWER:
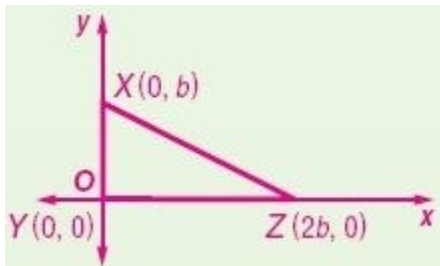
52. isosceles $\triangle QRT$ with base $\overline{QR}$ that is b units long

SOLUTION:

By placing Q at the origin and the base of the isosceles triangle can be positioned along the x-axis. Since $\overline{QR}$ is b units long, you can place point R along the x-axis and make the coordinates $(b,0)$. Since the vertex , point T, of the isosceles triangle is directly above the midpoint of the base, its x-value would be half the length of $\overline{QR}$ and its coordinates are $\left(\frac{b}{2}, c\right)$.

ANSWER:

Determine whether $\overleftrightarrow{RS}$ and $\overrightarrow{JK}$ are *parallel, perpendicular,* or *neither*. Graph each line to verify your answer.

53. *R*(5, −4), *S*(10, 0), *J*(9, −8), *K*(5, −13)

SOLUTION:

Slope of $\overleftrightarrow{RS}$: $m_1 = \dfrac{0+4}{10-5} = \dfrac{4}{5}$;

Slope of $\overrightarrow{JK}$: $m_2 = \dfrac{-13+8}{5-9} = \dfrac{5}{4}$;

Neither parallel nor perpendicular.
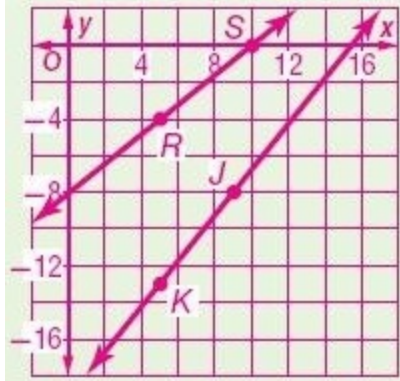
ANSWER:

neither
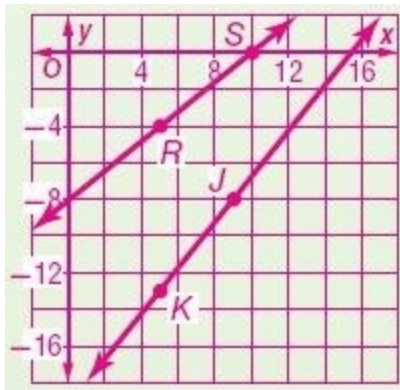
54. $R(1, 1)$, $S(9, 8)$, $J(-6, 1)$, $K(2, 8)$

   SOLUTION:

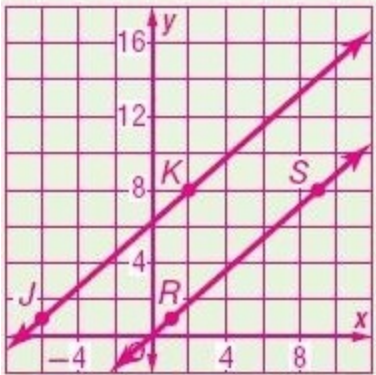   Slope of $\overrightarrow{RS}$: $m_1 = \dfrac{8-1}{9-1} = \dfrac{7}{8}$;
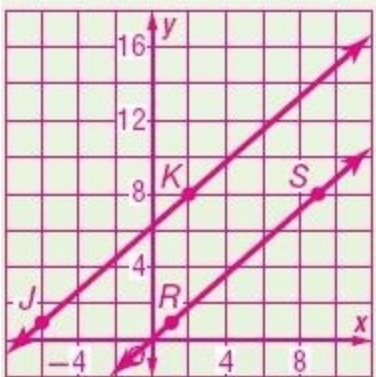
   Slope of $\overrightarrow{JK}$: $m_2 = \dfrac{8-1}{2+6} = \dfrac{7}{8}$;

   Since the slopes are equal, the lines are parallel.

   ANSWER:

   Parallel

55. **HIGHWAYS** Near the city of Hopewell, Virginia, Route 10 runs perpendicular to Interstate 95 and Interstate 295. Show that the angles at the intersections of Route 10 with Interstate 95 and Interstate 295 are congruent.
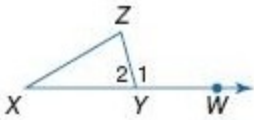

SOLUTION:

Because Route 10 is perpendicular to Interstate 295, $\angle 2$ is a right angle. The same is true for Route 10 and Interstate 95 and therefore $\angle 1$ is also a right angle. Since both $\angle 1$ and $\angle 2$ are right angles, then they are congruent to each other because all right angles are congruent. Therefore, $\angle 1$ is congruent to $\angle 2$.

ANSWER:

Because the lines are perpendicular, the angles formed are right angles. All right angles are congruent. Therefore, $\angle 1$ is congruent to $\angle 2$.

PROOF Write a flow proof of the Exterior Angle Theorem.

56. **Given:** $\triangle XYZ$
    Prove: $m\angle X + m\angle Z = m\angle 1$

SOLUTION:

This proof is based on two main relationships - the Triangle Angle-Sum Theorem and the Definition of a Linear Pair. Based on the diagram, write two equations using each of these relationships. If two angles form a linear pair, then we know that they are supplementary and consequently add up to 180 degrees. At this stage of the proof, you can set the two equations equal to each other and simplify to obtain the final conclusion of the proof.

△XYZ
Given

∠2 and ∠1 form a linear pair.

Definition of a Linear Pair

∠2 and ∠1 are supplementary.

If 2∡ form a linear pair, they are supplementary.

$m\angle X + m\angle Z + m\angle 2 = 180$

Triangle Angle-Sum Theorem

$m\angle 2 + m\angle 1 = 180$

Definition of supplementary

$m\angle X + m\angle Z + m\angle 2 = m\angle 2 + m\angle 1$

Substitution

$m\angle X + m\angle Z = m\angle 1$

Subtraction Property of Equality

ANSWER:

△XYZ
Given

∠2 and ∠1 form a linear pair.

Definition of a Linear Pair

∠2 and ∠1 are supplementary.

If 2∡ form a linear pair, they are supplementary.

$m\angle X + m\angle Z + m\angle 2 = 180$

Triangle Angle-Sum Theorem

$m\angle 2 + m\angle 1 = 180$

Definition of supplementary

$m\angle X + m\angle Z + m\angle 2 = m\angle 2 + m\angle 1$

Substitution

$m\angle X + m\angle Z = m\angle 1$

Subtraction Property of Equality