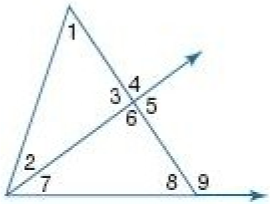
5-3 Inequalities in One Triangle

Use the Exterior Angle Inequality Theorem to list all of the angles that satisfy the stated condition.



1. measures less than $m\angle 4$

SOLUTION:

By the Exterior Angle Inequality Theorem, the exterior angle ($\angle 4$) is larger than either remote interior angle ($\angle 1$ and $\angle 2$).

Also, $m\angle 4 + m\angle 3 = 180$, and $(m\angle 1 + m\angle 2) + m\angle 3 = 180$. By substitution, $m\angle 4 = m\angle 1 + m\angle 2$. Therefore, must be larger than each individual angle. By the Exterior Angle Inequality Theorem, $m\angle 4 > m\angle 1$ and $m\angle 4 > m\angle 2$.

ANSWER:

$\angle 1$, $\angle 2$

2. measures greater than $m\angle 7$

SOLUTION:

By the Exterior Angle Inequality Theorem, the exterior angle ($\angle 5$) is larger than either remote interior angle ($\angle 7$ and $\angle 8$). Similarly, the exterior angle ($\angle 9$) is larger than either remote interior angle ($\angle 7$ and $\angle 6$). Therefore, $\angle 5$ and $\angle 9$ are both larger than $\angle 7$.

ANSWER:

$\angle 5$, $\angle 9$

3. measures greater than $m\angle 2$

SOLUTION:

By the Exterior Angle Inequality Theorem, the exterior angle ($\angle 4$) is larger than either remote interior angle ($\angle 1$ and $\angle 2$). Therefore, $\angle 4$ is larger than $\angle 2$.

ANSWER:

$\angle 4$

4. measures less than $m\angle 9$

SOLUTION:

By the Exterior Angle Inequality Theorem, the exterior angle ($\angle 9$) is larger than either remote interior angle ($\angle 6$ and $\angle 7$).

Also, $m\angle 9 + m\angle 8 = 180$, and ($(m\angle 6 + m\angle 7) + m\angle 8 = 180$.

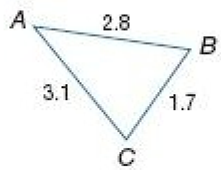
By substitution, $m\angle 9 = m\angle 6 + m\angle 7$.

Therefore, $\angle 9$ must be larger than each individual angle so $\angle 9$ is larger than both $\angle 6$ and $\angle 7$.

ANSWER:

$\angle 6$, $\angle 7$

List the angles and sides of each triangle in order from smallest to largest.

5. 

SOLUTION:

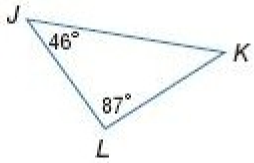
Here it is given that, $BC < AB < AC$. Therefore, from angle-side relationships in a triangle, we know that $m\angle A < m\angle C < m\angle B$.

Angle: $\angle A$, $\angle C$, $\angle B$

Side: $\overline{BC}, \overline{AB}, \overline{AC}$

ANSWER:

$\angle A$, $\angle C$, $\angle B$; $\overline{BC}, \overline{AB}, \overline{AC}$

6. 

SOLUTION:

By the Triangle Sum Theorem, $m\angle K = 180 - (46 + 87) = 47$.
So, $m\angle J < m\angle K < m\angle L$.

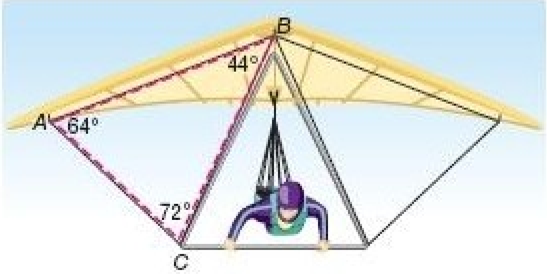
From angle-side relationships in a triangle, we know $KL < JL < JK$.

Angle: $\angle J$, $\angle K$, $\angle L$
Side: $\overline{KL}, \overline{JL}, \overline{JK}$

ANSWER:

$\angle J$, $\angle K$, $\angle L$; $\overline{KL}, \overline{JL}, \overline{JK}$

7. **HANG GLIDING** The supports on a hang glider form triangles like the one shown. Which is longer — the support represented by $\overline{AC}$ or the support represented by $\overline{BC}$? Explain your reasoning.



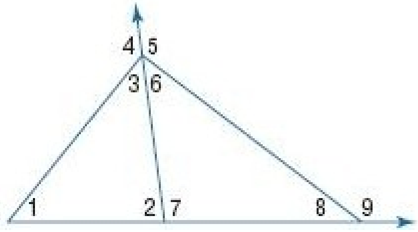
SOLUTION:

Since the angle across from segment $\overline{BC}$ is larger than the angle across from $\overline{AC}$, we can conclude that $\overline{BC}$ is longer than $\overline{AC}$ due to Theorem 5.9, which states that if one side of a triangle is longer than another, then the angle opposite the longer side has a greater measure than the angle opposite the shorter side.

ANSWER:

$\overline{BC}$; Sample answer: Since the angle across from segment $\overline{BC}$ is larger than the angle across from $\overline{AC}$, $\overline{BC}$ is longer.

CCSS SENSE-MAKING Use the Exterior Angle Inequality Theorem to list all of the angles that satisfy the stated condition.



8. measures greater than $m\angle 2$

SOLUTION:

By the Exterior Angle Inequality Theorem, the exterior angle ($\angle 4$) is larger than either remote interior angle ($\angle 1$ and $\angle 2$). Therefore, $m\angle 4 > m\angle 2$.

ANSWER:

$\angle 4$

9. measures less than $m\angle 4$

SOLUTION:

By the Exterior Angle Inequality Theorem, the exterior angle ($\angle 4$) is greater than either remote interior angle ($\angle 1$ and $\angle 2$).Therefore, $m\angle 1 < m\angle 4$ and $m\angle 2 < m\angle 4$.

ANSWER:

$\angle 1$, $\angle 2$

10. measures less than $m\angle 5$

SOLUTION:

By the Exterior Angle Inequality Theorem, the exterior angle ($\angle 5$) is larger than either remote interior angle ($\angle 7$ and $\angle 8$). Therefore, $m\angle 7 < m\angle 5$ and $m\angle 8 < m\angle 5$.

ANSWER:

$\angle 7$, $\angle 8$

11. measures less than $m\angle 9$

SOLUTION:

By the Exterior Angle Inequality Theorem, the exterior angle ($\angle 9$) is larger than either remote interior angle from two different triangles ($\angle 6$ and $\angle 7$ from one triangle and $\angle 1$ and $\angle 3$ from another. Therefore, $m\angle 6 < m\angle 9$, $m\angle 7 < m\angle 9$, $m\angle 1 < m\angle 9$, and $m\angle 3 < m\angle 9$.

ANSWER:

$\angle 1$, $\angle 3$, $\angle 6$, $\angle 7$

12. measures greater than $m\angle 8$

SOLUTION:

By the Exterior Angle Inequality Theorem, the exterior angle ($\angle 2$) is larger than either remote interior angle ($\angle 6$ and $\angle 8$). Similarly, the exterior angle ($\angle 5$) is larger than either remote interior angle ($\angle 7$ and $\angle 8$). Therefore, $m\angle 2 > m\angle 8$ and $m\angle 5 > m\angle 8$.

ANSWER:

$\angle 2$, $\angle 5$

13. measures greater than $m\angle 7$

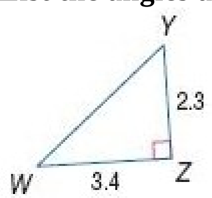
SOLUTION:

By the Exterior Angle Inequality Theorem, the exterior angle ($\angle 9$) is larger than either remote interior angle ($\angle 6$ and $\angle 7$). Similarly, the exterior angle ($\angle 5$) is larger than either remote interior angle ($\angle 7$ and $\angle 8$). Therefore, $m\angle 9 > m\angle 7$ and $m\angle 5 > m\angle 7$

ANSWER:

$\angle 5$, $\angle 9$

List the angles and sides of each triangle in order from smallest to largest.

14. 

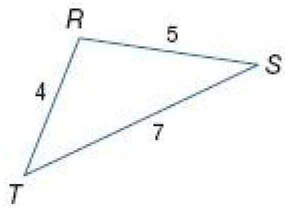
SOLUTION:

The hypotenuse of the right triangle must be greater than the other two sides.
Therefore, $YZ < WZ < WY$. By Theorem 5.9, the measure of the angle opposite the longer side has a greater measure tahn the angle opposite the shorter side, therefore $m\angle W < m\angle Y < m\angle Z$.

Angle: $\angle W$, $\angle Y$, $\angle Z$
Side: $\overline{YZ}$, $\overline{WZ}$, $\overline{WY}$

ANSWER:

$\angle W$, $\angle Y$, $\angle Z$; $\overline{YZ}, \overline{WZ}, \overline{WY}$

15. 

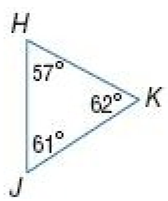
SOLUTION:

Based on the diagram, we see that $RT < RS < TS$. By Theorem 5.9, the measure of the angle opposite the longer side has a greater measure than the angle opposite the shorter side, therefore $m\angle S < m\angle T < m\angle R$.

Angle: $\angle S, \angle T, \angle R$
Side: $\overline{RT}, \overline{RS}, \overline{ST}$

ANSWER:

$\angle S$, $\angle T$, $\angle R$; $\overline{RT}, \overline{RS}, \overline{ST}$

16. 

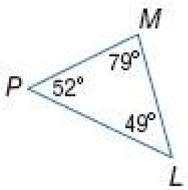
SOLUTION:

Here, $m\angle H < m\angle J < m\angle K$. Therefore, by Theorem 5.10 we know that the side opposite the greater angle is longer than the side opposite a lesser angle and $JK < HK < HJ$.

Angle: $\angle H, \angle J, \angle K$
Side: $\overline{JK}, \overline{HK}, \overline{HJ}$

ANSWER:

$\angle H$, $\angle J$, $\angle K$; $\overline{JK}, \overline{HK}, \overline{HJ}$

17. 

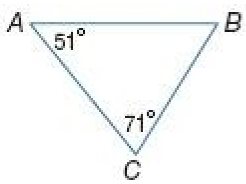
SOLUTION:

Here, $m\angle L < m\angle P < m\angle M$. Therefore, by Theorem 5.10 we know that the side opposite the greater angle is longer than the side opposite a lesser angle and $PM < ML < PL$.

Angle: $\angle L, \angle P, \angle M$
Side: $\overline{PM}, \overline{ML}, \overline{PL}$

ANSWER:

$\angle L, \angle P, \angle M$; $\overline{PM}, \overline{ML}, \overline{PL}$

18. 

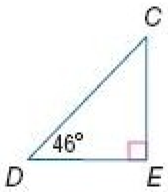
SOLUTION:

By the Triangle Sum Theorem, $m\angle B = 180 - (51 + 71) = 58.$
So, $m\angle A < m\angle B < m\angle C.$ Therefore, by Theorem 5.10 we know that the side opposite the greater angle is longer than the side opposite a lesser angle and $BC < AC < AB$.

Angle: $\angle A, \angle B, \angle C$
Side: $\overline{BC}, \overline{AC}, \overline{AB}$

ANSWER:

$\angle A, \angle B, \angle C$; $\overline{BC}, \overline{AC}, \overline{AB}$

19. 

SOLUTION:

By the Triangle Sum Theorem, $m\angle C = 180 - (90 + 46) = 44$. So, $m\angle C < m\angle D < m\angle E$. Therefore, by Theorem 5.10 we know that the side opposite the greater angle is longer than the side opposite a lesser angle and $DE < CE < CD$.

Angle: $\angle C, \angle D, \angle E$
Side: $\overline{DE}, \overline{CE}, \overline{CD}$

ANSWER:

$\angle C, \angle D, \angle E$; $\overline{DE}, \overline{CE}, \overline{CD}$

20. **SPORTS** Ben, Gilberto, and Hannah are playing Ultimate. Hannah is trying to decide if she should pass to Ben or Gilberto. Which player should she choose in order to have the shorter passing distance? Explain your reasoning.

SOLUTION:

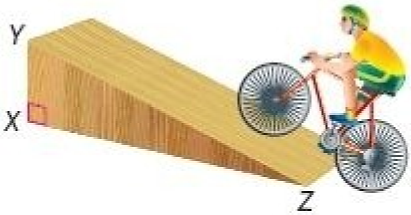
Based on the Triangle Sum Theorem, the measure of the missing angle next to Ben is $180 - (48 - 62) = 70$ degrees. Since 48 < 70, the side connecting Hannah to Ben is the shortest, based on Theorem 5.10. Therefore, Hannah should pass to Ben.

ANSWER:

Ben; sample answer: Using the Triangle Sum Theorem, the measure of the angle across from the segment between Hannah and Gilberto is 70. Since 48 < 70, the pass from Hannah to Ben would be shorter.

5-3 Inequalities in One Triangle

21. **RAMPS** The wedge below represents a bike ramp. Which is longer, the length of the ramp $\overline{XZ}$ or the length of the top surface of the ramp $\overline{YZ}$? Explain your reasoning using Theorem 5.9.



SOLUTION:

If $m\angle X = 90$, then , based on the Triangle Sum Theorem, $m\angle Y + m\angle Z = 90$, so $m\angle Y < 90$ by the definition of inequality. So $m\angle X > m\angle Y$.

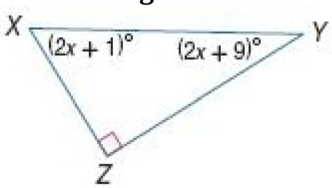
According to Theorem 5.10, if $m\angle X > m\angle Y$, then the length of the side opposite $\angle X$ must be greater than the length of the side opposite $\angle Y$. Since $\overline{YZ}$ is opposite $\angle X$, and $\overline{XZ}$ is opposite $\angle Y$, then $YZ > XZ$. Therefore YZ, the length of the top surface of the ramp, must be greater than the length of the ramp.

ANSWER:

If $m\angle X = 90$, then $m\angle Y + m\angle Z = 90$, so $m\angle Y < 90$ by the definition of inequality. So $m\angle X > m\angle Y$. According to Theorem 5.9, if $m\angle X > m\angle Y$, then the length of the side opposite $\angle X$ must be greater than the length of the side opposite $\angle Y$. Since $\overline{YZ}$ is opposite $\angle X$, and $\overline{XZ}$ is opposite $\angle Y$, then $YZ > XZ$. So YZ, the length of the top surface of the ramp, must be greater than the length of the ramp.

List the angles and sides of each triangle in order from smallest to largest.

22.



SOLUTION:

Using the Triangle Sum Theorem, we can solve for x, as shown below.

$$(2x+9)+(2x+1)+90=180$$
$$4x+100=180$$
$$4x=80$$
$$x=20$$

$m\angle X = 2(20)+1=41$ degrees and the $m\angle Y = 2(20)+9=49$ degrees. Therefore, $m\angle X < m\angle Y < m\angle Z$. By Theorem 5.10, we know that the lengths of sides across from larger angles are longer than those across from shorter angles so $YZ < XZ < XY$.

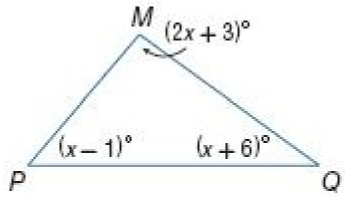
Angle: $\angle X$, $\angle Y$, $\angle Z$

Side: $\overline{YZ}, \overline{XZ}, \overline{XY}$

ANSWER:

$\angle X$, $\angle Y$, $\angle Z$; $\overline{YZ} < \overline{XZ} < \overline{XY}$

5-3 Inequalities in One Triangle

23. 

SOLUTION:

Using the Triangle Sum Theorem, we can solve for x, as shown below.

$$\begin{aligned}(2x+3)+(x-1)+(x+6)&=180\\4x+8&=180\\4x&=172\\x&=43\end{aligned}$$

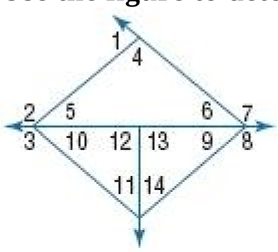
$m\angle M=2(43)+3=89$ degrees, $m\angle P=(43)-1=42$ degrees and the $m\angle Q=43+6=49$ degrees. Therefore, $m\angle P<m\angle Q<m\angle M$. By Theorem 5.10, we know that the lengths of sides across from larger angles are longer than those across from shorter angles so $MQ<PM<PQ$.

Angle: $\angle P$, $\angle Q$, $\angle M$
Side: $\overline{MQ}, \overline{PM}, \overline{PQ}$

ANSWER:

$\angle P$, $\angle Q$, $\angle M$; $\overline{MQ}, \overline{PM}, \overline{PQ}$

Use the figure to determine which angle has the greatest measure.



24. $\angle 1$, $\angle 5$, $\angle 6$

SOLUTION:

By the Exterior Angle Inequality, we know that the measure of the exterior angle of a triangle is greater than the measure of either of its corresponding remote interior angles. Therefore, since $\angle 1$ is the exterior angle and $\angle 5$ and $\angle 6$ are its remote interior angles, the $m\angle 1$ is greater than $m\angle 5$ and $m\angle 6$.

ANSWER:

$\angle 1$

25. $\angle 2$, $\angle 4$, $\angle 6$

SOLUTION:

By the Exterior Angle Inequality, we know that the measure of the exterior angle of a triangle is greater than the measure of either of its corresponding remote interior angles. Therefore, since $\angle 2$ is the exterior angle and $\angle 4$ and $\angle 6$ are its remote interior angles, the $m\angle 2$ is greater than $m\angle 4$ and $m\angle 6$.

ANSWER:

$\angle 2$

26. $\angle 7$, $\angle 4$, $\angle 5$

SOLUTION:

By the Exterior Angle Inequality, we know that the measure of the exterior angle of a triangle is greater than the measure of either of its corresponding remote interior angles. Therefore, since $\angle 7$ is the exterior angle and $\angle 5$ and $\angle 4$ are its remote interior angles, the $m\angle 7$ is greater than $m\angle 4$ and $m\angle 5$.

ANSWER:

$\angle 7$

27. $\angle 3$, $\angle 11$, $\angle 12$

SOLUTION:

By the Exterior Angle Inequality, we know that the measure of the exterior angle of a triangle is greater than the measure of either of its corresponding remote interior angles. Therefore, since $\angle 3$ is the exterior angle and $\angle 11$ and $\angle 12$ are its remote interior angles, the $m\angle 3$ is greater than $m\angle 11$ and $m\angle 12$.

ANSWER:

$\angle 3$

28. $\angle 3$, $\angle 9$, $\angle 14$

SOLUTION:

By the Exterior Angle Inequality, we know that the measure of the exterior angle of a triangle is greater than the measure of either of its corresponding remote interior angles. Therefore, since $\angle 3$ is the exterior angle and $\angle 9$ and the sum of $\angle 11$ and $\angle 14$ are its remote interior angles, the $m\angle 3$ is greater than $m\angle 9$ and the sum of the $m\angle 11$ and $m\angle 14$. If $m\angle 3$ is greater than the sum of $m\angle 11$ and $m\angle 14$, then it is greater than the measure of the individual angles. Therefore, the $m\angle 3$ is greater than $m\angle 9$ and $m\angle 14$.

ANSWER:

$\angle 3$

29. $\angle 8$, $\angle 10$, $\angle 11$

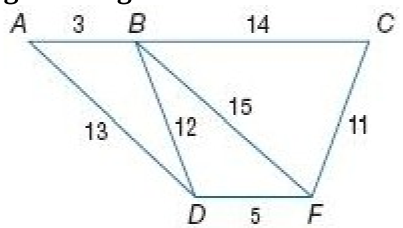
SOLUTION:

By the Exterior Angle Inequality, we know that the measure of the exterior angle of a triangle is greater than the measure of either of its corresponding remote interior angles. Therefore, since $\angle 8$ is the exterior angle and $\angle 10$ and the sum of $\angle 11$ and $\angle 14$ are its remote interior angles, the $m\angle 8$ is greater than $m\angle 10$ and the sum of the $m\angle 11$ and $m\angle 14$. If $m\angle 8$ is greater than the sum of $m\angle 11$ and $m\angle 14$, then it is greater than the measure of the individual angles. Therefore, the $m\angle 8$ is greater than $m\angle 10$ and $m\angle 11$.

ANSWER:

$\angle 8$

CCSS SENSE-MAKING Use the figure to determine the relationship between the measures of the given angles.



30. $\angle ABD$, $\angle BDA$

SOLUTION:

The side opposite $\angle ABD$ is $\overline{DA}$, which is of length 13.
The side opposite $\angle BDA$ is $\overline{BA}$, which is of length 3.
Since $\overline{DA} > \overline{BA}$, $m\angle ABD > m\angle BDA$ by Theorem 5.9.

ANSWER:

$m\angle ABD > m\angle BDA$

31. $\angle BCF$, $\angle CFB$

SOLUTION:

The side opposite $\angle BCF$ is $\overline{BF}$, which is of length 15.
The side opposite $\angle CFB$ is $\overline{BC}$, which is of length 14.
Since $\overline{BF} > \overline{BC}$ in ΔBCF, $m\angle BCF > m\angle CFB$ by Theorem 5.9.

ANSWER:

$m\angle BCF > m\angle CFB$

32. $\angle BFD$, $\angle BDF$

SOLUTION:

The side opposite $\angle BFD$ is $\overline{BD}$, which is of length 12.
The side opposite $\angle BDF$ is $\overline{BF}$, which is of length 15.
Since $\overline{BD} < \overline{BF}$ in ΔBDF, $m\angle BFD < m\angle BDF$ by Theorem 5.9.

ANSWER:

$m\angle BFD < m\angle BDF$

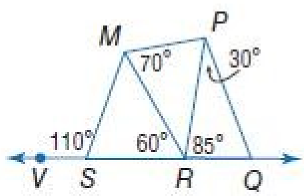
33. $\angle DBF$, $\angle BFD$

SOLUTION:

The side opposite $\angle DBF$ is $\overline{DF}$, which is of length 5.
The side opposite $\angle BFD$ is $\overline{BD}$, which is of length 12.
Since $\overline{DF} < \overline{BD}$ in ΔBDF, $m\angle DBF < m\angle BFD$ by Theorem 5.9.

ANSWER:
$m\angle DBF < m\angle BFD$

Use the figure to determine the relationship between the lengths of the given sides.



34. *SM*, *MR*

SOLUTION:

Since $\angle MSV$ and $\angle MSR$ are a linear pair, $m\angle MSR = 180 - 110 = 70.$
The side opposite $\angle SRM$ is $\overline{SM}$. The side opposite $\angle MSR$ is $\overline{MR}$. In ΔSRM, $m\angle SRM < m\angle MSR$, since 60 < 70. Therefore, by Theorem 5.10, $SM < MR$.

ANSWER:

$SM < MR$

35. *RP*, *MP*

SOLUTION:

Since $\angle SRM$, $\angle MRP$, and $\angle PRQ$ form a straight angle, $m\angle MRP = 180 - (60 + 85)$ or 35.
The side opposite $\angle PMR$ is $\overline{RP}$. The side opposite $\angle MRP$ is $\overline{MP}$. In ΔPRM, $m\angle PMR > m\angle MRP$, since 70 < 35. Therefore, by Theorem 5.10, $RP > MP$.

ANSWER:

$RP > MP$

36. *RQ*, *PQ*

SOLUTION:

The side opposite $\angle RPQ$ is $\overline{RQ}$. The side opposite $\angle PRQ$ is $\overline{PQ}$. In ΔPQR, $m\angle RPQ < m\angle PRQ$, since 30 < 85. Therefore, by Theorem 5.10, $RQ < PQ$.

ANSWER:

$RQ < PQ$

5-3 Inequalities in One Triangle

37. *RM, RQ*

SOLUTION:

By the Triangle Sum Theorem, $m\angle PQR = 180 - (85 + 30) = 65$. The side opposite $\angle PQR$ is $\overline{PR}$. The side opposite $\angle RPQ$ is $\overline{RQ}$. In ΔPQR, $m\angle PQR > m\angle RPQ$, since 65 > 30. Therefore, by Theorem 5.10, $PR > RQ$.

Also, $\angle SRM$, $\angle MRP$, and $\angle PRQ$ form a straight angle, $m\angle MRP = 180 - (60 + 85) = 35$ and, by the Triangle Sum Theorem, $m\angle MPR = 180 - (35 + 70) = 75$.

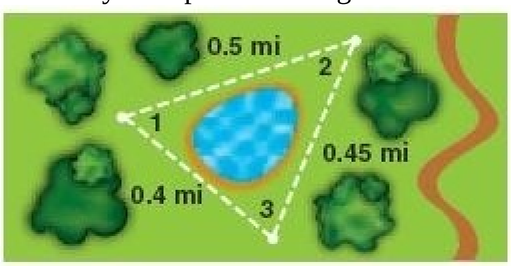
The side opposite $\angle MPR$ is $\overline{RM}$. The side opposite $\angle PMR$ is $\overline{PR}$. In ΔPRM, $m\angle MPR > m\angle PMR$, since 75 > 70. Therefore, by Theorem 5.10, $RM > PR$.

If $RM > PR$ and $PR > RQ$, then by the transitive property of inequality, $RM > RQ$.

Thus, $RM > RQ$.

ANSWER:
$RM > RQ$

38. **HIKING** Justin and his family are hiking around a lake as shown in the diagram. Order the angles of the triangle formed by their path from largest to smallest.



SOLUTION:

The side opposite $\angle 3$ is of length 0.5.
The side opposite $\angle 2$ is of length 0.4.
The side opposite $\angle 1$ is of length 0.45.
Since 0.5 > 0.45 > 0.4, by Theorem 5.9, $m\angle 3 > m\angle 1 > m\angle 2$.

ANSWER:
$m\angle 3 > m\angle 1 > m\angle 2$

COORDINATE GEOMETRY List the angles of each triangle with the given vertices in order from smallest to largest. Justify your answer.

39. $A(-4, 6)$, $B(-2, 1)$, $C(5, 6)$

SOLUTION:

Use the Distance Formula to find the side lengths.

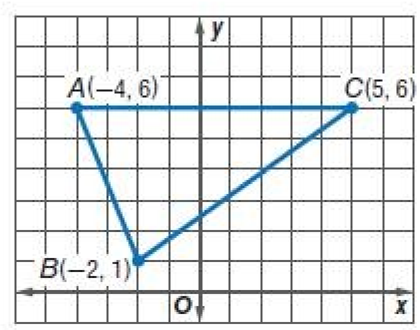
$AB = \sqrt{(-2+4)^2 + (1-6)^2} = \sqrt{29} \approx 5.3;$

$BC = \sqrt{(5+2)^2 + (6-1)^2} = \sqrt{74} \approx 8.6;$

$AC = \sqrt{(5+4)^2 + (6-6)^2} = 9;$

So, $\overline{AB} < \overline{BC} < \overline{AC}.$

Then, $\angle C < \angle A < \angle B$.



ANSWER:

$\angle C$, $\angle A$, $\angle B$, because $AB = \sqrt{29} \approx 5.4$, $BC = \sqrt{74} \approx 8.6$, and AC = 9.

40. $X(-3, -2), Y(3, 2), Z(-3, -6)$

SOLUTION:

Use the Distance Formula to find the side lengths.

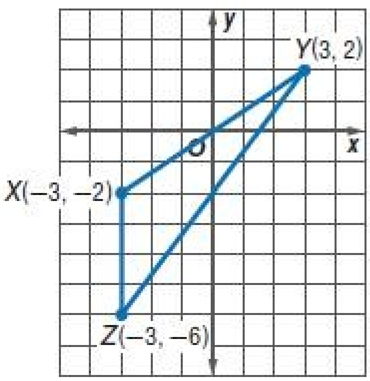
$XY = \sqrt{(3+3)^2 + (2+2)^2} = \sqrt{52} \approx 7.2;$

$YZ = \sqrt{(-3-3)^2 + (-6-2)^2} = 10;$

$XZ = \sqrt{(-3+3)^2 + (-6+2)^2} = 4;$

So, $\overline{XZ} < \overline{XY} < \overline{YZ}$.

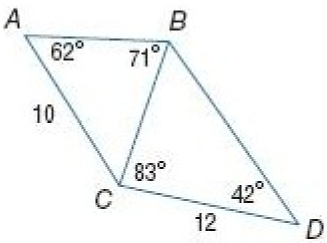
Then, $\angle Y < \angle Z < \angle X$.



ANSWER:

$\angle Y$, $\angle Z$, $\angle X$, because $XZ = 4$, $XY = \sqrt{52} \approx 7.2$, and $YZ = 10$.

41. List the side lengths of the triangles in the figure from shortest to longest. Explain your reasoning.



SOLUTION:

AB, *BC*, *AC*, *CD*, *BD*;

In ΔABC, $AB < BC < AC$ and in ΔBCD, $BC < CD < BD$.

By the figure $AC < CD$, so $BC < AC < CD$.

ANSWER:

AB, *BC*, *AC*, *CD*, *BD*; In ΔABC, $AB < BC < AC$ and in ΔBCD, $BC < CD < BD$. By the figure $AC < CD$, so $BC < AC < CD$.

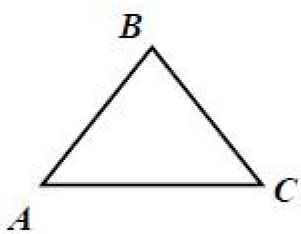
42. **MULTIPLE REPRESENTATIONS** In this problem, you will explore the relationship between the sides of a triangle.

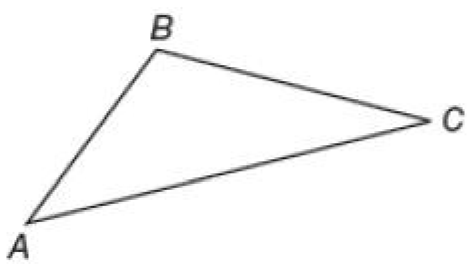
Triangle	AB	BC	AB + BC	CA
Acute				
Obtuse				
Right				

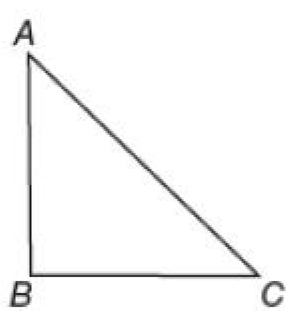
a. GEOMETRIC Draw three triangles, including one acute, one obtuse, and one right angle. Label the vertices of each triangle *A*, *B*, and *C*.
b. TABULAR Measure the length of each side of the three triangles. Then copy and complete the table.
c. TABULAR Create two additional tables like the one above, finding the sum of *BC* and *CA* in one table and the sum of *AB* and *CA* in the other.
d. ALGEBRAIC Write an inequality for each of the tables you created relating the measure of the sum of two of the sides to the measure of the third side of a triangle.
e. VERBAL Make a conjecture about the relationship between the measure of the sum of two sides of a triangle and the measure of the third side.

SOLUTION:

a.







b. It might be easier to use centimeters to measure the sides of your triangles and round to the nearest tenth place, as shown in the table below.
Sample answer:

5-3 Inequalities in One Triangle

Triangle	AB	BC	$AB + BC$	CA
Acute	2	2.4	4.4	3.2
Obtuse	2.6	3.4	6.0	5.0
Right	2.7	2.8	5.5	3.8

c. It might be easier to use centimeters to measure the sides of your triangles and round to the nearest tenth place, as shown in the table below.
Sample answer:

Triangle	BC	CA	$BC + CA$	AB
Acute	2.4	3.2	5.6	2
Obtuse	3.4	5.0	8.4	2.6
Right	2.8	3.8	6.6	2.7

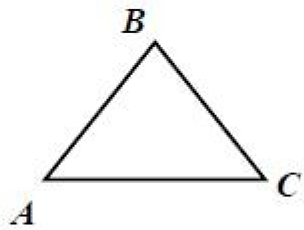
Triangle	AB	CA	$AB + CA$	BC
Acute	2	3.2	5.2	2.4
Obtuse	2.6	5.0	7.6	3.4
Right	2.7	3.8	6.5	2.8

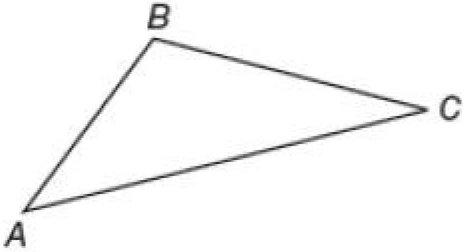
d. Compare the last two columns in each table and write an inequality comparing each of them.
$AB + BC > CA$, $BC + CA > AB$, $AB + CA > BC$

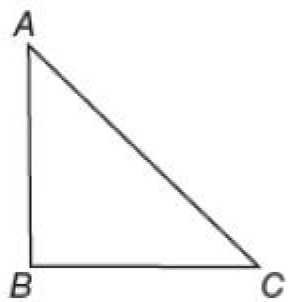
e. Sample answer: The sum of the measures of two sides of a triangle is always greater than the measure of the third side of the triangle.

ANSWER:

a.







b. Sample answer:

Triangle	AB	BC	AB + BC	CA
Acute	2	2.4	4.4	3.2
Obtuse	2.6	3.4	6.0	5.0
Right	2.7	2.8	5.5	3.8

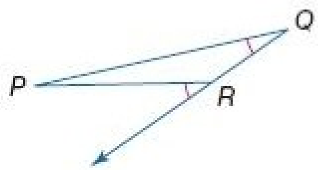
c. Sample answer:

Triangle	BC	CA	BC + CA	AB
Acute	2.4	3.2	5.6	2
Obtuse	3.4	5.0	8.4	2.6
Right	2.8	3.8	6.6	2.7

Triangle	AB	CA	AB + CA	BC
Acute	2	3.2	5.2	2.4
Obtuse	2.6	5.0	7.6	3.4
Right	2.7	3.8	6.5	2.8

d. $AB + BC > CA$, $BC + CA > AB$, $AB + CA > BC$
e. Sample answer: The sum of the measures of two sides of a triangle is greater than the measure of the third side of the triangle.

43. **WRITING IN MATH** Analyze the information given in the diagram and explain why the markings must be incorrect.



SOLUTION:

Sample answer: $\angle R$ is an exterior angle to ΔPQR, so, by the Exterior Angle Inequality, $m\angle R$ must be greater than $m\angle Q$, one of its corresponding remote interior angles. The markings indicate that $\angle R \cong \angle Q$, indicating that $m\angle R = m\angle Q$. This is a contradiction of the Exterior Angle Inequality Theorem since $\angle R$ can't be both equal to and greater than $\angle Q$ at the same time. Therefore, the markings are incorrect.

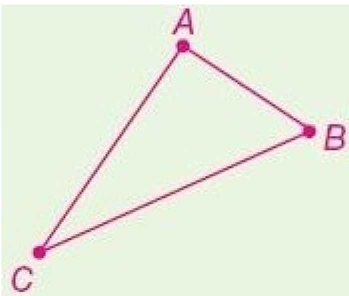
ANSWER:

Sample answer: $\angle R$ is an exterior angle to ΔPQR, so by the Exterior Angle Inequality, $m\angle R$ must be greater than $m\angle Q$. The markings indicate that $\angle R \cong \angle Q$, indicating that $m\angle R = m\angle Q$. This is a contradiction of the Exterior Angle Inequality Theorem, so the markings are incorrect.

44. **CHALLENGE** Using only a ruler, draw ΔABC such that $m\angle A > m\angle B > m\angle C$. Justify your drawing.

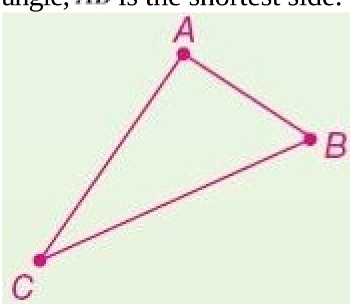
SOLUTION:

Due to the given information, that $m\angle A < m\angle B < m\angle C$, we know that this is a triangle with three different angle measures. Therefore, it also has three different side lengths. You might want to start your construction with the longest side, which would be across from the largest angle measure. Since $\angle A$ is the largest angle, the side opposite it, $\overline{CB}$, is the longest side. Then, make a short side across from the smallest angle measure. Since $\angle C$ is the smallest angle, $\overline{AB}$ is the shortest side. Connect points A to C and verify that the length of $\overline{AC}$ is between the other two sides.

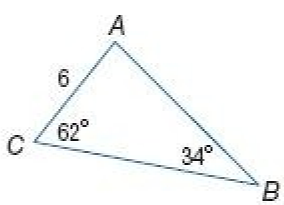


ANSWER:

Sample answer: Since $\angle A$ is the largest angle, the side opposite it, $\overline{CB}$, is the longest side. Since $\angle C$ is the smallest angle, $\overline{AB}$ is the shortest side.



45. **OPEN ENDED** Give a possible measure for $\overline{AB}$ in ΔABC shown. Explain your reasoning.



SOLUTION:

Since $m\angle C > m\angle B$, we know, according to Theorem 5.10, that the side opposite $\angle C$ $(\overline{AB})$ must be greater than the side opposite $\angle B$ $(\overline{AC})$. Therefore, if $AB > AC$, then $AB > 6$ so you need to choose a length for AB that is greater than 6.

Sample answer: 10; $m\angle C > m\angle B$ so $AB > AC$, Therefore, Theorem 5.10 is satisfied since 10 > 6.

ANSWER:

Sample answer: 10; $m\angle C > m\angle B$, so if $AB > AC$, Theorem 5.10 is satisfied. Since 10 > 6, $AB > AC$.

46. **CCSS ARGUMENTS** Is the base of an isosceles triangle *sometimes*, *always*, or *never* the longest side of the triangle? Explain.

SOLUTION:

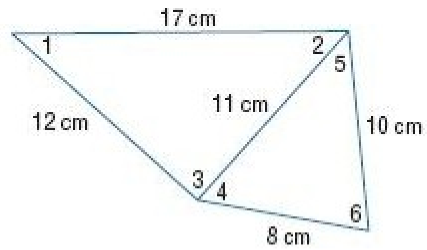
To reason through this answer, see if you can sketch isosceles triangles that have a base shorter than the two congruent legs, as well as longer than the two congruent legs.

Sometimes;
Sample answer: If the measures of the base angles are less than 60 degrees, then the base will be the longest leg. If the measures of the base angles are greater than 60 degrees, then the base will be the shortest leg.

ANSWER:

Sometimes; sample answer: If the measures of the base angles are less than 60 degrees, then the base will be the longest leg. If the measures of the base angles are greater than 60 degrees, then the base will be the shortest leg.

47. **CHALLENGE** Use the side lengths in the figure to list the numbered angles in order from smallest to largest given that $m\angle 2 = m\angle 5$. Explain your reasoning.



SOLUTION:

Walk through these angles one side at a time.
Given: $m\angle 2 = m\angle 5$

The side opposite $\angle 5$ is the smallest side in that triangle and $m\angle 2 = m\angle 5$, so we know that $m\angle 4$ and $m\angle 6$ are both greater than $m\angle 2$ and $m\angle 5$. Also, the side opposite $m\angle 6$ is greater than the side opposite $m\angle 4$.

So far, we have $(m\angle 2 = m\angle 5) < m\angle 4 < m\angle 6$.

Since the side opposite $\angle 2$ is greater than the side opposite $\angle 1$, we know that $m\angle 1$ is less than $m\angle 2$ and $m\angle 5$.

Now, we have $m\angle 1 < (m\angle 2 = m\angle 5) < m\angle 4 < m\angle 6$.

From the triangles, $m\angle 1 + m\angle 2 + m\angle 3 = 180$ and $m\angle 4 + m\angle 5 + m\angle 6 = 180$. Since $m\angle 2 = m\angle 5$, $m\angle 1 + m\angle 3$ must equal $m\angle 4 + m\angle 6$. Since $m\angle 1$ is less than $m\angle 4$, we know that $m\angle 3$ is must be greater than $m\angle 6$.

The side lengths list in order from smallest to largest is :$m\angle 1$, $m\angle 2 = m\angle 5$, $m\angle 4$, $m\angle 6$, $m\angle 3$.

ANSWER:

$m\angle 1$, $m\angle 2 = m\angle 5$, $m\angle 4$, $m\angle 6$, $m\angle 3$; Sample answer: The side opposite $\angle 5$ is the smallest side in that triangle and $m\angle 2 = m\angle 5$, so we know that $m\angle 4$ and $m\angle 6$ are both greater than $m\angle 2$ and $m\angle 5$. The side opposite $m\angle 6 >$ the side opposite $m\angle 4$. Since the side opposite $\angle 2 >$ the side opposite $\angle 1$, we know that $m\angle 1 < m\angle 2$ and $m\angle 5$. Since $m\angle 2 = m\angle 5$, $m\angle 1 + m\angle 3 = m\angle 4 + m\angle 6$. Since $m\angle 1 < m\angle 4$ then $m\angle 3 > m\angle 6$.

5-3 Inequalities in One Triangle

48. **WRITING IN MATH** Explain why the hypotenuse of a right triangle is always the longest side of the triangle.

SOLUTION:

Based on the Triangle Sum Theorem, we know that the sum of all the angles of a triangle add up to 180 degrees. If one of the angles is 90 degrees, then the sum of the other two angles must equal $180-(90)=90$ degrees. If two angles add up to 90 degrees, and neither is 0 degrees, then they each must equal less than 90 degrees, making the sides across from these two acute angles shorter than the side across from the 90 degree angle, according to Theorem 5.10. Because the hypotenuse is across from the 90 degree angle, the largest angle, then it must be the longest side of the triangle.

ANSWER:

Sample answer: Since the hypotenuse is across from the right angle and both of the other angles in a right triangle are always acute, the hypotenuse is always opposite the largest angle of the triangle and is always the longest side.

49. **STATISTICS** The chart shows the number and types of DVDs sold at three stores.

DVD Type	Store 1	Store 2	Store 3
Comedy	75	80	92
Action	54	37	65
Horror	30	48	62
Science Fiction	21	81	36
Total	180	246	255

According to the information in the chart, which of these statements is true?

A The mean number of DVDs sold per store was 56.
B Store 1 sold twice as many action and horror films as store 3 sold of science fiction.
C Store 2 sold fewer comedy and science fiction than store 3 sold.
D The mean number of science fiction DVDs sold per store was 46.

SOLUTION:

Mean of science fiction DVDs is $\left(\frac{21+81+36}{3}\right)$ or 46.

The correct choice is D.

ANSWER:

D

50. Two angles of a triangle have measures $45°$ and $92°$. What type of triangle is it?

F obtuse scalene
G obtuse isosceles
H acute scalene
J acute isosceles

SOLUTION:

By Triangle Sum Theorem, the measure of the third angle is $180-(92+45)°$ or $43°$.
Since no angle measures are equal, it is a scalene triangle. And one of the angles is obtuse; so, it is an Obtuse Scalene Triangle. The correct choice is F.

ANSWER:

F

51. **EXTENDED RESPONSE** At a five-star restaurant, a waiter earns a total of *t* dollars for working *h* hours in which he receives $198 in tips and makes $2.50 per hour.

a. Write an equation to represent the total amount of money the waiter earns.

b. If the waiter earned a total of $213, how many hours did he work?

c. If the waiter earned $150 in tips and worked for 12 hours, what is the total amount of money he earned?

SOLUTION:

a. The equation that represents the total amount of money the waiter earns is:

$t = 2.50(h) + 198$

$t = 2.5h + 198$

b. Substitute $t = 213$ in the equation and find h.

$213 = 2.5h + 198$

$2.5h = 15$

$h = 6$

So, the waiter worked for 6 hours.

c.

$t = 2.5(12) + 150$

$= 30 + 150$

$= 180$

The total amount he earned is $180.

ANSWER:

a. $t = 2.5h + 198$

b. 6

c. $180

52. **SAT/ACT** Which expression has the *least* value?

A $|-99|$

B $|45|$

C $|-39|$

D $|-28|$

E $|15|$

SOLUTION:

$|45| = 45;$

$|15| = 15;$

$|-28| = 28;$

$|-39| = 39;$

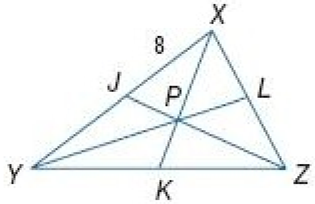
$|-99| = 99$

$|15|$ has the least value. So, the correct choice is E.

ANSWER:

E

In $\triangle XYZ$, *P* is the centroid, *KP* = 3, and *XJ* = 8. Find each length.



53. *XK*

SOLUTION:

Here, $XK = KP + XP$.

By Centroid Theorem, $XP = \frac{2}{3}XK$.

$$XK = 3 + \frac{2}{3}XK$$
$$\frac{XK}{3} = 3$$
$$XK = 9$$

ANSWER:

9

54. *YJ*

SOLUTION:

Since J is the midpoint of XY, $YJ = XJ = 8$.

ANSWER:

8

COORDINATE GEOMETRY Write an equation in slope-intercept form for the perpendicular bisector of the segment with the given endpoints. Justify your answer.

55. *D*(–2, 4) and *E*(3, 5)

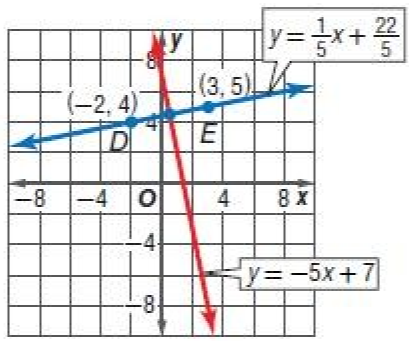
SOLUTION:

The slope of the segment *DE* is $\frac{5-4}{3+2}$ or $\frac{1}{5}$. So, the slope of the perpendicular bisector is –5.

The perpendicular bisector passes through the mid point of the segment *DE*. The midpoint of *DE* is $\left(\frac{-2+3}{2}, \frac{4+5}{2}\right)$ or $\left(\frac{1}{2}, \frac{9}{2}\right)$.

The slope-intercept form for the equation of the perpendicular bisector of *DE* is:

$$\left(y-\frac{9}{2}\right)=-5\left(x-\frac{1}{2}\right)$$
$$y-\frac{9}{2}=-5x+\frac{5}{2}$$
$$y=-5x+7$$



ANSWER:

y = –5*x* + 7; The perpendicular bisector bisects the segment at the midpoint of the segment. The midpoint is $\left(\frac{1}{2}, \frac{9}{2}\right)$.

The slope of the given segment is $\frac{1}{5}$, so the slope of the perpendicular bisector is –5.

56. $D(-2, -4)$ and $E(2, 1)$

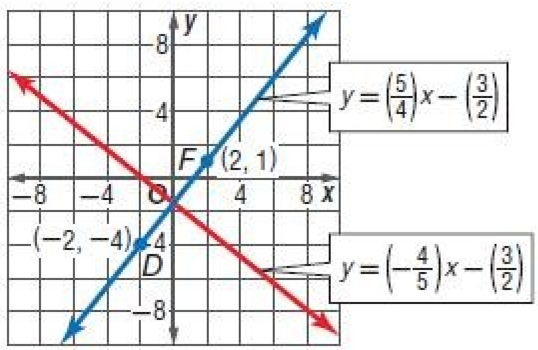
SOLUTION:

The slope of the segment *DE* is $\frac{1+4}{2+2}$ or $\frac{5}{4}$. So, the slope of the perpendicular bisector is $-\frac{4}{5}$.

The perpendicular bisector passes through the mid point of the segment *DE*. The midpoint of *DE* is $\left(\frac{-2+2}{2}, \frac{-4+1}{2}\right)$ or $\left(0, -\frac{3}{2}\right)$.

The slope-intercept form for the equation of the perpendicular bisector of *DE* is:

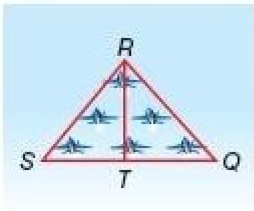
$$\left(y+\frac{3}{2}\right) = -\frac{4}{5}(x-0)$$
$$y+\frac{3}{2} = -\frac{4}{5}x$$
$$y = -\frac{4}{5}x-\frac{3}{2}$$



ANSWER:

$y = -\frac{4}{5}x-\frac{3}{2}$; The perpendicular bisector bisects the segment at the midpoint of the segment. The midpoint is $\left(0, -\frac{3}{2}\right)$. The slope of the given segment is $\frac{5}{4}$, so the slope of the perpendicular bisector is $-\frac{4}{5}$.

57. **JETS** The United States Navy Flight Demonstration Squadron, the Blue Angels, flies in a formation that can be viewed as two triangles with a common side. Write a two-column proof to prove that $\Delta SRT \cong \Delta QRT$ if T is the midpoint of $\overline{SQ}$ and $\overline{SR} \cong \overline{QR}$.



SOLUTION:

The given information in this proof can help make congruent corresponding sides for the two triangles. If T is the midpoint of $\overline{SQ}$, then it would form two congruent segments, $\overline{ST}$ and $\overline{TQ}$. Using this, along with the other given and the Reflexive Property for the third pair of sides, you can prove these triangles are congruent using SSS.

Given: T is the midpoint of $\overline{SQ}$. $\overline{SR} \cong \overline{QR}$.
Prove: $\Delta SRT \cong \Delta QRT$
Proof:
Statements(Reasons)
1. T is the midpoint of $\overline{SQ}$. (Given)
2. $\overline{ST} \cong \overline{TQ}$ (Definition of midpoint)
3. $\overline{SR} \cong \overline{QR}$ (Given)
4. $\overline{RT} \cong \overline{RT}$ (Reflexive Property)
5. $\Delta SRT \cong \Delta QRT$ (SSS)

ANSWER:

Given: T is the midpoint of $\overline{SQ}$. $\overline{SR} \cong \overline{QR}$.
Prove: $\Delta SRT \cong \Delta QRT$
Proof:
Statements(Reasons)
1. T is the midpoint of $\overline{SQ}$. (Given)
2. $\overline{ST} \cong \overline{TQ}$ (Definition of midpoint)
3. $\overline{SR} \cong \overline{QR}$. (Given)
4. $\overline{RT} \cong \overline{RT}$ (Reflexive Property)
5. $\Delta SRT \cong \Delta QRT$ (SSS)

58. **POOLS** A rectangular pool is 20 feet by 30 feet. The depth of the pool is 60 inches, but the depth of the water is $\frac{3}{4}$ of the depth of the pool. Find each measure to the nearest tenth. (Lesson 1-7)

a. the surface area of the pool

b. the volume of water in the pool

SOLUTION:

12 inches = 1 foot. Therefore, 60 inches = 5 feet.

a. Surface area of the pool *S* is:

$$S = 2(20 \cdot 30 + 30 \cdot 5 + 5 \cdot 20)$$
$$= 2(850)$$
$$= 1700 \text{ ft}^2$$

b. The depth of the water is $\left(5 \cdot \frac{3}{4}\right)$ or 3.75ft.

The volume of the water in the pool is:

$$V = 20(30)(3.75)$$
$$= 2250 \text{ ft}^3$$

ANSWER:

a. 1700 ft^2

b. 2250 ft^3

Determine whether each equation is true or false if $x = 8$, $y = 2$, and $z = 3$.

59. $z(x - y) = 13$

SOLUTION:

$$3(8 - 2) \stackrel{?}{=} 13$$
$$24 \neq 13$$

The equation is false for these values.

ANSWER:

false

60. $2x = 3yz$

SOLUTION:

$$2(8) \stackrel{?}{=} 3(2)(3)$$
$$16 \neq 18$$

The equation is false for these values.

ANSWER:

false

5-3 Inequalities in One Triangle

61. $x + y > z + y$

SOLUTION:

$$8 + 2 \overset{?}{>} 3 + 2$$
$$6 > 5$$

The equation is true for these values.

ANSWER:

true