5-5 The Triangle Inequality

Is it possible to form a triangle with the given side lengths? If not, explain why not.

1. 5 cm, 7 cm, 10 cm

SOLUTION:

The sum of the lengths of any two sides of a triangle must be greater than the length of the third side.

Yes; 5 + 7 > 10, 5 + 10 > 7, and 7 + 10 > 5

ANSWER:

Yes; 5 + 7 > 10, 5 + 10 > 7, and 7 + 10 > 5

2. 3 in., 4 in., 8 in.

SOLUTION:

No; $3+4 \ngtr 8$. The sum of the lengths of any two sides of a triangle must be greater than the length of the third side.

ANSWER:

No; $3+4 \ngtr 8$

3. 6 m, 14 m, 10 m

SOLUTION:

Yes; 6 + 14 > 10, 6 + 10 > 14, and 10 + 14 > 6 .
The sum of the lengths of any two sides of a triangle must be greater than the length of the third side.

ANSWER:

Yes; 6 + 14 > 10, 6 + 10 > 14, and 10 + 14 > 6

4. **MULTIPLE CHOICE** If the measures of two sides of a triangle are 5 yards and 9 yards, what is the least possible measure of the third side if the measure is an integer?

A 4 yd
B 5 yd
C 6 yd
D 14 yd

SOLUTION:

Let x represents the length of the third side. Next, set up and solve each of the three triangle inequalities.
$5 + 9 > x$, $5 + x > 9$, and $9 + x > 5$
That is, $14 > x$, $x > 4$, and $x > –4$.

Notice that $x > –4$ is always true for any whole number measure for x. Combining the two remaining inequalities, the range of values that fit both inequalities is $x > 4$ and $x < 14$, which can be written as $4 < x < 14$. So, the least possible measure of the third side could be 5 yd.

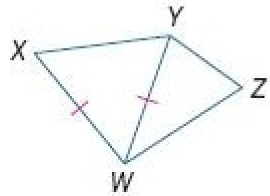
The correct option is B.

ANSWER:

B

PROOF Write a two-column proof.

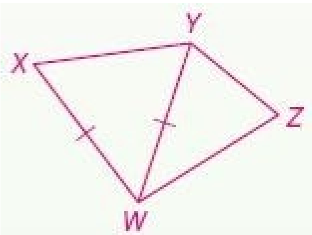
5. **Given:** $\overline{XW} \cong \overline{YW}$
Prove: $YZ + ZW > XW$



SOLUTION:

Think backwards when considering this proof. Notice that what you are trying to prove is an inequality statement. However, it isn't exactly related to ΔYZW, except for instead of side $\overline{WY}$ being used, it is $\overline{WX}$. Since it is given that $\overline{XW} \cong \overline{YW}$, you can easily use this in a substitution step.

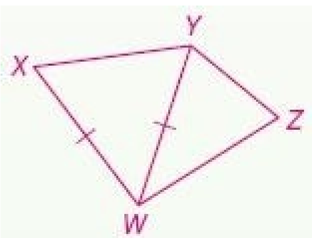
Given: $\overline{XW} \cong \overline{YW}$
Prove: $YZ + ZW > XW$



Statements (Reasons)
1. $\overline{XW} \cong \overline{YW}$ (Given)
2. $XW = YW$ (Def. of $\cong$ segments)
3. $YZ + ZW > YW$ (Δ Inequal. Thm.)
4. $YZ + ZW > XW$ (Substitution Property.)

ANSWER:

Given: $\overline{XW} \cong \overline{YW}$
Prove: $YZ + ZW > XW$



Statements (Reasons)
1. $\overline{XW} \cong \overline{YW}$ (Given)
2. $XW = YW$ (Def. of $\cong$ segments)
3. $YZ + ZW > YW$ (Δ Inequal. Thm.)
4. $YZ + ZW > XW$ (Subst.)

Is it possible to form a triangle with the given side lengths? If not, explain why not.

6. 4 ft, 9 ft, 15 ft

SOLUTION:

No; $4+9 \ngtr 15$. The sum of the lengths of any two sides of a triangle must be greater than the length of the third side.

ANSWER:

No; $4+9 \ngtr 15$

7. 11 mm, 21 mm, 16 mm

SOLUTION:

Yes; 11 + 21 > 16, 11 + 16 > 21, and 16 + 21 > 11 .
The sum of the lengths of any two sides of a triangle must be greater than the length of the third side.

ANSWER:

Yes; 11 + 21 > 16, 11 + 16 > 21, and 16 + 21 > 11

8. 9.9 cm, 1.1 cm, 8.2 cm

SOLUTION:

No; $1.1+8.2 \ngtr 9.9$ The sum of the lengths of any two sides of a triangle must be greater than the length of the third side.

ANSWER:

No; $1.1+8.2 \ngtr 9.9$

9. 2.1 in., 4.2 in., 7.9 in.

SOLUTION:

No; $2.1+4.2 \ngtr 7.9$ The sum of the lengths of any two sides of a triangle must be greater than the length of the third side.

ANSWER:

No; $2.1+4.2 \ngtr 7.9$

10. $2\frac{1}{2}\text{m}, 1\frac{3}{4}\text{m}, 5\frac{1}{8}\text{m}$

SOLUTION:

No; $2\frac{1}{2}+1\frac{3}{4} \ngtr 5\frac{1}{8}$ The sum of the lengths of any two sides of a triangle must be greater than the length of the third side.

ANSWER:

No; $2\frac{1}{2}+1\frac{3}{4} \ngtr 5\frac{1}{8}$

5-5 The Triangle Inequality

11. $1\frac{1}{5}\text{km}, 4\frac{1}{2}\text{km}, 3\frac{3}{4}\text{km}$

SOLUTION:

Yes; $1\frac{1}{5}+4\frac{1}{2}>3\frac{3}{4}, 1\frac{1}{5}+3\frac{3}{4}>4\frac{1}{2}, 4\frac{1}{2}+3\frac{3}{4}>1\frac{1}{5}$

The sum of the lengths of any two sides of a triangle must be greater than the length of the third side.

ANSWER:

Yes; $1\frac{1}{5}+4\frac{1}{2}>3\frac{3}{4}, 1\frac{1}{5}+3\frac{3}{4}>4\frac{1}{2}, 4\frac{1}{2}+3\frac{3}{4}>1\frac{1}{5}$

Find the range for the measure of the third side of a triangle given the measures of two sides.

12. 4 ft, 8 ft

SOLUTION:

Let n represent the length of the third side.

According to the Triangle Inequality Theorem, the largest side cannot be greater than the sum of the other two sides.

If n is the largest side, then n must be less than 4 + 8. Therefore, $n < 12$.
If n is not the largest side, then 8 is the largest and 8 must be less than $4 + n$. Therefore, $4 < n$.

Combining these two inequalities, we get $4 < n < 12$.

ANSWER:

4 ft < n < 12 ft

13. 5 m, 11 m

SOLUTION:

Let n represent the length of the third side.

According to the Triangle Inequality Theorem, the largest side cannot be greater than the sum of the other two sides.

If n is the largest side, then n must be less than 5 +11. Therefore, $n < 16$.
If n is not the largest side, then 11 is the largest and 11 must be less than $5 + n$. Therefore, $6 < n$.

Combining these two inequalities, we get $6 < n < 16$.

ANSWER:

6 m < n < 16 m

14. 2.7 cm, 4.2 cm

SOLUTION:
Let *n* represent the length of the third side.

According to the Triangle Inequality Theorem, the largest side cannot be greater than the sum of the other two sides.

If *n* is the largest side, then *n* must be less than 2.7 + 4.2. Therefore, $n < 6.9$.
If *n* is not the largest side, then 4.2 is the largest and 4.2 must be less than $2.7 + n$. Therefore, $1.5 < n$.

Combining these two inequalities, we get $1.5 < n < 6.9$.

ANSWER:
1.5 cm < *n* < 6.9 cm

15. 3.8 in., 9.2 in.

SOLUTION:
Let *n* represent the length of the third side.

According to the Triangle Inequality Theorem, the largest side cannot be greater than the sum of the other two sides.

If *n* is the largest side, then *n* must be less than 3.8 + 9.2. Therefore, $n < 13$.
If *n* is not the largest side, then 9.2 is the largest and 9.2 must be less than $3.8 + n$. Therefore, $5.4 < n$.

Combining these two inequalities, we get $5.4 < n < 13$.

ANSWER:
5.4 in. < *n* < 13 in.

16. $\frac{1}{2}\text{km}, 3\frac{1}{4}\text{km}$

SOLUTION:
Let *n* represent the length of the third side.

According to the Triangle Inequality Theorem, the largest side cannot be greater than the sum of the other two sides.

If *n* is the largest side, then *n* must be less than $\frac{1}{2}+3\frac{1}{4}$. Therefore, $n<3\frac{3}{4}$.
If *n* is not the largest side, then $3\frac{1}{4}$ is the largest and $3\frac{1}{4}$ must be less than $\frac{1}{2}+n$. Therefore, $2\frac{3}{4}<n$.

Combining these two inequalities, we get $2\frac{3}{4}<n<3\frac{3}{4}$.

ANSWER:
$2\frac{3}{4}\text{km}<n<3\frac{3}{4}\text{ km}$

17. $2\frac{1}{3}\text{yd}, 7\frac{2}{3}\text{yd}$

SOLUTION:
Let *n* represent the length of the third side.

According to the Triangle Inequality Theorem, the largest side cannot be greater than the sum of the other two sides.

If *n* is the largest side, then *n* must be less than $2\frac{1}{3}+7\frac{2}{3}$. Therefore, $n<10$.
If *n* is not the largest side, then $7\frac{2}{3}$ is the largest and $7\frac{2}{3}$ must be less than $2\frac{1}{3}+n$. Therefore, $5\frac{1}{3}<n$.

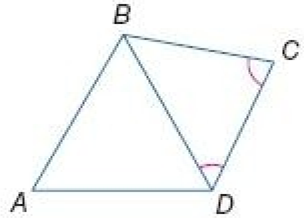
Combining these two inequalities, we get $5\frac{1}{3}<n<10$.

ANSWER:
$5\frac{1}{3}\text{yd}<n<10\text{yd}$

PROOF Write a two-column proof.

18. **Given:** $\angle BCD \cong \angle CDB$

Prove: $AB + AD > BC$



SOLUTION:

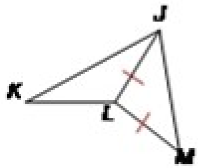
The key to this proof is to figure out some way to get *BC*=*BD* so that you can substitute one in for the other using the Triangle Inequality Theorem. Consider the given statement, if two angles of a triangle are congruent, what kind of triangle is it and , therefore, how do you know that *BC* must equal *BD*?

Proof:
Statements (Reasons)
1. $\angle BCD \cong \angle CDB$ (Given)
2. $\overline{BC} \cong \overline{BD}$ (Converse of Isosceles Δ Thm.)
3. $BC = BD$ (Def. of $\cong$ segments)
4. $AB + AD > BD$ (Δ Inequality Thm.)
5. $AB + AD > BC$ (Substitution Property.)

ANSWER:

Proof:
Statements (Reasons)
1. $\angle BCD \cong \angle CDB$ (Given)
2. $\overline{BC} \cong \overline{BD}$ (Conv. Isos. Δ Thm.)
3. $BC = BD$ (Def. of $\cong$ segments)
4. $AB + AD > BD$ (Δ Inequal. Thm.)
5. $AB + AD > BC$ (Subst.)

19. **Given:** $\overline{JL} \cong \overline{LM}$
Prove: *KJ* +*KL*> *LM*

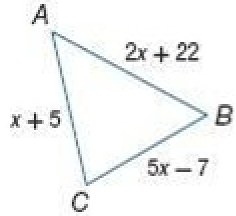


SOLUTION:
Think backwards when considering this proof. Notice that what you are trying to prove is an inequality statement. However, it isn't exactly related to ΔKJL, except for instead of side $\overline{JL}$ being used, it is $\overline{LM}$. Since it is given that $\overline{JL} \cong \overline{LM}$, you can easily use this in a substitution step

Proof:
Statements (Reasons)
1. $\overline{JL} \cong \overline{LM}$ (Given)
2. *JL* = *LM* (Def. of $\cong$ segments)
3. *KJ* + *KL* > *JL* (Δ Inequality Thm.)
4. *KJ* + *KL* > *LM* (Substitution Property)

ANSWER:
Proof:
Statements (Reasons)
1. $\overline{JL} \cong \overline{LM}$ (Given)
2. *JL* = *LM* (Def. of $\cong$ segments)
3. *KJ* + *KL* > *JL* (Δ Inequal. Thm.)
4. *KJ* + *KL* > *LM* (Subst.)

CCSS SENSE-MAKING Determine the possible values of *x*.

20. 

SOLUTION:

Set up and solve each of the three triangle inequalities.

$2x+22+x+5>5x-7$,
$2x+22+5x-7>x+5$,
and $x+5+5x-7>2x+22$

$$\begin{aligned}2x+22+x+5&>5x-7\\3x+27&>5x-7\\-2x&>-34\\x&<17\end{aligned}$$

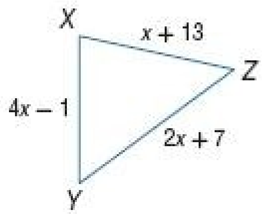
$$\begin{aligned}2x+22+5x-7&>x+5\\7x+15&>x+5\\6x&>-10\\x&>-\frac{5}{3}\end{aligned}$$

$$\begin{aligned}x+5+5x-7&>2x+22\\6x-2&>2x+22\\4x&>24\\x&>6\end{aligned}$$

Notice that $x>-\frac{5}{3}$ is always true for any whole number measure for *x*.The range of values that would be true for the other two inequalities is $x<17$ and $x>6$, which can be written as $6<x<17$.

ANSWER:

$6 < x < 17$

21. 

SOLUTION:

Set up and solve each of the three triangle inequalities.

$4x - 1 + x + 13 > 2x + 7$,
$4x - 1 + 2x + 7 > x + 13$,
and $x + 13 + 2x + 7 > 4x - 1$

$$\begin{aligned} 4x - 1 + x + 13 &> 2x + 7 \\ 5x + 12 &> 2x + 7 \\ 3x &> -5 \\ x &> -\frac{5}{3} \end{aligned}$$

$$\begin{aligned} 4x - 1 + 2x + 7 &> x + 13 \\ 6x + 6 &> x + 13 \\ 5x &> 7 \\ x &> \frac{7}{5} \end{aligned}$$

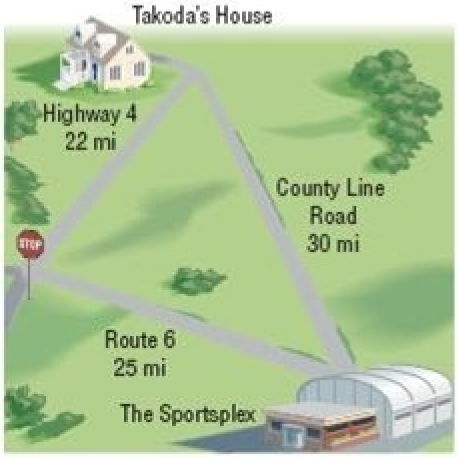
$$\begin{aligned} x + 13 + 2x + 7 &> 4x - 1 \\ 3x + 20 &> 4x - 1 \\ -x &> -21 \\ x &< 21 \end{aligned}$$

Notice that $x > -\frac{5}{3}$ is always true for any whole number measure for *x*. Combining the two remaining inequalities, the range of values that fit both inequalities is $x < 21$ and $x > \frac{7}{5}$, which can be written as $\frac{7}{5} < x < 21$.

ANSWER:

$\frac{7}{5} < x < 21$

22. **DRIVING** Takoda wants to take the most efficient route from his house to a soccer tournament at The Sportsplex. He can take County Line Road or he can take Highway 4 and then Route 6 to the get to The Sportsplex.
a. Which of the two possible routes is the shortest? Explain your reasoning.
b. Suppose Takoda always drives below the speed limit. If the speed limit on County Line Road is 30 miles per hour and on both Highway 4 and Route 6 it is 55 miles per hour, which route will be faster? Explain.



SOLUTION:

a. County Line Road; sample answer: In a triangle, the sum of two of the sides is always greater than the third side, so the sum of the distance on Highway 4 and the distance on Route 6 is greater than the distance on County Line Road. Or you can add the distances using Highway 4 and Route 6 and compare their sum to the 30 miles of County Line Road. Since 47 miles is greater than 30 miles, County Line Road is the shortest distance.

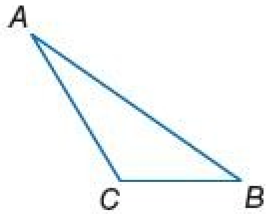
b. Highway 4 to Route 6; sample answer: Since Takoda drives below the 30 mph speed limit on County Line Road and the distance is 30 miles, it will take him about 30/30 = 1 hour to get to The Sportsplex. He has to drive 47 miles on Highway 4 and Route 6, and the speed limit is 55 miles per hour, so it will take him 45/55 = 0.85 hour or about 51 minutes. The route on Highway 4 and Route 6 will take less time than the route on County Line Road.

ANSWER:

a. County Line Road; sample answer: In a triangle, the sum of two of the sides is always greater than the third side, so the sum of the distance on Highway 4 and the distance on Route 6 is greater than the distance on County Line Road.
b. Highway 4 to Route 6; sample answer: Since Takoda can drive 30 miles per hour on County Line Road and the distance is 30 miles, it will take him 1 hour. He has to drive 47 miles on Highway 4 and Route 6, and the speed limit is 55 miles per hour, so it will take him 0.85 hour or about 51 minutes. The route on Highway 4 and Route 6 will take less time than the route on County Line Road.

PROOF Write a two-column proof.

23. PROOF Write a two-column proof.
Given: ΔABC
Prove: $AC + BC > AB$ (Triangle Inequality Theorem)
(Hint: Draw auxiliary segment $\overline{CD}$, so that C is between B and D and $\overline{CD} \cong \overline{AC}$.)



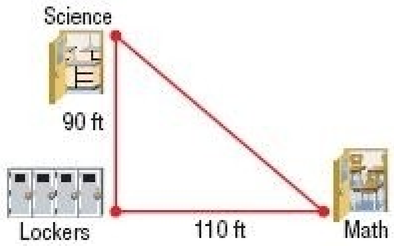
SOLUTION:
Proof:
Statements (Reasons)

1. Construct $\overline{CD}$ so that C is between B and D and $\overline{CD} \cong \overline{AC}$. (Ruler Postulate)
2. $CD = AC$ (Definition of congruence)
3. $\angle CAD \cong \angle ADC$ (Isosceles Triangle Theorem)
4. $m\angle CAD = m\angle ADC$ (Definition. of congruence angles)
5. $m\angle BAC + m\angle CAD = m\angle BAD$ ($\angle$ Addition Postulate)
6. $m\angle BAC + m\angle ADC = m\angle BAD$ (Substitution)
7. $m\angle ADC < m\angle BAD$ (Definition of inequality)
8. $AB < BD$ (Angle–Side Relationships in Triangles)
9. $BD = BC + CD$ (Segment Addition Postulate)
10. $AB < BC + CD$ (Substitution)
11. $AB < BC + AC$ (Substitution (Steps 2, 10))

ANSWER:
Proof:
Statements (Reasons)

1. Construct $\overline{CD}$ so that C is between B and D and $\overline{CD} \cong \overline{AC}$.(Ruler Post.)
2. $CD = AC$ (Def. of $\cong$)
3. $\angle CAD \cong \angle ADC$ (Isos. Δ Thm)
4. $m\angle CAD = m\angle ADC$ (Def. of $\cong$ $\angle s$)
5. $m\angle BAC + m\angle CAD = m\angle BAD$ ($\angle$ Add. Post.)
6. $m\angle BAC + m\angle ADC = m\angle BAD$ (Subst.)
7. $m\angle ADC < m\angle BAD$ (Def. of inequality)
8. $AB < BD$ (Angle–Side Relationships in Triangles)
9. $BD = BC + CD$ (Seg. Add. Post.)
10. $AB < BC + CD$ (Subst.)
11. $AB < BC + AC$ (Subst. (Steps 2, 10))

24. **SCHOOL** When Toya goes from science class to math class, she usually stops at her locker. The distance from her science classroom to her locker is 90 feet, and the distance from her locker to her math classroom is 110 feet. What are the possible distances from science class to math class if she takes the hallway that goes directly between the two classrooms?



SOLUTION:

Let n represent the length of the third side.

According to the Triangle Inequality Theorem, the largest side cannot be greater than the sum of the other two sides.

If n is the largest side, then n must be less than 90 + 110. Therefore, $n < 90+110$ or $n < 200$.
If n is not the largest side, then 110 is the largest and 110 must be less than $90 + n$. Therefore, $90 + n > 110$ or $n > 20$.

Combining these two inequalities, we get $20 < n < 200$. So, the distance is greater than 20 ft and less than 200 ft.

ANSWER:

The distance is greater than 20 ft and less than 200 ft.

Find the range of possible measures of x if each set of expressions represents measures of the sides of a triangle.

25. x, 4, 6

SOLUTION:

According to the Triangle Inequality Theorem, the largest side cannot be greater than the sum of the other two sides.

If x is the largest side, then x must be less than 4 + 6. Therefore, $x < 4+6$ or $x < 10$.
If x is not the largest side, then 6 is the largest and 6 must be less than $4 + x$. Therefore, $4 + x > 6$ or $x > 2$.

Combining these two inequalities, we get $2 < x < 10$.

ANSWER:

$2 < x < 10$

26. 8, x, 12

SOLUTION:

According to the Triangle Inequality Theorem, the largest side cannot be greater than the sum of the other two sides.

If x is the largest side, then x must be less than 8 + 12. Therefore, $x < 8 + 12$ or $x < 20$.
If x is not the largest side, then 12 is the largest and 12 must be less than $8 + x$. Therefore, $8 + x > 12$ or $x > 4$.

Combining these two inequalities, we get $4 < x < 20$.

ANSWER:

$4 < x < 20$

27. $x + 1$, 5, 7

SOLUTION:

According to the Triangle Inequality Theorem, the largest side cannot be greater than the sum of the other two sides.

If $x + 1$ is the largest side, then $x + 1$ must be less than 5 + 7. Therefore, $x + 1 < 5 + 7$ or $x + 1 < 12$ or $x < 11$.
If $x + 1$ is not the largest side, then 7 is the largest and 7 must be less than $5 + (x + 1)$. Therefore, $5 + x + 1 > 7$ or $6 + x > 7$ or $x > 1$.

Combining these two inequalities, we get $1 < x < 11$.

ANSWER:

$1 < x < 11$

28. $x – 2$, 10, 12

SOLUTION:

According to the Triangle Inequality Theorem, the largest side cannot be greater than the sum of the other two sides.

If $x – 2$ is the largest side, then $x – 2$ must be less than 10 + 12. Therefore, $x – 2 < 10 + 12$ or $x – 2 < 22$ or $x < 24$.
If $x – 2$ is not the largest side, then 12 is the largest and 12 must be less than $10 + (x – 2)$. Therefore, $10 + x – 2 > 12$ or $8 + x > 12$ or $x > 4$.

Combining these two inequalities, we get $4 < x < 24$.

ANSWER:

$4 < x < 24$

29. $x + 2$, $x + 4$, $x + 6$

SOLUTION:

Set up and solve each of the three triangle inequalities.

$x + 2 + x + 4 > x + 6$,
$x + 2 + x + 6 > x + 4$,
and $x + 4 + x + 6 > x + 2$

$$\begin{aligned} x + 2 + x + 4 &> x + 6 \\ 2x + 6 &> x + 6 \\ x &> 0 \end{aligned}$$

$$\begin{aligned} x + 2 + x + 6 &> x + 4 \\ 2x + 8 &> x + 4 \\ x &> -4 \end{aligned}$$

$$\begin{aligned} x + 4 + x + 6 &> x + 2 \\ 2x + 10 &> x + 2 \\ x &> -8 \end{aligned}$$

Notice that $x > -4$ and $x > -8$ are always true for any whole number measure for x. So, the only required inequality is $x > 0$.

ANSWER:

$x > 0$

30. x, $2x + 1$, $x + 4$

SOLUTION:

Set up and solve each of the three triangle inequalities.

$x + 2x + 1 > x + 4$,
$x + 4 + 2x + 1 > x$,
and $x + 4 + x > 2x + 1$

$$\begin{aligned} x + 2x + 1 &> x + 4 \\ 3x + 1 &> x + 4 \\ 2x &> 3 \\ x &> \frac{3}{2} \end{aligned}$$

$$\begin{aligned} x + 4 + 2x + 1 &> x \\ 3x + 5 &> x \\ 2x &> -5 \\ x &> -\frac{5}{2} \end{aligned}$$

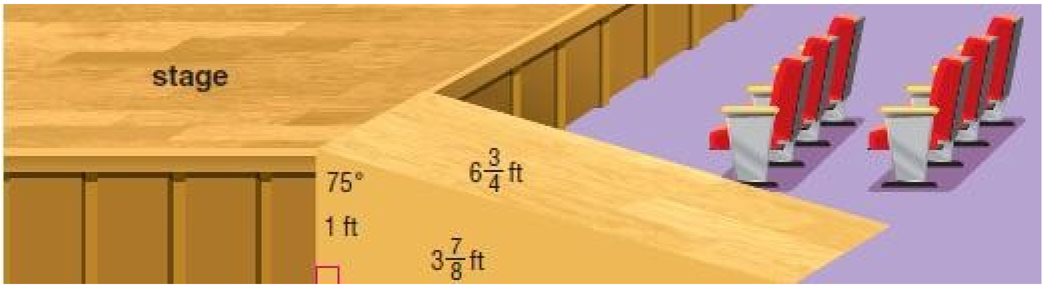
$$\begin{aligned} x + 4 + x &> 2x + 1 \\ 2x + 4 &> 2x + 1 \\ 4 &> 1 \end{aligned}$$

Notice that $x > -\frac{5}{2}$ is always true for any whole number measure for *x* and $4 > 1$ is always true. So, the required inequality is $x > \frac{3}{2}$.

ANSWER:

$x > \frac{3}{2}$

31. **Drama Club** Anthony and Catherine are working on a ramp up to the stage for the drama club's next production. Anthony's sketch of the ramp is shown below. Catherine is concerned about the measurements and thinks they should recheck the measures before they start cutting the wood. Is Catherine's concern valid? Explain your reasoning.



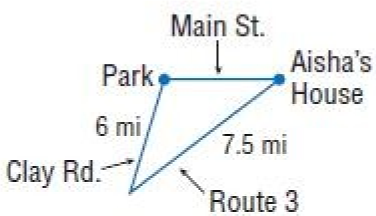
SOLUTION:

Yes; sample answer: The measurements on the drawing do not form a triangle. According to the Triangle Inequality Theorem, the sum of the lengths of any two sides of a triangle is greater than the length of the third side. The lengths in the drawing are 1 ft, $3\frac{7}{8}$ ft, and $6\frac{3}{4}$ ft. Since $1 + 3\frac{7}{8} \ngtr 6\frac{7}{8}$, the triangle is impossible. They should recalculate their measurements before they cut the wood.

ANSWER:

Yes; sample answer: The measurements on the drawing do not form a triangle. According to the Triangle Inequality Theorem, the sum of the lengths of any two sides of a triangle is greater than the length of the third side. The lengths in the drawing are 1 ft, $3\frac{7}{8}$ ft, and $6\frac{3}{4}$ ft. Since $1 + 3\frac{7}{8} \ngtr 6\frac{7}{8}$, the triangle is impossible. They should recalculate their measurements before they cut the wood.

32. **CCSS SENSE-MAKING** Aisha is riding her bike to the park and can take one of two routes. The most direct route from her house is to take Main Street, but it is safer to take Route 3 and then turn right on Clay Road as shown. The additional distance she will travel if she takes Route 3 to Clay Road is between what two number of miles?



SOLUTION:

The distance from Aisha's house to the park via Main St. represents the third side of a triangle.

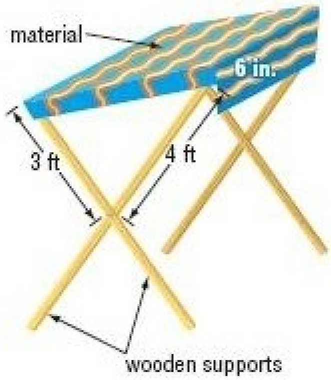
From the Triangle Inequality Theorem the length of this side must also be greater than 7.5 – 6 or 1.5 miles and must be less than 6 + 7.5 or 13.5 miles. Therefore, the distance *d* from her house to the park via Main St. can be represented by $1.5 < d < 13.5$. The distance to the park by taking Route 3 to Clay Road is 7.5 + 6 or 13.5 miles.

The least additional number of miles she would travel would be greater than 13.5 –13.5 or 0. The greatest number of additional miles she would travel would be less than 13.5 – 1.5 or 12. Therefore, the additional distance she will travel if she takes Route 3 to Clay Road is between 0 and 12 miles.

ANSWER:

0 and 12

33. **DESIGN** Carlota designed an awning that she and her friends could take to the beach. Carlota decides to cover the top of the awning with material that will drape 6 inches over the front. What length of material should she buy to use with her design so that it covers the top of the awning, including the drape, when the supports are open as far as possible? Assume that the width of the material is sufficient to cover the awning.



SOLUTION:

Let x be the length of the material needed.

According to the Triangle Inequality Theorem, the largest side cannot be greater than the sum of the other two sides.

If x is the largest side, then x must be less than 4 + 3. Therefore, $n < 7$. Since the material will drape 6 inches or 0.5 feet over the front, the minimum length of material she should buy is 7 + 0.5 or 7.5 feet at the most.

ANSWER:

She should buy no more than 7.5 ft.

ESTIMATION Without using a calculator, determine if it is possible to form a triangle with the given side lengths. Explain.

34. $\sqrt{8}$ ft, $\sqrt{2}$ ft, $\sqrt{35}$ ft

SOLUTION:

Estimate each side length by comparing the values to perfect squares.

Since $\sqrt{9} = 3$, $\sqrt{8} \approx 2.9$.
$\sqrt{2}$ is between $\sqrt{1}$ and $\sqrt{4}$. Since $\sqrt{1}$ = 1 and $\sqrt{4}$ = 2, $\sqrt{2} \approx 1.5$.

Since $\sqrt{36} = 6$, $\sqrt{35} \approx 5.9$.

5.9 > 2.9 + 1.5, so it is not possible to form a triangle with the given side lengths.

ANSWER:

No; $\sqrt{8} \approx 2.9$ since $\sqrt{9} = 3$,
$\sqrt{2} \approx 1.5$ since it is between $\sqrt{1}$ or 1 and $\sqrt{4}$ or 2, and $\sqrt{35} \approx 5.9$ since $\sqrt{36} = 6$. So, $2.9 + 1.5 \ngtr 5.9$.

35. $\sqrt{99}$ yd, $\sqrt{48}$ yd, $\sqrt{65}$ yd

SOLUTION:
Estimate each side length by comparing the values to perfect squares.

Since $\sqrt{100} = 10$, then $\sqrt{99} \approx 9.9$.

Also, since $\sqrt{49} = 7$, then $\sqrt{48} \approx 6.9$.

And since $\sqrt{64} = 8$, then $\sqrt{65} \approx 8.1$.

9.9 < 6.9 + 8.1, so yes, it is possible to form a triangle with the given side lengths.

ANSWER:
Yes. $\sqrt{99} \approx 9.9$ since $\sqrt{100} = 10$, $\sqrt{48} \approx 6.9$ since $\sqrt{49} = 7$, and $\sqrt{65} \approx 8.1$ since $\sqrt{64} = 8$. 6.9 + 8.1 > 9.9, so it is possible.

36. $\sqrt{3}$ m, $\sqrt{15}$ m, $\sqrt{24}$ m

SOLUTION:
Estimate each side length by comparing the values to perfect squares.

Since $\sqrt{4} = 2$, then $\sqrt{3} \approx 1.9$.
Also, since $\sqrt{16} = 4$, then $\sqrt{15} \approx 3.9$.
And since $\sqrt{25} = 5$, then $\sqrt{24} \approx 4.9$.

4.9< 3.9 + 1.9, so yes, it is possible to form a triangle with the given side lengths.

ANSWER:
Yes. $\sqrt{3} \approx 1.9$ since $\sqrt{4} = 2$, $\sqrt{15} \approx 3.9$ since $\sqrt{16} = 4$, and $\sqrt{24} \approx 4.9$ since $\sqrt{25} = 5$.
1.9 + 3.9 > 4.9, so it is possible.

37. $\sqrt{122}$ in, $\sqrt{5}$ in, $\sqrt{26}$ in .

SOLUTION:

Estimate each side length by comparing the values to perfect squares.

Since $\sqrt{121} = 11$, then $\sqrt{122} \approx 11.1$.
Also, since $\sqrt{4} = 2$, then $\sqrt{5} \approx 2.1$.
And since $\sqrt{25} = 5$, then $\sqrt{26} \approx 5.1$.

11.1> 2.1 + 5.1 so no, it is not possible to form a triangle with the given side lengths.

ANSWER:

No; $\sqrt{122} \approx 11.1$ since $\sqrt{121} = 11$, $\sqrt{5} \approx 2.1$ since $\sqrt{4} = 2$, and $\sqrt{26} \approx 5.1$ since $\sqrt{25} = 5$. So, 2.1 + 5.1 $\ngtr$ 11.1.

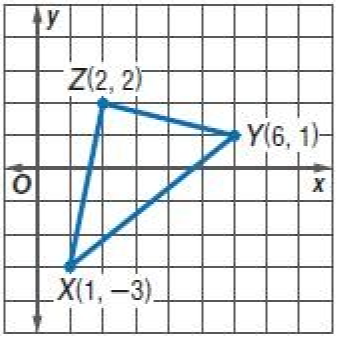
CCSS REASONING Determine whether the given coordinates are the vertices of a triangle. Explain.

5-5 The Triangle Inequality

38. $X(1, -3), Y(6, 1), Z(2, 2)$

SOLUTION:

We can graphically show that given coordinates form a triangle by graphing them, as shown below.



We can algebraically prove that the given coordinates form a triangle by proving that the length of the longest side is greater than the sum of the two shorter sides.

Use the distance formula. $\overline{XY}$ has endpoints $X(1,-3)$ and $Y(6, 1)$.

$$\begin{aligned} XY &= \sqrt{(6-1)^2 + (1-(-3))^2} \\ &= \sqrt{(5)^2 + (4)^2} \\ &= \sqrt{25+16} \\ &= \sqrt{41} \\ &\approx 6.4 \end{aligned}$$

$\overline{YZ}$ has endpoints $Y(6, 1)$ and $Z(2, 2)$.

$$\begin{aligned} YZ &= \sqrt{(2-6)^2 + (2-1)^2} \\ &= \sqrt{(-4)^2 + (1)^2} \\ &= \sqrt{16+1} \\ &= \sqrt{17} \\ &\approx 4.1 \end{aligned}$$

$\overline{ZX}$ has endpoints $Z(2, 2)$ and $X(1, -3)$.

$$\begin{aligned} ZX &= \sqrt{(1-2)^2 + (-3-2)^2} \\ &= \sqrt{(-1)^2 + (-5)^2} \\ &= \sqrt{1+25} \\ &= \sqrt{26} \\ &\approx 5.1 \end{aligned}$$

Here, $XY + YZ > XZ$, $XY + XZ > YZ$, and $XZ + YZ > XY$.
Thus, the given coordinates form a triangle.

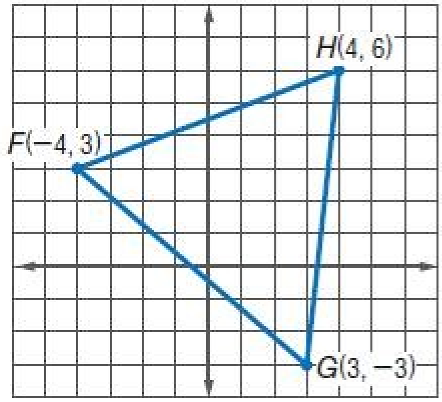
ANSWER:

Yes; $XY + YZ > XZ$, $XY + XZ > YZ$, and $XZ + YZ > XY$

39. $F(-4, 3)$, $G(3, -3)$, $H(4, 6)$

SOLUTION:

We can graphically show that given coordinates form a triangle by graphing them, as shown below.



We can algebraically prove that the given coordinates form a triangle by proving that the length of the longest side is greater than the sum of the two shorter sides.

Use the distance formula. $\overline{FG}$ has endpoints $F(-4,3)$ and $G(3, 3)$.

$$\begin{aligned} FG &= \sqrt{(3-(-4))^2+(3-3)^2} \\ &= \sqrt{(7)^2+(0)^2} \\ &= \sqrt{49} \\ &= 7 \end{aligned}$$

$\overline{GH}$ has endpoints $G(3, 3)$ and $H(4, 6)$.

$$\begin{aligned} GH &= \sqrt{(4-3)^2+(6-3)^2} \\ &= \sqrt{(1)^2+(3)^2} \\ &= \sqrt{1+9} \\ &= \sqrt{10} \\ &\approx 3.2 \end{aligned}$$

$\overline{HF}$ has endpoints $H(4, 6)$ and $F(-4, 3)$.

$$\begin{aligned} HF &= \sqrt{(-4-4)^2+(3-6)^2} \\ &= \sqrt{(-8)^2+(-3)^2} \\ &= \sqrt{64+9} \\ &= \sqrt{73} \\ &\approx 8.5 \end{aligned}$$

Here, $FG + GH > FH$, $FG + FH > GH$, and $GH + FH > FG$.
Thus, the coordinated form a triangle.

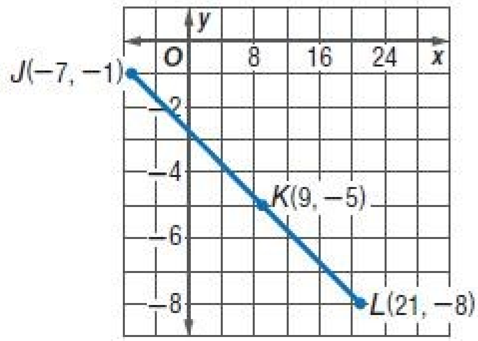
ANSWER:

Yes; $FG + GH > FH$, $FG + FH > GH$, and $GH + FH > FG$

40. $J(-7, -1)$, $K(9, -5)$, $L(21, -8)$

SOLUTION:

We can graphically determine if the given coordinates form a triangle by graphing them, as shown below.



We can algebraically prove that the these three points are collinear and therefore, by showing that the sum of the two shorter segments is equal to the longest segment.

Use the distance formula. $\overline{JK}$ has endpoints $J(-7,-1)$ and $K(9, -5)$.

$$
\begin{aligned}
JK &= \sqrt{(9-(-7))^2 + (-5-(-1))^2} \\
&= \sqrt{(16)^2 + (-4)^2} \\
&= \sqrt{256+16} \\
&= \sqrt{272} \\
&\approx 16.5
\end{aligned}
$$

$\overline{KL}$ has endpoints $K(9, -5)$ and $L(21, -8)$.

$$
\begin{aligned}
KL &= \sqrt{(21-9)^2 + (-8-(-5))^2} \\
&= \sqrt{(12)^2 + (-3)^2} \\
&= \sqrt{144+9} \\
&= \sqrt{153} \\
&\approx 12.4
\end{aligned}
$$

$\overline{LJ}$ has endpoints $L(21, -8)$ and $J(-7, -1)$.

$$
\begin{aligned}
LJ &= \sqrt{(-7-21)^2 + (-1-(-8))^2} \\
&= \sqrt{(-28)^2 + (7)^2} \\
&= \sqrt{784+49} \\
&= \sqrt{833} \\
&\approx 28.9
\end{aligned}
$$

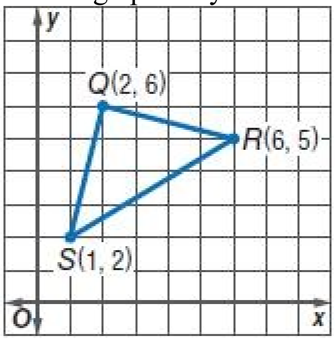
Here $JK + KL = JL$. You can also confirm this by using your calculator. Compute $\sqrt{833} - \sqrt{153} - \sqrt{272}$ to confirm that it equals 0.
Thus the given coordinates do not form a triangle.

ANSWER:
No; $JK + KL = JL$

41. $Q(2, 6)$, $R(6, 5)$, $S(1, 2)$

SOLUTION:

We can graphically determine if the given coordinates form a triangle by graphing them, as shown below.



We can algebraically prove that the given coordinates form a triangle by proving that the length of one of the sides equals zero.

Use the distance formula.

$\overline{QR}$ has endpoints $Q(2,6)$ and $R(6, 5)$.

$$\begin{aligned} QR &= \sqrt{(6-2)^2 + (5-6)^2} \\ &= \sqrt{(4)^2 + (-1)^2} \\ &= \sqrt{16+1} \\ &= \sqrt{17} \end{aligned}$$

$\overline{RS}$ has endpoints $R(6, 5)$ and $S(1, 2)$.

$$\begin{aligned} RS &= \sqrt{(1-6)^2 + (2-5)^2} \\ &= \sqrt{(-5)^2 + (-3)^2} \\ &= \sqrt{25+9} \\ &= \sqrt{34} \end{aligned}$$

$\overline{SQ}$ has endpoints $S(1, 2)$ and $Q(2, 6)$.

$$\begin{aligned} SQ &= \sqrt{(2-1)^2 + (6-2)^2} \\ &= \sqrt{(1)^2 + (4)^2} \\ &= \sqrt{1+16} \\ &= \sqrt{17} \end{aligned}$$

Then $QR + RS > QS$, $QR + QS > RS$, and $QS + RS > QR$.
Thus the coordinates form a triangle.

ANSWER:

Yes; $QR + QS > RS$, $QR + RS > QS$, *and* $QS + RS > QR$

42. **MULTIPLE REPRESENTATIONS** In this problem, you will use inequalities to make comparisons between the

sides and angles of two triangles.

a. GEOMETRIC Draw three pairs of triangles that have two pairs of congruent sides and one pair of sides that is not congruent. Mark each pair of congruent sides. Label each triangle pair *ABC* and *DEF*, where $\overline{AB} \cong \overline{DE}$ and $\overline{AC} \cong \overline{DF}$.

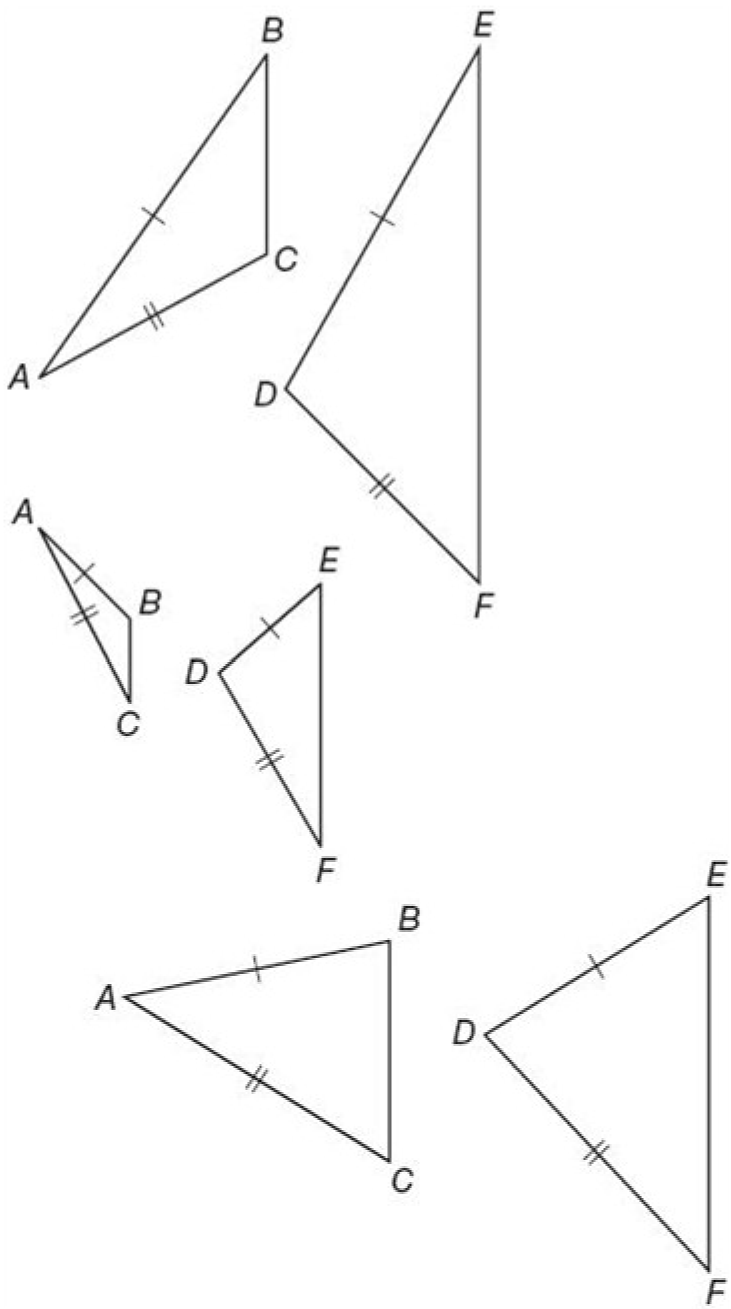
b. TABULAR Copy the table below. Measure and record the values of *BC*, $m\angle A$, *EF*, and $m\angle D$ for each triangle pair.

Triangle Pair	BC	$m\angle A$	EF	$m\angle D$
1				
2				
3				

c. VERBAL Make a conjecture about the relationship between the angles opposite the noncongruent sides of a pair of triangles that have two pairs of congruent legs.

SOLUTION:

a. Using a ruler, compass, or drawing tool, make sure that $\overline{AB} \cong \overline{DE}$ and $\overline{AC} \cong \overline{DF}$, in each of the triangle pairs made.



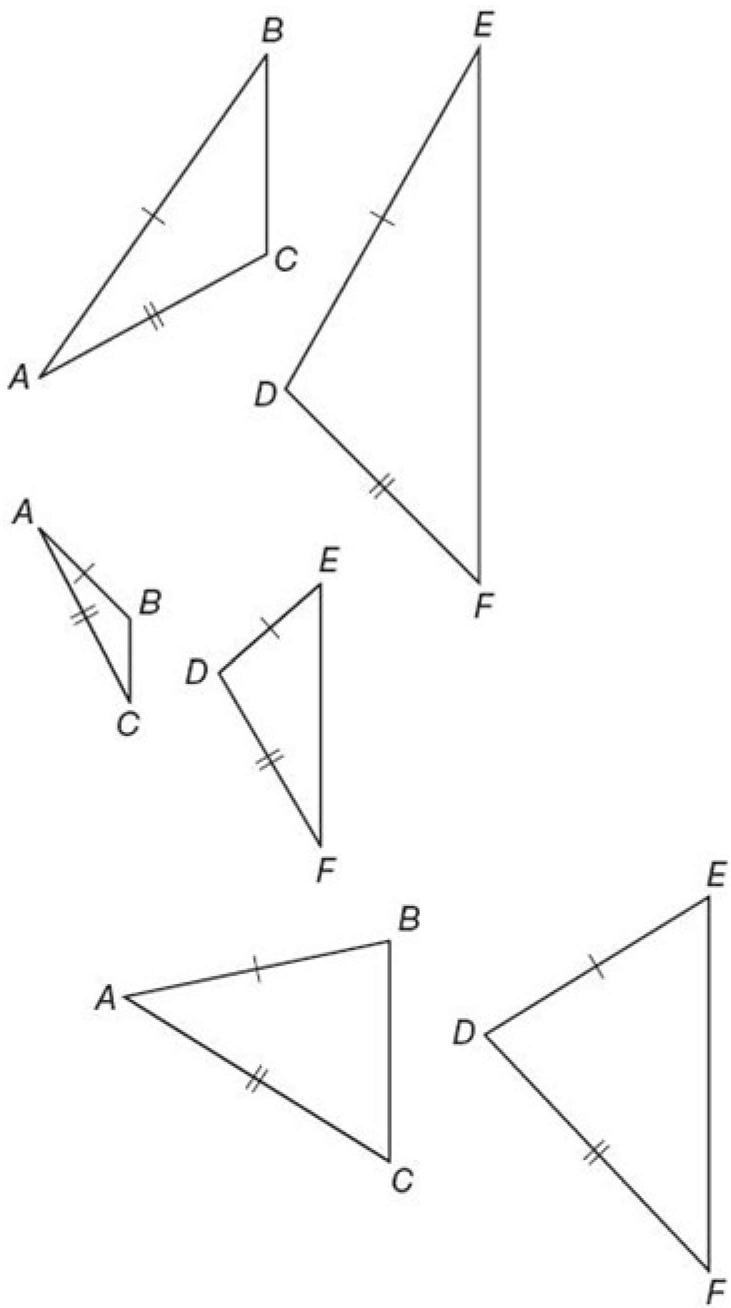
b. Use a protractor and ruler to carefully measure the indicated lengths and angle measures in the table below. Look for a pattern when comparing $m\angle A$ to $m\angle D$.

Triangle Pair	BC	$m\angle A$	EF	$m\angle D$
1	0.75	26	2	105
2	0.3	15	1	97
3	0.8	44	1.4	101

c. Sample answer: The angle opposite the longer of the two noncongruent sides is greater than the angle opposite the shorter of the two noncongruent sides.

ANSWER:

a.

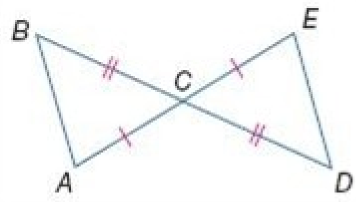


b.

Triangle Pair	BC	$m\angle A$	EF	$m\angle D$
1	0.75	26	2	105
2	0.3	15	1	97
3	0.8	44	1.4	101

c. Sample answer: The angle opposite the longer of the two noncongruent sides is greater than the angle opposite the shorter of the two noncongruent sides.

43. **CHALLENGE** What is the range of possible perimeters for figure *ABCDE* if *AC* = 7 and *DC* = 9? Explain your reasoning.



SOLUTION:

The perimeter is greater than 36 and less than 64. Sample answer: From the diagram we know that $\overline{AC} \cong \overline{EC}$ and $\overline{DC} \cong \overline{BC}$, and $\angle ACB \cong \angle ECD$ because vertical angles are congruent, so $\Delta ACB \cong \Delta ECD$.

Using the Triangle Inequality Theorem, if 9 is the longest length of the triangle, then the minimum length of $\overline{AB}$ or $\overline{ED}$ is 9 – 7 = 2. If $\overline{AB}$ or $\overline{ED}$ is the longest length of the triangle, then the maximum value is 9+7=16. Therefore, the minimum value of the total perimeter, p, of the two triangles is greater than 2(2 + 7 + 9) or 36, and the maximum value of the perimeter is less than 2(16 + 7 + 9) or 64 or, expressed as an inequality, $36 < p < 64$.

ANSWER:

The perimeter is greater than 36 and less than 64. Sample answer: From the diagram we know that $\overline{AC} \cong \overline{EC}$ and $\overline{DC} \cong \overline{BC}$, and $\angle ACB \cong \angle ECD$ because vertical angles are congruent, so $\Delta ACB \cong \Delta ECD$. Using the Triangle Inequality Theorem, the minimum value of *AB* and *ED* is 2 and the maximum value is 16. Therefore, the minimum value of the perimeter is greater than 2(2 + 7 + 9) or 36, and the maximum value of the perimeter is less than 2(16 + 7 + 9) or 64.

44. **REASONING** What is the range of lengths of each leg of an isosceles triangle if the measure of the base is 6 inches? Explain.

SOLUTION:

Each leg must be greater than 3 inches. According to the Triangle Inequality Theorem, the sum of any two sides of a triangle must be greater than the sum of the third side. Therefore. if you consider an isosceles triangle with lengths *x*, *x*, and 6, we know three inequalities must hold true: $x + x > 6$, $.x + 6 > x$, $\text{and}\, x + 6 > x$. Since the last two inequalities are the same, we will only consider the solutions of the first two.

$$\begin{aligned} x + x &> 6 \\ 2x &> 6 \qquad x + 6 > x \\ x &> 3 \qquad\quad 6 > 0 \end{aligned}$$

Since 6 >0 is always true, the solution for the lengths of the legs of the isosceles triangle is greater than 3. There is no maximum value.

ANSWER:

Each leg must be greater than 3 inches. Sample answer: When you use the Triangle Inequality Theorem to find the minimum leg length, the solution is greater than 3 inches. When you use it to find the maximum leg length, the inequality is 0 < 6, which is always true. Therefore, there is no maximum length.

45. **WRITING IN MATH** What can you tell about a triangle when given three side lengths? Include at least two items.

SOLUTION:

Sample answers: whether or not the side lengths actually form a triangle, what the smallest and largest angles are, whether the triangle is equilateral, isosceles, or scalene

ANSWER:

Sample answers: whether or not the side lengths actually form a triangle, what the smallest and largest angles are, whether the triangle is equilateral, isosceles, or scalene

46. **CHALLENGE** The sides of an isosceles triangle are whole numbers, and its perimeter is 30 units. What is the probability that the triangle is equilateral?

SOLUTION:

Let *x* be the length of the congruent sides of an isosceles triangle. Based on the Triangle Inequality Theorem and properties of isosceles triangles, we know that the following inequality can be written and solved:

$$\begin{aligned} x + x &> 30 - 2x \\ 2x &> 30 - 2x \\ 4x &> 30 \\ x &> 7.5 \end{aligned}$$

Therefore, based on the given information that the two congruent sides are whole numbers greater than 7.5 and the perimeter of the triangle is 30 units, we can create a list of possible side lengths for this triangle:

$8, 8, 14$
$9, 9, 12$
$10, 10, 10$ *
$11, 11, 8$
$12, 12, 6$
$13, 13, 4$
$14, 14, 2$

* 10,10 10 is equilateral so the probability of the triangle being equilateral is $\frac{1}{7}$.

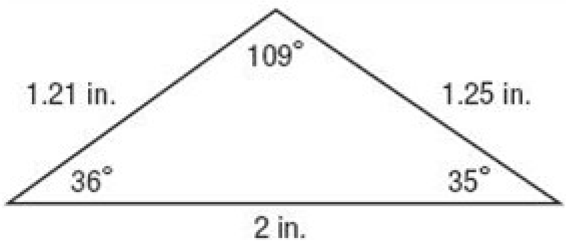
ANSWER:

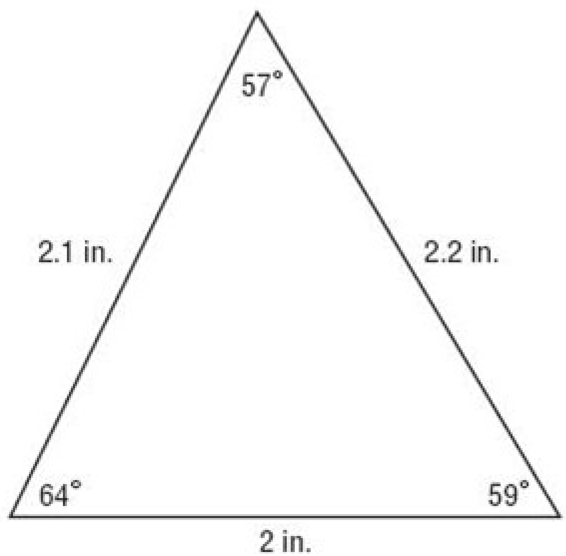
$\frac{1}{7}$

47. **OPEN ENDED** The length of one side of a triangle is 2 inches. Draw a triangle in which the 2-inch side is the shortest side and one in which the 2-inch side is the longest side. Include side and angle measures on your drawing.

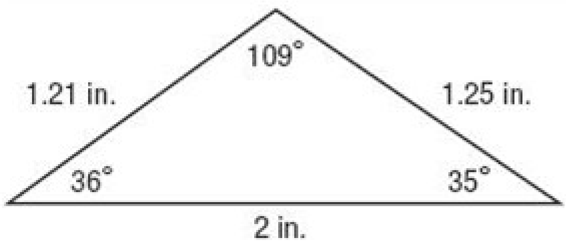
SOLUTION:

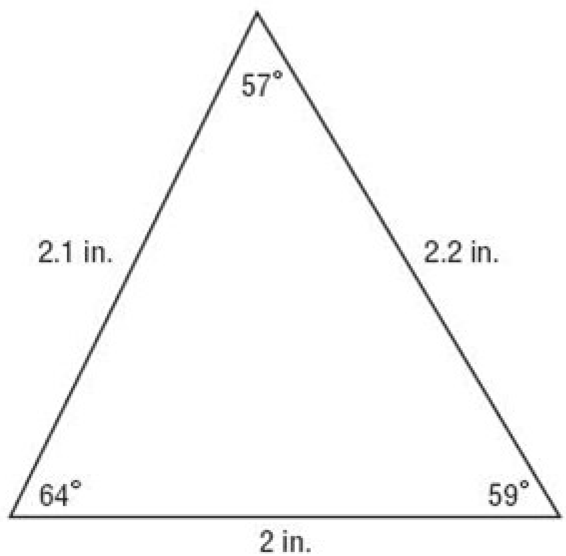
When drawing your triangles, be sure to choose side lengths that follow the conditions of the Triangle Inequality Theorem. For the triangle where 2 is the longest side length, the other two sides must each be less than 2, however, their sum must be greater than 2. For the triangle where 2 is the shortest side, one of the other sides plus 2 must have a greater sum than the length of the third side. Sample sketches are provided below.





ANSWER:


109°
1.21 in.
1.25 in.
36°
35°
2 in.


57°
2.1 in.
2.2 in.
64°
59°
2 in.

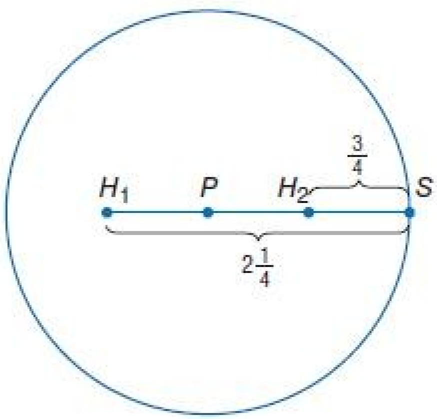
48. **WRITING IN MATH** Suppose your house is $\frac{3}{4}$ mile from a park and the park is 1.5 miles from a shopping center.

a. If your house, the park, and the shopping center are noncollinear, what do you know about the distance from your house to the shopping center? Explain your reasoning.
b. If the three locations are collinear, what do you know about the distance from your house to the shopping center? Explain your reasoning.

SOLUTION:

a. Sample answer: By the Triangle Inequality Theorem, the distance from my house to the shopping center is greater than $\frac{3}{4}$ mile and less than $2\frac{1}{4}$ miles.

b. Sample answer: The park (P) can be between my house (H_1) and the shopping center (S), which means that the distance from my house to the shopping center is $2\frac{1}{4}$ miles, or my house (H_2) can be between the park (P) and the shopping center (S), which means that the distance from my house to the shopping center is 3/4 mile.

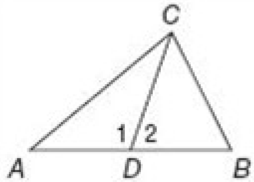


ANSWER:

a. Sample answer: By the Triangle Inequality Theorem, the distance from my house to the shopping center is greater than $\frac{3}{4}$ mile and less than $2\frac{1}{4}$ miles.
b. Sample answer: The park can be between my house and the shopping center, which means that the distance from my house to the shopping center is $2\frac{1}{4}$ miles, or my house can be between the park and the shopping center, which means that the distance from my house to the shopping center is 3/4 mile.

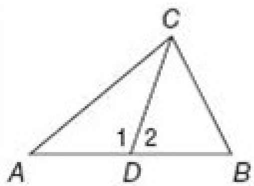
5-5 The Triangle Inequality

49. If $\overline{DC}$ is a median of $\triangle ABC$ and $m\angle 1 > m\angle 2$, which of the following statements is *not* true?



A $AD = BD$
B $m\angle ADC = m\angle BDC$
C $AC > BC$
D $m\angle 1 > m\angle B$

SOLUTION:



A $AD = BD$ This is true because D is the median of $\overline{AB}$, which means that D is the midpoint of $\overline{AB}$.
B $m\angle ADC = m\angle BDC$ This is not true because it is given that $m\angle ADC > m\angle BDC$.
C $AC > BC$ This is true. Since $m\angle 1 > m\angle 2$ and we know that $AD = DB$, then $AC > BC$.
D $m\angle 1 > m\angle B$ This is true, based on the Exterior Angle Theorem.

B is the answer.

ANSWER:
B

50. **SHORT RESPONSE** A high school soccer team has a goal of winning at least 75% of their 15 games this season. In the first three weeks, the team has won 5 games. How many more games must the team win to meet their goal?

SOLUTION:
75% of 15 is 11.25. The number of games should not be in decimals, so the team has to win at least 12 games in this season. They already won 5 games, so they must win 12 – 5 or 7 games to meet their goal.

ANSWER:
7

51. Which of the following is a logical conclusion based on the statement and its converse below?
Statement: If a polygon is a rectangle, then it has four sides.
Converse: If a polygon has four sides, then it is a rectangle.
F The statement and its converse are both true.
G The statement and its converse are both false.
H The statement is true; the converse is false.
J The statement is false; the converse is true.

SOLUTION:
The statement is correct because there exists no contradiction. All rectangles are four-sided polygons.
The converse is false because there exists a contradiction. A trapezoid is a four-sided polygon that is not a rectangle.

Thus, H is the answer.

ANSWER:
H

52. **SAT/ACT** When 7 is subtracted from $14w$, the result is z. Which of the following equations represents this statement?
A $7 - 14w = z$
B $z = 14w + 7$
C $7 - z = 14w$
D $z = 14w - 7$
E $7 + 14w = 7z$

SOLUTION:
A $7 - 14w = z$ This is not correct because $14w$ is subtracted from 7, not the other way around.
B $z = 14w + 7$ This is not correct because $14w$ and 7 are added, not subtracted.
C $7 - z = 14w$ This is not correct because the difference of 7 and z is considered, not $14w$ and 7.
D $z = 14w - 7$ This is correct.
E $7 + 14w = 7z$ This is not correct because $14w$ and 7 are added, not subtracted.

Thus, the correct answer is D.

ANSWER:
D

State the assumption you would make to start an indirect proof of each statement.

53. If $4y + 17 = 41$, then $y = 6$.

SOLUTION:
In an indirect proof or proof by contradiction, you temporarily assume that what you are trying to prove is false. By showing this assumption to be logically impossible, you prove your assumption false and the original conclusion true. For this problem, assume that $y > 6$ or $y < 6$.

$y > 6$ or $y < 6$

ANSWER:
$y > 6$ or $y < 6$

54. If two lines are cut by a transversal and a pair of alternate interior angles are congruent, then the two lines are parallel.

SOLUTION:

In an indirect proof or proof by contradiction, you temporarily assume that what you are trying to prove is false. By showing this assumption to be logically impossible, you prove your assumption false and the original conclusion true. For this problem, assume that the two lines are not parallel..

The two lines are not parallel.

ANSWER:

The two lines are not parallel.

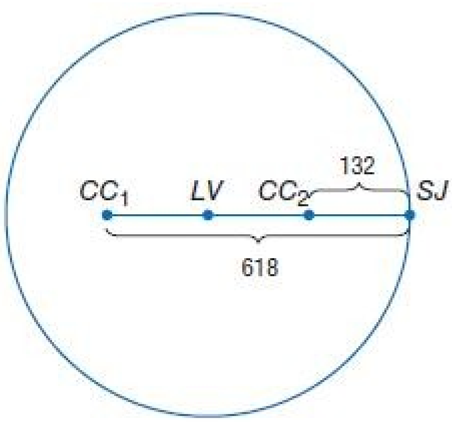
55. **GEOGRAPHY** The distance between San Jose, California, and Las Vegas, Nevada, is about 375 miles. The distance from Las Vegas to Carlsbad, California, is about 243 miles. Use the Triangle Inequality Theorem to find the possible distance between San Jose and Carlsbad.

SOLUTION:

To determine the distance between San Jose and Carlsbad, there are two cases to consider: Case 1 is that the three cities form a triangle with Las Vegas and Case 2 is that the three cities are collinear.

Case 1: If the three cities form a triangle, then we can use the Triangle Inequality Theorem to find the possible lengths for the third side.
Let d represent the distance from Carlsbad to San Jose. Based on the Triangle Inequality Theorem, the sum of the lengths of any two sides of a triangle must be greater than the third side. Therefore, $d + 375 > 243$ or $d + 243 > 375$ and combining these inequalities results in the range of values $132 < d < 618$.

However, Case 2 considers that the cities may be collinear. In this case, the distance from Carlsbad to San Jose could be determined by the sum of the distances from San Jose (SJ) to Las Vegas (LV) and Las Vegas (LV) to Carlsbad (CC 1), which makes the maximum distance 375+243= 618 miles.Or the distance from Carlsbad to San Jose could be determined by the difference of the distances from San Jose (SJ) to Las Vegas (LV) and Las Vegas (LV) to Carlsbad (CC 1), which makes the minimum distance 375 – 243 = 132 miles.

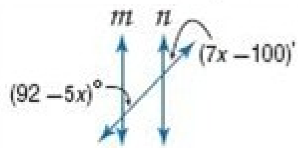


Therefore, the answer is $132 \le d \le 618$ miles.

ANSWER:

$132 \le d \le 618$ miles

Find x so that $m \parallel n$. Identify the postulate or theorem you used.

56. 

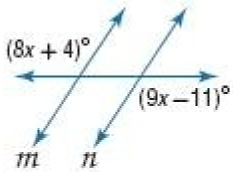
SOLUTION:

By the Corresponding Angles Postulate, $7x - 100 = 92 - 5x$.
Solve for x.

$$\begin{aligned} 7x - 100 &= 92 - 5x \\ 7x - 100 + 5x &= 92 - 5x + 5x \\ 12x - 100 &= 92 \\ 12x - 100 + 100 &= 92 + 100 \\ 12x &= 192 \\ x &= 16 \end{aligned}$$

ANSWER:

16; Corr. $\angle s$ Post.

57. 

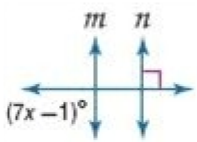
SOLUTION:

By the Alternate Exterior Angles Theorem, $9x - 11 = 8x + 4$.
Solve for x.

$$\begin{aligned} 9x - 11 &= 8x + 4 \\ 9x - 11 - 8x &= 8x + 4 - 8x \\ x - 11 &= 4 \\ x - 11 + 11 &= 4 + 11 \\ x &= 15 \end{aligned}$$

ANSWER:

15; Alt. Ext. $\angle s$ Thm.

58. 

SOLUTION:

By the Alternate Exterior Angles Theorem, $7x-1=90.$
Solve for x.

$$7x-1=90$$
$$7x-1+1=90+1$$
$$7x=91$$
$$x=13$$

ANSWER:

13; Alt. Ext. $\angle s$ Thm.

ALGEBRA Find *x* and *JK* if *J* is between *K* and *L*.

59. $KJ = 3x$, $JL = 6x$, and $KL = 12$

SOLUTION:

J is between *K* and *L*. So, $KJ + JL = KL$.
We have $KJ = 3x$, $JL = 6x$, and $KL = 8$.

Substitute.

$$3x+6x=12$$
$$9x=12$$
$$x=\frac{4}{3}$$

Find *JK*.

$$JK=3\left(\frac{4}{3}\right)$$
$$=4$$

ANSWER:

$x=\frac{4}{3}\approx 1.3$;$JK = 4$

60. $KJ = 3x - 6$, $JL = x + 6$, and $KL = 24$

SOLUTION:

J is between K and L. So, $KJ + JL = KL$.
We have $KJ = 3x - 6$, $JL = x + 6$, and $KL = 24$.

Substitute.

$$\begin{aligned} 3x - 6 + x + 6 &= 24 \\ 4x &= 24 \\ x &= 6 \end{aligned}$$

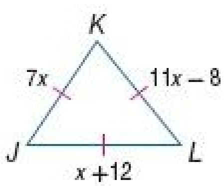
Find JK.

$$\begin{aligned} JK &= 3x - 6 \\ &= 3(6) - 6 \\ &= 18 = 6 \\ &= 12 \end{aligned}$$

ANSWER:

$x = 6$; $JK = 12$

Find x and the measures of the unknown sides of each triangle.

61. 

SOLUTION:

In the figure, $\overline{JK} \cong \overline{KL} \cong \overline{LJ}$.
So, $7x = x + 12$.

Solve for x.

$$\begin{aligned} 7x &= x + 12 \\ 7x - x &= x + 12 - x \\ 6x &= 12 \\ x &= 2 \end{aligned}$$

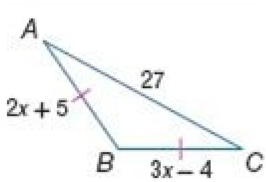
Substitute $x = 2$ in JK.

$$\begin{aligned} JK &= 7x \\ &= 7(2) \\ &= 14 \end{aligned}$$

Since all the sides are congruent, $JK = KL = LJ = 14$.

ANSWER:

$x = 2$; $JK = KL = JL = 14$

62. 

SOLUTION:

In the figure, $\overline{AB} \cong \overline{BC}$.
So, $3x - 4 = 2x + 5$.

Solve for *x*.

$$\begin{aligned} 3x - 4 &= 2x + 5 \\ 3x - 4 - 2x &= 2x + 5 - 2x \\ x - 4 &= 5 \\ x - 4 + 4 &= 5 + 4 \\ x &= 9 \end{aligned}$$

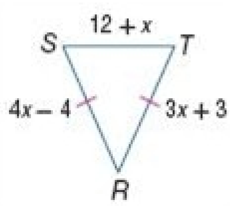
Substitute $x = 9$ in *AB* and *BC*.

$$\begin{aligned} BC &= 3x - 4 \\ &= 3(9) - 4 \\ &= 27 - 4 \\ &= 23 \end{aligned}$$

$$\begin{aligned} AB &= 2x + 5 \\ &= 2(9) + 5 \\ &= 18 + 5 \\ &= 23 \end{aligned}$$

ANSWER:

$x = 9$; $AB = BC = 23$

63. 

SOLUTION:

In the figure, $\overline{SR} \cong \overline{RT}$.
So, $4x - 4 = 3x + 3$.

Solve for *x*.

$$\begin{aligned} 4x - 4 &= 3x + 3 \\ 4x - 4 - 3x &= 3x + 3 - 3x \\ x - 4 &= 3 \\ x - 4 + 4 &= 3 + 4 \\ x &= 7 \end{aligned}$$

Substitute $x = 7$ in *SR*, *RT* , and *ST*.

$$\begin{aligned} SR &= 4x - 4 \\ &= 4(7) - 4 \\ &= 28 - 4 \\ &= 24 \end{aligned}$$

$$\begin{aligned} RT &= 3x + 3 \\ &= 3(7) + 3 \\ &= 21 + 3 \\ &= 24 \end{aligned}$$

$$\begin{aligned} ST &= 12 + x \\ &= 12 + 7 \\ &= 19 \end{aligned}$$

ANSWER:

x = 7; *SR* = *RT* = 24, *ST* = 19