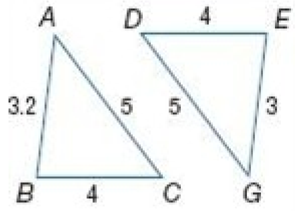
5-6 Inequalities in Two Triangles

Compare the given measures.

1. $m\angle ACB$ and $m\angle GDE$



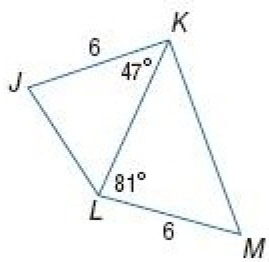
SOLUTION:

In $\triangle ABC$ and $\triangle GDE$, $BC \cong DE, AC \cong DG,$ and $AB > EG$. By the converse of the Hinge Theorem, $m\angle ACB > m\angle GDE.$

ANSWER:

$m\angle ACB > m\angle GDE$

2. JL and KM



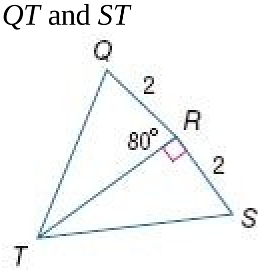
SOLUTION:

In $\triangle JKL$ and $\triangle KLM$, $JK \cong LM, KL \cong KL,$ and $m\angle JKL < m\angle KLM$. By the Hinge Theorem, $JL < KM$.

ANSWER:

$JL < KM$

3. QT and ST



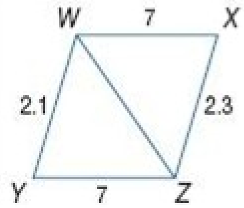
SOLUTION:

In $\triangle QRT$ and $\triangle SRT$, $QR \cong RS, RT \cong RT,$ and $m\angle QRT < m\angle SRT$. By the Hinge Theorem, $QT < ST$.

ANSWER:

$QT < ST$

4. $m\angle XWZ$ and $m\angle YZW$



SOLUTION:

In ΔXWZ and ΔYZW **,** $WZ \cong WZ, WX \cong YZ,$ and $XZ > WY$. By the converse of the Hinge Theorem, $m\angle XWZ > m\angle YZW$.

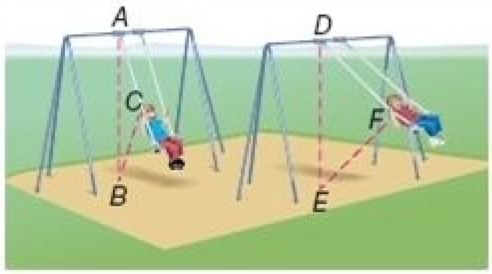
ANSWER:

$m\angle XWZ > m\angle YZW$

5. **SWINGS** The position of the swing changes based on how hard the swing is pushed.

a. Which pairs of segments are congruent?

b. Is the measure of $\angle A$ or the measure of $\angle D$ greater? Explain.



SOLUTION:

a. Since the height of each swing is the same, we know that $\overline{AB} \cong \overline{DE}$ and since the length of each chain is the same, we know that $\overline{AC} \cong \overline{DF}$.

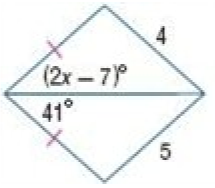
b. $\angle D$; Sample answer: Since $EF > BC$, according to the Converse of the Hinge Theorem, the included angle measure of the larger triangle is greater than the included angle measure of the smaller triangle, so since $\angle D$ is across from $\overline{EF}$ and $\angle A$ is across from $\overline{BC}$, then $m\angle D > m\angle A$.

ANSWER:

a. $\overline{AB} \cong \overline{DE}$, $\overline{AC} \cong \overline{DF}$

b. $\angle D$; Sample answer: Since $EF > BC$, according to the Converse of the Hinge Theorem, $m\angle D > m\angle A$.

Find the range of possible values for *x*.

6. 

SOLUTION:

In this figure, we have two pairs of congruent sides and the side opposite from the 41-degree angle is greater than the side opposite the (2*x* – 7) degree angle. By the converse of the Hinge Theorem, $41 > 2x - 7$.

$$\begin{aligned} 41 + 7 &> 2x - 7 + 7 \\ 48 &> 2x \\ 24 &> x \end{aligned}$$

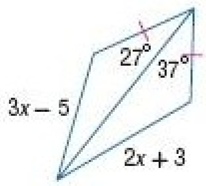
Using the fact that the measure of any angle is greater than 0, we can write a second inequality:

$$\begin{aligned} 2x - 7 &> 0 \\ 2x - 7 + 7 &> 0 + 7 \\ 2x &> 7 \\ x &> \frac{7}{2} \end{aligned}$$

Write $x > \frac{7}{2}$ and $x < 24$ as the compound inequality $\frac{7}{2} < x < 24$.

ANSWER:

$\frac{7}{2} < x < 24$.



7.

SOLUTION:

In this figure, we have two pairs of congruent sides, by the Hinge Theorem, we know that the side opposite the 37° angle is greater than the side across from the 27° angle. We can set up and solve an inequality:

$$\begin{aligned} 2x+3 &> 3x-5 \\ 2x+3-2x &> 3x-5-2x \\ 3 &> x-5 \\ 3+5 &> x-5+5 \\ 8 &> x \end{aligned}$$

Using the fact that the length of any side is greater than 0, we can write two more inequalities:

$$\begin{aligned} 2x+3 &> 0 \\ 2x &> -3 \\ x &> -\frac{3}{2} \end{aligned} \qquad\qquad \begin{aligned} 3x-5 &> 0 \\ 3x &> 5 \\ x &> \frac{5}{3} \end{aligned}$$

Since $x > -\frac{3}{2}$ is always true for any whole number measure for x, we only need to consider the second inequality $x > \frac{5}{3}$ as part of our solution. Write $x > \frac{5}{3}$ and $x < 8$ as the compound inequality $\frac{5}{3} < x < 8$.

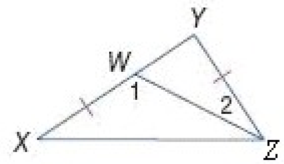
ANSWER:

$\frac{5}{3} < x < 8$.

CCSS ARGUMENTS Write a two-column proof.

8. **Given:** ΔYZX $\overline{YZ} \cong \overline{XW}$

Prove: $ZX > YW$

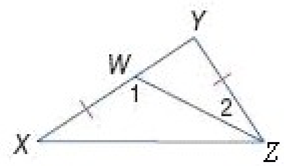


SOLUTION:

Think backwards on this proof. How can you prove that that one side of one triangle is greater than another side of a different triangle? You can show that the angles across from the longer side is greater than the angle across from the shorter side. This can be accomplished using the Exterior Angle Theorem, showing that $m\angle 1 > m\angle 2$. Start the proof by showing that you have two pairs of congruent sides so that you can use the SAS Inequality Theorem to bring everything together at the end.

Given: In ΔYZX ,$\overline{YZ} \cong \overline{XW}$

Prove: *ZX* > *YW*



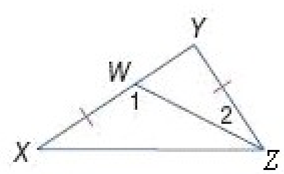
Statements (Reasons)

1. In ΔYZX ,$\overline{YZ} \cong \overline{XW}$ (Given)
2. $\overline{ZW} \cong \overline{ZW}$ (Reflexive Property)
3. $\angle 1$ is an exterior angle of ΔYZW . (Def. of ext. angle $\angle$)
4. $m\angle 1 > m\angle 2$ (Exterior Angle Inequality Theorem)
5. *ZX* > *YW* (SAS Inequality)

ANSWER:

Given: In ΔYZX , $\overline{YZ} \cong \overline{XW}$

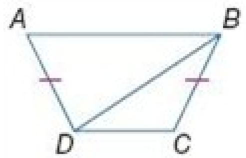
Prove: *ZX* > *YW*



Statements (Reasons)

1. ΔYZX $\overline{YZ} \cong \overline{XW}$ (Given)
2. $\overline{ZW} \cong \overline{ZW}$ (Reflexive Property)
3. $\angle 1$ is an exterior angle of ΔYZW . (Def. of ext. angle)
4. $m\angle 1 > m\angle 2$ (Exterior Angle Inequality Theorem)
5. *ZX* > *YW* (SAS Inequality)

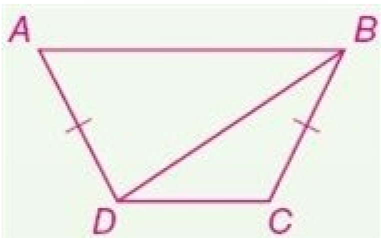
9. **Given:** $\overline{AD} \cong \overline{CB}$ $DC < AB$
Prove: $m \angle CBD < m \angle ADB$



SOLUTION:

Think backwards on this proof. How can you prove that that one angle of one triangle is less than another another angle in a different triangle? You can show that the sides across from the smaller angle is less than the angle across from the longer side. This can be accomplished using the SSS Inequality Theorem.

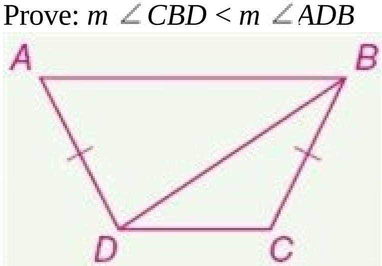
Given: $\overline{AD} \cong \overline{CB}$, $DC < AB$
Prove: $m\angle CBD < m\angle ADB$



<u>Statements (Reasons)</u>
1. $\overline{AD} \cong \overline{CB}$ (Given)
2. $\overline{DB} \cong \overline{DB}$ (Reflexive Property)
3. $DC < AB$ (Given)
4. $m\angle CBD < m\angle ADB$ (SSS Inequality)

ANSWER:

Given: $\overline{AD} \cong \overline{CB}$, $DC < AB$
Prove: $m \angle CBD < m \angle ADB$

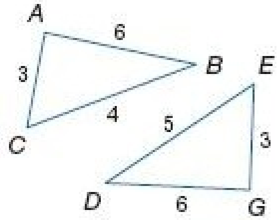


<u>Statements (Reasons)</u>
1. $\overline{AD} \cong \overline{CB}$ (Given)
2. $\overline{DB} \cong \overline{DB}$ (Reflexive Property)
3. $DC < AB$ (Given)
4. $m \angle CBD < m \angle ADB$ (SSS Inequality)

5-6 Inequalities in Two Triangles

Compare the given measures.

10. $m\angle BAC$ and $m\angle DGE$



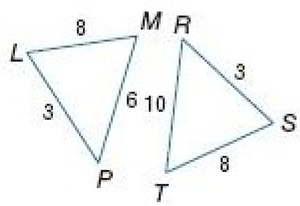
SOLUTION:

In ΔABC and ΔGDE, $AB \cong DG, AC \cong GE,$ and $BC < DE$. By the converse of the Hinge Theorem, $m\angle BAC < m\angle DGE.$

ANSWER:

$m\angle BAC < m\angle DGE$

11. $m\angle MLP$ and $m\angle TSR$



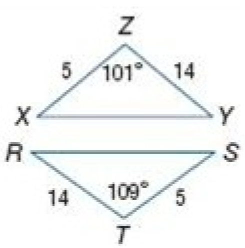
SOLUTION:

In ΔMLP and ΔRST, $LP \cong RS, LM \cong ST,$ and $MP < RT$. By the converse of the Hinge Theorem, $m\angle MLP < m\angle TSR.$

ANSWER:

$m\angle MLP < m\angle TSR$

12. SR and XY



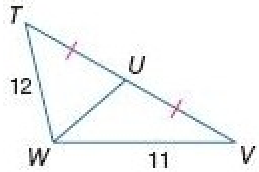
SOLUTION:

In ΔXYZ and ΔTSR, $ST \cong XZ, RT \cong ZY,$ and $m\angle RTS > m\angle XZY$. By the Hinge Theorem, $SR > XY$.

ANSWER:

$SR > XY$

13. $m \angle TUW$ and $m \angle VUW$



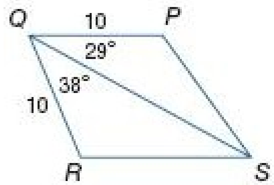
SOLUTION:

In ΔTUW and ΔVUW , $TU \cong UV, UW \cong UW,$ and $TW > WV$. By the converse of the Hinge Theorem, $m\angle TUW < m\angle VUW$.

ANSWER:

$m \angle TUW > m \angle VUW$

14. PS and SR



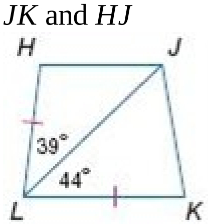
SOLUTION:

In ΔQPS and ΔQRS , $QP \cong QR, QS \cong QS,$ and $m\angle RQS > m\angle SQP$. By the Hinge Theorem, $PS < SR$.

ANSWER:

$PS < SR$

15. JK and HJ



SOLUTION:

In ΔHJL and ΔLJK , $LK \cong HL, JL \cong JL,$ and $m\angle KLJ > m\angle JLH$. By the Hinge Theorem, $JK < HJ$.

ANSWER:

$JK > HJ$

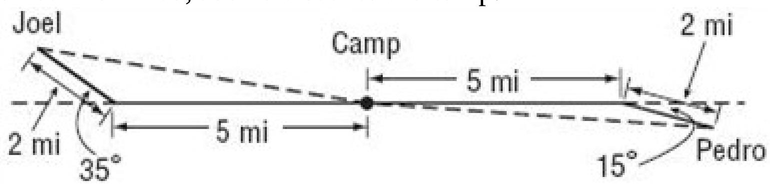
16. **CAMPING** Pedro and Joel are camping in a national park. One morning, Pedro decides to hike to the waterfall. He leaves camp and goes 5 miles east then turns $15°$ south of east and goes 2 more miles. Joel leaves the camp and travels 5 miles west, then turns $35°$ north of west and goes 2 miles to the lake for a swim.
a. When they reach their destinations, who is closer to the camp? Explain your reasoning. Include a diagram.
b. Suppose instead of turning $35°$ north of west, Joel turned $10°$ south of west. Who would then be farther from the camp? Explain your reasoning. Include a diagram.

SOLUTION:

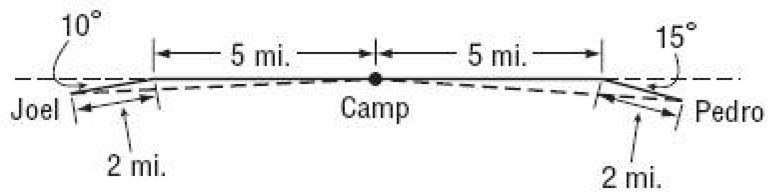
Sketch a diagram to create two triangles, one representing Pedro and another for Joel. Label the diagrams carefully, using the given information. The two triangles share two pairs of congruent sides that measure 2 and 5 miles.

Compare the angle measures created by these two sides of each triangle. The relationship between the included angles will help answer these questions. The relatively smaller angle measure will result in an opposite side length that is shorter than the one across from the relatively bigger angle measure, due to the Hinge Theorem.

a. Joel; sample answer: Pedro turned $15°$ south, so the measure of the angle across from the side of the triangle that represents his distance from the camp is 180 – 15 or 165. Joel turned $35°$ north, so the measure of the angle across from the side of the triangle that represents his distance from the camp is 180 – 35 or 145. By the Hinge Theorem, since 145 < 165, Joel is closer to the camp.

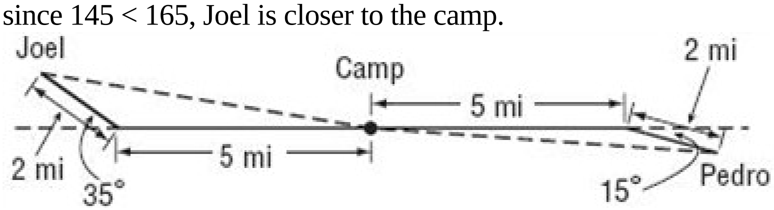


b. Joel; sample answer: Pedro turned $15°$ south, so the measure of the angle across from the side of the triangle that represents his distance from the camp is 180 – 15 or $165°$. Joel turned $10°$ south, so the measure of the angle across from the side of the triangle that represents his distance from the camp is 180 – 10 or 170. By the Hinge Theorem, since 170 > 165, Joel is farther from the camp.

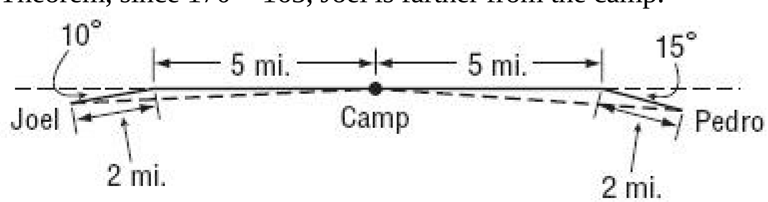


ANSWER:

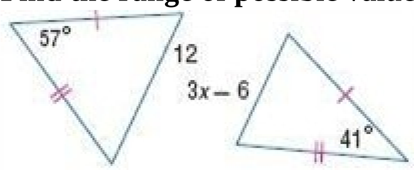
a. Joel; sample answer: Pedro turned $15°$ south, so the measure of the angle across from the side of the triangle that represents his distance from the camp is 180 – 15 or 165. Joel turned $35°$ north, so the measure of the angle across from the side of the triangle that represents his distance from the camp is 180 – 35 or 145. By the Hinge Theorem, since 145 < 165, Joel is closer to the camp.



b. Joel; sample answer: Pedro turned $15°$ south, so the measure of the angle across from the side of the triangle that represents his distance from the camp is 180 – 15 or $165°$. Joel turned $10°$ south, so the measure of the angle across from the side of the triangle that represents his distance from the camp is 180 – 10 or 170. By the Hinge Theorem, since 170 > 165, Joel is farther from the camp.



Find the range of possible values for *x*.

17. 

SOLUTION:

Since we have two pairs of congruent sides, by the Hinge Theorem, we can state that the side opposite the 57° angle is greater than the 41° angle. Therefore, we can write the inequality $12 > 3x - 6$.

$$\begin{aligned} 12 + 6 &> 3x - 6 + 6 \\ 18 &> 3x \\ 6 &> x \end{aligned}$$

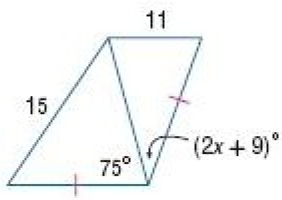
Using the fact that the measure of any side is greater than 0, we can write a second inequality.

$$\begin{aligned} 3x - 6 &> 0 \\ 3x - 6 + 6 &> 0 + 6 \\ 3x &> 6 \\ x &> 2 \end{aligned}$$

Write $x > 2$ and $x < 6$ as the compound inequality $2 < x < 6$.

ANSWER:

$2 < x < 6$

18. 

SOLUTION:

Since we have two pairs of congruent sides, by the Converse of the Hinge Theorem, we can state that the angle opposite the side with a length of 15 is greater than the angle opposite the side with a length of 11. Therefore, we can write the inequality $75 > 2x + 9$.

$$\begin{aligned} 75 &> 2x + 9 \\ 75 - 9 &> 2x + 9 - 9 \\ 66 &> 2x \\ 33 &> x \end{aligned}$$

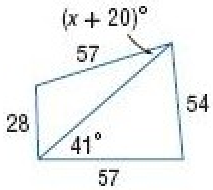
Using the fact that the measure of any angle in a polygon is greater than 0, we can write a second inequality:

$$\begin{aligned} 2x + 9 &> 0 \\ 2x + 9 - 9 &> 0 - 9 \\ x &> -\frac{9}{2} \\ x &> -4.5 \end{aligned}$$

Write $x > -4.5$ and $x < 33$ as the compound inequality $-4.5 < x < 33$.

ANSWER:

$-4.5 < x < 33$

19. 

SOLUTION:

Since we have two pairs of congruent sides, by the Converse of the Hinge Theorem, we can state that the angle opposite the side with a length of 57 is greater than the angle opposite the side with a length of 54. Therefore, we can write and solve the inequality $41 > x + 20.$

$$\begin{aligned} 41 &> x + 20 \\ 41 - 20 &> x + 20 - 20 \\ 21 &> x \end{aligned}$$

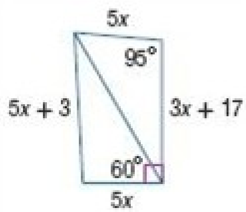
Using the fact that the measure of any angle is greater than 0, we can write a second inequality:

$$\begin{aligned} x + 20 &> 0 \\ x + 20 - 20 &> 0 - 20 \\ x &> -20 \end{aligned}$$

Write $x > -20$ and $x < 21$ as the compound inequality $-20 < x < 21.$

ANSWER:

$-20 < x < 21$

20. 

SOLUTION:

First, find the missing angle measures in the diagram. Notice that the 60° angle is part of a right angle, which makes the angle adjacent to the 60° angle equal 30° Since you already know that another angle in this triangle is 95°, you can find the missing angle, across from the $3x + 17$ side, measures 55°, using the Triangle Sum Theorem.

Since we have two pairs of congruent sides, by the Hinge Theorem, we can state that the side opposite the 60-degree angle is greater than the 55-degree angle. Therefore, we can write the inequality $5x + 3 > 3x + 17$.

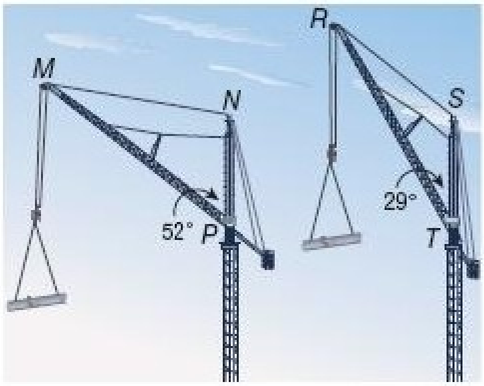
$$\begin{aligned} 5x+3 &> 3x+17 \\ 5x+3-3x &> 3x+17-3x \\ 2x+3 &> 17 \\ 2x &> 14 \\ x &> 7 \end{aligned}$$

Using the fact that any value of x greater than 7 will result in side lengths that are greater than zero, we can conclude the answer is $x < 7$.

ANSWER:

$x > 7$

21. **CRANES** In the diagram, a crane is shown lifting an object to two different heights. The length of the crane's arm is fixed, and $\overline{MP} \cong \overline{RT}$. Is $\overline{MN}$ or $\overline{RS}$ shorter? Explain your reasoning.



SOLUTION:

Use the Hinge Theorem to compare sides across from included angles of congruent sides of two triangles. We know that the cranes are the same height and the crane's arm is fixed at the same length, so we can use this to compare their included angles.

$\overline{RS}$; sample answer: The height of the crane is the same and the length of the crane arm is fixed, so according to the Hinge Theorem, the side opposite the smaller angle is shorter. Since $29° < 52°$, $\overline{RS} < \overline{MN}$.

ANSWER:

$\overline{RS}$; sample answer: The height of the crane is the same and the length of the crane arm is fixed, so according to the Hinge Theorem, the side opposite the smaller angle is shorter. Since $29° < 52°$, $\overline{RS} < \overline{MN}$.

22. **LOCKERS** Neva and Shawn both have their lockers open as shown in the diagram. Whose locker forms a larger angle? Explain your reasoning.

SOLUTION:

Use the Converse of the Hinge Theorem to compare the measures of the included angles formed by congruent side lengths of two different triangles. Since the lockers are the same width and their doors are the same length, we are able to compare the angles they form and, consequently, their opposite sides;

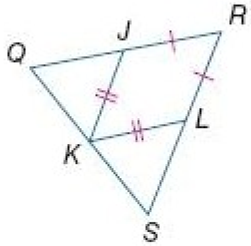
Shawn; sample answer: Since the lengths of the openings of the lockers and the lengths of the doors of the lockers are equal, use the Converse of the Hinge Theorem to determine that, since 17 in. > 12 in., the angle of the opening of Shawn's locker is greater than the angle of the opening of Neva's.

ANSWER:

Shawn; sample answer: Since the lengths of the openings of the lockers and the lengths of the doors of the lockers are equal, use the Converse of the Hinge Theorem to determine that, since 17 in. > 12 in., the angle of the opening of Shawn's locker is greater than the angle of the opening of Neva's.

CCSS ARGUMENTS Write a two-column proof.

23. **Given:** $\overline{LK} \cong \overline{JK}$, $\overline{RL} \cong \overline{RJ}$ K is the midpoint of $\overline{QS}$.
$m \angle SKL > m \angle QKJ$
Prove: $RS > QR$



SOLUTION:

There are many true statements that you can make based on this diagram and given information. Sort through the different relationships and make a list of what you see. You can write some congruent statements, such as $SK=QK$ by using the definition of midpoint, $RJ=RL$, using the definition of congruent segments, and two statements about RS and OR, using the Segment Addition Postulate. Also, you can write an inequality statement about SL and QJ, using the Hinge Theorem. Now, think about how you can put all of these relationships together to arrive at that final statement.

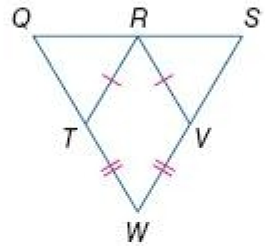
Proof:
Statements (Reasons)
1. $\overline{LK} \cong \overline{JK}$, $\overline{RL} \cong \overline{RJ}$, K is the midpoint of $\overline{QS}$, $m\angle SKL > m\angle QKJ$. (Given)
2. $SK = QK$ (Def. of midpoint)
3. $SL > QJ$ (Hinge Thm.)
4. $RL = RJ$ (Def. of $\cong$ segs.)
5. $SL + RL > RL + RJ$ (Add. Prop.)
6. $SL + RL > QJ + RJ$ (Substitution.)
7. $RS = SL + RL$, $QR = QJ + RJ$ (Seg. Add. Post.)
8. $RS > QR$ (Subst.)

ANSWER:

Proof:
Statements (Reasons)
1. $\overline{LK} \cong \overline{JK}$, $\overline{RL} \cong \overline{RJ}$, K is the midpoint of $\overline{QS}$,
$m \angle SKL > m \angle QKJ$ (Given)
2. $SK = QK$ (Def. of midpoint)
3. $SL > QJ$ (Hinge Thm.)
4. $RL = RJ$ (Def. of $\cong$ segs.)
5. $SL + RL > RL + RJ$ (Add. Prop.)
6. $SL + RL > QJ + RJ$ (Subst.)
7. $RS = SL + RL$, $QR = QJ + RJ$ (Seg. Add. Post.)
8. $RS > QR$ (Subst.)

24. **Given:** $\overline{VR} \cong \overline{RT}$, $\overline{WV} \cong \overline{WT}$ $m\angle SRV > m\angle QRT$ R is the midpoint of $\overline{SQ}$.

Prove: $WS > WQ$



SOLUTION:

There are many true statements that you can make based on this diagram and given information. Sort through the different relationships and make a list of what you see. You can write some congruent statements, such as $SR=QR$ by using the definition of midpoint, $WV=WT$, using the definition of congruent segments, and two statements about WS and WQ, using the Segment Addition Postulate. Also, you can write an inequality statement about VS and WQ, using the SAS Inequality Theorem. Now, think about how you can put all of these relationships together to arrive at that final statement.

Proof:

Statements (Reasons)

1. $\overline{VR} \cong \overline{RT}$; R is the midpoint of $\overline{SQ}$. (Given)
2. $SR = QR$ (Def. of midpoint)
3. $\overline{SR} \cong \overline{QR}$ (Def. of $\cong$ segs)
4. $m\angle SRV > m\angle QRT$ (Given)
5. $VS > TQ$ (SAS Inequality)
6. $\overline{WV} \cong \overline{WT}$ (Given)
7. $WV = WT$ (Def. of $\cong$ segs)
8. $WV + VS > WV + TQ$ (Add. Prop.)
9. $WV + VS > WT + TQ$ (Subst.)
10. $WV + VS = WS$, $WT + TQ = WQ$ (Seg. Add. Post.)
11. $WS > WQ$ (Subst.)

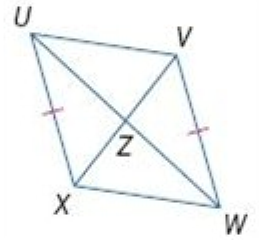
ANSWER:

Proof:

Statements (Reasons)

1. $\overline{VR} \cong \overline{RT}$; R is the midpoint of $\overline{SQ}$. (Given)
2. $SR = QR$ (Def. of midpoint)
3. $\overline{SR} \cong \overline{QR}$ (Def. of $\cong$ segs)
4. $m\angle SRV > m\angle QRT$ (Given)
5. $VS > TQ$ (SAS Inequality)
6. $\overline{WV} \cong \overline{WT}$ (Given)
7. $WV = WT$ (Def. of $\cong$ segs)
8. $WV + VS > WV + TQ$ (Add. Prop.)
9. $WV + VS > WT + TQ$ (Subst.)
10. $WV + VS = WS$, $WT + TQ = WQ$ (Seg. Add. Post.)
11. $WS > WQ$ (Subst.)

25. **Given:** $\overline{XU} \cong \overline{VW}$, $VW > XW$, $\overline{XU} \parallel \overline{VW}$
Prove: $m\angle XZU > m\angle UZV$

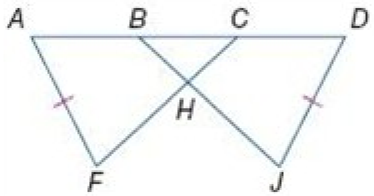


SOLUTION:
The key to this proof is to set the parallel lines to use the congruent alternate interior angles relationship to prove that $\Delta XZU \cong \Delta VZW$. Use the given $VW > XW$ and the Converse of the Hinge Theorem to prove that $m\angle VZW > m\angle XZW$. Then, because vertical angles are congruent, you can do some substitution relationships to finish this proof.

Proof:
Statements (Reasons)
1. $\overline{XU} \cong \overline{VW}$, $\overline{XU} \parallel \overline{VW}$ (Given)
2. $\angle UXV \cong \angle XVW$, $\angle XUW \cong \angle UWV$ (Alt. Int.angles Thm.)
3. $\Delta XZU \cong \Delta VZW$ (ASA)
4. $\overline{XZ} \cong \overline{WZ}$ (CPCTC)
5. $\overline{WZ} \cong \overline{WZ}$ (Refl. Prop.)
6. $VW > XW$ (Given)
7. $m\angle VZW > m\angle XZW$ (Converse of Hinge Thm.)
8. $\angle VZW \cong \angle XZU$, $\angle XZW \cong \angle VZU$ (Vert.angles are $\cong$)
9. $m\angle VZW = m\angle XZU$, $m\angle XZW \cong m\angle VZU$ (Def. of $\cong$angles)
10. $m\angle XZU > m\angle UZV$ (Subst.)

ANSWER:
Proof:
Statements (Reasons)
1. $\overline{XU} \cong \overline{VW}$, $\overline{XU} \parallel \overline{VW}$ (Given)
2. $\angle UXV \cong \angle XVW$, $\angle XUW \cong \angle UWV$ (Alt. Int. $\angle s$ Thm.)
3. $\Delta XZU \cong \Delta VZW$ (ASA)
4. $\overline{XZ} \cong \overline{WZ}$ (CPCTC)
5. $\overline{WZ} \cong \overline{WZ}$ (Refl. Prop.)
6. $VW > XW$ (Given)
7. $m\angle VZW > m\angle XZW$ (Converse of Hinge Thm.)
8. $m\angle VZW = m\angle XZU$, $m\angle XZW \cong m\angle VZU$ (Vert. $\angle s$ are $\cong$)
9. $m\angle VZW = m\angle XZU$, $m\angle XZW = m\angle VZU$ (Def. of $\cong$ $\angle s$)
10. $m\angle XZU > m\angle UZV$ (Subst.)

26. **Given:** $\overline{AF} \cong \overline{DJ}$, $\overline{FC} \cong \overline{JB}$, $AB > DC$
Prove: $m\angle AFC > m\angle DJB$



SOLUTION:

A good way to approach this proof is by thinking backwards. In order to prove that $m\angle AFC > m\angle DJB$, what side would have to be greater than what other side? Using the Converse of the Hinge Theorem, $AC > DB$ would be all you would need to prove this. So, how can you show this? You already know that $AB > DC$, so you just need to add BC to both segments, when using Segment Addition Postulate, and you're ready to start!

Proof:
Statements (Reasons)
1. $\overline{AF} \cong \overline{DJ}$, $\overline{FC} \cong \overline{JB}$, $AB > DC$ (Given)
2. $\overline{BC} \cong \overline{BC}$ (Refl. Prop.)
3. $BC = BC$ (Def. of $\cong$ segs.)
4. $AB + BC = AC$, $DC + CB = DB$ (Seg. Add. Post.)
5. $AB + BC > DC + CB$ (Add. Prop.)
6. $AC > DB$ (Subst.)
7. $m\angle AFC > m\angle DJB$ (Converse of Hinge Thm.)

ANSWER:

Proof:
Statements (Reasons)
1. $\overline{AF} \cong \overline{DJ}$, $\overline{FC} \cong \overline{JB}$, $AB > DC$ (Given)
2. $\overline{BC} \cong \overline{BC}$ (Refl. Prop.)
3. $BC = BC$ (Def. of $\cong$ segs.)
4. $AB + BC = AC$, $DC + CB = DB$ (Seg. Add. Post.)
5. $AB + BC > DC + CB$ (Add. Prop.)
6. $AC > DB$ (Subst.)
7. $m\angle AFC > m\angle DJB$ (Converse of Hinge Thm.)

27. **EXERCISE** Anica is doing knee-supported bicep curls as part of her strength training.

a. Is the distance from Anica's fist to her shoulder greater in Position 1 or Position 2? Justify your answer using measurement.
b. Is the measure of the angle formed by Anica's elbow greater in Position 1 or Position 2? Explain your reasoning.

SOLUTION:

As directed, measure the length from Anica's fist to her shoulder for part a. Then, according to the Converse of the Hinge Theorem, the angle across from the longer side will be greater than the angle across from the shorter side. We know her forearm and her upper arm are the same length in both diagrams, so the angle formed by her elbow would be considered the included angle in this problem.

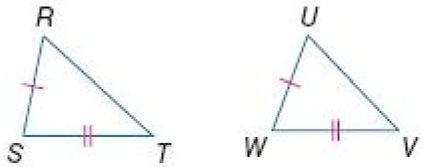
a. Position 2; sample answer: If you measure the distance from her elbow to her fist for each position, it is 1.6 cm for Position 1 and 2 cm for Position 2. Therefore, the distance from her shoulder to her fist is greater in position 2.
b. Position 2; sample answer: Using the measurements in part a and the Converse of the Hinge Theorem, you know that the measure of the angle opposite the larger side is larger, so the angle formed by Anica's elbow is greater in Position 2.

ANSWER:

a. Position 2; sample answer: If you measure the distance from her elbow to her fist for each position, it is 1.6 cm for Position 1 and 2 cm for Position 2. Therefore, the distance from her shoulder to her fist is greater in position 2.
b. Position 2; sample answer: Using the measurements in part a and the Converse of the Hinge Theorem, you know that the measure of the angle opposite the larger side is larger, so the angle formed by Anica's elbow is greater in Position 2.

PROOF Use an indirect proof to prove the SSS Inequality Theorem (Theorem 5.14).

28. **Given:** $\overline{RS} \cong \overline{UW}$ $\overline{ST} \cong \overline{WV}$, $RT > UV$
Prove: $m\angle S > m\angle W$



SOLUTION:
In an indirect proof or proof by contradiction, you temporarily assume that what you are trying to prove is false. By showing this assumption to be logically impossible, you prove your assumption false and the original conclusion true. For this problem, assume that $m\angle S \le m\angle W$, or, in other words, that either $m\angle S < m\angle W$ or $m\angle S = m\angle W$ is true.

Indirect Proof
Step 1: Assume that $m\angle S \le m\angle W$.

Step 2: If $m\angle S \le m\angle W$, then either $m\angle S < m\angle W$ or $m\angle S = m\angle W$.

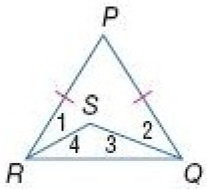
Case 1: If $m\angle S < m\angle W$,then $RT < UV$ by the SAS Inequality.

Case 2: If $m\angle S = m\angle W$, then $\triangle RST \cong \triangle UVW$ by SAS, and $\overline{RT} \cong \overline{UV}$ by CPCTC. Thus $RT = UV$.

Step 3: Both cases contradict the given $RT > UV$. Therefore, the assumption must be false, and the conclusion, $m\angle S > m\angle W$, must be true.

ANSWER:
Indirect Proof
Step 1: Assume that $m\angle S \le m\angle W$.
Step 2: If $m\angle S \le m\angle W$, then either $m\angle S < m\angle W$ or $m\angle S = m\angle W$.
Case 1: If $m\angle S < m\angle W$, then $RT < UV$ by the SAS Inequality.
Case 2: If $m\angle S = m\angle W$, then $\triangle RST \cong \triangle UVW$ by SAS, and $\overline{RT} \cong \overline{UV}$ by CPCTC. Thus $RT = UV$.
Step 3: Both cases contradict the given $RT > UV$. Therefore, the assumption must be false, and the conclusion, $m\angle S > m\angle W$, must be true.

29. **PROOF** If $\overline{PR} \cong \overline{PQ}$ and *SQ* > *SR*, write a two-column proof to prove $m\angle 1 < m\angle 2$.



SOLUTION:

The progression of this proof starts with proving that $m\angle 1 + m\angle 4 = m\angle 2 + m\angle 3$, by using the given information $(\angle PRQ \cong \angle PQR)$ and Angle Addition Postulate (showing that each individual angle is made up of the two parts). Then, since you know that *SQ* > *SR*, you know that $m\angle 4 > m\angle 3$, and consequently, $m\angle 4 = m\angle 3 + x$, because, if the measure of $\angle 4$ is greater than the measure of $\angle 3$, then it must be equal to the measure of $\angle 3$ and some other measure (Def. of inequality). Now, it's up to you to finish, using some substitution steps from the relationships you have established.

<u>Statements (Reasons)</u>

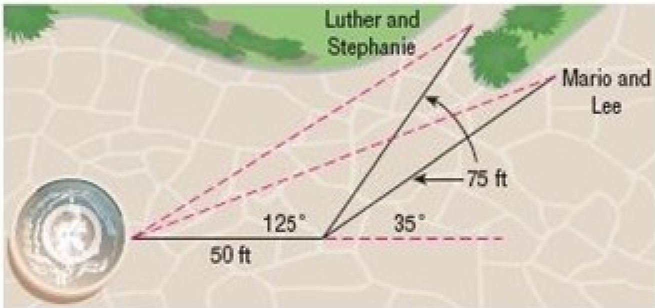
1. $\overline{PR} \cong \overline{PQ}$ (Given)
2. $\angle PRQ \cong \angle PQR$ (Isos. Δ Thm.)
3. $m\angle PRQ \cong m\angle 1 + m\angle 4$, $m\angle PQR \cong m\angle 2 + m\angle 3$ (Angle Add. Post.)
4. $m\angle PRQ = m\angle PQR$ (Def. of $\cong$ angles)
5. $m\angle 1 + m\angle 4 = m\angle 2 + m\angle 3$ (Subst.)
6. *SQ* > *SR* (Given)
7. $m\angle 4 > m\angle 3$ (Angle-Side Relationship Thm.)
8. $m\angle 4 = m\angle 3 + x$ (Def. of inequality)
9. $m\angle 1 + m\angle 4 - m\angle 4 = m\angle 2 + m\angle 3 - (m\angle 3 + x)$ (Subt. Prop.)
10. $m\angle 1 = m\angle 2 - x$ (Subtraction prop.)
11. $m\angle 1 + x = m\angle 2$ (Add. Prop.)
12. $m\angle 1 < m\angle 2$ (Def. of inequality)

ANSWER:

<u>Statements (Reasons)</u>

1. $\overline{PR} \cong \overline{PQ}$ (Given)
2. $\angle PRQ \cong \angle PQR$ (Isos. Δ Thm.)
3. $m\angle PRQ = m\angle 1 + m\angle 4$, $m\angle PQR = m\angle 2 + m\angle 3$ (Angle Add. Post.)
4. $m\angle PRQ = m\angle PQR$ (Def. of $\cong$ $\angle s$)
5. $m\angle 1 + m\angle 4 = m\angle 2 + m\angle 3$ (Subst.)
6. *SQ* > *SR* (Given)
7. $m\angle 4 > m\angle 3$ (Angle-Side Relationship Thm.)
8. $m\angle 4 = m\angle 3 + x$ (Def. of inequality)
9. $m\angle 1 + m\angle 4 - m\angle 4 = m\angle 2 + m\angle 3 - (m\angle 3 + x)$ (Subt. Prop.)
10. $m\angle 1 = m\angle 2 - x$ (Subtraction prop.)
11. $m\angle 1 + x = m\angle 2$ (Add. Prop.)
12. $m\angle 1 < m\angle 2$ (Def. of inequality)

30. **SCAVENGER HUNT** Stephanie, Mario, Lee, and Luther are participating in a scavenger hunt as part of a geography lesson. Their map shows that the next clue is 50 feet due east and then 75 feet $35°$ east of north starting from the fountain in the school courtyard. When they get ready to turn and go 75 feet $35°$ east of north, they disagree about which way to go, so they split up and take the paths shown in the diagram below.



a. Which pair chose the correct path? Explain your reasoning.
b. Which pair is closest to the fountain when they stop? Explain your reasoning.

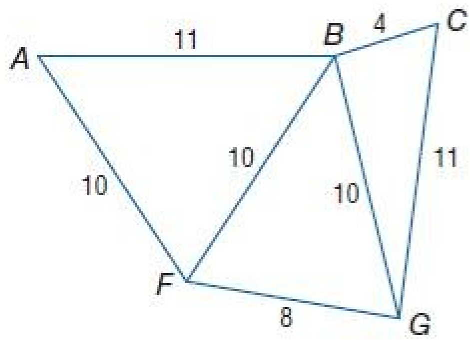
SOLUTION:

a. Luther and Stephanie; sample answer: The directions from the map were $35°$ east of north, which makes a $125°$ angle with the direction due west. Since their angle is $125°$ with the direction due west, they chose the path $35°$ east of north. Mario and Lee were incorrect because they went $35°$ *north of east*, instead of $35°$ *east of north*.
b. Luther and Stephanie; sample answer: Luther and Stephanie create a path leaving a $125°$ angle while Mario and Lee create an angle of $145°$.

ANSWER:

a. Luther and Stephanie; sample answer: The directions from the map were $35°$ east of north, which makes a $125°$ angle with the direction due west. Since their angle is $125°$ with the direction due west, they chose the path $35°$ east of north.
b. Luther and Stephanie; sample answer: Luther and Stephanie create a path leaving a $125°$ angle while Mario and Lee create an angle of $145°$.

CCSS SENSE-MAKING Use the figure to write an inequality relating the given pair of angles or segment measures.



31. *CB* and *AB*

SOLUTION:

Since $CB=4$ and $AB=11$, then $CB < AB$.

ANSWER:

$CB < AB$

32. $m\angle FBG$ and $m\angle BFA$

SOLUTION:

From the diagram, we can see that $AF = BG$, $BF = BF$, and $BA > FG$. Since $\angle FBG$ is across from $\overline{FG}$ and $\angle BFA$ is across from *BA*, then $m\angle FBG < m\angle BFA$

ANSWER:

$m\angle FBG > m\angle BFA$

33. $m\angle BGC$ and $m\angle FBA$

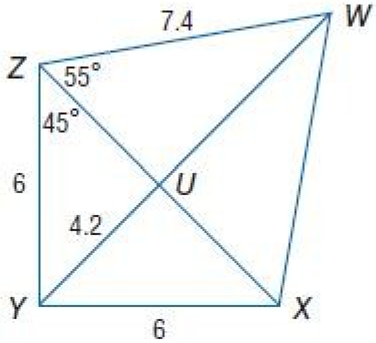
SOLUTION:

From the diagram, we can see that $AB = CG$, $BF = BG$, and BC<AF. Since $\angle BCG$ is across from $\overline{BC}$ and $\angle FBA$ is across from AF, then $m\angle BGC < m\angle FBA$.

ANSWER:

$m\angle BGC < m\angle FBA$

Use the figure to write an inequality relating the given pair of angles or segment measures.



34. $m\angle ZUY$ and $m\angle ZUW$

SOLUTION:

Since ΔYZW is an isosceles triangle, with $\angle Z$ as the vertex angle, we know that $m\angle Z = 45 + 55$ or 100 and $m\angle ZYW = m\angle ZWY = \frac{180-100}{2}$ or 40. Therefore, $m\angle ZUY = 180 - (45 + 40)$ or 95 and $m\angle ZUW = 180 - (55 + 40)$ or 85. Since $95 > 85$, then $m\angle ZUY > m\angle ZUW$.

ANSWER:

$m\angle ZUY > m\angle ZUW$

35. WU and YU

SOLUTION:

Since $\overline{YZ} \cong \overline{ZW}$, $\overline{ZU} \cong \overline{ZU}$, and 55º > 45 º, by the Hinge Theorem $WU > YU$.

ANSWER:

$WU > YU$

36. WX and XY

SOLUTION:

We know that $\overline{ZW} \cong \overline{ZY}$, $\overline{ZX} \cong \overline{ZX}$ and $m\angle WZX > m\angle YZX$. Therefore, by the Hinge Theorem, $WX > XY$.

ANSWER:

$WX > XY$

37. **MULTIPLE REPRESENTATIONS** In this problem, you will investigate properties of polygons.
a. GEOMETRIC Draw a three-sided, a four-sided, and a five-sided polygon. Label the 3-sided polygon *ABC*, the four-sided polygon *FGHJ*, and the five-sided polygon *PQRST*. Use a protractor to measure and label each angle.
b. TABULAR Copy and complete the table below.

Number of sides	Angle Measures				Sum of Angles
3	$m\angle A$		$m\angle C$		
	$m\angle B$				
4	$m\angle F$		$m\angle H$		
	$m\angle G$		$m\angle J$		
5	$m\angle P$		$m\angle S$		
	$m\angle Q$		$m\angle T$		
	$m\angle R$				

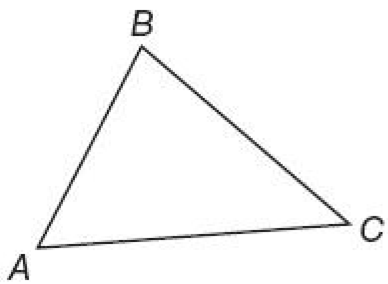
c. VERBAL Make a conjecture about the relationship between the number of sides of a polygon and the sum of the measures of the angles of the polygon.

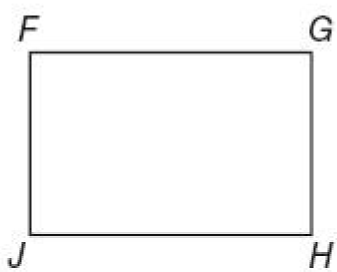
d. LOGICAL What type of reasoning did you use in part c? Explain.

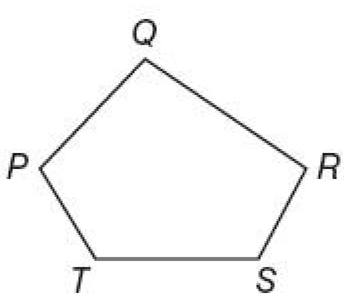
e. ALGEBRAIC Write an algebraic expression for the sum of the measures of the angles for a polygon with *n* sides.

SOLUTION:

a. To aid in measuring the angles in these polygons, it may help to draw them a bit larger. Be sure to label them as directed.







b. Measure the indicated angles and record their measures in the table below.

Number of sides	Angle Measures				Sum of Angles
3	$m\angle A$	59	$m\angle C$	45	180
	$m\angle B$	76			
4	$m\angle F$	90	$m\angle H$	90	360
	$m\angle G$	90	$m\angle J$	90	
5	$m\angle P$	105	$m\angle S$	116	540
	$m\angle Q$	100	$m\angle T$	123	
	$m\angle R$	96			

c. Refer to the last column of the table and see if you notice any pattern.
Sample answer: The sum of the angles of the polygon is equal to 180 times two less than the number of sides of the polygon.

d. Inductive reasoning is based on patterns, whereas deductive reasoning is based on facts, laws, and rules.

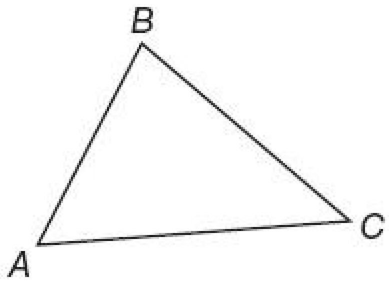
Inductive; sample answer: Since I used a pattern to determine the relationship, the reasoning I used was inductive.

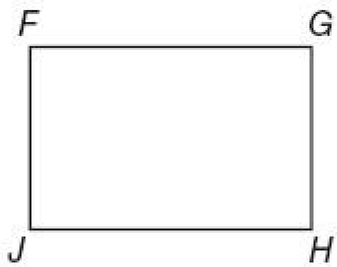
e. Compare the numbers in the "sum of angles" column. They appear to be increasing by a set amount, as the sides increase. How can you derive a formula for a polygon with *n*-number of sides? What is the smallest number of sides a polygon can have?

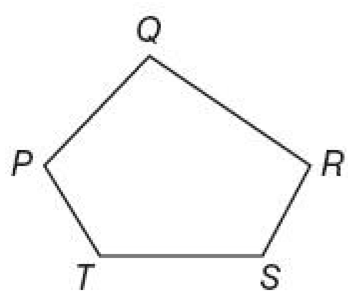
$(n - 2)180$

ANSWER:

a.







b.

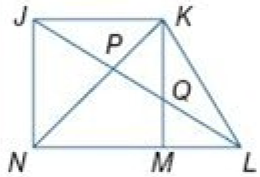
Number of sides	Angle Measures				Sum of Angles
3	$m\angle A$	59	$m\angle C$	45	180
	$m\angle B$	76			
4	$m\angle F$	90	$m\angle H$	90	360
	$m\angle G$	90	$m\angle J$	90	
5	$m\angle P$	105	$m\angle S$	116	540
	$m\angle Q$	100	$m\angle T$	123	
	$m\angle R$	96			

c. Sample answer: The sum of the angles of the polygon is equal to 180 times two less than the number of sides of the polygon.

d. Inductive; sample answer: Since I used a pattern to
determine the relationship, the reasoning I used was inductive.

e. $(n - 2)180$

38. **CHALLENGE** If $m\angle LJN > m\angle KJL$, $KJ \cong JN$, and $JN \perp NL$, which is greater, $m\angle LKN$ or $m\angle LNK$? Explain your reasoning.



SOLUTION:

In ΔJKL and ΔJNL, it is given that $\overline{KJ} \cong \overline{JN}$ and $\overline{JL} \cong \overline{JL}$. We also know that $m\angle LJN > m\angle KJL$ in these two triangles, so ,according to the Converse of the Hinge Theorem, $LN > LK$. In ΔLKN, since $LN > LK$ which means that the angles opposite those sides are related in a similar manner. Therefore, according to the Angle- Side Relationship Theorem, we can conclude that $m\angle LKN > m\angle LNK$.

ANSWER:

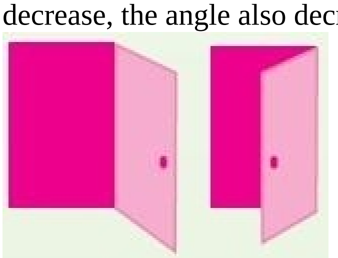
In ΔJKL and ΔJNL, it is given that $\overline{KJ} \cong \overline{JN}$ and $m\angle LJN >$ $m\angle KJL$, and $\overline{JL} \cong \overline{JL}$, so according to the Converse of the Hinge Theorem, $LN > LK$. In ΔLKN, $LN > LK$ which means that $m\angle LKN > m\angle LNK$.

39. **OPEN ENDED** Give a real-world example of an object that uses a hinge. Draw two sketches in which the hinge on your object is adjusted to two different positions. Use your sketches to explain why Theorem 5.13 is called the Hinge Theorem.

SOLUTION:

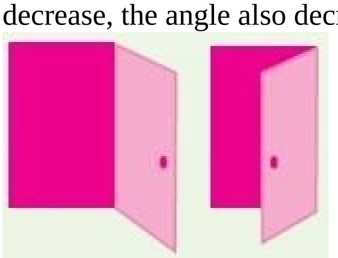
Consider what items you use daily that work using a hinge.

A door; as the door opens, the door opening increases as the angle made by the hinge increases. As the door closes, the
door opening decreases as the angle made by the hinge decreases. This is similar to the side opposite the angle in a triangle, because as the side opposite an angle increases the measure of the angle also increases. As the side decrease, the angle also decreases.



ANSWER:

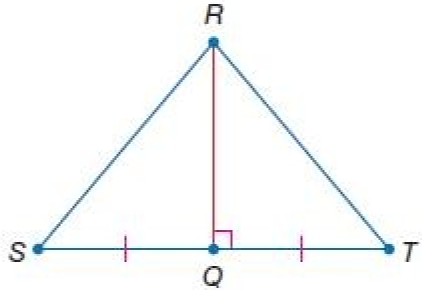
A door; as the door opens, the door opening increases as the angle made by the hinge increases. As the door closes, the door opening decreases as the angle made by the hinge decreases. This is similar to the side opposite the angle in a triangle, because as the side opposite an angle increases the measure of the angle also increases. As the side decrease, the angle also decreases.



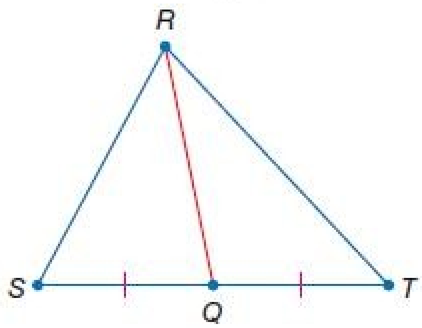
40. **CHALLENGE** Given ΔRST with median $\overline{RQ}$, if *RT* is greater than or equal to *RS*, what are the possible classifications of ΔRQT ? Explain your reasoning.

SOLUTION:
ΔRQT is either right or obtuse.

Case 1: If *RT* = *RS*, then the triangle is isosceles. Therefore, we can prove that the median is perpendicular to $\overline{TS}$. using SSS. If $m\angle Q = 90$, then ΔRQT is a right triangle.

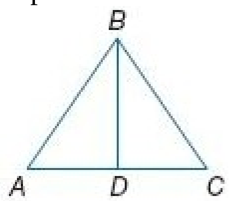


Case 2: Based on the given information, we know that *RQ* = *RQ* (Reflexive Property) and *SQ* = *QT* (Def. of median).
Therefore, we can use the Hinge Theorem to prove that, if *RT* > *RS*, then $m\angle RQT > m\angle RQS$.
Since they form a linear pair and the sum of the angles measures must be 180, $m\angle RQT$ must be greater than 90. Therefore, ΔRQT is obtuse.



ANSWER:
Right or obtuse; sample answer: If *RT* = *RS*, then the triangle is isosceles, and the median is also perpendicular to $\overline{TS}$. That would mean that both triangles formed by the median, ΔRQT and ΔRQS , are right. If *RT* > *RS*, that means that $m\angle RQT > m\angle RQS$. Since they are a linear pair and the sum of the angles measures must be 180, $m\angle RQT$ must be greater than
90 and ΔRQT is obtuse.

41. **CCSS PRECISION** If $\overline{BD}$ is a median and *AB* < *BC*, then $\angle BDC$ is sometimes, always, or never an acute angle. Explain.



SOLUTION:

We know that BD=BD (Reflexive prop) and AD=DC (Defn of median/defn of midpoint). Considering the Converse of the Hinge Theorem, if AB<BC, then we know that the angles opposite these two sides are also related in a similar way.

Never; from the converse of the Hinge Theorem, $m\angle ADB < m\angle BDC$. $\angle ADB$ and $\angle BDC$ form a linear pair. So, $m\angle ADB + m\angle BDC = 180$. Since, $m\angle ADB < m\angle BDC$, $m\angle BDC$ must be greater than 90 and $m\angle ADB$ must be smaller than 90. So, by the definition of obtuse and acute angles, $m\angle BDC$ is always obtuse and $m\angle ADB$ is always acute.

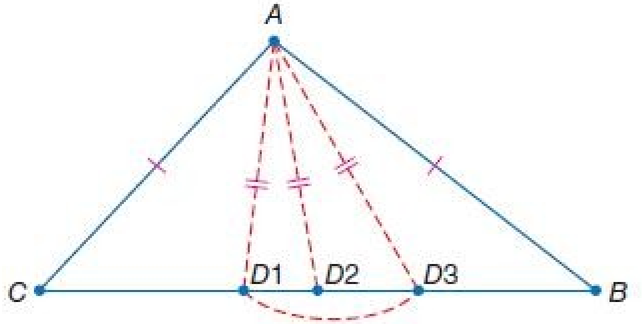
ANSWER:

Never; from the Converse of the Hinge Theorem, $\boldsymbol{m\angle ADB < m\angle BDC}$. $\boldsymbol{\angle ADB}$ **and** $\boldsymbol{\angle BDC}$ form a linear pair. So, $m\angle ADB + m\angle BDC = 180$. Since, $m\angle BDC > m\angle ADB$, $m\angle BDC$ must be greater than 90 and $m\angle ADB$ must be smaller than 90. So, by the definition of obtuse and acute angles, $m\angle BDC$ is always obtuse and $m\angle ADB$ is always acute.

42. **WRITING IN MATH** Compare and contrast the Hinge Theorem to the SAS Postulate for triangle congruence.

SOLUTION:

To answer this question, make a list of items that both SAS Postulate for triangle congruence and the Hinge Theorem have in common, as well as how they are different. Consider $\triangle ABD$ and $\triangle CBD$, where point D move to different places on $\overline{AC}$. Point D_2 is the location where the triangles are congruent.

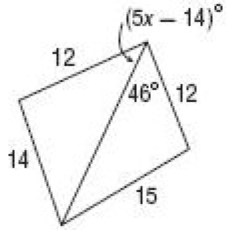


Sample answer: Both the SAS Postulate and the Hinge Theorem require that you have two pairs of corresponding side congruent ($\overline{AC} \cong \overline{AB}$ and $\overline{AD} \cong \overline{AD}$) and consider the included angle ($\angle CAD$ and $\angle BAD$). Using the SAS Postulate for triangle congruence, if the corresponding included angles are congruent (at point D_2), then the two triangles are congruent. Using the Hinge Theorem, if the one of the included angles is greater than the corresponding angle in the other triangle (at points D_1 or D_3), then the side opposite the greater angle is longer than the side opposite the lesser angle in the other triangle ($\overline{CD}_1$ or $\overline{CD}_3$)).

ANSWER:

Both the SAS Postulate and the Hinge Theorem require that you have two pairs of corresponding side congruent and consider the included angle. Using the SAS Postulate for triangle congruence, if the corresponding included angles are congruent, then the two triangles are congruent. Using the Hinge Theorem, if the one of the included angles is greater than the corresponding angle in the other triangle, then the side opposite the greater angle is longer than the side opposite the lesser angle in the other triangle.

43. **SHORT RESPONSE** Write an inequality to describe the possible range of values for *x*.



SOLUTION:

Since we have two pairs of congruent sides, by the Converse of the Hinge Theorem, we can state that the angle opposite the side with length of 15 is greater than the angle across from the side with length 14. Therefore, we can write the inequality $46 > 5x - 14$.

$$46 + 14 > 5x - 14 + 14$$
$$60 > 5x$$
$$12 > x$$

Using the fact that the measure of any angle in a polygon is greater than 0, we can write a second inequality:

$$5x - 14 > 0$$
$$5x - 14 + 14 > 0 + 14$$
$$5x > 14$$
$$x > 2.8$$

Write $x > 2.8$ and $x < 12$ as the compound inequality and we have the answer $2.8 < x < 12$.

ANSWER:

$2.8 < x < 12$

44. Which of the following is the inverse of the statement *If it is snowing, then Steve wears his snow boots*?

A If Steve wears his snow boots, then it is snowing.
B If it is not snowing, then Steve does not wear his snow boots.
C If it is not snowing, then Steve wears his snow boots.
D If it never snows, then Steve does not own snow boots.

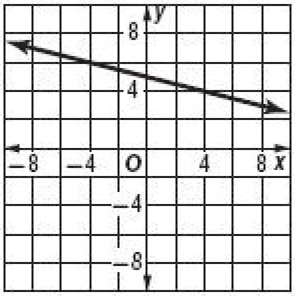
SOLUTION:

The inverse of a conditional statement is formed by negating both the hypothesis and conclusion.
So, the correct choice is B.

ANSWER:

B

45. **ALGEBRA** Which linear function best describes the graph shown below?



F $y=-\frac{1}{4}x+5$

G $y=-\frac{1}{4}x-5$

H $y=\frac{1}{4}x+5$

J $y=\frac{1}{4}x-5$

SOLUTION:

Points (4, 4) and (0, 5) are on the line.
Use the slope formula.

$$m=\frac{5-4}{0-4}$$
$$=-\frac{1}{4}$$

The y-intercept is 5.
Use the slope-intercept formula.

$$y=mx+c$$

Substitute.

$$y=-\frac{1}{4}x+5$$

So, the correct choice is F.

ANSWER:

F

46. **SAT/ACT** If the side of a square is $x + 3$, then the diagonal of the square is
A $x^2 + 1$
B $x\sqrt{2} + 3\sqrt{2}$
C $2x + 6$
D $x^2\sqrt{2} + 6$
E $x^2 + 9$

SOLUTION:
Use the Pythagorean Theorem to find the diagonal of the square.
Let d be the diagonal of the square.

$$(x+3)^2 + (x+3)^2 = d^2$$
$$2(x+3)^2 = d^2$$
$$\pm\left(\sqrt{2}(x+3)\right) = d$$

The length of a side cannot be negative, so omit the negative value.

$$\sqrt{2}(x+3) = d$$
$$x\sqrt{2} + 3\sqrt{2} = d$$

So, the correct choice is B.

ANSWER:
B

Find the range for the measure of the third side of a triangle given the measures of two sides.
47. 3.2 cm, 4.4 cm

SOLUTION:
Let n represent the length of the third side. Next, set up and solve each of the three triangle inequalities.

$$3.2 + 4.4 > n$$
$$7.6 > n$$

$$3.2 + n > 4.4$$
$$n > 1.2$$

$$4.4 + n > 3.2$$
$$n > -1.2$$

Notice that $n > -1.2$ is always true for any whole number measure for n. Combining the two remaining inequalities, the range of values that fit both inequalities is $n > 1.2$ and $n < 7.6$, which can be written as $1.2 \text{ cm} < n < 7.6 \text{ cm}$.

ANSWER:
$1.2 \text{ cm} < n < 7.6 \text{ cm}$

48. 5 ft, 10 ft

SOLUTION:

Let *n* represent the length of the third side. Next, set up and solve each of the three triangle inequalities.

$$\begin{aligned} 5+10 &> n \\ 15 &> n \end{aligned}$$

$$\begin{aligned} 5+n &> 10 \\ n &> 5 \end{aligned}$$

$$\begin{aligned} 10+n &> 5 \\ n &> -5 \end{aligned}$$

Notice that $n > -5$ is always true for any whole number measure for *n*. Combining the two remaining inequalities, the range of values that fit both inequalities is $n > 5$ and $n < 15$, which can be written as 5 ft < *n* < 15 ft.

ANSWER:

5 ft < *n* < 15 ft

49. 3 m, 9 m

SOLUTION:

Let *n* represent the length of the third side. Next, set up and solve each of the three triangle inequalities.

$$\begin{aligned} 6+12 &> n \\ 18 &> n \end{aligned}$$

$$\begin{aligned} 6+n &> 12 \\ n &> 6 \end{aligned}$$

$$\begin{aligned} 12+n &> 6 \\ n &> -6 \end{aligned}$$

Notice that $n > -6$ is always true for any whole number measure for *n*. Combining the two remaining inequalities, the range of values that fit both inequalities is $n > 6$ and $n < 18$, which can be written as 6 m < *n* < 18 m.

ANSWER:

6 m < *n* < 12 m

50. **CRUISES** Ally asked Tavia the cost of a cruise she and her best friend went on after graduation. Tavia could not remember how much it cost per person, but she did remember that the total cost was over $500. Use indirect reasoning to show that the cost for one person was more than $250.

SOLUTION:

In an indirect proof or proof by contradiction, you temporarily assume that what you are trying to prove is false. By showing this assumption to be logically impossible, you prove your assumption false and the original conclusion true. For this problem, assume that $x \le 250$ and $y \le 250$ is true.

Let the cost of Tavia's cruise be x and the other be y.

Step 1 Given: $x + y > 500$
Prove: $x > 250$ or $y > 250$
Indirect Proof:
Assume that $x \le 250$ and $y \le 250$.

Step 2 If $x \le 250$ and $y \le 250$, then $x + y \le 250 + 250$ or $x + y \le 500$. This is a contradiction because we know that $x + y > 500$.

Step 3 Since the assumption that $x \le 250$ and $y \le 250$ leads to a contradiction of a known fact, the assumption must be false.

Therefore, the conclusion that $x > 250$ or $y > 250$ must be true. Thus, the cost of at least one cruise had to cost more than $250.

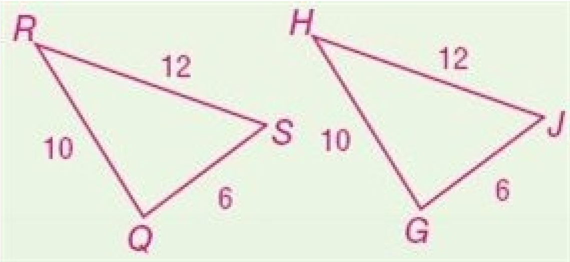
ANSWER:

Let the cost of Tavia's cruise be x and the other be y.
Step 1 Given: $x + y > 500$
Prove: $x > 250$ or $y > 250$
Indirect Proof:
Assume that $x \le 250$ and $y \le 250$
Step 2 If $x \le 250$ and $y \le 250$, then $x + y \le 250 + 250$ or $x + y \le 500$. This is a contradiction because we know that $x + y > 500$.
Step 3 Since the assumption that $x \le 250$ and $y \le 250$ leads to a
contradiction of a known fact, the assumption must be false.
Therefore, the conclusion that $x > 250$ or $y > 250$ must be true.
Thus, the cost of at least one cruise had to cost more than $250.

51. $\Delta QRS \cong \Delta GHJ$, $RS = 12$, $QR = 10$, $QS = 6$, and $HJ = 2x - 4$.

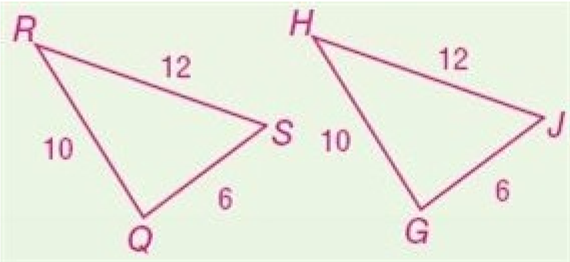
SOLUTION:

By CPCTC, $2x - 4 = 12$.
So, $x = 8$.



ANSWER:

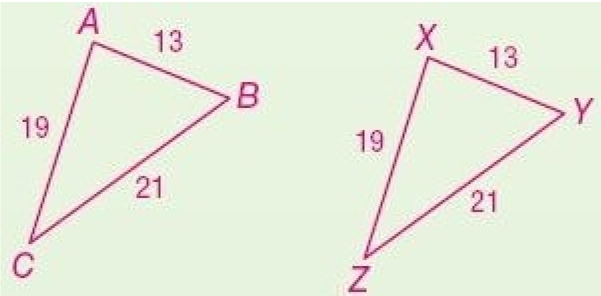
$x = 8$



52. $\Delta ABC \cong \Delta XYZ$, $AB = 13$, $AC = 19$, $BC = 21$, and $XY = 3x + 7$.

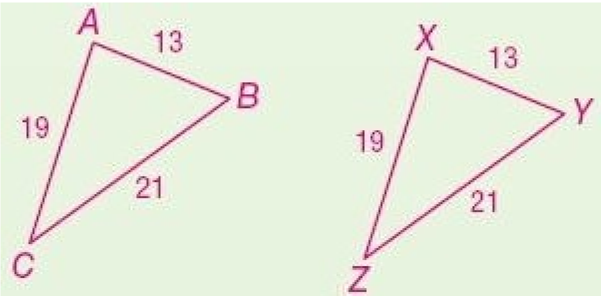
SOLUTION:

By CPCTC, $3x + 7 = 13$.
So, $x = 2$.

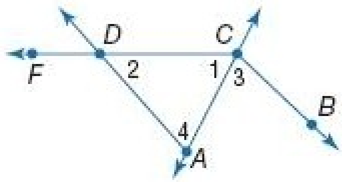


ANSWER:

$x = 2$



Use the figure.



53. Name the vertex of $\angle 4$.

SOLUTION:
A

ANSWER:
A

54. What is another name for $\angle 2$?

SOLUTION:
$\angle CDA, \angle ADC$

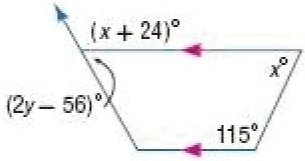
ANSWER:
$\angle CDA$, $\angle ADC$

55. What is another name for $\angle BCA$?

SOLUTION:
$\angle 3, \angle ACB$

ANSWER:
$\angle 3$, $\angle ACB$

Find the value of the variable(s) in each figure. Explain your reasoning.

56. 

SOLUTION:

By the Consecutive Interior Angles Theorem, $x + 115 = 180$.
So, $x = 65$.

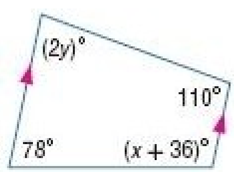
Then the exterior angle is $x + 24 = 65 + 24$ or 89.

89 and $(2y - 56)$ are supplementary angles. Therefore, by Supplementary Theorem

$$\begin{aligned} 89 + 2y - 56 &= 180 \\ 2y + 33 &= 180 \\ 2y &= 147 \\ y &= 73.5. \end{aligned}$$

ANSWER:

$x = 65$ by the Consecutive Interior Angles Theorem; $y = 73.5$ by the Supplement Theorem

57. 

SOLUTION:

By the Consecutive Interior Angles Theorem, $x + 36 + 78 = 180$.
Solve for x.

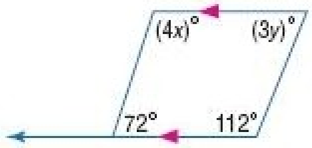
$$\begin{aligned} x + 36 + 78 &= 180 \\ x + 114 &= 180 \\ x &= 66 \end{aligned}$$

By the Consecutive Interior Angles Theorem, $2y + 110 = 180$.
Solve for y.

$$\begin{aligned} 2y + 110 &= 180 \\ 2y &= 70 \\ y &= 35 \end{aligned}$$

ANSWER:

$x = 66$ by the Consecutive Interior Angles Theorem; $y = 35$ by the Consecutive Interior Angles Theorem

58. 

SOLUTION:

By the Consecutive Interior Angles Theorem, $4x + 72 = 180$.
Solve for *x*.

$$\begin{aligned} 4x + 72 &= 180 \\ 4x &= 108 \\ x &= 27 \end{aligned}$$

By the Consecutive Interior Angles Theorem, $3y + 112 = 180$.
Solve for y.

$$\begin{aligned} 3y + 112 &= 180 \\ 3y &= 68 \\ y &= 22\frac{2}{3} \end{aligned}$$

ANSWER:

x = 27 by the Consecutive Interior Angles Theorem; $y = 22\frac{2}{3}$ by the Consecutive Interior Angles Theorem