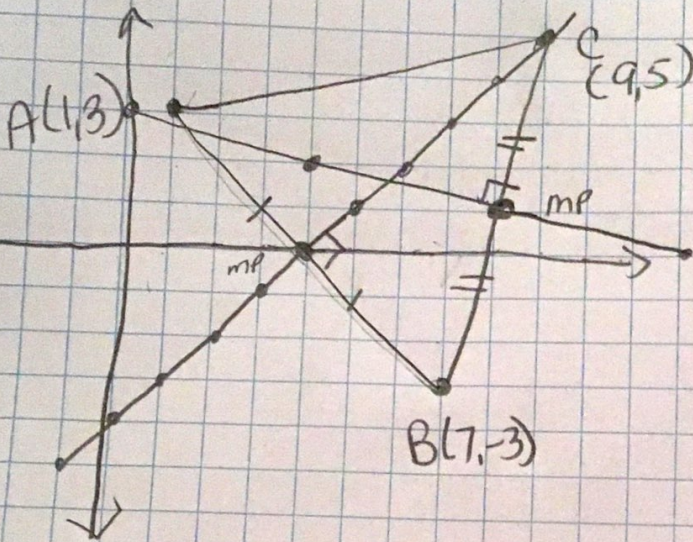


More Practice 5.2

① Circumcenter \Rightarrow point of intersection of all perpendicular bisectors



$$\text{Slope } AB = -\frac{6}{6} = \textcircled{-1}$$

$$\text{Midpoint } AB = \left(\frac{1+7}{2}, \frac{3+(-3)}{2}\right) = \textcircled{(4, 0)}$$

Equation of perpendicular bisector
 $m = 1$ through $(4, 0)$

$$y - 0 = 1(x - 4)$$

$$\boxed{y = x - 4}$$

$$\text{Slope } BC = \frac{8}{2} = \textcircled{4}$$

$$\text{Midpoint } BC = \left(\frac{7+9}{2}, \frac{-3+5}{2}\right) = \textcircled{(8, 1)}$$

Equation of perpendicular bisector
 $m = -\frac{1}{4}$ through $(8, 1)$

$$y - 1 = -\frac{1}{4}(x - 8)$$

$$y - 1 = -\frac{1}{4}x + 2$$

$$\boxed{y = -\frac{1}{4}x + 3}$$

Intersection of

$$\begin{cases} y = x - 4 \\ y = -\frac{1}{4}x + 3 \end{cases}$$

$$x - 4 = -\frac{1}{4}x + 3$$

$$\frac{4}{5} \cdot \frac{5}{4}x = \frac{7 \cdot 4}{1 \cdot 5}$$

$$\textcircled{x = \frac{28}{5}}$$

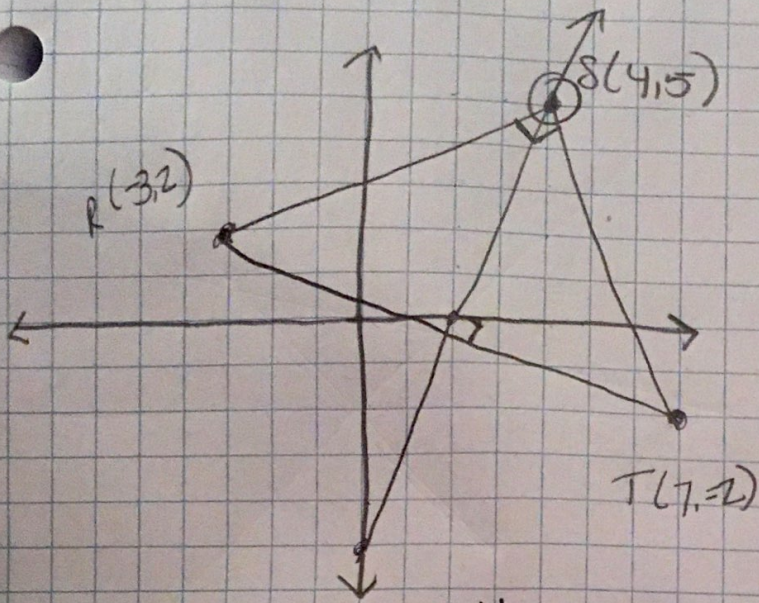
$$y = \frac{28}{5} - 4$$

$$y = \frac{28}{5} - \frac{20}{5}$$

$$\textcircled{y = \frac{8}{5}}$$

$$\boxed{\text{Circumcenter: } \left(\frac{28}{5}, \frac{8}{5}\right)}$$

② Orthocenter \rightarrow Point of intersection of all altitudes



perpendicular to opp. side

$$\text{Slope } RS = \frac{3}{7}$$

$$\perp \text{ Slope} = -\frac{7}{3}$$

opp. vertex T(-7,-2)

Equation

$$y + 2 = -\frac{7}{3}(x - 7)$$

$$y + 2 = -\frac{7}{3}x + \frac{49}{3}$$

$$\begin{array}{r} -2 \\ -2 \end{array} \qquad \begin{array}{r} -\frac{6}{3} \\ -\frac{6}{3} \end{array}$$

$$y = -\frac{7}{3}x + \frac{43}{3}$$

$$\text{Slope } RT = \frac{-4}{10} = -\frac{2}{5}$$

$$\perp \text{ Slope} = \frac{5}{2}$$

opp. vertex = S(4,5)

Equation

$$y - 5 = \frac{5}{2}(x - 4)$$

$$y - 5 = \frac{5}{2}x - 10$$

$$\begin{array}{r} y + 5 \\ +5 \end{array} \qquad \begin{array}{r} \frac{5}{2}x \\ \frac{15}{2} \end{array}$$

$$y = \frac{5}{2}x - 5$$

Intersection

$$\begin{cases} y = \frac{5}{2}x - 5 \\ y = -\frac{7}{3}x + \frac{43}{3} \end{cases}$$

$$\frac{5}{2}x - 5 = -\frac{7}{3}x + \frac{43}{3}$$

$$\begin{array}{r} +\frac{7}{3}x \\ +\frac{7}{3}x \end{array} \qquad \begin{array}{r} +\frac{15}{3} \\ +\frac{15}{3} \end{array}$$

$$\left(\frac{6}{29}\right) \frac{29}{6}x = \frac{58}{3} \left(\frac{6}{29}\right)$$

$$x = 4$$

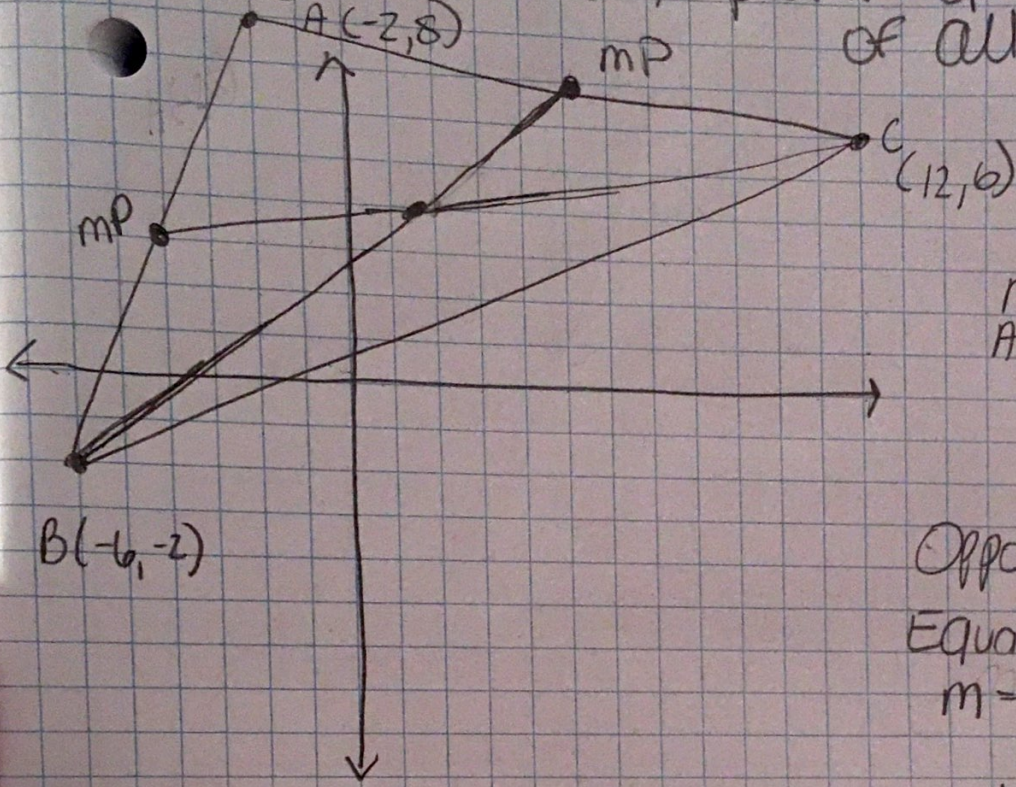
$$y = \frac{5}{2}(4) - 5$$

$$y = 5$$

Orthocenter:

(4,5)

③ Centroid \Rightarrow point of intersection of all medians



midpoint of AB $\left(\frac{-6+(-2)}{2}, \frac{8+(-2)}{2} \right)$
 $(-4, 3)$

Opposite vertex C (12, 6)

Equation of Line

$$m = \frac{3}{16}$$

$$y - 6 = \frac{3}{16}(x - 12)$$

$$y - 6 = \frac{3}{16}x - \frac{9}{4}$$

$$+6 \qquad + \frac{24}{4}$$

$$\boxed{y = \frac{3}{16}x + \frac{15}{4}}$$

midpoint of AC $\left(\frac{-2+12}{2}, \frac{8+6}{2} \right) = (5, 7)$

opposite vertex B (-6, -2)

Equation of Line

$$m = \frac{9}{11}$$

$$y + 2 = \frac{9}{11}(x + 6)$$

$$y + 2 = \frac{9}{11}x + \frac{54}{11}$$

$$-2 \qquad - \frac{22}{11}$$

$$\boxed{y = \frac{9}{11}x + \frac{32}{11}}$$

Centroid: $\left(\frac{4}{3}, 4 \right)$

Intersection

$$\begin{cases} y = \frac{3}{16}x + \frac{15}{4} \\ y = \frac{9}{11}x + \frac{32}{11} \end{cases}$$

$$\frac{3}{16}x + \frac{15}{4} = \frac{9}{11}x + \frac{32}{11}$$

$$- \frac{9}{11}x \quad - \frac{15}{4} \quad - \frac{9}{11}x \quad - \frac{15}{4}$$

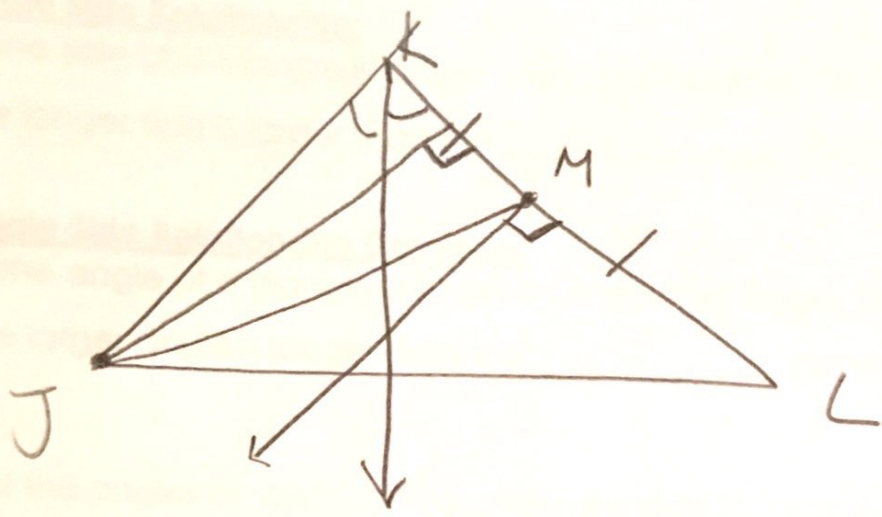
$$\frac{176}{-111} \left(\frac{-111}{176} \right) x = \frac{-37}{44} \left(\frac{-176}{111} \right)$$

$$x = \frac{4}{3}$$

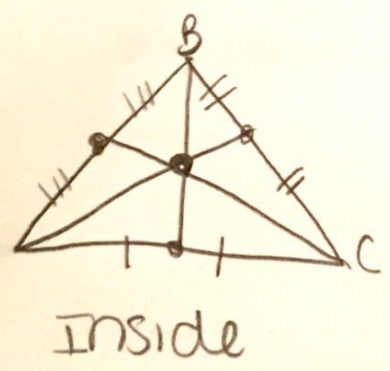
$$y = \frac{3}{16} \left(\frac{4}{3} \right) + \frac{15}{4}$$

$$y = 4$$

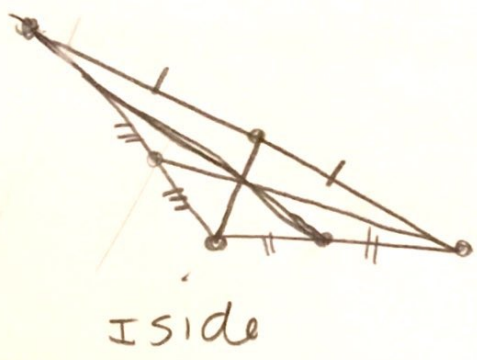
4



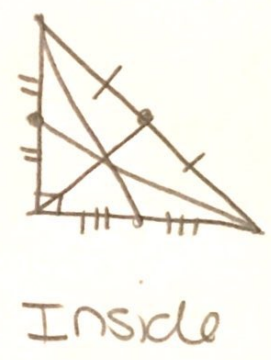
5 Acute



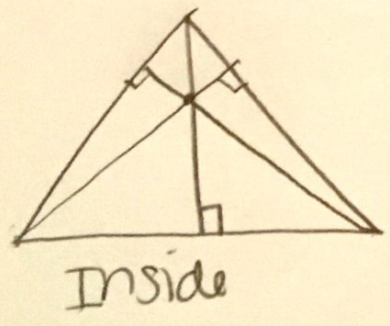
obtuse



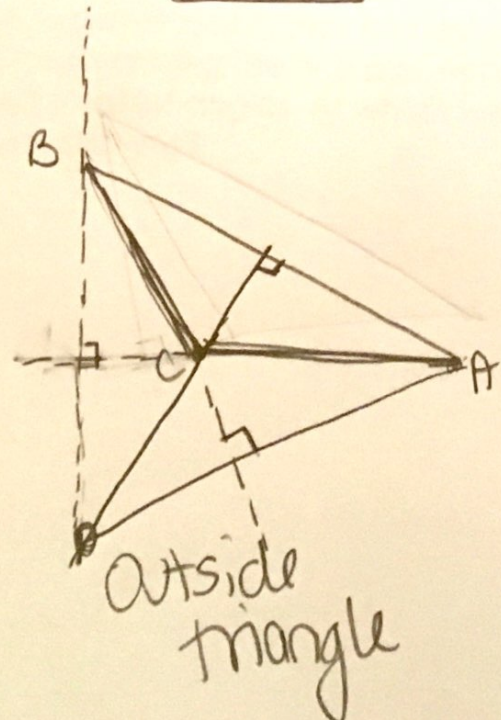
Right



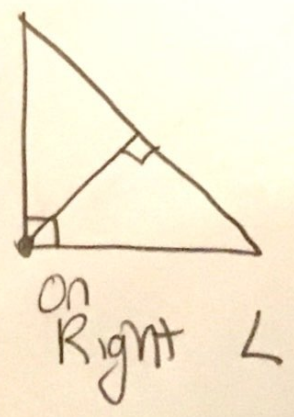
6 Acute



obtuse



Right



| object | point of concurrency | significance of point of concurrency | special distances | location of point of concurrency |
|------------------------|----------------------|--------------------------------------|--|---|
| Perpendicular Bisector | Circumcenter | center of circumscribing circle | equidistant from vertices | Acute - in Right - on hypotenuse Obtuse - outside |
| Medians | centroid | balancing point of triangle | $\frac{2}{3}$ of way from vertex to midpoint | ALL inside |
| angle Bisector | incenter | center of inscribing circle | equidistant from sides | ALL inside |
| altitudes | Orthocenter | N/A | N/A | Acute - inside Right - on Obtuse - outside |