6-1 Angles of Polygons

Find the sum of the measures of the interior angles of each convex polygon.

1. **decagon**

SOLUTION:

A decagon has ten sides. Use the Polygon Interior Angles Sum Theorem to find the sum of its interior angle measures.

Substitute $n = 10$ in $(n-2)180$.

$$\begin{aligned}(n-2)180 &= (10-2)180\\ &= 8 \cdot 180\\ &= 1440\end{aligned}$$

ANSWER:

1440

2. **pentagon**

SOLUTION:

A pentagon has five sides. Use the Polygon Interior Angles Sum Theorem to find the sum of its interior angle measures.

Substitute $n = 5$ in $(n-2)180$.

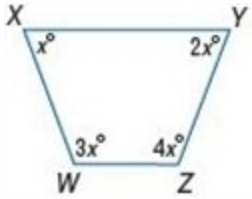
$$\begin{aligned}(n-2)180 &= (5-2)180\\ &= 3 \cdot 180\\ &= 540\end{aligned}$$

ANSWER:

540

6-1 Angles of Polygons

Find the measure of each interior angle.

3. 

SOLUTION:

The sum of the interior angle measures is $(4-2)180$ or 360.

$$m\angle X + m\angle Y + m\angle W + m\angle Z = 360$$
$$x + 2x + 3x + 4x = 360$$
$$10x = 360$$
$$x = 36$$

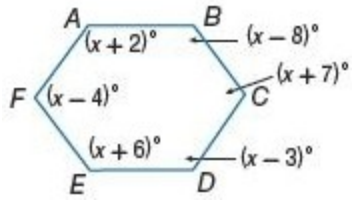
Use the value of x to find the measure of each angle.

$$m\angle X = x$$
$$= 36$$
$$m\angle Y = 2(x)$$
$$= 2(36)$$
$$= 72$$
$$m\angle W = 3(x)$$
$$= 3(36)$$
$$= 108$$
$$m\angle Z = 4(x)$$
$$= 4(36)$$
$$= 144$$

ANSWER:

$m\angle X = 36, m\angle Y = 72, m\angle Z = 144, m\angle W = 108$

6-1 Angles of Polygons

4. 

SOLUTION:

The sum of the interior angle measures is $(6-2)180$ or 720.

$$m\angle A + m\angle B + m\angle C + m\angle D + m\angle E + m\angle F = 720$$
$$(x+2)+(x-8)+(x+7)+(x-3)+(x+6)+(x-4) = 720$$
$$x+2+x-8+x+7+x-3+x+6+x-4 = 720$$
$$6x = 720$$
$$x = 120$$

Use the value of x to find the measure of each angle.

$$m\angle A = x+2 = 122$$
$$m\angle B = x-8 = 120-8 = 112$$
$$m\angle C = x+7 = 120+7 = 127$$
$$m\angle D = x-3 = 120-3 = 117$$
$$m\angle E = x+6 = 120+6 = 126$$
$$m\angle F = x-4 = 120-4 = 116$$

ANSWER:

$m\angle A = 122, m\angle B = 112, m\angle C = 127, m\angle D = 117,$
$m\angle E = 126, m\angle F = 116$

5. **AMUSEMENT** The Wonder Wheel at Coney Island in Brooklyn, New York, is a regular polygon with 16 sides. What is the measure of each interior angle of the polygon?
Refer to the photo on page 397.

SOLUTION:

The sum of the interior angle measures is $(16-2)180$ or 2520. Since this is a regular polygon, it has congruent angles and congruent sides. Let x be the measure of each interior angle of a regular polygon with 16 sides.

$$16x = 2520$$
$$x = 157.5$$

ANSWER:

157.5

The measure of an interior angle of a regular polygon is given. Find the number of sides in the polygon.

6. 150

SOLUTION:

Let n be the number of sides in the polygon. Since all angles of a regular polygon are congruent, the sum of the interior angle measures is $150n$. By the Polygon Interior Angles Sum Theorem, the sum of the interior angle measures can also be expressed as $(n-2)180$.

$$150n = (n-2)180$$
$$150n = 180n - 360$$
$$-30n = -360$$
$$n = 12$$

ANSWER:

12

7. 170

SOLUTION:

Let n be the number of sides in the polygon. Since all angles of a regular polygon are congruent, the sum of the interior angle measures is $170n$. By the Polygon Interior Angles Sum Theorem, the sum of the interior angle measures can also be expressed as $(n-2)180$.

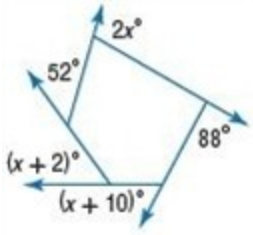
$$170n = (n-2)180$$
$$170n = 180n - 360$$
$$-10n = -360$$
$$n = 36$$

ANSWER:

36

6-1 Angles of Polygons

Find the value of x in each diagram.

8. 

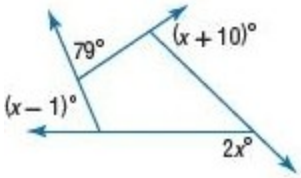
SOLUTION:

Use the Polygon Exterior Angles Sum Theorem to write an equation. Then solve for x.

$$2x+88+(x+10)+(x+2)+52=360$$
$$2x+88+x+10+x+2+52=360$$
$$4x+152=360$$
$$4x=208$$
$$x=52$$

ANSWER:

52

9. 

SOLUTION:

Use the Polygon Exterior Angles Sum Theorem to write an equation. Then solve for x.

$$79+(x+10)+2x+(x-1)=360$$
$$4x+88=360$$
$$4x=272$$
$$x=68$$

ANSWER:

68

Find the measure of each exterior angle of each regular polygon.

10. quadrilateral

SOLUTION:

A regular quadrilateral has 4 congruent sides and 4 congruent interior angles. The exterior angles are also congruent, since angles supplementary to congruent angles are congruent. Let n be the measure of each exterior angle. Use the Polygon Exterior Angles Sum Theorem to write an equation.

$4n = 360$

Solve for n.

$n = 90$

The measure of each exterior angle of a regular quadrilateral is 90.

ANSWER:

90

11. octagon

SOLUTION:

A regular octagon has 8 congruent sides and 8 congruent interior angles. The exterior angles are also congruent, since angles supplementary to congruent angles are congruent. Let n be the measure of each exterior angle.

Use the Polygon Exterior Angles Sum Theorem to write an equation.

$8n = 360$

Solve for n.

$n = 45$

The measure of each exterior angle of a regular octagon is 45.

ANSWER:

45

Find the sum of the measures of the interior angles of each convex polygon.

12. dodecagon

SOLUTION:

A dodecagon has twelve sides. Use the Polygon Interior Angles Sum Theorem to find the sum of its interior angle measures.

Substitute $n = 12$ in $(n-2)180$.

$$\begin{aligned}(n-2)180 &= (12-2)180\\ &= 10 \cdot 180\\ &= 1800\end{aligned}$$

ANSWER:

1800

13. 20-gon

SOLUTION:

A 20-gon has twenty sides. Use the Polygon Interior Angles Sum Theorem to find the sum of its interior angle measures.

Substitute $n = 20$ in $(n-2)180$.

$$\begin{aligned}(n-2)180 &= (20-2)180\\ &= 18 \cdot 180\\ &= 3240\end{aligned}$$

ANSWER:

3240

14. 29-gon

SOLUTION:

A 29-gon has twenty nine sides. Use the Polygon Interior Angles Sum Theorem to find the sum of its interior angle measures.

Substitute $n = 29$ in $(n-2)180$.

$$\begin{aligned}(n-2)180 &= (29-2)180\\ &= 27 \cdot 180\\ &= 4860\end{aligned}$$

ANSWER:

4860

15. 32-gon

SOLUTION:

A 32-gon has thirty two sides. Use the Polygon Interior Angles Sum Theorem to find the sum of its interior angle measures.

Substitute $n = 32$ in $(n-2)180$.

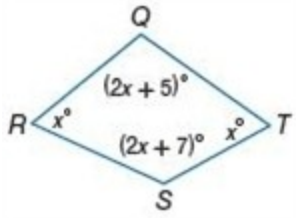
$$\begin{aligned}(n-2)180 &= (32-2)180\\ &= 30 \cdot 180\\ &= 5400\end{aligned}$$

ANSWER:

5400

Find the measure of each interior angle.

16.



SOLUTION:

The sum of the interior angle measures is $(4-2)180$ or 360.

$$m\angle Q + m\angle R + m\angle S + m\angle T = 360$$
$$(2x+5) + x + (2x+7) + x = 360$$
$$6x + 12 = 360$$
$$6x = 348$$
$$x = 58$$

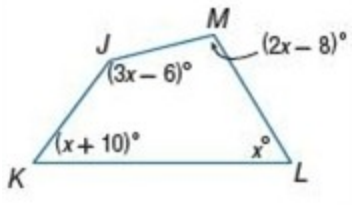
Use the value of x to find the measure of each angle.

$$m\angle Q = 2x + 5$$
$$= 2(58) + 5$$
$$= 116 + 5$$
$$= 121$$
$$m\angle R = x$$
$$= 58$$
$$m\angle S = 2x + 7$$
$$= 2(58) + 7$$
$$= 116 + 7$$
$$= 123$$
$$m\angle T = x$$
$$= 58$$

ANSWER:

$m\angle Q = 121, m\angle R = 58, m\angle S = 123, m\angle T = 58$

6-1 Angles of Polygons

17. 

SOLUTION:

The sum of the interior angle measures is $(4-2)180$ or 360.

$$m\angle J + m\angle K + m\angle L + m\angle M = 360$$
$$(3x-6)+(x+10)+x+(2x-8)=360$$
$$7x-4=360$$
$$7x=364$$
$$x=52$$

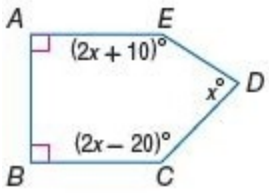
Use the value of x to find the measure of each angle.

$$m\angle J = 3x-6 = 3(52)-6 = 156-6 = 150$$
$$m\angle K = x+10 = 52+10 = 62$$
$$m\angle L = x = 52$$
$$m\angle M = 2x-8 = 2(52)-8 = 104-8 = 96$$

ANSWER:

$m\angle J = 150, m\angle K = 62, m\angle L = 52, m\angle M = 96$

6-1 Angles of Polygons

18. 

SOLUTION:

The sum of the interior angle measures is $(5-2)180$ or 540.

$$m\angle A + m\angle B + m\angle C + m\angle D + m\angle E = 540$$
$$90 + 90 + (2x - 20) + x + (2x + 10) = 540$$
$$5x + 170 = 540$$
$$5x = 370$$
$$x = 74$$

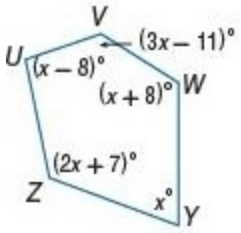
Use the value of x to find the measure of each angle.

$$m\angle A = 90$$
$$m\angle B = 90$$
$$m\angle C = 2x - 20$$
$$= 2(74) - 20$$
$$= 148 - 20$$
$$= 128$$
$$m\angle D = x$$
$$= 74$$
$$m\angle E = 2x + 10$$
$$= 2(74) + 10$$
$$= 148 + 10$$
$$= 158$$

ANSWER:

$m\angle A = 90, m\angle B = 90, m\angle C = 128, m\angle D = 74, m\angle E = 158$

6-1 Angles of Polygons

19. 

SOLUTION:

The sum of the interior angle measures is $(5-2)180$ or 540.

$$m\angle U + m\angle V + m\angle W + m\angle Y + m\angle Z = 540$$
$$(x-8) + (3x-11) + (x+8) + x + (2x+7) = 540$$
$$x - 8 + 3x - 11 + x + 8 + x + 2x + 7 = 540$$
$$8x - 4 = 540$$
$$8x = 544$$
$$x = 68$$

Use the value of x to find the measure of each angle.

$$m\angle U = x - 8 = 68 - 8 = 60$$
$$m\angle V = 3x - 11 = 3(68) - 11 = 204 - 11 = 193$$
$$m\angle W = x + 8 = 68 + 8 = 76$$
$$m\angle Y = x = 68$$
$$m\angle Z = 2x + 7 = 2(68) + 7 = 136 + 7 = 143$$

ANSWER:

$m\angle U = 60, m\angle V = 193, m\angle W = 76, m\angle Y = 68, m\angle Z = 143$

20. **BASEBALL** In baseball, home plate is a pentagon. The dimensions of home plate are shown. What is the sum of the measures of the interior angles of home plate?

SOLUTION:

A pentagon has five sides. Use the Polygon Interior Angles Sum Theorem to find the sum of its interior angle measures.

Substitute $n = 5$ in $(n-2)180$.

$$(n-2)180 = (5-2)180$$
$$= 3 \cdot 180$$
$$= 540$$

ANSWER:

540

Find the measure of each interior angle of each regular polygon.

21. dodecagon

SOLUTION:

Let n be the number of sides in the polygon and x be the measure of each interior angle of a regular polygon with 12 sides. Since all angles of a regular dodecagon are congruent, the sum of the interior angle measures is $12x$. By the Polygon Interior Angles Sum Theorem, the sum of the interior angle measures can also be expressed as $(n-2)180$.

$$12x = (n-2)180$$
$$12x = (12-2)180$$
$$12x = (10)180$$
$$12x = 1800$$
$$x = 150$$

The measure of each interior angle of a regular dodecagon is 150.

ANSWER:

150

22. pentagon

SOLUTION:

Let *n* be the number of sides in the polygon and *x* be the measure of each interior angle of a regular polygon with 5 sides. Since all angles of a regular pentagon are congruent, the sum of the interior angle measures is 5*x*. By the Polygon Interior Angles Sum Theorem, the sum of the interior angle measures can also be expressed as $(n-2)180$.

$5x = (n-2)180$

$5x = (5-2)180$

$5x = (3)180$

$5x = 540$

$x = 108$

The measure of each interior angle of a regular pentagon is 108.

ANSWER:

108

23. decagon

SOLUTION:

Let *n* be the number of sides in the polygon and *x* be the measure of each interior angle of a regular polygon with 10 sides. Since all angles of a regular decagon are congruent, the sum of the interior angle measures is 10*x*. By the Polygon Interior Angles Sum Theorem, the sum of the interior angle measures can also be expressed as $(n-2)180$.

$10x = (n-2)180$

$10x = (10-2)180$

$10x = (8)180$

$10x = 1440$

$x = 144$

The measure of each interior angle of a regular decagon is 144.

ANSWER:

144

24. nonagon

SOLUTION:

Let *n* be the number of sides in the polygon and *x* be the measure of each interior angle of a regular polygon with 9 sides. Since all angles of a regular nonagon are congruent, the sum of the interior angle measures is 9*x*. By the Polygon Interior Angles Sum Theorem, the sum of the interior angle measures can also be expressed as $(n-2)180$.

$9x = (9-2)180$

$9x = (9-2)180$

$9x = (7)180$

$9x = 1260$

$x = 140$

The measure of each interior angle of a regular nonagon is 140.

ANSWER:

140

25. **CCSS MODELING** Hexagonal chess is played on a regular hexagonal board comprised of 92 small hexagons in three colors. The chess pieces are arranged so that a player can move any piece at the start of a game.
a. What is the sum of the measures of the interior angles of the chess board?
b. Does each interior angle have the same measure? If so, give the measure. Explain your reasoning.

SOLUTION:

a. A hexagon has six sides. Use the Polygon Interior Angles Sum Theorem to find the sum of its interior angle measures.
Substitute $n = 6$ in $(n-2)180$.

$$\begin{aligned}(n-2)180 &= (6-2)180\\ &= 4 \cdot 180\\ &= 720\end{aligned}$$

b. Yes, 120; sample answer: Since the hexagon is regular, the measures of the angles are equal. That means each angle is 720 ÷ 6 or 120.

ANSWER:

a. 720
b. Yes, 120; sample answer: Since the hexagon is regular, the measures of the angles are equal. That means each angle is 720 ÷ 6 or 120.

The measure of an interior angle of a regular polygon is given. Find the number of sides in the polygon.

26. 60

SOLUTION:

Let n be the number of sides. Since all angles of a regular polygon are congruent, the sum of the interior angle measures is $60n$. By the Polygon Interior Angles Sum Theorem, the sum of the interior angle measures can also be expressed as $(n-2)180$.

$$\begin{aligned}60n &= (n-2)180\\ 60n &= 180n - 360\\ -120n &= -360\\ n &= 3\end{aligned}$$

ANSWER:

3

27. 90

SOLUTION:

Let *n* be the number of sides. Since all angles of a regular polygon are congruent, the sum of the interior angle measures is 90*n*. By the Polygon Interior Angles Sum Theorem, the sum of the interior angle measures can also be expressed as $(n-2)180$.

$$\begin{aligned} 90n &= (n-2)180 \\ 90n &= 180n - 360 \\ -90n &= -360 \\ n &= 4 \end{aligned}$$

ANSWER:

4

28. 120

SOLUTION:

Let *n* be the number of sides. Since all angles of a regular polygon are congruent, the sum of the interior angle measures is 120*n*. By the Polygon Interior Angles Sum Theorem, the sum of the interior angle measures can also be expressed as $(n-2)180$.

$$\begin{aligned} 120n &= (n-2)180 \\ 120n &= 180n - 360 \\ -60n &= -360 \\ n &= 6 \end{aligned}$$

ANSWER:

6

29. 156

SOLUTION:

Let *n* be the number of sides. Since all angles of a regular polygon are congruent, the sum of the interior angle measures is 156*n*. By the Polygon Interior Angles Sum Theorem, the sum of the interior angle measures can also be expressed as $(n-2)180$.

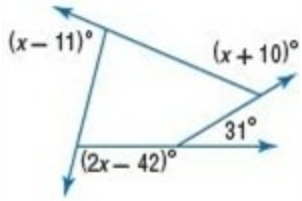
$$\begin{aligned} 156n &= (n-2)180 \\ 156n &= 180n - 360 \\ -24n &= -360 \\ n &= 15 \end{aligned}$$

ANSWER:

15

6-1 Angles of Polygons

Find the value of x in each diagram.

30. 

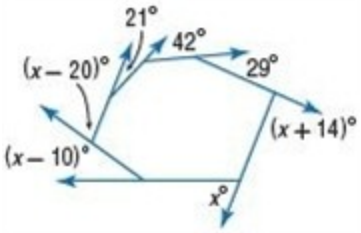
SOLUTION:

Use the Polygon Exterior Angles Sum Theorem to write an equation. Then solve for x.

$$31 + (2x - 42) + (x - 11) + (x + 10) = 360$$
$$31 + 2x - 42 + x - 11 + x + 10 = 360$$
$$4x - 12 = 360$$
$$4x = 372$$
$$x = 93$$

ANSWER:

93

31. 

SOLUTION:

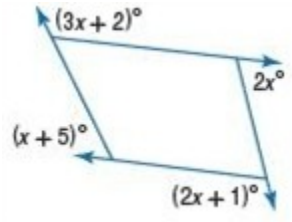
Use the Polygon Exterior Angles Sum Theorem to write an equation. Then solve for x.

$$21 + 42 + 29 + (x + 14) + x + (x - 10) + (x - 20) = 360$$
$$21 + 42 + 29 + x + 14 + x + x - 10 + x - 20 = 360$$
$$4x + 76 = 360$$
$$4x = 284$$
$$x = 71$$

ANSWER:

71

6-1 Angles of Polygons

32. 

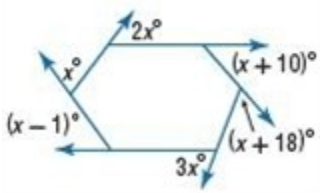
SOLUTION:

Use the Polygon Exterior Angles Sum Theorem to write an equation. Then solve for x.

$$(3x+2)+2x+(2x+1)+(x+5)=360$$
$$3x+2+2x+2x+1+x+5=360$$
$$8x+8=360$$
$$8x=352$$
$$x=44$$

ANSWER:

44

33. 

SOLUTION:

Use the Polygon Exterior Angles Sum Theorem to write an equation. Then solve for x.

$$x+2x+(x+10)+(x+18)+3x+(x-1)=360$$
$$x+2x+x+10+x+18+3x+x-1=360$$
$$9x+27=360$$
$$9x=333$$
$$x=37$$

ANSWER:

37

Find the measure of each exterior angle of each regular polygon.

34. decagon

SOLUTION:

A regular decagon has 10 congruent sides and 10 congruent interior angles. The exterior angles are also congruent, since angles supplementary to congruent angles are congruent. Let n be the measure of each exterior angle.
Use the Polygon Exterior Angles Sum Theorem to write an equation.
$10n = 360$
Solve for n.
$n = 36$
The measure of each exterior angle of a regular decagon is 36.

ANSWER:

36

35. pentagon

SOLUTION:

A regular pentagon has 5 congruent sides and 5 congruent interior angles. The exterior angles are also congruent, since angles supplementary to congruent angles are congruent. Let n be the measure of each exterior angle.
Use the Polygon Exterior Angles Sum Theorem to write an equation.
$5n = 360$
Solve for n.
$n = 72$
The measure of each exterior angle of a regular pentagon is 72.

ANSWER:

72

36. hexagon

SOLUTION:

A regular hexagon has 6 congruent sides and 6 congruent interior angles. The exterior angles are also congruent, since angles supplementary to congruent angles are congruent. Let n be the measure of each exterior angle.
Use the Polygon Exterior Angles Sum Theorem to write an equation.
$6n = 360$
Solve for n.
$n = 60$
The measure of each exterior angle of a regular hexagon is 60.

ANSWER:

60

37. 15-gon

SOLUTION:

A regular15-gon has 15 congruent sides and 15 congruent interior angles. The exterior angles are also congruent, since angles supplementary to congruent angles are congruent. Let n be the measure of each exterior angle and write and solve an equation.

$15n = 360$

$n = 24$

The measure of each exterior angle of a regular 15-gon is 24.

ANSWER:

24

38. **COLOR GUARD** During the halftime performance for a football game, the color guard is planning a new formation in which seven members stand around a central point and stretch their flag to the person immediately to their left as shown.

a. What is the measure of each exterior angle of the formation?

b. If the perimeter of the formation is 38.5 feet, how long is each flag?

SOLUTION:

a. The given formation is in the shape of a regular heptagon. A regular heptagon has 7 congruent sides and 7 congruent interior angles. The exterior angles are also congruent, since angles supplementary to congruent angles are congruent. Let n be the measure of each exterior angle.

Use the Polygon Exterior Angles Sum Theorem to write an equation.

$7n = 360$

Solve for n.

$n \approx 51.4$

The measure of each exterior angle of the formation is about 51.4.

b. To find the perimeter of a polygon, add the lengths of its sides. This formation is in the shape of a regular heptagon. Let x be the length of each flag. The perimeter of the formation is $7x$, that is, 38.5 feet.

$7x = 38.5$

$x = 5.5$

The length of each flag is 5.5 ft.

ANSWER:

a. about 51.4

b. 5.5 ft

Find the measures of an exterior angle and an interior angle given the number of sides of each regular polygon. Round to the nearest tenth, if necessary.

39. 7

SOLUTION:

The given regular polygon has 7 congruent sides and 7 congruent interior angles. The exterior angles are also congruent, since angles supplementary to congruent angles are congruent. Let n be the measure of each exterior angle.

Use the Polygon Exterior Angles Sum Theorem to write an equation.

$7n = 360$

Solve for n.

$n \approx 51.4$

The measure of each exterior angle of a 7-sided regular polygon is about 51.4.

Let n be the number of sides in the polygon and x be the measure of each interior angle of a regular polygon with 7 sides. Since all angles of a regular polygon are congruent, the sum of the interior angle measures is $7x$. By the Polygon Interior Angles Sum Theorem, the sum of the interior angle measures can also be expressed as $(n-2)180$.

$7x = (n-2)180$

$7x = (7-2)180$

$7x = (5)180$

$7x = 900$

$x \approx 128.6$

The measure of each interior angle of a regular polygon with 7 sides is about 128.6.

ANSWER:

51.4, 128.6

40. 13

SOLUTION:

The given regular polygon has 13 congruent sides and 13 congruent interior angles. The exterior angles are also congruent, since angles supplementary to congruent angles are congruent. Let n be the measure of each exterior angle.

Use the Polygon Exterior Angles Sum Theorem to write an equation.

$13n = 360$

Solve for n.

$n \approx 27.7$

The measure of each exterior angle of a 13-sided regular polygon is about 27.7.

Let n be the number of sides in the polygon and x be the measure of each interior angle of a regular polygon with 13 sides. Since all angles of a regular polygon are congruent, the sum of the interior angle measures is $13x$. By the Polygon Interior Angles Sum Theorem, the sum of the interior angle measures can also be expressed as $(n-2)180$.

$13x = (n-2)180$

$13x = (13-2)180$

$13x = (11)180$

$13x = 1980$

$x \approx 152.3$

The measure of each interior angle of a regular polygon with 13 sides is about 152.3.

ANSWER:

27.7, 152.3

41. 14

SOLUTION:

The given regular polygon has 14 congruent sides and 14 congruent interior angles. The exterior angles are also congruent, since angles supplementary to congruent angles are congruent. Let n be the measure of each exterior angle.

Use the Polygon Exterior Angles Sum Theorem to write an equation.

$14n = 360$

Solve for n.

$n \approx 25.7$

The measure of each exterior angle of a 14-sided regular polygon is about 25.7.

Let n be the number of sides in the polygon and x be the measure of each interior angle of a regular polygon with 14 sides. Since all angles of a regular polygon are congruent, the sum of the interior angle measures is $14x$. By the Polygon Interior Angles Sum Theorem, the sum of the interior angle measures can also be expressed as $(n-2)180$.

$14x = (n-2)180$

$14x = (14-2)180$

$14x = (12)180$

$13x = 2160$

$x \approx 154.3$

The measure of each interior angle of a regular polygon with 14 sides is about 154.3.

ANSWER:

25.7, 154.3

42. **PROOF** Write a paragraph proof to prove the Polygon Interior Angles Sum Theorem for octagons.

SOLUTION:

The Polygon Interior Angles Sum Theorem states that the sum of the interior angle measures of an n-sided polygon is $(n - 2)180$. So for an octagon, we need to prove that the sum of the interior angle measures is $(8 - 2)(180)$ or 1080. First, draw an octagon with all the diagonals from one vertex.

Notice that the polygon is divided up in to 6 triangles. The sum of the measures of the interior angles of each triangle is 180, so the sum of the measures of the interior angles of the octagon is $6 \cdot 180 = 1080 = (8 - 2) \cdot 180$ or $(n - 2) \cdot 180$ if n = the number of sides of the polygon.

ANSWER:

Draw all the diagonals from one vertex in an octagon.

Notice that the polygon is divided up in to 6 triangles. The sum of the measures of the interior angles of each triangle is 180, so the sum of the measures of the interior angles of the octagon is $6 \cdot 180 = 1080 = (8 - 2) \cdot 180$ or $(n - 2) \cdot 180$ if n = the number of sides of the polygon.

43. **PROOF** Use algebra to prove the Polygon Exterior Angle Sum Theorem.

SOLUTION:

The Polygon Exterior Angles Sum Theorem states that the sum of the exterior angle measures of a convex polygon is 360. So, we need to prove that the sum of the exterior angle measures of an n-gon is 360. Begin by listing what we know.

- The sum of the interior angle measures is $(n - 2)(180)$.
- Each interior angle forms a linear pair with its exterior angle.
- The sum of the measures of each linear pair is 180.

We can find the sum of the exterior angles by subtracting the sum of the interior angles from the sum of the linear pairs.

Consider the sum of the measures of the exterior angles N for an n-gon.

N = sum of measures of linear pairs – sum of measures of interior angles

$= 180n - 180(n - 2)$

$= 180n - 180n + 360$

$= 360$

So, the sum of the exterior angle measures is 360 for any convex polygon.

ANSWER:

Consider the sum of the measures of the exterior angles N for an n-gon.

N = sum of measures of linear pairs – sum of measures of interior angles

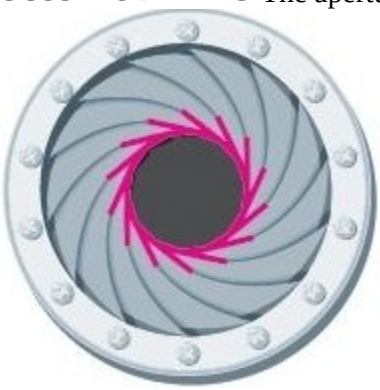
$= 180n - 180(n - 2)$

$= 180n - 180n + 360$

$= 360$

So, the sum of the exterior angle measures is 360 for any convex polygon.

44. **CCSS MODELING** The aperture on the camera lens shown is a regular 14-sided polygon.



a. What is the measure of each interior angle of the polygon?
b. What is the measure of each exterior angle of the polygon?

SOLUTION:

a. Let x be the measure of each interior angle. Since all angles of a regular polygon are congruent, the sum of the interior angle measures is $14x$. By the Polygon Interior Angles Sum Theorem, the sum of the interior angle measures can also be expressed as $(n-2)180$.

$$14x = (n-2)180$$
$$14x = (14-2)180$$
$$14x = (12)180$$
$$14x = 2160$$
$$x \approx 154.3$$

The measure of each interior angle of a regular polygon with 14 sides is about 154.3.

b. The given regular polygon has 14 congruent sides and 14 congruent interior angles. The exterior angles are also congruent, since angles supplementary to congruent angles are congruent. Let n = the measure of each exterior angle and write and solve an equation.

$$14n = 360$$
$$n \approx 25.7$$

The measure of each exterior angle of a 14-sided regular polygon is about 25.7.

ANSWER:

a. about 154.3
b. about 25.7

ALGEBRA Find the measure of each interior angle.

45. decagon, in which the measures of the interior angles are $x + 5$, $x + 10$, $x + 20$, $x + 30$, $x + 35$, $x + 40$, $x + 60$, $x + 70$, $x + 80$, and $x + 90$

SOLUTION:

A decagon has ten sides. Use the Polygon Interior Angles Sum Theorem to find the sum of its interior angle measures.

Substitute $n = 10$ in $(n-2)180$.

$$\begin{aligned}(n-2)180 &= (10-2)180\\ &= 8 \cdot 180\\ &= 1440\end{aligned}$$

$$\left.\begin{aligned}&(x+5)+(x+10)+(x+20)+(x+30)+(x+35)+\\&(x+40)+(x+60)+(x+70)+(x+80)+(x+90)\end{aligned}\right\} = 1440$$

$$\left.\begin{aligned}&x+5+x+10+x+20+x+30+x+35+\\&x+40+x+60+x+70+x+80+x+90\end{aligned}\right\} = 1440$$

$$\begin{aligned}10x + 440 &= 1440\\ 10x &= 1000\\ x &= 100\end{aligned}$$

Use the value of x to find the measure of each angle.

the measures of the interior angles are 105, 110, 120, 130, 135, 140, 160, 170, 180, and 190.

ANSWER:

105, 110, 120, 130, 135, 140, 160, 170, 180, 190

46. polygon *ABCDE*, in which the measures of the interior angles are $6x$, $4x + 13$, $x + 9$, $2x - 8$, $4x - 1$

SOLUTION:

A pentagon has five sides. Use the Polygon Interior Angles Sum Theorem to find the sum of its interior angle measures.

Substitute $n = 5$ in $(n-2)180$.

$$\begin{aligned}(n-2)180 &= (5-2)180\\ &= 3\cdot 180\\ &= 540\end{aligned}$$

$$\begin{aligned}6x+(4x+13)+(x+9)+(2x-8)+(4x-1) &= 540\\ 6x+4x+13+x+9+2x-8+4x-1 &= 540\\ 13+9-8+17x-1 &= 540\\ 17x+13 &= 540\\ 17x &= 527\\ x &= 31\end{aligned}$$

Use the value of x to find the measure of each angle.

$$\begin{aligned}m\angle A &= 6x\\ &= 6(31)\\ &= 186\end{aligned}$$

$$\begin{aligned}m\angle B &= 4x+13\\ &= 4(31)+13\\ &= 124+13\\ &= 137\end{aligned}$$

$$\begin{aligned}m\angle C &= x+9\\ &= 32+9\\ &= 40\end{aligned}$$

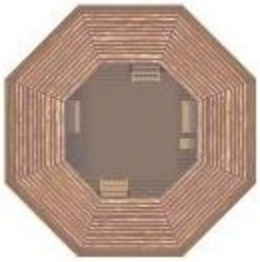
$$\begin{aligned}m\angle D &= 2x-8\\ &= 2(31)-8\\ &= 62-8\\ &= 54\end{aligned}$$

$$\begin{aligned}m\angle E &= 4x-1\\ &= 4(31)-1\\ &= 124-1\\ &= 123\end{aligned}$$

ANSWER:

$m\angle A = 186,\ m\angle B = 137,\ m\angle C = 40,\ m\angle D = 54,\ m\angle E = 123$

47. **THEATER** The drama club would like to build a theater in the round so the audience can be seated on all sides of the stage for its next production.



a. The stage is to be a regular octagon with a total perimeter of 60 feet. To what length should each board be cut to form the sides of the stage?
b. At what angle should each board be cut so that they will fit together as shown? Explain your reasoning.

SOLUTION:

a. Let x be the length of each side. The perimeter of the regular octagon is $8x$, that is, 60 feet.

$$8x = 60$$

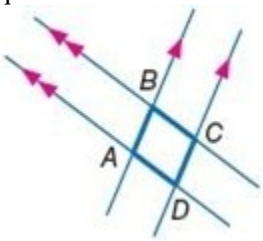
$$x = 7.5$$

The length of each side is 7.5 ft.
b. First find the measure of each interior angle of a regular octagon. Since each interior angle is comprised of two boards, divide by 2 to find the angle of each board. The measure of each angle of a regular octagon is 135, so if each side of the board makes up half of the angle, each one measures 135 ÷ 2 or 67.5.

ANSWER:

a. 7.5 ft
b. 67.5; Sample answer: The measure of each angle of a regular octagon is 135, so if each side of the board makes up half of the angle, each one measures 135 ÷ 2 or 67.5.

48. **MULTIPLE REPRESENTATIONS** In this problem, you will explore angle and side relationships in special quadrilaterals.



a. GEOMETRIC Draw two pairs of parallel lines that intersect like the ones shown. Label the quadrilateral formed by *ABCD*. Repeat these steps to form two additional quadrilaterals, *FGHJ* and *QRST*.
b. TABULAR Copy and complete the table below.

Quadrilateral	Lengths and Measures							
ABCD	$m\angle A$		$m\angle B$		$m\angle C$		$m\angle D$	
	AB		*BC*		*CD*		*DA*	
FGHJ	$m\angle F$		$m\angle G$		$m\angle H$		$m\angle J$	
	FG		*GH*		*HJ*		*JF*	
QRST	$m\angle Q$		$m\angle R$		$m\angle S$		$m\angle T$	
	QR		*RS*		*ST*		*TQ*	

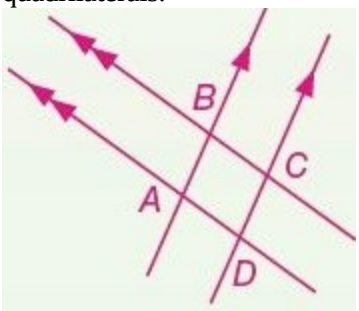
c. VERBAL Make a conjecture about the relationship between two consecutive angles in a quadrilateral formed by two pairs of parallel lines.
d. VERBAL Make a conjecture about the relationship between the angles adjacent to each other in a quadrilateral

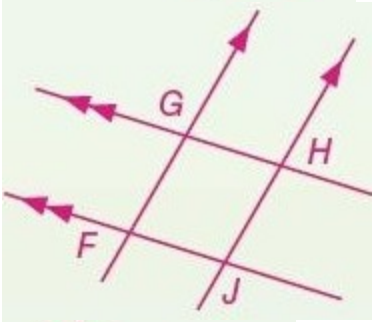
formed by two pairs of parallel lines.

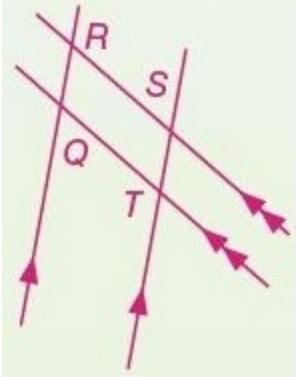
e. VERBAL Make a conjecture about the relationship between the sides opposite each other in a quadrilateral formed by two pairs of parallel lines.

SOLUTION:

a. Use a straightedge to draw each pair of parallel lines. Label the intersections on each figure to form 3 quadrilaterals.







b. Using a protractor and a ruler to measure each side and angle, complete the table.

Quadrilateral	Lengths and Measures							
ABCD	$m\angle A$	101	$m\angle B$	79	$m\angle C$	101	$m\angle D$	79
	AB	0.6 cm	BC	0.6 cm	CD	0.6 cm	DA	0.6 cm
FGHJ	$m\angle F$	76	$m\angle G$	104	$m\angle H$	76	$m\angle J$	104
	FG	1 cm	GH	0.9 cm	HJ	1 cm	JF	0.9 cm
QRST	$m\angle Q$	121	$m\angle R$	59	$m\angle S$	121	$m\angle T$	59
	QR	0.5 cm	RS	1.2 cm	ST	0.5 cm	TQ	1.2 cm

c. Each of the quadrilaterals was formed by 2 pairs of parallel lines. From the table it is shown that the measures of the angles that are opposites are the same. So, the angles opposite each other in a quadrilateral formed by two pairs of parallel lines are congruent.

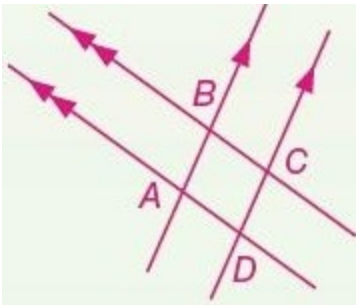
d. Each of the quadrilaterals was formed by 2 pairs of parallel lines. From the table it is shown that the measures of the consecutive angles in each quadrilateral add to 180. So, the angles adjacent to each other in a quadrilateral formed by two pairs of parallel lines are supplementary.

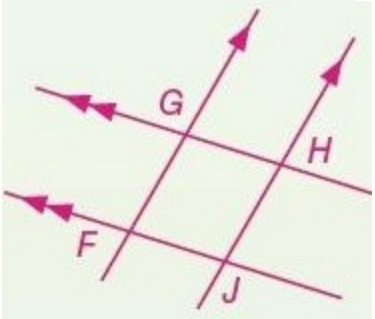
e. Each of the quadrilaterals was formed by 2 pairs of parallel lines. From the table it is shown that the measures

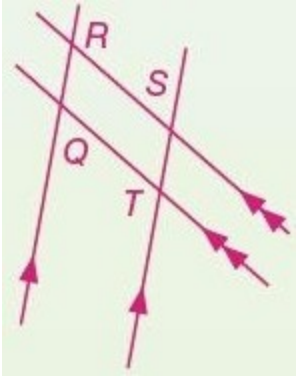
of the sides that are opposites are the same. So, the sides opposite each other in a quadrilateral formed by two pairs of parallel lines are congruent.

ANSWER:

a.







b.

Quadrilateral	Lengths and Measures							
ABCD	$m\angle A$	101	$m\angle B$	79	$m\angle C$	101	$m\angle D$	79
	AB	0.6 cm	BC	0.6 cm	CD	0.6 cm	DA	0.6 cm
FGHJ	$m\angle F$	76	$m\angle G$	104	$m\angle H$	76	$m\angle J$	104
	FG	1 cm	GH	0.9 cm	HJ	1 cm	JF	0.9 cm
QRST	$m\angle Q$	121	$m\angle R$	59	$m\angle S$	121	$m\angle T$	59
	QR	0.5 cm	RS	1.2 cm	ST	0.5 cm	TQ	1.2 cm

c. Sample answer: The angles opposite each other in a quadrilateral formed by two pairs of parallel lines are congruent.

d. Sample answer: The angles adjacent to each other in a quadrilateral formed by two pairs of parallel lines are supplementary.

e. Sample answer: The sides opposite each other in a quadrilateral formed by two pairs of parallel lines are congruent.

49. **ERROR ANALYSIS** Marcus says that the sum of the exterior angles of a decagon is greater than that of a heptagon because a decagon has more sides. Liam says that the sum of the exterior angles for both polygons is the same. Is either of them correct?
Explain your reasoning.

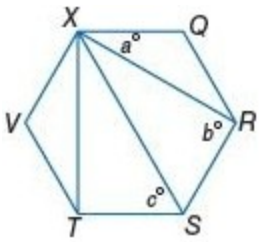
SOLUTION:

The Exterior Angle Sum Theorem states that the sum of the measures of any convex polygon is 360 regardless of how many sides it has. Liam is correct.

ANSWER:

Liam; by the Exterior Angle Sum Theorem, the sum of the measures of any convex polygon is 360.

50. **CHALLENGE** Find the values of *a*, *b*, and *c* if *QRSTVX* is a regular hexagon. Justify your answer.



SOLUTION:

We need to find the values of angles *a*, *b*, and *c*, which are all parts of interior angles of the hexagon.

What information are we given?

We are given that the figure is a regular hexagon, so we know that all of the interior angles are equal. We can find the measure of these angles using the interior Angle Sum Theorem. We can then use this information to find the values of *a*, *b*, and *c*.

30, 90, 60; By the Interior Angle Sum Theorem, the sum of the interior angles is 720. Since polygon *QRSTVX* is regular, there are 6 congruent angles. Each angle has a measure of 120. So, $m\angle XQR \text{ and } m\angle XVT = 120$. Since polygon *QRSTVX* is regular, *XQ* = *QR*. By the Isosceles Δ Theorem, $m\angle QXR \cong m\angle QRX$ .The interior angles of a triangle add up to 180, so $m\angle QXR + m\angle QRX + m\angle XQR = 180$. By substitution, *a* + *a* + 120 =180. So, 2*a* = 60 and *a* = 30. $m\angle QRS = m\angle QRX + m\angle XRS,$ by angle addition. By substitution, $120 = 30 + m\angle XRS$. From subtraction, $m\angle XRS = 90$. So, *b* = 90. By SAS, $\Delta XVT \cong \Delta XQR$ and $\Delta XTS \cong \Delta XRS$. By angle addition, $m\angle VXQ = m\angle VXT + m\angle TXS + m\angle SXR + 30$. By substitution, $120 = 30 + m\angle TXS + m\angle SXR + 30$. So, $m\angle TXS + m\angle SXR = 60$ and since $m\angle TXS \cong m\angle SXR$ by CPCTC, $m\angle TXS = m\angle SXR = 30$. In $\Delta XTS, m\angle XTS + m\angle TSX + m\angle SXT = 180$. By substitution, 90 + *c* + 30 = 180. So *c* = 60.

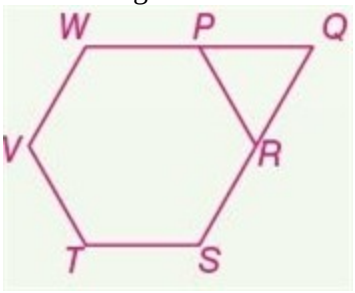
ANSWER:

30, 90, 60; By the Interior Angle Sum Theorem, the sum of the interior angles is 720. Since polygon *QRSTVX* is regular, there are 6 congruent angles. Each angle has a measure of 120. So, $m\angle XQR \text{ and } m\angle XVT = 120$. Since polygon *QRSTVX* is regular, *XQ* = *QR*. By the Isosceles Δ Theorem, $m\angle QXR \cong m\angle QRX$ .The interior angles of a triangle add up to 180, so $m\angle QXR + m\angle QRX + m\angle XQR = 180$. By substitution, *a* + *a* + 120 =180. So, 2*a* = 60 and *a* = 30. $m\angle QRS = m\angle QRX + m\angle XRS,$ by angle addition. By substitution, $120 = 30 + m\angle XRS$. From subtraction, $m\angle XRS = 90$. So, *b* = 90. By SAS, $\Delta XVT \cong \Delta XQR$ and $\Delta XTS \cong \Delta XRS$. By angle addition, $m\angle VXQ = m\angle VXT + m\angle TXS + m\angle SXR + 30$. By substitution, $120 = 30 + m\angle TXS + m\angle SXR + 30$. So, $m\angle TXS + m\angle SXR = 60$ and since $m\angle TXS \cong m\angle SXR$ by CPCTC, $m\angle TXS = m\angle SXR = 30$. In $\Delta XTS, m\angle XTS + m\angle TSX + m\angle SXT = 180$. By substitution, 90 + *c* + 30 = 180. So *c* = 60.

51. **CCSS ARGUMENTS** If two sides of a regular hexagon are extended to meet at a point in the exterior of the polygon, will the triangle formed *sometimes*, *always* or *never* be equilateral? Justify your answer.

SOLUTION:

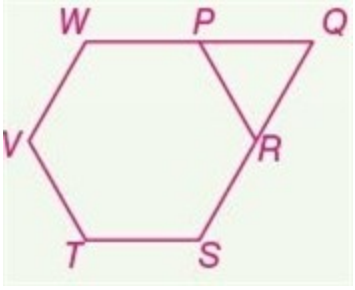
Draw a diagram first.



In order to determine whether triangle *PQR* is equilateral, find the measures of the exterior angles of the hexagon. By the Exterior Angle Sum Theorem, $m\angle QPR = 60$ and $m\angle QRP = 60$. Since the sum of the interior angle measures is 180, the measure of $\angle PQR = 180 - m\angle QPR - m\angle QRP = 180 - 60 - 60 = 60$. So, ΔPQR is always an equilateral triangle.

ANSWER:

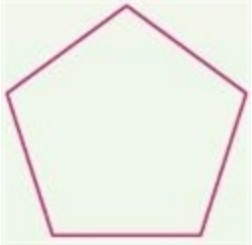
Always; by the Exterior Angle Sum Theorem, $m\angle QPR = 60$ and $m\angle QRP = 60$. Since the sum of the interior angle measures is 180, the measure of $\angle PQR = 180 - m\angle QPR - m\angle QRP = 180 - 60 - 60 = 60$. So, ΔPQR is an equilateral triangle.



52. **OPEN ENDED** Sketch a polygon and find the sum of its interior angles. How many sides does a polygon with twice this interior angles sum have? Justify your answer.

SOLUTION:

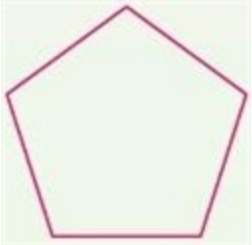
Sample answer: Draw a regular pentagon and find the sum of the interior angle measures.



Interior angles sum = $(n - 2) \cdot 180$
Interior angles sum = $(5 - 2) \cdot 180$ or 540.
Twice this sum is 2(540) or 1080. To find a polygon with this interior angles sum, write the equation: $(n - 2) \cdot 180 = 1080$ and solve for n; $n = 8$.

ANSWER:

8; Sample answer: Interior angles sum = $(5 - 2) \cdot 180$ or 540. Twice this sum is 2(540) or 1080. A polygon with this interior angles sum is the solution to $(n - 2) \cdot 180 = 1080$, $n = 8$.



53. **WRITING IN MATH** Explain how triangles are related to the Interior Angles Sum Theorem.

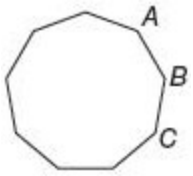
SOLUTION:

The Interior Angles Sum Theorem is derived by drawing all of the possible diagonals from one vertex of a polygon. This forms $(n - 2)$ triangles in the interior of the polygon with n sides. Since the sum of the measures of a triangle is 180, the sum of the interior angle measures of a convex polygon is $(n - 2)180$.

ANSWER:

The Interior Angles Sum Theorem is derived from the pattern between the number of sides in a polygon and the number of triangles. The formula is the product of the sum of the measures of the angles in a triangle, 180, and the number of triangles in the polygon.

54. If the polygon shown is regular, what is $m\angle ABC$?



A 140
B 144
C 162
D 180

SOLUTION:

Since the given regular polygon has 9 congruent sides, it is a regular nonagon.
Let x be the measure of each interior angle of a regular polygon with 9 sides. Since all angles of a regular nonagon are congruent, the sum of the interior angle measures is $9x$. By the Polygon Interior Angles Sum Theorem, the sum of the interior angle measures can also be expressed as $(n-2)180$.

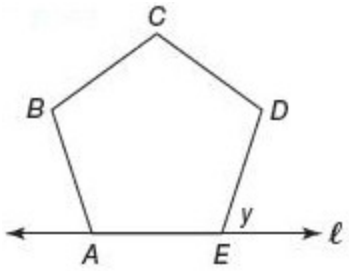
$$9x = (9-2)180$$
$$9x = (9-2)180$$
$$9x = (7)180$$
$$9x = 1260$$
$$x = 140$$

The measure of each interior angle of a regular nonagon is 140.
So, the correct option is A.

ANSWER:

A

55. **SHORT RESPONSE** Figure *ABCDE* is a regular pentagon with line ℓ passing through side *AE*. What is $m\angle y$?



SOLUTION:

Let *x* be the measure of each interior angle of a regular polygon with 5 sides. Since all angles of a regular pentagon are congruent, the sum of the interior angle measures is 5*x*. By the Polygon Interior Angles Sum Theorem, the sum of the interior angle measures can also be expressed as $(n-2)180$.

$5x = (5-2)180$

$5x = (5-2)180$

$5x = (3)180$

$5x = 540$

$x = 108$

The measure of each interior angle of a regular pentagon is 108.

Here, angle *y* and angle *E* form a linear pair.

$m\angle E + m\angle y = 180$

$108 + m\angle y = 180$

$m\angle y = 72$

ANSWER:

72

56. **ALGEBRA** $\frac{3^2 \cdot 4^5 \cdot 5^3}{5^3 \cdot 3^3 \cdot 4^6} =$

F $\frac{1}{60}$

G $\frac{1}{12}$

H $\frac{3}{4}$

J 12

SOLUTION:

$$\frac{3^2 \cdot 4^5 \cdot 5^3}{5^3 \cdot 3^3 \cdot 4^6} = \frac{3^{\overset{1}{2}} \cdot 4^{\overset{1}{5}} \cdot 5^{\overset{1}{3}}}{5^{\overset{1}{3}} \cdot 3^{3-1} \cdot 4^{6-1}}$$

$$= \frac{1}{3 \cdot 4}$$

The correct option is G.

ANSWER: $\frac{1}{12}$

G

57. **SAT/ACT** The sum of the measures of the interior angles of a polygon is twice the sum of the measures of its exterior angles. What type of polygon is it?

A square

B pentagon

C hexagon

D octagon

E nonagon

SOLUTION:

The sum of the exterior angle measures of a convex polygon, one angle at each vertex, is 360. The sum of the interior angle measures of an *n*-sided convex polygon is $(n-2)180$. By the given information, $(n-2)180 = 2(360)$. Solve for *n*.

$$(n-2)180 = 720$$

$$n-2 = 4$$

$$n = 6$$

If a polygon has 6 sides, then it is a hexagon.

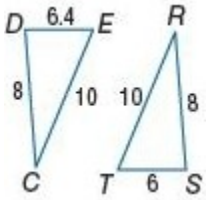
The correct option is C.

ANSWER:

C

Compare the given measures.

58. $m\angle DCE$ and $m\angle SRT$



SOLUTION:

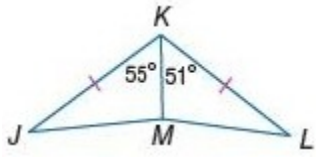
$\overline{CD} \cong \overline{RS}$, $\overline{CE} \cong \overline{RT}$, and $\overline{DE} \cong \overline{ST}$

By converse of the Hinge Theorem, $m\angle DCE > m\angle SRT$.

ANSWER:

$m\angle DCE > m\angle SRT$

59. JM and ML



SOLUTION:

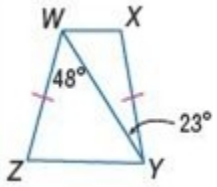
$\overline{JK} \cong \overline{LK}$, $\angle JKM > \angle LKM$

By the Hinge Theorem, $ML < JM$.

ANSWER:

$ML < JM$

60. WX and ZY



SOLUTION:

$\overline{WZ} \cong \overline{YZ}$, $\angle ZWY > \angle XYW$

By the Hinge Theorem, $WX < ZY$.

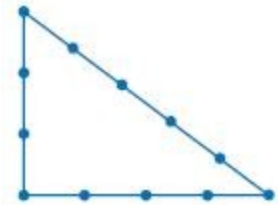
ANSWER:

$WX < ZY$

61. **HISTORY** The early Egyptians used to make triangles by using a rope with knots tied at equal intervals. Each vertex of the triangle had to occur at a knot. How many different triangles can be formed using the rope below?

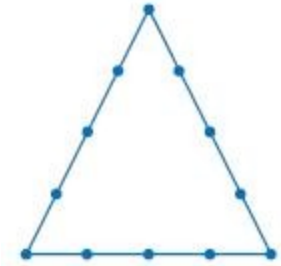
SOLUTION:

Use the Triangle Inequality theorem to determine how many different triangles can be made from the rope shown. the rope has 13 knots and 12 segments.
One is a right triangle with sides of 3, 4, and 5 segments.



One is an isosceles triangle with sides of 5, 5, and 2 segments.

And an equilateral triangle can be formed with sides 4 segments long.

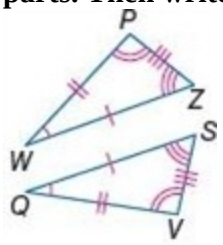


So there are 3 different triangles formed from the rope.

ANSWER:

3

Show that the triangles are congruent by identifying all congruent corresponding parts. Then write a congruence statement.

62. 

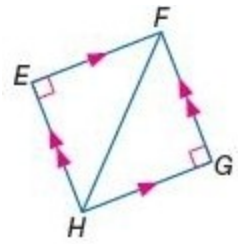
SOLUTION:

Sides and angles are identified as congruent if there are an equal number of tick marks through them. When two triangles share a common side, then those sides are considered congruent.

$\angle W \cong \angle Q; \angle P \cong \angle V; \angle Z \cong \angle S; \overline{WP} \cong \overline{QV}; \overline{WZ} \cong \overline{QS};$
$\overline{PZ} \cong \overline{SV}; \Delta WPZ \cong \Delta QVS$

ANSWER:

$\angle W \cong \angle Q; \angle P \cong \angle V; \angle Z \cong \angle S; \overline{WP} \cong \overline{QV}; \overline{WZ} \cong \overline{QS};$
$\overline{PZ} \cong \overline{SV}; \Delta WPZ \cong \Delta QVS$

63. 

SOLUTION:

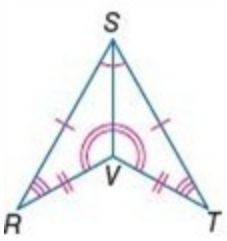
From the figure shown, angles *E* and *G* are each right angles and each pair of opposite sides is parallel.
$\overline{FH}$ is a transversal through $\overline{EF}$ and $\overline{GH}$, and $\overline{EH}$ and $\overline{GF}$. Since alternate interior angles are congruent, $\angle EFH \cong \angle GHF$ and $\angle EHF \cong \angle GFH$. By the Reflexive Property, $\overline{FH} \cong \overline{HF}$.
The distance between two parallel lines is the perpendicular distance between one line and any point on the other line. So, $\overline{EF} \cong \overline{GH}$, $\overline{EH} \cong \overline{GF}$, and $\overline{FH} \cong \overline{HF}$.

$\angle E \cong \angle G; \angle EFH \cong \angle GHF; \angle EHF \cong \angle GFH; \overline{EF} \cong \overline{GH}; \overline{EH} \cong \overline{GF};$
$\overline{FH} \cong \overline{HF}; \Delta EFH \cong \Delta GHF$

ANSWER:

$\angle E \cong \angle G; \angle EFH \cong \angle GHF; \angle EHF \cong \angle GFH; \overline{EF} \cong \overline{GH};$
$\overline{EH} \cong \overline{GF}; \overline{FH} \cong \overline{HF}; \Delta EFH \cong \Delta GHF$

6-1 Angles of Polygons

64. 

SOLUTION:

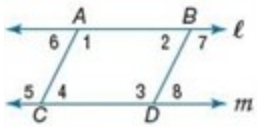
Sides and angles are identified as congruent if there are an equal number of tick marks through them. When two triangles share a common side, then those sides are considered congruent.

$\angle R \cong \angle T; \angle RSV \cong \angle TSV; \angle RVS \cong \angle TVS; \overline{RS} \cong \overline{TS}; \overline{SV} \cong \overline{SV};$
$\overline{RV} \cong \overline{TV}; \Delta RSV \cong \Delta TSV$

ANSWER:

$\angle R \cong \angle T; \angle RSV \cong \angle TSV; \angle RVS \cong \angle TVS; \overline{RS} \cong \overline{TS}; \overline{SV} \cong \overline{SV};$
$\overline{RV} \cong \overline{TV}; \Delta RSV \cong \Delta TSV$

In the figure, $\ell \parallel m$ and $\overline{AC} \parallel \overline{BD}$. Name all pairs of angles for each type indicated.



65. alternate interior angles

SOLUTION:

Alternate interior angles are nonadjacent interior angles that lie on opposite sides of a transversal.

$\angle 1$ and $\angle 5$, $\angle 4$ and $\angle 6$, $\angle 2$ and $\angle 8$, $\angle 3$ and $\angle 7$

ANSWER:

$\angle 1$ and $\angle 5$, $\angle 4$ and $\angle 6$, $\angle 2$ and $\angle 8$, $\angle 3$ and $\angle 7$

66. consecutive interior angles

SOLUTION:

Consecutive interior angles are interior angles that lie on the same side of a transversal.

$\angle 1$ and $\angle 4$, $\angle 2$ and $\angle 3$, $\angle 1$ and $\angle 2$, $\angle 3$ and $\angle 4$

ANSWER:

$\angle 1$ and $\angle 4$, $\angle 2$ and $\angle 3$, $\angle 1$ and $\angle 2$, $\angle 3$ and $\angle 4$