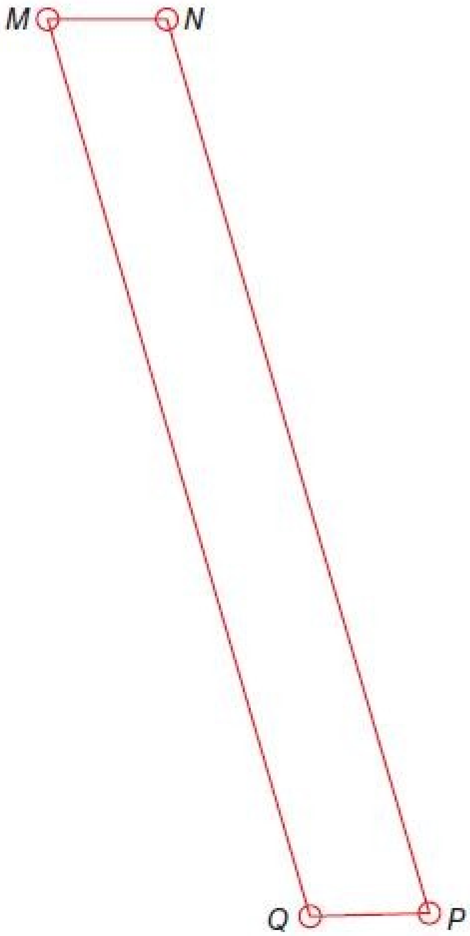
6-2 Parallelograms

1. **NAVIGATION** To chart a course, sailors use a *parallel ruler*. One edge of the ruler is placed along the line representing the direction of the course to be taken. Then the other ruler is moved until its edge reaches the compass rose printed on the chart. Reading the compass determines which direction to travel. The rulers and the crossbars of the tool form $\square MNPQ$.



Refer to Page 407.

a. If $m\angle NMQ = 32$, find $m\angle MNP$.

b. If $m\angle MQP = 125$, find $m\angle MNP$

c. If $MQ = 4$, what is NP?

SOLUTION:

a. Angles *NMQ* and *MNP* are consecutive angles. Consecutive angles in a parallelogram are supplementary.

So, $m\angle NMQ + m\angle MNP = 180$.

Substitute.

$$32 + m\angle MNP = 180$$
$$m\angle MNP = 148$$

b. Angles *MQP* and *MNP* are opposite angles. Opposite angles of a parallelogram are congruent.

So, $m\angle MQP = m\angle MNP = 125$.

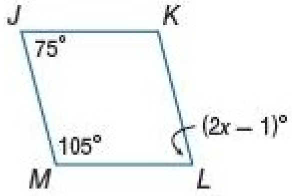
c. $\overline{MQ}$ and $\overline{NP}$ are opposite sides. Opposite sides of a parallelogram are congruent.

So, $MQ = NP = 4$.

ANSWER:

a. 148
b. 125
c. 4

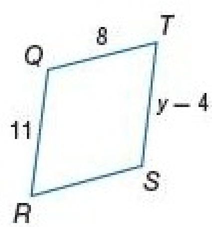
ALGEBRA Find the value of each variable in each parallelogram.

2. 

SOLUTION:
Opposite angles of a parallelogram are congruent.
So, $2x-1=75$.
Solve for x.

$$2x-1=75$$
$$2x=76$$
$$x=38$$

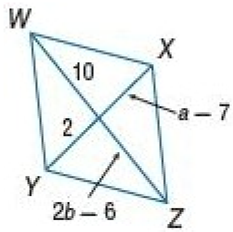
ANSWER:
38

3. 

SOLUTION:
Opposite sides of a parallelogram are congruent.
So, $y-4=11$.
Solve for y.

$$y-4=11$$
$$y=15$$

ANSWER:
15

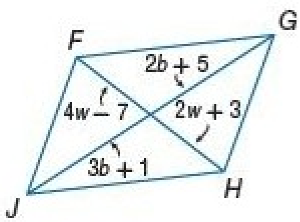
4. 

SOLUTION:

Diagonals of a parallelogram bisect each other.
So, $a - 7 = 2$ and $2b - 6 = 10$.
Solve for a and b.
So, $a = 9$ and $b = 8$.

ANSWER:

$a = 9,\ b = 8$

5. 

SOLUTION:

Diagonals of a parallelogram bisect each other.
So, $2b + 5 = 3b + 1$ and $4w - 7 = 2w + 3$.
Solve for b.
$2b + 5 = 3b + 1$
$2b = 3b - 4$
$-b = -4$
$b = 4$
Solve for w.
$4w - 7 = 2w + 3$
$4w = 2w + 10$
$2w = 10$
$w = 5$

ANSWER:

$w = 5,\ b = 4$

6. **COORDINATE GEOMETRY** Determine the coordinates of the intersection of the diagonals of $\square ABCD$ with vertices $A(-4, 6)$, $B(5, 6)$, $C(4, -2)$, and $D(-5, -2)$.

SOLUTION:

Since the diagonals of a parallelogram bisect each other, their intersection point is the midpoint of $\overline{AC}$ or the midpoint of $\overline{BD}$. Find the midpoint of $\overline{AC}$. Use the Midpoint Formula

$$\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right).$$

Substitute.

$$\left(\frac{-4+4}{2}, \frac{6-2}{2}\right)=(0,2)$$

The coordinates of the intersection of the diagonals of parallelogram *ABCD* are (0, 2).

ANSWER:

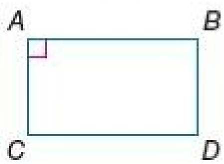
(0, 2)

PROOF Write the indicated type of proof.

7. paragraph

Given: $\square ABCD, \angle A$ is a right angle.

Prove: $\angle B, \angle C,$ *and* $\angle D$ are right angles. (Theorem 6.6)



SOLUTION:

Begin by listing what is known. Since *ABCD* is a parallelogram, the properties of parallelograms apply. Each pair of opposite sides are parallel and congruent and each pair of opposite angles are congruent. It is given that $\angle A$ is a right angle so $\overline{AC} \perp \overline{AB}$.

Given: $\square ABCD, \angle A$ is a right angle.

Prove: $\angle B, \angle C,$ and $\angle D$ are right angles. (Theorem 6.6)

Proof: By definition of a parallelogram, $\overline{AB} \parallel \overline{CD}$. Since $\angle A$ is a right angle, $\overline{AC} \perp \overline{AB}$. By the Perpendicular Transversal Theorem, $\overline{AC} \perp \overline{CD}$. $\angle C$ is a right angle, because perpendicular lines form a right angle. $\angle B \cong \angle C$ and $\angle A \cong \angle D$ because opposite angles in a parallelogram are congruent. $\angle C$ and $\angle D$ are right angles, since all right angles are congruent.

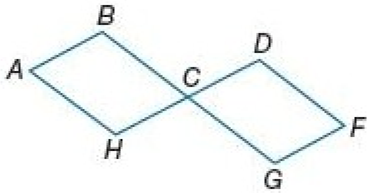
ANSWER:

Given: $\square ABCD, \angle A$ is a right angle.

Prove: $\angle B, \angle C,$ and $\angle D$ are right angles. (Theorem 6.6)

Proof: By definition of a parallelogram, $\overline{AB} \parallel \overline{CD}$. Since $\angle A$ is a right angle, $\overline{AC} \perp \overline{AB}$. By the Perpendicular Transversal Theorem, $\overline{AC} \perp \overline{CD}$. $\angle C$ is a right angle, because perpendicular lines form a right angle. $\angle B \cong \angle C$ and $\angle A \cong \angle D$ because opposite angles in a parallelogram are congruent. $\angle C$ and $\angle D$ are right angles, since all right angles are congruent.

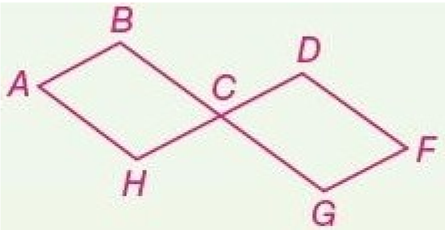
8. two-column
Given: *ABCH* and *DCGF* are parallelograms.
Prove: $\angle A \cong \angle F$



SOLUTION:

First list what is known. Since *ABCH* and *DCGF* are parallelograms, the properties of parallelograms apply. From the figure, $\angle BCH$ and $\angle DCG$ are vertical angles.

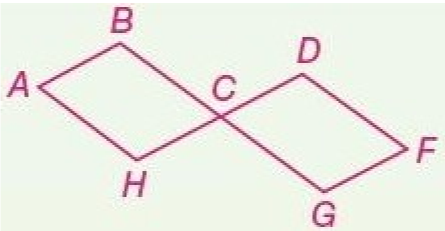
Given: *ABCH* and *DCGF* are parallelograms.
Prove: $\angle A \cong \angle F$



Proof:
Statements (Reasons)
1. *ABCH* and *DCGF* are parallelograms.(Given)
2. $\angle BCH \cong \angle DCG$ (Vert. $\angle s$ are $\cong$)
3. $\angle A \cong \angle BCH$ and $\angle DCG \cong \angle F$ (Opp. $\angle s$ of a $\square$ are $\cong$)
4. $\angle A \cong \angle F$ (Substitution.)

ANSWER:

Given: *ABCH* and *DCGF* are parallelograms.
Prove: $\angle A \cong \angle F$



Proof:
Statements (Reasons)
1. *ABCH* and *DCGF* are parallelograms.(Given)
2. $\angle BCH \cong \angle DCG$ (Vert. $\angle s$ are $\cong$)
3. $\angle A \cong \angle BCH$ and $\angle DCG \cong \angle F$ (Opp. $\angle s$ of a $\square$ are $\cong$)
4. $\angle A \cong \angle F$ (Substitution.)

Use $\square PQRS$ to find each measure.

9. $m\angle R$

SOLUTION:

Consecutive angles in a parallelogram are supplementary.

So, $m\angle Q + m\angle R = 180.$

Substitute.

$$128 + m\angle R = 180$$
$$m\angle R = 52$$

ANSWER:

52

10. *QR*

SOLUTION:

Opposite sides of a parallelogram are congruent.

So, $PS = QR = 3.$

ANSWER:

3

11. *QP*

SOLUTION:

Opposite sides of a parallelogram are congruent.

So, $RS = QP = 5.$

ANSWER:

5

12. $m\angle S$

SOLUTION:

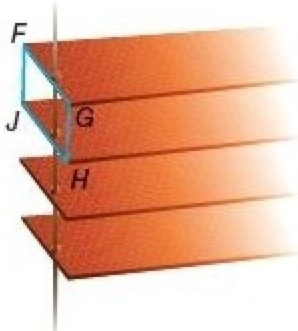
Opposite angles of a parallelogram are congruent.

So, $m\angle Q = m\angle S = 128.$

ANSWER:

128

13. **HOME DECOR** The slats on Venetian blinds are designed to remain parallel in order to direct the path of light coming in a widow. In $\square FGHJ, FJ = \frac{3}{4}$ inch, $FG = 1$ inch, and $\angle JHG = 62$. Find each measure.



a. *JH*
b. *GH*
c. $m\angle JFG$
d. $m\angle FJH$

SOLUTION:

a. Opposite sides of a parallelogram are congruent.
So, $FG = JH = 1$ inch.

b. Opposite sides of a parallelogram are congruent.
So, $GH = FJ = \frac{3}{4}$ in.

c. Opposite angles of a parallelogram are congruent.
So, $m\angle JFG = m\angle JHG = 62$.

d. Consecutive angles in a parallelogram are supplementary.
So, $m\angle FJH + m\angle JHG = 180$.
Substitute.

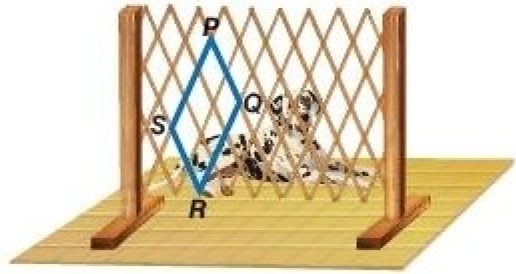
$$m\angle FJH + 62 = 180$$
$$m\angle FJH = 118$$

ANSWER:

a. 1 in.
b. $\frac{3}{4}$in
c. 62
d. 118

6-2 Parallelograms

14. **CCSS MODELING** Wesley is a member of the kennel club in his area. His club uses accordion fencing like the section shown at the right to block out areas at dog shows.



a. Identify two pairs of congruent segments.
b. Identify two pairs of supplementary angles.

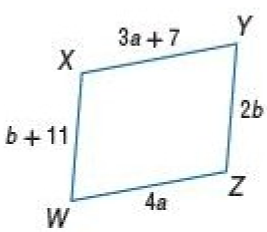
SOLUTION:
a. Opposite sides of a parallelogram are congruent.

$\overline{PS} \cong \overline{QR}, \overline{PQ} \cong \overline{SR}$

b. Consecutive angles of a parallelogram are supplementary.
Sample answer: $\angle P$ and $\angle Q, \angle S$ and $\angle R$

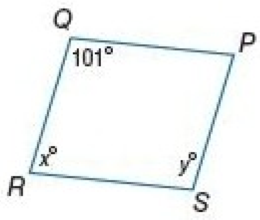
ANSWER:
a. $\overline{PS} \cong \overline{QR}, \overline{PQ} \cong \overline{SR}$
b. Sample answer: $\angle P$ *and* $\angle Q, \angle S$ *and* $\angle R$

ALGEBRA Find the value of each variable in each parallelogram.

15. 

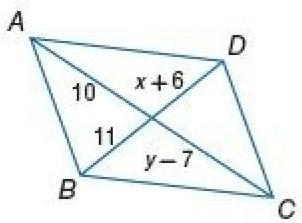
SOLUTION:
Opposite sides of a parallelogram are congruent.
So, $4a = 3a + 7$ and $2b = b + 11$.
Solve for a.
$4a = 3a + 7$
$a = 7$
Solve for b.
$2b = b + 11$
$b = 11$

ANSWER:
$a = 7, b = 11$

16. 

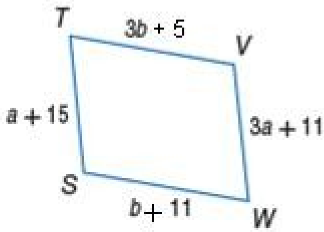
SOLUTION:
Consecutive angles in a parallelogram are supplementary.
So, $101 + x = 180$.
Solve for x.
$101 + x = 180$
$x = 79$
Opposite angles of a parallelogram are congruent.
So, $y = m\angle Q = 101$.

ANSWER:
$x = 79$, $y = 101$

17. 

SOLUTION:
Diagonals of a parallelogram bisect each other.
So, $y - 7 = 10$ and $x + 6 = 11$.
Solve for y.
$y - 7 = 10$
$y = 17$
Solve for x.
$x + 6 = 11$
$x = 5$

ANSWER:
$x = 5$, $y = 17$

18. 

SOLUTION:

Opposite sides of a parallelogram are congruent.
So, $3b + 5 = b + 11$ and $a + 15 = 3a + 11$.
Solve for *a*.

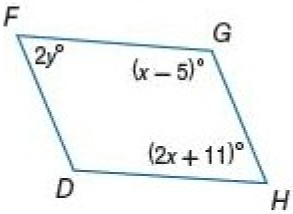
$$a + 15 = 3a + 11$$
$$-2a = -4$$
$$a = 2$$

Solve for *b*.

$$3b + 5 = b + 11$$
$$2b = 6$$
$$b = 3$$

ANSWER:

a = 2, *b* = 3

19. 

SOLUTION:

Consecutive angles in a parallelogram are supplementary.

So, $(x-5)+(2x+11)=180.$

Solve for *x*.

$$\begin{aligned}(x-5)+(2x+11)&=180\\ x-5+2x+11&=180\\ 3x+6&=180\\ 3x&=174\\ x&=58\end{aligned}$$

Substitute $x=58$ in $m\angle H$.

$$\begin{aligned}m\angle H&=2x+11\\ &=2(58)+11\\ &=116+11\\ &=127\end{aligned}$$

Opposite angles of a parallelogram are congruent.

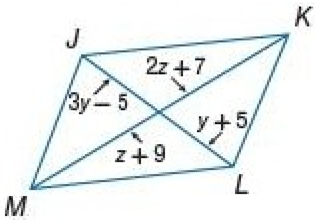
So, $m\angle F=m\angle H$.

Substitute.

$$\begin{aligned}2y&=127\\ y&=63.5\end{aligned}$$

ANSWER:

$x = 58$, $y = 63.5$

20. 

SOLUTION:

Diagonals of a parallelogram bisect each other. So, $2z+7=z+9$ and $3y-5=y+5$.

Solve for z.

$$2z+7=z+9$$
$$z=2$$

Solve for y.

$$3y-5=y+5$$
$$2y=10$$
$$y=5$$

ANSWER:

$z = 2$, $y = 5$

COORDINATE GEOMETRY Find the coordinates of the intersection of the diagonals of ▱$WXYZ$ with the given vertices.

21. $W(-1, 7)$, $X(8, 7)$, $Y(6, -2)$, $Z(-3, -2)$

SOLUTION:

Since the diagonals of a parallelogram bisect each other, their intersection point is the midpoint of $\overline{WY}$ or the midpoint of $\overline{XZ}$. Find the midpoint of $\overline{WY}$. Use the Midpoint Formula $\left(\frac{x_1+x_2}{2},\frac{y_1+y_2}{2}\right)$.

Substitute.

$$\left(\frac{-1+6}{2},\frac{7-2}{2}\right)=(2.5,2.5)$$

The coordinates of the intersection of the diagonals of parallelogram $WXYZ$ are (2.5, 2.5).

ANSWER:

(2.5, 2.5)

22. $W(-4, 5)$, $X(5, 7)$, $Y(4, -2)$, $Z(-5, -4)$

SOLUTION:

Since the diagonals of a parallelogram bisect each other, their intersection point is the midpoint of $\overline{WY}$ or the midpoint of $\overline{XZ}$. Find the midpoint of $\overline{WY}$.

Use the Midpoint Formula

$$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right).$$

Substitute.

$$\left(\frac{-4 + 4}{2}, \frac{5 - 2}{2}\right) = (0, 1.5)$$

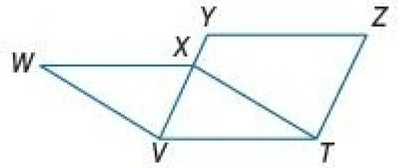
The coordinates of the intersection of the diagonals of parallelogram *WXYZ* are (0, 1.5).

ANSWER:

(0, 1.5)

PROOF Write a two-column proof.

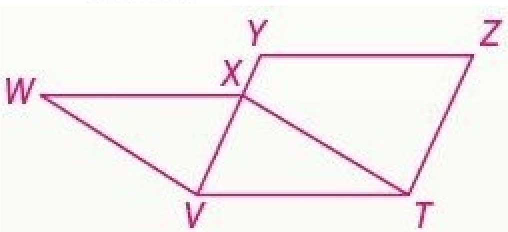
23. **Given:** *WXTV* and *ZYVT* are parallelograms.
Prove: $\overline{WX} \cong \overline{ZY}$



SOLUTION:

Begin by listing the known information. It is given that *WXTV* and *ZYVT* are parallelograms so the properties of parallelograms apply. From the figure, these two parallelograms share a common side, $\overline{VT}$.

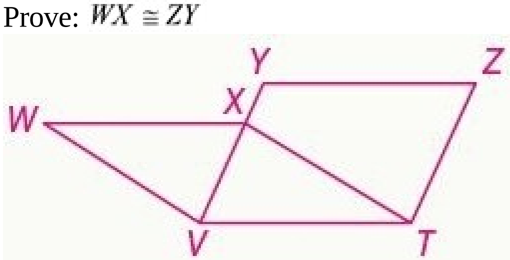
Given: *WXTV* and *ZYVT* are parallelograms.
Prove: $\overline{WX} \cong \overline{ZY}$



Proof:
Statements (Reasons)
1. *WXTV* and *ZYVT* are parallelograms. (Given)
2. $\overline{WX} \cong \overline{VT}, \overline{VT} \cong \overline{YZ}$ (Opp. Sides of a ▱ are ≅.)
3. $\overline{WX} \cong \overline{ZY}$ (Trans. Prop.)

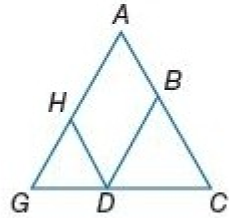
ANSWER:

Given: *WXTV* and *ZYVT* are parallelograms.
Prove: $\overline{WX} \cong \overline{ZY}$



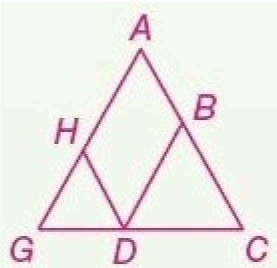
Proof:
Statements (Reasons)
1. *WXTV* and *ZYVT* are parallelograms. (Given)
2. $\overline{WX} \cong \overline{VT}, \overline{VT} \cong \overline{YZ}$ (Opp. Sides of a ▱ are ≅.)
3. $\overline{WX} \cong \overline{ZY}$ (Trans. Prop.)

24. **Given:** $\square BDHA, \overline{CA} \cong \overline{CG}$
Prove: $\angle BDH \cong \angle G$



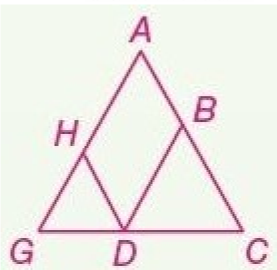
SOLUTION:
Begin by listing the known information. It is given that BDHA is a parallelogram so the properties of a parallelogram apply. It is also given that $\overline{CA} \cong \overline{CG}$. From the figure, $\triangle ACG$ has sides $\overline{CA}$ and $\overline{CG}$. Since two sides are congruent, $\triangle ACG$ must be an isosceles triangle.

Given: $\square BDHA, \overline{CA} \cong \overline{CG}$
Prove: $\angle BDH \cong \angle G$



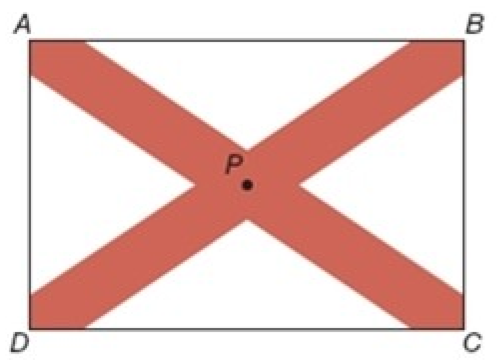
Proof:
Statements (Reasons)
1. $\square BDHA, \overline{CA} \cong \overline{CG}$ (Given)
2. $\angle A \cong \angle BDH$ (Opp. $\angle s$ of a $\square$ are $\cong$.)
3. $\angle A \cong \angle G$ (Isos. Δ Thm.)
4. $\angle BDH \cong \angle G$ (Trans. Prop.)

ANSWER:
Given: $\square BDHA, \overline{CA} \cong \overline{CG}$
Prove: $\angle BDH \cong \angle G$



Proof:
Statements (Reasons)
1. $\square BDHA, \overline{CA} \cong \overline{CG}$ (Given)
2. $\angle A \cong \angle BDH$ (Opp. $\angle s$ of a $\square$ are $\cong$.)
3. $\angle A \cong \angle G$ (Isos. Δ Thm.)
4. $\angle BDH \cong \angle G$ (Trans. Prop.)

25. **FLAGS** Refer to the Alabama state flag at the right.
Given: $\Delta ACD \cong \Delta CAB$
Prove: $\overline{DP} \cong \overline{PB}$



SOLUTION:

Begin by listing the known information. It is given that $\Delta ACD \cong \Delta CAB$. From CPCTC, $\angle ACD \cong \angle CAB$, $\angle D \cong \angle B$, $\overline{AD} \cong \overline{CB}$, and $\overline{AB} \cong \overline{CD}$. In order to prove that $\overline{DP} \cong \overline{PB}$, show that $\Delta ABP \cong \Delta CDP$.

Proof:
Statements (Reasons):
1. $\Delta ACD \cong \Delta CAB$ (Given)
2. $\angle ACD \cong \angle CAB$ (CPCTC)
3. $\angle DPC \cong \angle BPA$ (Vert. $\angle s$ are $\cong$.)
4. $\overline{AB} \cong \overline{CD}$ (CPCTC)
5. $\Delta ABP \cong \Delta CDP$ (AAS)
6. $\overline{DP} \cong \overline{PB}$ (CPCTC)

ANSWER:
Proof:
Statements (Reasons):
1. $\Delta ACD \cong \Delta CAB$ (Given)
2. $\angle ACD \cong \angle CAB$ (CPCTC)
3. $\angle DPC \cong \angle BPA$ (Vert. $\angle s$ are $\cong$.)
4. $\overline{AB} \cong \overline{CD}$ (CPCTC)
5. $\Delta ABP \cong \Delta CDP$ (AAS)
6. $\overline{DP} \cong \overline{PB}$ (CPCTC)

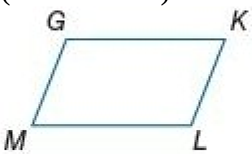
CCSS ARGUMENTS Write the indicated type of proof.

26. two-column

Given: $\square GKLM$

Prove: $\angle G$ and $\angle K, \angle K$ and $\angle L, \angle L$ and $\angle M, \angle M$ and $\angle G$ are supplementary.

(Theorem 6.5)



SOLUTION:

Begin by listing what is known. It is given the *GKLM* is a parallelogram. So, opposite sides are parallel and congruent. If two parallel lines are cut by a transversal, consecutive interior angles are supplementary.

Proof:

Statements (Reasons)

1. $\square GKLM$ (Given)
2. $\overline{GK} \parallel \overline{ML}, \overline{GM} \parallel \overline{KL}$ (Opp. sides of a $\square$ are $\parallel$.)
3. $\angle G$ and $\angle K$ are supplementary, $\angle K$ and $\angle L$ are supplementary, $\angle L$ and $\angle M$ are supplementary, and $\angle M$ and $\angle G$ are supplementary. (Cons. int. $\angle s$ are suppl.)

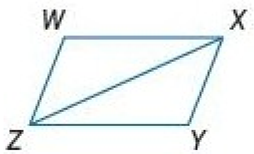
ANSWER:

Proof:

Statements (Reasons)

1. $\square GKLM$ (Given)
2. $\overline{GK} \parallel \overline{ML}, \overline{GM} \parallel \overline{KL}$ (Opp. sides of a $\square$ are $\parallel$.)
3. $\angle G$ and $\angle K$ are supplementary, $\angle K$ and $\angle L$ are supplementary, $\angle L$ and $\angle M$ are supplementary, and $\angle M$ and $\angle G$ are supplementary. (Cons. int. $\angle s$ are suppl.)

27. two-column
Given: $\square WXYZ$
Prove: $\Delta WXZ \cong \Delta YZX$
(Theorem 6.8)



SOLUTION:
Begin by listing what is known. It is given the *WXYZ* is a parallelogram. So, opposite sides are parallel and congruent and opposite angles are congruent. This information can be used to prove $\Delta WXZ \cong \Delta YZX$.

Proof:
Statements (Reasons)
1. $\square WXYZ$ (Given)
2. $\overline{WX} \cong \overline{ZY}, \overline{WZ} \cong \overline{XY}$ (Opp. sides of a $\square$ are $\cong$.)
3. $\angle ZWX \cong \angle XYZ$ (Opp. $\angle s$ of a $\square$ are $\cong$.)
4. $\Delta WXZ \cong \Delta YZX$ (SAS)

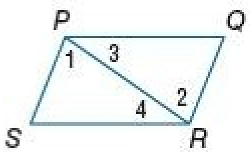
ANSWER:
Proof:
Statements (Reasons)
1. $\square WXYZ$ (Given)
2. $\overline{WX} \cong \overline{ZY}, \overline{WZ} \cong \overline{XY}$ (Opp. sides of a $\square$ are $\cong$.)
3. $\angle ZWX \cong \angle XYZ$ (Opp. $\angle s$ of a $\square$ are $\cong$.)
4. $\Delta WXZ \cong \Delta YZX$ (SAS)

28. two-column

Given: $\square PQRS$

Prove: $\overline{PQ} \cong \overline{RS}, \overline{QR} \cong \overline{SP}$

(Theorem 6.3)



SOLUTION:

Begin by listing what is known. It is given the *GKLM* is a parallelogram. We are proving that opposite sides are congruent but we can use the other properties of parallelograms such as opposite sides are parallel. If two parallel lines are cut by a transversal, alternate interior angles are congruent. If we can show that $\Delta QPR \cong \Delta SRP$ then $\overline{PQ} \cong \overline{RS}, \overline{QR} \cong \overline{SP}$ by CPCTC.

Proof:

Statements (Reasons)

1. $\square PQRS$ (Given)
2. Draw an auxiliary segment $\overline{PR}$ and label angles 1, 2, 3, and 4 as shown. (Diagonal of *PQRS*)
3. $\overline{PQ} \parallel \overline{SR}, \overline{PS} \parallel \overline{QR}$ (Opp. sides of a $\square$ are $\parallel$.)
4. $\angle 1 \cong \angle 2$, and $\angle 3 \cong \angle 4$ (Alt. int. $\angle s$ Thm.)
5. $\overline{PR} \cong \overline{RP}$ (Refl. Prop.)
6. $\Delta QPR \cong \Delta SRP$ (ASA)
7. $\overline{PQ} \cong \overline{RS}, \overline{QR} \cong \overline{SP}$ (CPCTC)

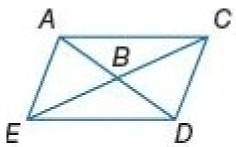
ANSWER:

Proof:

Statements (Reasons)

1. $\square PQRS$ (Given)
2. Draw an auxiliary segment $\overline{PR}$ and label angles 1, 2, 3, and 4 as shown. (Diagonal of *PQRS*)
3. $\overline{PQ} \parallel \overline{SR}, \overline{PS} \parallel \overline{QR}$ (Opp. sides of a $\square$ are $\parallel$.)
4. $\angle 1 \cong \angle 2$, and $\angle 3 \cong \angle 4$ (Alt. int. $\angle s$ Thm.)
5. $\overline{PR} \cong \overline{RP}$ (Refl. Prop.)
6. $\Delta QPR \cong \Delta SRP$ (ASA)
7. $\overline{PQ} \cong \overline{RS}, \overline{QR} \cong \overline{SP}$ (CPCTC)

29. paragraph
Given: $\square ACDE$ is a parallelogram.
Prove: $\overline{EC}$ bisects $\overline{AD}$
(Theorem 6.7)

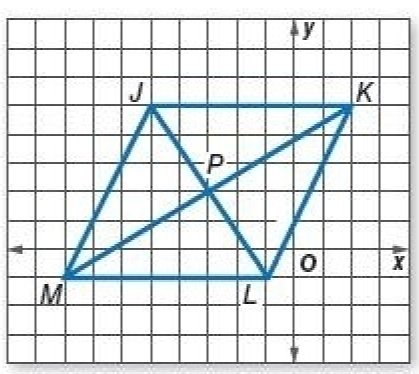


SOLUTION:
Begin by listing what is known. It is given that $ACDE$ is a parallelogram. So, opposite sides are parallel and congruent. If two parallel lines are cut by a transversal, alternate interior angles are congruent. To prove that $\overline{EC}$ bisects $\overline{AD}$ and $\overline{AD}$ bisects $\overline{EC}$ we must show that $\overline{EB} \cong \overline{BC}$ and $\overline{AB} \cong \overline{BD}$. This can be done if it is shown that $\Delta EBA \cong \Delta CBD$

Proof: It is given that $ACDE$ is a parallelogram. Since opposite sides of a parallelogram are congruent, $\overline{EA} \cong \overline{DC}$. By definition of a parallelogram, $\overline{EA} \parallel \overline{DC}$.$\angle AEB \cong \angle DCB$ and $\angle EAB \cong \angle CDB$. because alternate interior angles are congruent. $\Delta EBA \cong \Delta CBD$ by ASA. $\overline{EB} \cong \overline{BC}$ and $\overline{AB} \cong \overline{BD}$ by CPCTC. By the definition of segment bisector, $\overline{EC}$ bisects $\overline{AD}$ and $\overline{AD}$ bisects $\overline{EC}$.

ANSWER:
Proof: It is given that $ACDE$ is a parallelogram. Since opposite sides of a parallelogram are congruent, $\overline{EA} \cong \overline{DC}$. By definition of a parallelogram, $\overline{EA} \parallel \overline{DC}$.$\angle AEB \cong \angle DCB$ and $\angle EAB \cong \angle CDB$. because alternate interior angles are congruent. $\Delta EBA \cong \Delta CBD$ by ASA. $\overline{EB} \cong \overline{BC}$ and $\overline{AB} \cong \overline{BD}$ by CPCTC. By the definition of segment bisector, $\overline{EC}$ bisects $\overline{AD}$ and $\overline{AD}$ bisects $\overline{EC}$

30. **COORDINATE GEOMETRY** Use the graph shown.
a. Use the Distance Formula to determine if the diagonals of JKLM bisect each other. Explain.
b. Determine whether the diagonals are congruent. Explain.
c. Use slopes to determine if the consecutive sides are perpendicular. Explain.



SOLUTION:
a. Identify the coordinates of each point using the given graph.
$K(2, 5)$, $L(–1, –1)$, $M(–8, –1)$, $J(–5, 5)$ and $P(–3, 2)$
Use the distance formula.

$$\begin{aligned} JP &= \sqrt{(x_2-x_1)^2+(y_2-y_1)^2} \\ &= \sqrt{(-3-(-5))^2+(2-5)^2} \\ &= \sqrt{(2)^2+(-3)^2} \\ &= \sqrt{4+9} \\ &= \sqrt{13} \end{aligned}$$

$$\begin{aligned} LP &= \sqrt{(x_2-x_1)^2+(y_2-y_1)^2} \\ &= \sqrt{(-3-(-1))^2+(2-(-1))^2} \\ &= \sqrt{(-2)^2+(3)^2} \\ &= \sqrt{4+9} \\ &= \sqrt{13} \end{aligned}$$

$$\begin{aligned} MP &= \sqrt{(x_2-x_1)^2+(y_2-y_1)^2} \\ &= \sqrt{(-3-(-8))^2+(2-(-1))^2} \\ &= \sqrt{(5)^2+(3)^2} \\ &= \sqrt{25+9} \\ &= \sqrt{34} \end{aligned}$$

$$\begin{aligned} KP &= \sqrt{(x_2-x_1)^2+(y_2-y_1)^2} \\ &= \sqrt{(-3-2)^2+(2-5)^2} \\ &= \sqrt{(-5)^2+(-3)^2} \\ &= \sqrt{25+9} \\ &= \sqrt{34} \end{aligned}$$

$JP = \sqrt{13}, LP = \sqrt{13}, MP = \sqrt{34}, KP = \sqrt{34}$; since $JP = LP$ and $MP = KP$, the diagonals bisect each other.

b. The diagonals are congruent if they have the same measure. Since $JP + LP \neq MP + KP$, $JL \neq MK$ so they are not congruent.

c. The slopes of two perpendicular lines are negative reciprocals of each other. Find the slopes of *JK* and *JM*.

Find the slope of $\overline{JK}$ using slope formula.

$$\begin{aligned} m &= \frac{y_2-y_1}{x_2-x_1} \\ &= \frac{5-5}{2-(-5)} \\ &= 0 \end{aligned}$$

Find the slope of $\overline{JM}$ using slope formula.

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-1-5}{-8-(-5)} = 2$$

The slope of $\overline{JK} = 0$ and the slope of $\overline{JM} = 2$. The slopes are not negative reciprocals of each other. So, the answer is "No".

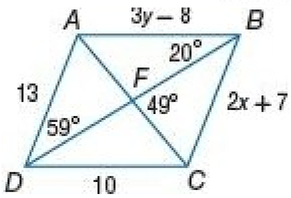
ANSWER:

a. $JP = \sqrt{13}, LP = \sqrt{13}, MP = \sqrt{34}, KP = \sqrt{34}$; since $JP = LP$ and $MP = KP$, the diagonals bisect each other.

b. No; $JP + LP \neq MP + KP$.

c. No; the slope of $\overline{JK} = 0$ and the slope of $\overline{JM} = 2$. The slopes are not negative reciprocals of each other.

ALGEBRA Use $\square ABCD$ to find each measure or value.



31. x

SOLUTION:

Opposite sides of a parallelogram are congruent.

So, $2x + 7 = 13$.

Solve for x.

$$2x + 7 = 13$$
$$2x = 6$$
$$x = 3$$

ANSWER:

3

32. y

SOLUTION:

Opposite sides of a parallelogram are congruent.

So, $3y - 8 = 10$.

Solve for y.

$$3y - 8 = 10$$
$$3y = 18$$
$$y = 6$$

ANSWER:

6

33. $m\angle AFB$

SOLUTION:

In the figure, angle *AFB* and $49°$ angle form a linear pair. So, $m\angle AFB + 49 = 180.$
$m\angle AFB = 131$

ANSWER:

131

34. $m\angle DAC$

SOLUTION:

Since the vertical angles are congruent, $m\angle AFD = m\angle BFC = 49$.
The sum of the measures of interior angles in a triangle is 180 .
Here, $m\angle AFD + m\angle DAF + m\angle ADF = 180.$
Substitute.

$$49 + m\angle DAF + 59 = 180$$
$$m\angle DAF + 108 = 180$$
$$m\angle DAF = 72$$

$\angle DAF$ and $\angle DAC$ represent the same angle. So, $m\angle DAC = 72.$

ANSWER:

72

35. $m\angle ACD$

SOLUTION:

Since the vertical angles are congruent, $m\angle AFD = m\angle BFC = 49$.
The sum of the measures of interior angles in a triangle is 180 .
Here, $m\angle AFD + m\angle DAF + m\angle ADF = 180.$
Substitute.

$$49 + m\angle DAF + 59 = 180$$
$$m\angle DAF + 108 = 180$$
$$m\angle DAF = 72$$

So, $m\angle DAC = 72.$
By the Alternate Interior Angles Theorem, $m\angle BDC = 20.$
So, $m\angle ADC = 20 + 59 = 79.$
In ΔADC , $m\angle ADC + m\angle ACD + m\angle DAC = 180.$
Substitute.

$$79 + m\angle ACD + 72 = 180$$
$$m\angle ACD + 151 = 180$$
$$m\angle ACD = 29$$

ANSWER:

29

36. $m\angle DAB$

SOLUTION:

The sum of the measures of interior angles in a triangle is 180.
Here, $m\angle DAB + m\angle ABD + m\angle BDA = 180.$
Substitute.

$$m\angle DAB + 20 + 59 = 180$$
$$m\angle DAB + 79 = 180$$
$$m\angle DAB = 101$$

ANSWER:
101

37. **COORDINATE GEOMETRY** $\square ABCD$ has vertices $A(-3, 5)$, $B(1, 2)$, and $C(3, -4)$. Determine the coordinates of vertex D if it is located in Quadrant III.

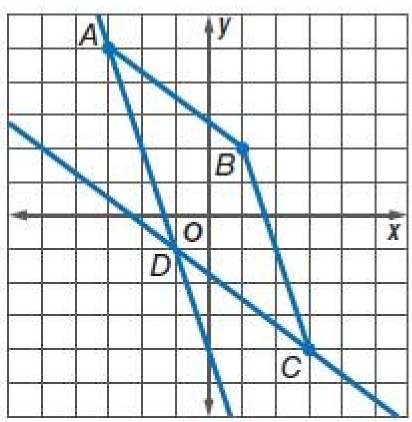
SOLUTION:
Opposite sides of a parallelogram are parallel.

Since the slope of $\overline{BC} = \frac{-4-2}{3-1} = \frac{-6}{2}$, the slope of $\overline{AD}$ must also be $\frac{-6}{2}$.

To locate vertex D, make a line from A, starting from vertex A and moving down 6 and right 2.

The slope of $\overline{AB}$ is $\frac{5-2}{-3-1} = -\frac{3}{4}$. The slope of $\overline{DC}$ must also be $-\frac{3}{4}$. Draw a line from vertex C.

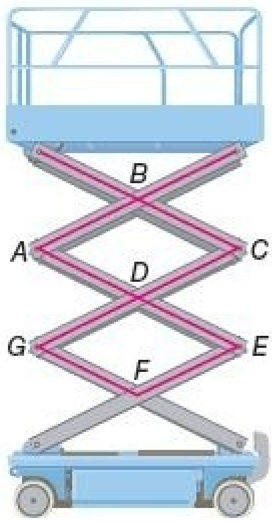
The intersection of the two lines is (–1, –1). This is vertex D.



ANSWER:
(–1, –1)

6-2 Parallelograms

38. **MECHANICS** Scissor lifts are variable elevation work platforms. In the diagram, *ABCD* and *DEFG* are congruent parallelograms.



a. List the angle(s) congruent to $\angle A$. Explain your reasoning.
b. List the segment(s) congruent to $\overline{BC}$. Explain your reasoning.
c. List the angle(s) supplementary to $\angle C$. Explain your reasoning.

SOLUTION:

Use the properties of parallelograms that you learned in this lesson.

a. $\angle C, \angle E, \angle G$; sample answer: $\angle C$ is congruent to $\angle A$ because opposite angles of parallelograms are congruent. $\angle E$ is congruent to $\angle A$ because the parallelograms are congruent, and $\angle G$ is congruent to $\angle E$ because opposite angles of parallelograms are congruent and congruent to $\angle A$ by the Transitive Property.
b. $\overline{AD}, \overline{DE}, \overline{GF}$; sample answer: $\overline{AD}$ is congruent to $\overline{BC}$ because opposite sides of parallelograms are congruent. $\overline{DE}$ is congruent to $\overline{BC}$ because the parallelograms are congruent, and $\overline{GF}$ is congruent to $\overline{DE}$ because opposite sides of parallelograms are congruent and congruent to $\overline{BC}$ by the Transitive Property.
c. $\angle ABC, \angle ADC, \angle EDG, \angle EFG$; sample answer: Angles *ABC* and *ADC* are supplementary to $\angle C$ because consecutive angles of parallelograms are supplementary. $\angle EDG$ is supplementary to $\angle C$ because it is congruent to $\angle ADC$ by the Vertical angles Theorem and supplementary to $\angle C$ by substitution. $\angle EFG$ is congruent to $\angle EDG$ because opposite angles of parallelograms are congruent and it is supplementary to $\angle C$ by substitution.

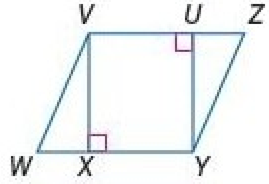
ANSWER:

a. $\angle C, \angle E, \angle G$; sample answer: $\angle C$ is congruent to $\angle A$ because opposite angles of parallelograms are congruent. $\angle E$ is congruent to $\angle A$ because the parallelograms are congruent, and $\angle G$ is congruent to $\angle E$ because opposite angles of parallelograms are congruent and congruent to $\angle A$ by the Transitive Property.
b. $\overline{AD}, \overline{DE}, \overline{GF}$; sample answer: $\overline{AD}$ is congruent to $\overline{BC}$ because opposite sides of parallelograms are congruent. $\overline{DE}$ is congruent to $\overline{BC}$ because the parallelograms are congruent, and $\overline{GF}$ is congruent to $\overline{DE}$ because opposite sides of parallelograms are congruent and congruent to $\overline{BC}$ by the Transitive Property.
c. $\angle ABC, \angle ADC, \angle EDG, \angle EFG$; sample answer: Angles *ABC* and *ADC* are supplementary to $\angle C$ because consecutive angles of parallelograms are supplementary. $\angle EDG$ is supplementary to $\angle C$ because it is congruent to $\angle ADC$ by the Vertical angles Theorem and supplementary to $\angle C$ by substitution. $\angle EFG$ is congruent to $\angle EDG$ because opposite angles of parallelograms are congruent and it is supplementary to $\angle C$ by substitution

PROOF Write a two-column proof.

39. **Given:** $\square YWVZ, \overline{VX} \perp \overline{WY}, \overline{YU} \perp \overline{VZ}$

Prove: $\Delta YUZ \cong \Delta VXW$

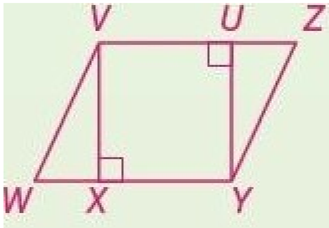


SOLUTION:

Begin by listing what is known. It is given that $\square YWVZ, \overline{VX} \perp \overline{WY}, \overline{YU} \perp \overline{VZ}$. Opposite sides and angles are congruent in parallelograms.

Given: $\square YWVZ, \overline{VX} \perp \overline{WY}, \overline{YU} \perp \overline{VZ}$

Prove: $\Delta YUZ \cong \Delta VXW$



Proof:

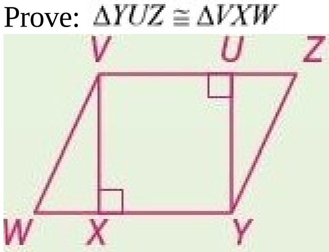
Statements (Reasons)

1. $\square YWVZ, \overline{VX} \perp \overline{WY}, \overline{YU} \perp \overline{VZ}$ (Given)
2. $\angle Z \cong \angle W$ (Opp. $\angle s$ of a $\square$ are $\cong$.)
3. $\overline{WV} \cong \overline{ZY}$ (Opp. sides of a $\square$ are $\cong$.)
4. $\angle VXW$ and $\angle YUZ$ are rt. $\angle s$ ($\perp$ lines form rt. $\angle s$.)
5. ΔVXW *and* ΔYUZ are rt. triangles. (Def. of rt. triangles)
6. $\Delta YUZ \cong \Delta VXW$ (HA)

ANSWER:

Given: $\square YWVZ, \overline{VX} \perp \overline{WY}, \overline{YU} \perp \overline{VZ}$

Prove: $\Delta YUZ \cong \Delta VXW$



Proof:

Statements (Reasons)

1. $\square YWVZ, \overline{VX} \perp \overline{WY}, \overline{YU} \perp \overline{VZ}$ (Given)
2. $\angle Z \cong \angle W$ (Opp. $\angle s$ of a $\square$ are $\cong$.)
3. $\overline{WV} \cong \overline{ZY}$ (Opp. sides of a $\square$ are $\cong$.)
4. $\angle VXW$ and $\angle YUZ$ are rt. $\angle s$ ($\perp$ lines form rt. $\angle s$.)
5. ΔVXW *and* ΔYUZ are rt. triangles. (Def. of rt. triangles)

6. $\Delta YUZ \cong \Delta VXW$ (HA)

40. **MULTIPLE REPRESENTATIONS** In this problem, you will explore tests for parallelograms.

a. GEOMETRIC Draw three pairs of segments that are both congruent and parallel and connect the endpoints to form quadrilaterals. Label one quadrilateral *ABCD,* one *MNOP,* and one *WXYZ.* Measure and label the sides and angles of the quadrilaterals.

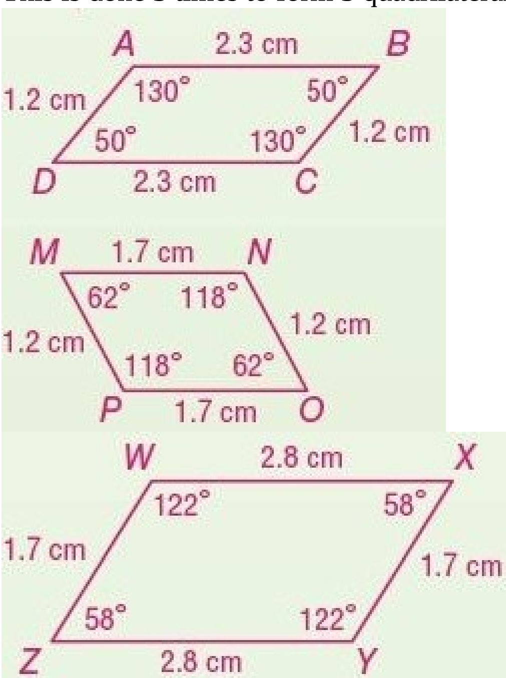
b. TABULAR Complete the table below for each quadrilateral.

Quadrilateral	Opposite Sides Congruent?	Opposite Angles Congruent?	Parallelogram
ABCD			
MNOP			
WXYZ			

c. VERBAL Make a conjecture about quadrilaterals with one pair of segments that are both congruent and parallel.

SOLUTION:

a. Sample answer: Draw a pair of congruent parallel segments and connect the endpoints to form a quadrilateral. This is done 3 times to form 3 quadrilaterals. Use a ruler and protractor to find the measure of each side and angle.



b. Compare the measures of each pair of opposite sides and angles. By the definition of congruence, pairs of sides or angles with equal measures are congruent.

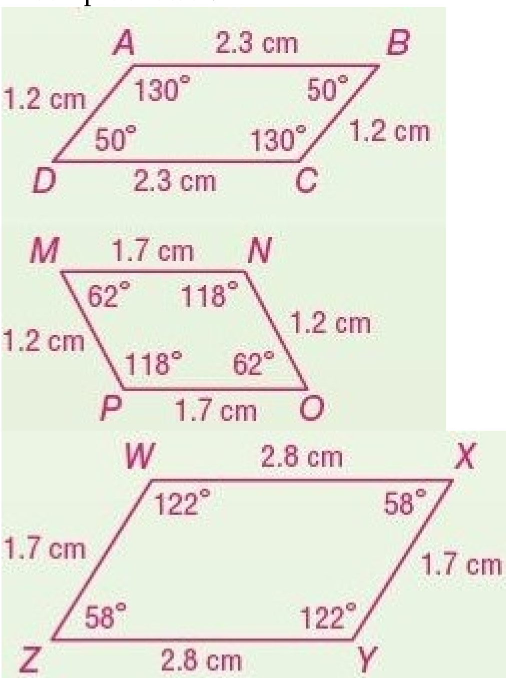
Quadrilateral	Opposite Sides Congruent?	Opposite Angles Congruent?	Parallelogram
ABCD	yes	yes	yes
MNOP	yes	yes	yes
WXYZ	yes	yes	yes

c. Sample answer:

The quadrilaterals were drawn from pairs of segments that are congruent and parallel. When the measures of each side and angle were taken and recorded in the table it was discovered that each pair of opposite sides and angles is congruent. This is the definition of a parallelogram. Therefore, if a quadrilateral has a pair of sides that are congruent and parallel, then the quadrilateral is a parallelogram.

ANSWER:

a. Sample answer:

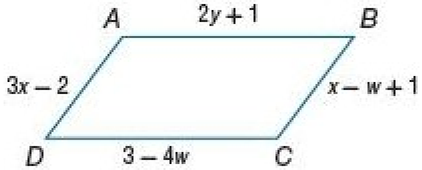


b.

Quadrilateral	Opposite Sides Congruent?	Opposite Angles Congruent?	Parallelogram
ABCD	yes	yes	yes
MNOP	yes	yes	yes
WXYZ	yes	yes	yes

c. Sample answer: If a quadrilateral has a pair of sides that are $\cong and \parallel$, then the quadrilateral is a parallelogram.

41. **CHALLENGE** *ABCD* is a parallelogram with side lengths as indicated in the figure. The perimeter of *ABCD* is 22. Find *AB*.



SOLUTION:

We know that opposite sides of a parallelogram are congruent. So, $2y + 1 = 3 - 4w$ and $3x - 2 = x - w + 1$. The perimeter is 22.

First solve this equation for x in terms of w: $3x - 2 = x - w + 1$.

$$\begin{aligned} 3x - 2 &= x - w + 1 && AD = BC \\ 3x - x - 2 &= -w + 1 && \text{Subtract } x \text{ from each side.} \\ 2x - 2 &= -w + 1 && \text{Simplify.} \\ 2x &= -w + 3 && \text{Add 2 to each side.} \\ x &= -\tfrac{1}{2}w + \tfrac{3}{2} && \text{Divide each side by 2.} \end{aligned}$$

Next, set up an equation for the perimeter. Since opposite sides are congruent, $AB = DC$ and $AD = BC$.

$$\begin{aligned} AB + DC + AD + BC &= 22 && \text{Perimeter of parallelogram} \\ DC + DC + AD + AD &= 22 && \text{Substitute.} \\ 2DC + 2AD &= 22 && \text{Simplify.} \\ 2(3 - 4w) + 2(3x - 2) &= 22 && \text{Substitute.} \\ 2(3 - 4w) + 2\left[3\left(-\tfrac{1}{2}w + \tfrac{3}{2}\right) - 2\right] &= 22 && \text{Substitute.} \\ 6 - 8w + 2\left[-\tfrac{3}{2}w + \tfrac{9}{2} - 2\right] &= 22 && \text{Distributive Property} \\ 6 - 8w - 3w + 9 - 4 &= 22 && \text{Distributive Property} \\ -11w + 11 &= 22 && \text{Combine like terms.} \\ -11w &= 11 && \text{Subtract 11 from each side.} \\ w &= -1 && \text{Divide each side by -11.} \end{aligned}$$

Substitute $w = -1$ in *CD*.

$$\begin{aligned} CD &= 3 - 4w \\ &= 3 - 4(-1) \\ &= 3 + 4 \\ &= 7 \end{aligned}$$

Since the opposite sides of a parallelogram are congruent, $AB = CD = 7$.

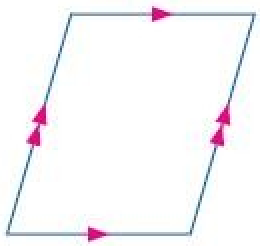
ANSWER:

7

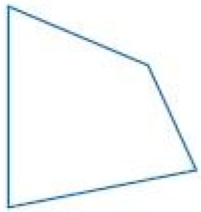
42. **WRITING IN MATH** Explain why parallelograms are *always* quadrilaterals, but quadrilaterals are *sometimes* parallelograms.

SOLUTION:

A parallelogram is a polygon with four sides in which the opposite sides and angles are congruent. Quadrilaterals are defined as four-sided polygons. Since a parallelogram always has four sides, it is always a quadrilateral. A quadrilateral is only a parallelogram when the opposite sides and angles of the polygon are congruent.
A quadrilateral that is also a parallelogram:



A quadrilateral that is not a parallelogram:



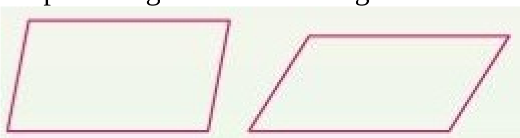
ANSWER:

A parallelogram is a polygon with four sides in which the opposite sides and angles are congruent. Quadrilaterals are defined as four-sided polygons. Since a parallelogram always has four sides, it is always a quadrilateral. A quadrilateral is only a parallelogram when the opposite sides and angles of the polygon are congruent.

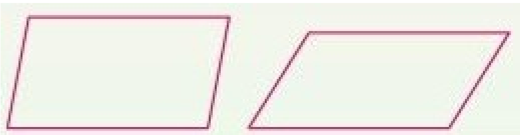
43. **OPEN ENDED** Provide a counterexample to show that parallelograms are not always congruent if their corresponding sides are congruent.

SOLUTION:

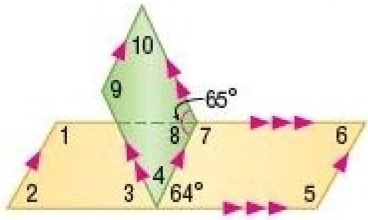
Sample answer:
These parallelograms each have corresponding congruent sides but the corresponding angles are not congruent so the parallelograms are not congruent.



ANSWER:



44. **CCSS REASONING** Find $m\angle 1$ and $m\angle 10$ in the figure. Explain.



SOLUTION:

$\angle 10$ is supplementary to the 65 degree angle because consecutive angles in a parallelogram are supplementary, so $m\angle 10$ is 180 − 65 or 115.

$\angle 1$ is supplementary to $\angle 8$ because consecutive angles in a parallelogram are supplementary, $m\angle 8 = 64$ because alternate interior angles are congruent., so $m\angle 1$ is 116.

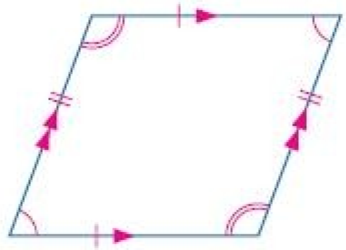
ANSWER:

$m\angle 1 = 116$, $m\angle 10 = 115$; sample answer: $m\angle 8 = 64$ because alternate interior angles are congruent. $\angle 1$ is supplementary to $\angle 8$ because consecutive angles in a parallelogram are supplementary, so $m\angle 1$ is 116. $\angle 10$ is supplementary to the 65 degree angle because consecutive angles in a parallelogram are supplementary, so $m\angle 10$ is 180 − 65 or 115.

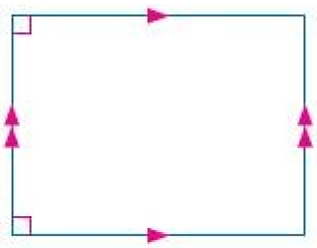
45. **WRITING IN MATH** Summarize the properties of the sides, angles, and diagonals of a parallelogram.

SOLUTION:

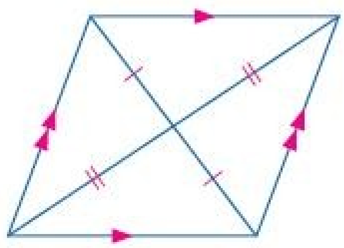
Sample answer: In a parallelogram, the opp. sides and $\angle s$ are $\cong$.



Two consecutive $\angle s$ in a ▱ are supplementary. If one angle of a ▱ is right, then all the angles are right.



The diagonals of a parallelogram bisect each other.



ANSWER:

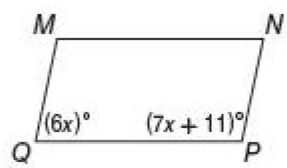
Sample answer: In a parallelogram, the opp. sides and $\angle s$ *are* $\cong$ .Two consecutive $\angle s$ *in a* ▱ *are* supplementary. If one angle of a ▱ is right, then all the angles are right. The diagonals of a parallelogram bisect each other.

46. Two consecutive angles of a parallelogram measure 3*x* + 42 and 9*x* – 18. What are the measures of the angles?
A 13, 167
B 58.5, 31.5
C 39, 141
D 81, 99

SOLUTION:
Consecutive angles in a parallelogram are supplementary.
So, $(3x+42)+(9x-18)=180.$
Solve for *x*.
$$\begin{aligned}(3x+42)+(9x-18)&=180\\ 12x+24&=180\\ 12x&=156\\ x&=13\end{aligned}$$
Substitute $x=13$ in $3x+42$ and $9x-18$.
$$\begin{aligned}3x+42&=3(13)+42\\ &=39+42\\ &=81\end{aligned}$$
$$\begin{aligned}9x-18&=9x-18\\ &=9(13)-18\\ &=117-18\\ &=99\end{aligned}$$
So, the correct option is D.

ANSWER:
D

47. **GRIDDED RESPONSE** Parallelogram *MNPQ* is shown. What is the value of *x*?



SOLUTION:
Consecutive angles in a parallelogram are supplementary.
So, $(6x)+(7x+11)=180.$
Solve for *x*.
$$\begin{aligned}(6x)+(7x+11)&=180\\ 13x+11&=180\\ 13x&=169\\ x&=13\end{aligned}$$

ANSWER:
13

48. **ALGEBRA** In a history class with 32 students, the ratio of girls to boys is 5 to 3. How many more girls are there than boys?

F 2 **H** 12
G 8 **J** 15

SOLUTION:

The ratio of girls to boys is 5 to 3. There are 5 girls in each group of 8 students. $\frac{\text{Number of girls}}{\text{total in group}} = \frac{5}{8}$ Let g be the number of girls in a history class. So, there are g girls to 32 students.

$$\frac{g}{32} = \frac{5}{8}$$
$$g = 20$$

Therefore, the number of boys in a class is 32 – 20 or 12.
There are 8 more girls than boys.
So, the correct option is G.

ANSWER:
G

49. **SAT/ACT** The table shows the heights of the tallest buildings in Kansas City, Missouri. To the nearest tenth, what is the positive difference between the median and the mean of the data?

Name	Height (m)
One Kansas City Place	193
Town Pavillion	180
Hyatt Regency	154
Power and Light Building	147
City Hall	135
1201 Walnut	130

A 5
B 6
C 7
D 8
E 10

SOLUTION:

$$\text{mean} = \frac{193 + 180 + 154 + 147 + 135 + 130}{6}$$
$$= 156.5$$
$$\text{median} = \frac{154 + 147}{2}$$
$$= 150.5$$

Difference = 156.5 – 150.5 = 6
So, the correct option is B.

ANSWER:
B

The measure of an interior angle of a regular polygon is given. Find the number of sides in the polygon.

50. 108

SOLUTION:

Let *n* be the number of sides. Since all angles of a regular polygon are congruent, the sum of the interior angle measures is 108*n*. By the Polygon Interior Angles Sum Theorem, the sum of the interior angle measures can also be expressed as $(n-2)180$.

$$108n = (n-2)180$$
$$108n = 180n - 360$$
$$-72n = -360$$
$$n = 5$$

ANSWER:

5

51. 140

SOLUTION:

Let *n* be the number of sides. Since all angles of a regular polygon are congruent, the sum of the interior angle measures is 140*n*. By the Polygon Interior Angles Sum Theorem, the sum of the interior angle measures can also be expressed as $(n-2)180$.

$$140n = (n-2)180$$
$$140n = 180n - 360$$
$$-40n = -360$$
$$n = 9$$

ANSWER:

9

52. ≈ 147.3

SOLUTION:

Let *n* be the number of sides. Since all angles of a regular polygon are congruent, the sum of the interior angle measures is 147.3*n*. By the Polygon Interior Angles Sum Theorem, the sum of the interior angle measures can also be expressed as $(n-2)180$.

$$147.3n = (n-2)180$$
$$147.3n = 180n - 360$$
$$-32.7n = -360$$
$$n \approx 11$$

So, the polygon has 11 sides.

ANSWER:

11

53. 160

SOLUTION:

Let n be the number of sides. Since all angles of a regular polygon are congruent, the sum of the interior angle measures is $160n$. By the Polygon Interior Angles Sum Theorem, the sum of the interior angle measures can also be expressed as $(n-2)180$.

$$160n = (n-2)180$$
$$160n = 180n - 360$$
$$-20n = -360$$
$$n = 18$$

ANSWER:

18

54. 135

SOLUTION:

Let n be the number of sides. Since all angles of a regular polygon are congruent, the sum of the interior angle measures is $135n$. By the Polygon Interior Angles Sum Theorem, the sum of the interior angle measures can also be expressed as $(n-2)180$.

$$135n = (n-2)180$$
$$135n = 180n - 360$$
$$-45n = -360$$
$$n = 8$$

ANSWER:

8

55. 176.4

SOLUTION:

Let n be the number of sides. Since all angles of a regular polygon are congruent, the sum of the interior angle measures is $176.4n$. By the Polygon Interior Angles Sum Theorem, the sum of the interior angle measures can also be expressed as $(n-2)180$.

$$176.4n = (n-2)180$$
$$176.4n = 180n - 360$$
$$-3.6n = -360$$
$$n = 100$$

ANSWER:

100

56. **LANDSCAPING** When landscapers plant new trees, they usually brace the tree using a stake tied to the trunk of the tree. Use the SAS or SSS Inequality to explain why this is an effective method for keeping a newly planted tree perpendicular to the ground. Assume that the tree does not lean forward or backward.

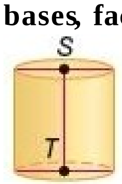
SOLUTION:

By the SAS Inequality Theorem, if the tree started to lean, one of the angles of the triangle formed by the tree, the ground, and the stake would change, and the side opposite that angle would change as well. However, with one end of the long wooden stake in the ground and the other end tied to the tree, no side of the triangle can change length. Thus, no angle can change. This ensures that if the tree were perpendicular to the ground when the stake was attached, then it will remain perpendicular to the ground.

ANSWER:

By the SAS Inequality Theorem, if the tree started to lean, one of the angles of the triangle formed by the tree, the ground, and the stake would change, and the side opposite that angle would change as well. However, with the stake in the ground and fixed to the tree, no side of the triangle can change length. Thus, no angle can change. This ensures that the tree will stay perpendicular to the ground.

Determine whether the solid is a polyhedron. Then identify the solid. If it is a polyhedron, name the bases, faces, edges, and vertices.

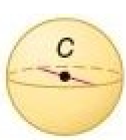
57. 

SOLUTION:

The figure has a curved surface, so it is not a polyhedron. It is a cylinder.

ANSWER:

not a polyhedron; cylinder

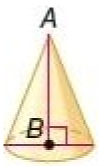
58. 

SOLUTION:

The figure has a curved surface, so it is not a polyhedron. It is a sphere.

ANSWER:

not a polyhedron; sphere

59. 

SOLUTION:

The figure has a curved surface, so it is not a polyhedron. It is a cone.

ANSWER:

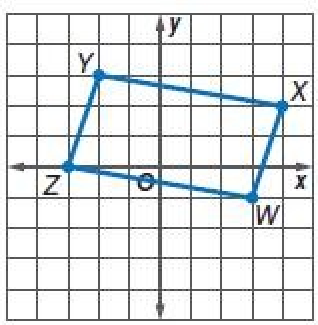
not a polyhedron; cone

The vertices of a quadrilateral are $W(3, -1)$, $X(4, 2)$, $Y(-2, 3)$ and $Z(-3, 0)$. Determine whether each segment is a side or diagonal of the quadrilateral, and find the slope of each segment.

60. $\overline{YZ}$

SOLUTION:

First graph the four points.



$\overline{YZ}$ is a side of the quadrilateral. The slope is 3.

ANSWER:

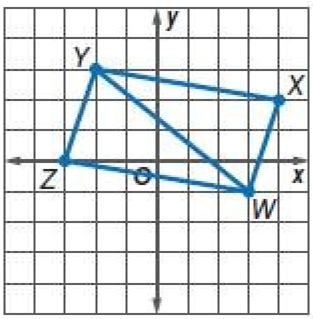
side; 3

6-2 Parallelograms

61. $\overline{YW}$

SOLUTION:

First graph the quadrilateral and $\overline{YW}$.



$\overline{YW}$ is a diagonal with $\text{slope} = -\frac{4}{5}$.

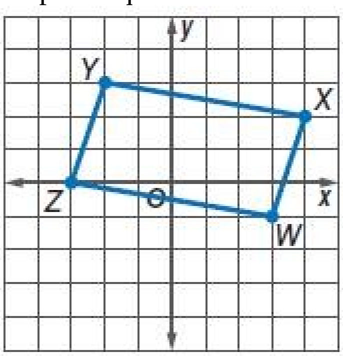
ANSWER:

diagonal; $-\frac{4}{5}$

62. $\overline{ZW}$

SOLUTION:

Graph the quadrilateral.



$\overline{ZW}$ is a side with $\text{slope} = -\frac{1}{6}$.

ANSWER:

side; $-\frac{1}{6}$