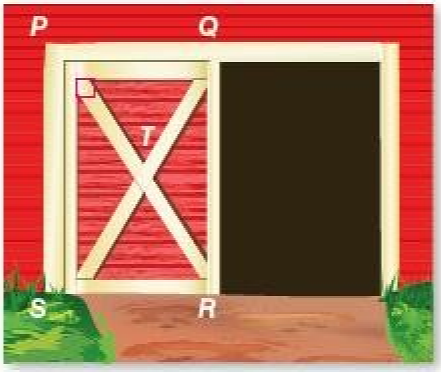
FARMING An X-brace on a rectangular barn door is both decorative and functional. It helps to prevent the door from warping over time. If $ST = 3\frac{13}{16}$ feet, PS = 7 feet, and $m\angle PTQ = 67$, find each measure.



1. *QR*

SOLUTION:
The opposite sides of a rectangle are parallel and congruent. Therefore, *QR* = *PS* = 7 ft.

ANSWER:
7 ft

2. *SQ*

SOLUTION:
The diagonals of a rectangle bisect each other. So,

$$\begin{aligned} SQ &= 2(ST) \\ &= 2\left(\frac{61}{16}\right) \\ &= \frac{61}{8} \\ &= 7\frac{5}{8} \end{aligned}$$

ANSWER:
$7\frac{5}{8}$ ft

3. $m\angle TQR$

SOLUTION:
The diagonals of a rectangle are congruent and bisect each other. So, ΔTQR is an isosceles triangle.
Then, $\angle TQR \cong \angle TRQ$.
By the Exterior Angle Theorem, $m\angle TQR + m\angle TRQ = m\angle PTQ = 67$.
Therefore, $m\angle TQR = \frac{1}{2}(m\angle PTQ) = \frac{1}{2}(67) = 33.5$.

ANSWER:
33.5

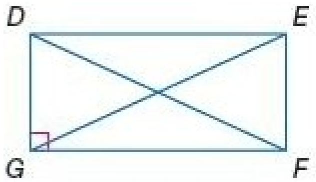
4. $m\angle TSR$

SOLUTION:

The diagonals of a rectangle are congruent and bisect each other. So, ΔTSR is an isosceles triangle.
Then, $\angle TSR \cong \angle TRS.$
By the Vertical Angle Theorem, $m\angle STR = m\angle PTQ = 67.$
Therefore, $m\angle TSR = \frac{1}{2}(180 - m\angle STR) = \frac{1}{2}(113) = 56.5.$

ANSWER:
56.5

ALGEBRA Quadrilateral *DEFG* is a rectangle.



5. If $FD = 3x - 7$ and $EG = x + 5$, find *EG*.

SOLUTION:
The diagonals of a rectangle are congruent to each other. So, *FD* = *EG*.

$$3x - 7 = x + 5$$
$$2x = 12$$
$$x = 6$$

Use the value of *x* to find *EG*.
EG = 6 + 5 = 11

ANSWER:
11

6. If $m\angle EFD = 2x - 3$ and $m\angle DFG = x + 12$,
find $m\angle EFD$.

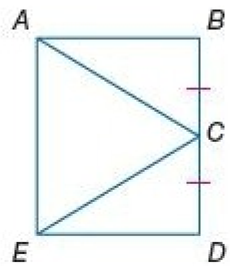
SOLUTION:
All four angles of a rectangle are right angles. So, $m\angle EFD + m\angle DFG = 90.$

$$2x - 3 + x + 12 = 90$$
$$3x = 81$$
$$x = 27$$
$$m\angle EFD = 2(27) - 3$$
$$= 51$$

ANSWER:
51

7. **PROOF** If *ABDE* is a rectangle and $\overline{BC} \cong \overline{DC}$, prove that $\overline{AC} \cong \overline{EC}$.

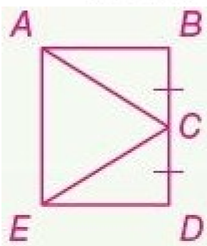
6-4 Rectangles



SOLUTION:

You need to walk through the proof step by step. Look over what you are given and what you need to prove. You are given *ABDE* is a rectangle and $\overline{BC} \cong \overline{DC}$. You need to prove $\overline{AC} \cong \overline{EC}$. Use the properties that you have learned about rectangles to walk through the proof.

Given: *ABDE* is a rectangle; $\overline{BC} \cong \overline{DC}$
Prove: $\overline{AC} \cong \overline{EC}$

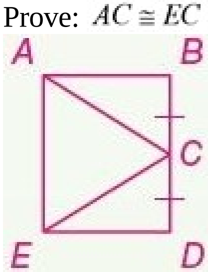


Statements(Reasons)

1. *ABDE* is a rectangle; $\overline{BC} \cong \overline{DC}$. (Given)
2. *ABDE* is a parallelogram. (Def. of rectangle)
3. $\overline{AB} \cong \overline{DE}$ (Opp. sides of a ▱ are ≅.)
4. $\angle B$ and $\angle D$ are right angles.(Def. of rectangle)
5. $\angle B \cong \angle D$ (All rt $\angle$s are ≅.)
6. $\Delta ABC \cong \Delta EDC$ (SAS)
7. $\overline{AC} \cong \overline{EC}$ (CPCTC)

ANSWER:

Given: *ABDE* is a rectangle; $\overline{BC} \cong \overline{DC}$
Prove: $\overline{AC} \cong \overline{EC}$



Statements(Reasons)

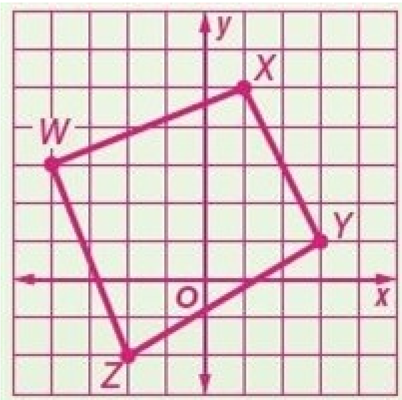
1. *ABDE* is a rectangle; $\overline{BC} \cong \overline{DC}$. (Given)
2. *ABDE* is a parallelogram. (Def. of rectangle)

3. $\overline{AB} \cong \overline{DE}$ (Opp. sides of a ▭ are ≅.)
4. $\angle B$ and $\angle D$ are right angles.(Def. of rectangle)
5. $\angle B \cong \angle D$ (All rt $\angle$s are ≅.)
6. $\Delta ABC \cong \Delta EDC$ (SAS)
7. $\overline{AC} \cong \overline{EC}$ (CPCTC)

COORDINATE GEOMETRY Graph each quadrilateral with the given vertices. Determine whether the figure is a rectangle. Justify your answer using the indicated formula.

8. *W*(–4, 3), *X*(1, 5), *Y*(3, 1), *Z*(–2, –2); Slope Formula

SOLUTION:



Use the slope formula to find the slope of the sides of the quadrilateral.

$$m_{WX} = \frac{5-3}{1-(-4)} = \frac{2}{5}$$

$$m_{XY} = \frac{1-5}{3-1} = -2$$

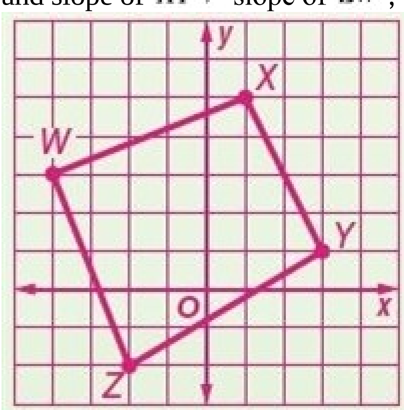
$$m_{YZ} = \frac{-2-1}{-2-3} = \frac{3}{5}$$

$$m_{WZ} = \frac{-2-3}{-2-(-4)} = -\frac{5}{2}$$

There are no parallel sides, so *WXYZ* is not a parallelogram. Therefore, *WXYZ* is not a rectangle.

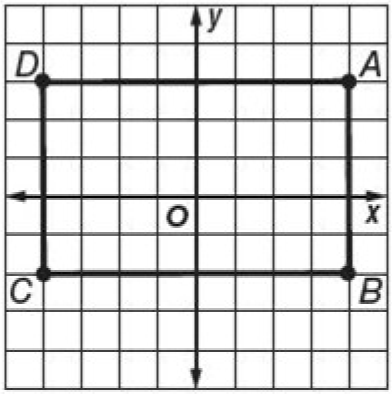
ANSWER:

No; slope of $\overline{WX} = \frac{2}{5}$, slope of $\overline{XY} = -2$, slope of $\overline{YZ} = \frac{3}{5}$, and slope of $\overline{ZW} = -\frac{5}{2}$. Slope of $\overline{WX} \neq$ slope of $\overline{YZ}$, and slope of $\overline{XY} \neq$ slope of $\overline{ZW}$, so *WXYZ* is not a parallelogram. Therefore, *WXYZ* is not a rectangle.



9. $A(4, 3)$, $B(4, –2)$, $C(–4, –2)$, $D(–4, 3)$; Distance Formula

SOLUTION:



First, Use the Distance formula to find the lengths of the sides of the quadrilateral.

$$AB = \sqrt{(4-4)^2 + (-2-3)^2} = \sqrt{0 + (-5)^2} = \sqrt{25} = 5$$

$$BC = \sqrt{(-4-4)^2 + (-2-(-2))^2} = \sqrt{(-8)^2 + 0} = \sqrt{64} = 8$$

$$CD = \sqrt{(-4-(-4))^2 + (3-(-2))^2} = \sqrt{0 + 5^2} = \sqrt{25} = 5$$

$$AD = \sqrt{(-4-4)^2 + (3-3)^2} = \sqrt{(-8)^2 + 0} = \sqrt{64} = 8$$

$AB = 5 = CD, BC = 8 = AD,$ so *ABCD* is a parallelogram.

A parallelogram is a rectangle if the diagonals are congruent. Use the Distance formula to find the lengths of the diagonals.

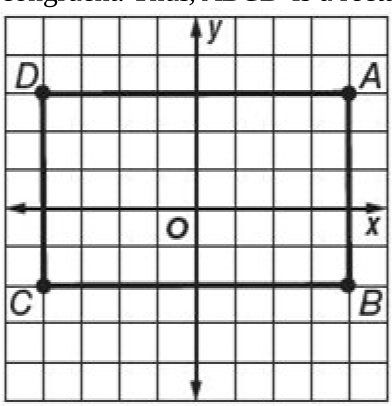
$$AC = \sqrt{(-4-4)^2 + (-2-3)^2} = \sqrt{\left(-8\right)^2 + (-5)^2} = \sqrt{64+25} = \sqrt{89}$$

$$BD = \sqrt{(-4-4)^2 + (3-(-2))^2} = \sqrt{\left(-8\right)^2 + (5)^2} = \sqrt{64+25} = \sqrt{89}$$

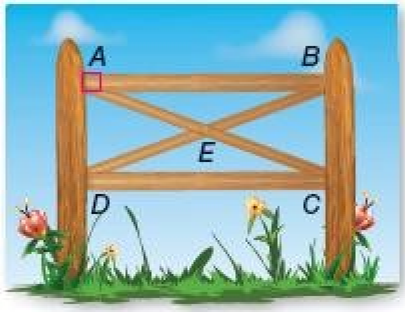
So the diagonals are congruent. Thus, *ABCD* is a rectangle.

ANSWER:

Yes; AB = 5 = CD and BC = 8 = AD. So, *ABCD* is a parallelogram. $BD = \sqrt{89} = AC$, so the diagonals are congruent. Thus, *ABCD* is a rectangle.



FENCING X-braces are also used to provide support in rectangular fencing. If *AB* = 6 feet, *AD* = 2 feet, and $m\angle DAE = 65$, find each measure.



10. *BC*

SOLUTION:

The opposite sides of a rectangle are parallel and congruent. Therefore, *BC* = *AD* = 2 ft.

ANSWER:

2 ft

11. *DB*

SOLUTION:

All four angles of a rectangle are right angles. So, ΔBCD is a right triangle. By the Pythagorean Theorem, $DB^2 = BC^2 + DC^2$.

BC = *AD* = 2

DC = *AB* = 6

$$DB^2 = (2)^2 + (6)^2 = 40$$

$$DB = \sqrt{40} \approx 6.3$$

ANSWER:

6.3 ft

12. $m\angle CEB$

SOLUTION:

The diagonals of a rectangle are congruent and bisect each other. So, ΔAED is an isosceles triangle.

Then, $\angle DAE \cong \angle ADE$.

$$m\angle AED = 180 - (m\angle DAE + m\angle ADE)$$
$$= 180 - (65 + 65)$$
$$= 50$$

By the Vertical Angle Theorem, $m\angle CEB = m\angle AED = 50$.

ANSWER:

50

13. $m\angle EDC$

SOLUTION:

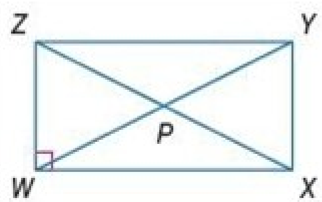
The diagonals of a rectangle are congruent and bisect each other. So, $\triangle AED$ is an isosceles triangle.
Then, $m\angle ADE = m\angle DAE = 50.$
All four angles of a rectangle are right angles.

$$\begin{aligned} m\angle EDC &= 90 - m\angle ADE \\ &= 90 - 65 \\ &= 25 \end{aligned}$$

ANSWER:

25

CCSS REGULARITY Quadrilateral *WXYZ* is a rectangle.



14. If $ZY = 2x + 3$ and $WX = x + 4$, find WX.

SOLUTION:

The opposite sides of a rectangle are parallel and congruent. So, $ZY = WX$.
$2x + 3 = x + 4$
$x = 1$
Use the value of x to find WX.
$WX = 1 + 4 = 5$.

ANSWER:

5

15. If $PY = 3x - 5$ and $WP = 2x + 11$, find ZP.

SOLUTION:

The diagonals of a rectangle bisect each other. So, $PY = WP$.

$$\begin{aligned} 3x - 5 &= 2x + 11 \\ x &= 16 \end{aligned}$$

The diagonals of a rectangle are congruent and bisect each other. So, ΔZPW is an isosceles triangle.

$$\begin{aligned} ZP &= WP \\ &= 2(16) + 11 \\ &= 43 \end{aligned}$$

ANSWER:

43

16. If $m\angle ZYW = 2x - 7$ and $m\angle WYX = 2x + 5$, find $m\angle ZYW$.

SOLUTION:

All four angles of a rectangle are right angles. So, $m\angle ZYW + m\angle WYX = 90.$

$2x - 7 + 2x + 5 = 90$

$4x = 92$

$x = 23$

$m\angle ZYW = 2(23) - 7$

$= 39$

ANSWER:

39

17. If *ZP* = 4*x* – 9 and *PY* = 2*x* + 5, find *ZX*.

SOLUTION:

The diagonals of a rectangle are congruent and bisect each other. So, *ZP* = *PY*.

$4x - 9 = 2x + 5$

$2x = 14$

$x = 7$

Then *ZP* = 4(7) – 9 = 19.
Therefore, *ZX* = 2(*ZP*) = 38.

ANSWER:

38

18. If $m\angle XZY = 3x + 6$ and $m\angle XZW = 5x - 12$, find $m\angle YXZ$.

SOLUTION:

All four angles of a rectangle are right angles. So, $m\angle YZX + m\angle XZW = 90.$

$3x + 6 + 5x - 12 = 90$

$8x = 96$

$x = 12$

$m\angle YXZ = m\angle XZW = 5(12) - 12 = 48$

ANSWER:

48

6-4 Rectangles

19. If $m\angle ZXW = x - 11$ and $m\angle WZX = x - 9$, find $m\angle ZXY$.

SOLUTION:

All four angles of a rectangle are right angles. So, ΔZWX is a right triangle. Then,
$m\angle ZXW + m\angle WZX = 90.$

$x - 11 + x - 9 = 90$

$2x = 110$

$x = 55$

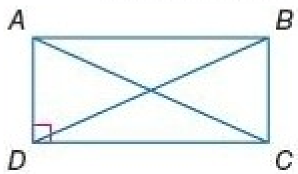
Therefore, $m\angle WZX = 55 - 9$ or $46.$ Since $\overline{ZX}$ is a transversal of parallel sides $\overline{ZY}$ and $\overline{WX}$, alternate interior angles $\angle WZX$ and $\angle ZXY$ are congruent. So, $m\angle ZXY = m\angle WZX = 46$.

ANSWER:

46

PROOF Write a two-column proof.

20. **Given:** *ABCD* is a rectangle.
Prove: $\Delta ADC \cong \Delta BCD$



SOLUTION:

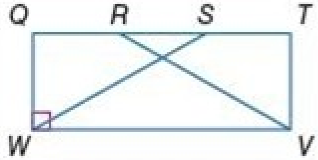
You need to walk through the proof step by step. Look over what you are given and what you need to prove. Here, you are given *ABCD* is a rectangle. You need to prove $\Delta ADC \cong \Delta BCD$. Use the properties that you have learned about rectangles to walk through the proof.

Given: *ABCD* is a rectangle.
Prove: $\Delta ADC \cong \Delta BCD$
Proof:
<u>Statements (Reasons)</u>
1. *ABCD* is a rectangle. (Given)
2. *ABCD* is a parallelogram. (Def. of rectangle)
3. $\overline{AD} \cong \overline{BC}$ (Opp. sides of a ▭ are ≅.)
4. $\overline{DC} \cong \overline{CD}$ (Refl. Prop.)
5. $\overline{AC} \cong \overline{BD}$ (Diagonals of a rectangle are ≅.)
6. $\Delta ADC \cong \Delta BCD$ (SSS)

ANSWER:

Proof:
<u>Statements (Reasons)</u>
1. *ABCD* is a rectangle. (Given)
2. *ABCD* is a parallelogram. (Def. of rectangle)
3. $\overline{AD} \cong \overline{BC}$ (Opp. sides of a ▭ are ≅.)
4. $\overline{DC} \cong \overline{CD}$ (Refl. Prop.)
5. $\overline{AC} \cong \overline{BD}$ (Diagonals of a rectangle are ≅.)
6. $\Delta ADC \cong \Delta BCD$ (SSS)

21. **Given:** *QTVW* is a rectangle.
$\overline{QR} \cong \overline{ST}$

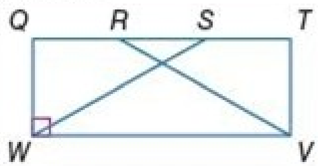


Prove: $\Delta SWQ \cong \Delta RVT$

SOLUTION:

You need to walk through the proof step by step. Look over what you are given and what you need to prove. Here, you are given *QTVW* is a rectangle and $\overline{QR} \cong \overline{ST}$. You need to prove $\Delta SWQ \cong \Delta RVT$. Use the properties that you have learned about rectangles to walk through the proof.

Given: *QTVW* is a rectangle.
$\overline{QR} \cong \overline{ST}$



Prove: $\Delta SWQ \cong \Delta RVT$
Proof:
Statements (Reasons)
1. *QTVW* is a rectangle; $\overline{QR} \cong \overline{ST}$.(Given)
2. *QTVW* is a parallelogram. (Def. of rectangle)
3. $\overline{WQ} \cong \overline{VT}$ (Opp sides of a ▱ are ≅.)
4. $\angle Q$ and $\angle T$ are right angles.(Def. of rectangle)
5. $\angle Q \cong \angle T$ (All rt ∠s are ≅.)
6. $QR = ST$ (Def. of ≅ segs.)
7. $\overline{RS} \cong \overline{RS}$ (Refl. Prop.)
8. $RS = RS$ (Def. of ≅ segs.)
9. $QR + RS = RS + ST$ (Add. prop.)
10. $QS = QR + RS$, $RT = RS + ST$ (Seg. Add. Post.)
11. $QS = RT$ (Subst.)
12. $\overline{QS} \cong \overline{RT}$ (Def. of ≅ segs.)
13. $\Delta SWQ \cong \Delta RVT$ (SAS)

ANSWER:

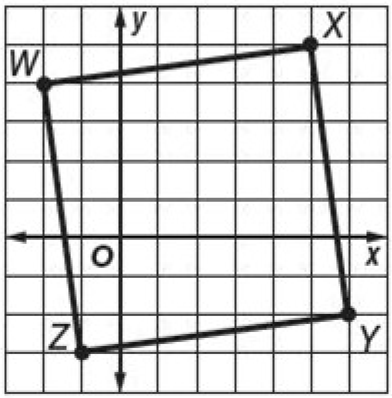
Proof:
Statements (Reasons)
1. *QTVW* is a rectangle; $\overline{QR} \cong \overline{ST}$.(Given)
2. *QTVW* is a parallelogram. (Def. of rectangle)
3. $\overline{WQ} \cong \overline{VT}$ (Opp sides of a ▱ are ≅.)
4. $\angle Q$ and $\angle T$ are right angles.(Def. of rectangle)
5. $\angle Q \cong \angle T$ (All rt ∠s are ≅.)
6. $QR = ST$ (Def. of ≅ segs.)

7. $\overline{RS} \cong \overline{RS}$ (Refl. Prop.)
8. $RS = RS$ (Def. of $\cong$ segs.)
9. $QR + RS = RS + ST$ (Add. prop.)
10. $QS = QR + RS$, $RT = RS + ST$ (Seg. Add. Post.)
11. $QS = RT$ (Subst.)
12. $\overline{QS} \cong \overline{RT}$ (Def. of $\cong$ segs.)
13. $\Delta SWQ \cong \Delta RVT$ (SAS)

COORDINATE GEOMETRY Graph each quadrilateral with the given vertices. Determine whether the figure is a rectangle. Justify your answer using the indicated formula.

22. *W*(–2, 4), *X*(5, 5), *Y*(6, –2), *Z*(–1, –3); Slope Formula

SOLUTION:



Use the slope formula to find the slope of the sides of the quadrilateral.

$$m_{WX} = \frac{5-4}{5-(-2)} = \frac{1}{7}$$

$$m_{XY} = \frac{-2-5}{6-5} = -7$$

$$m_{YZ} = \frac{-3-(-2)}{-1-6} = \frac{1}{7}$$

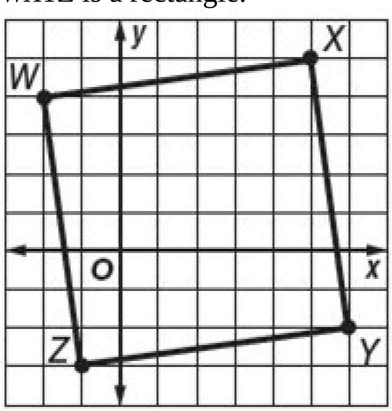
$$m_{WZ} = \frac{-3-4}{-1-(-2)} = -7$$

The slopes of each pair of opposite sides are equal. So, the two pairs of opposite sides are parallel. Therefore, the quadrilateral *WXYZ* is a parallelogram.

The products of the slopes of the adjacent sides are –1. So, any two adjacent sides are perpendicular to each other. That is, all the four angles are right angles. Therefore, *WXYZ* is a rectangle.

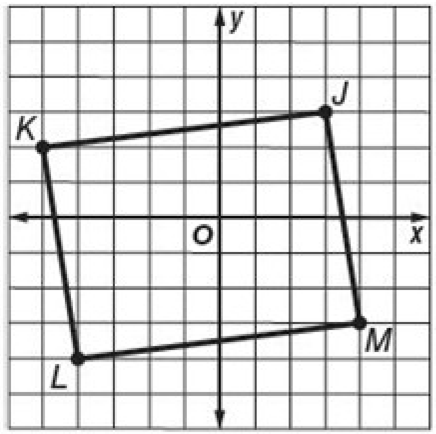
ANSWER:

Yes; slope of $\overline{WX} = \frac{1}{7} =$ slope of $\overline{YZ}$, slope of $\overline{XY} = -7 =$ slope of $\overline{ZW}$. So *WXYZ* is a parallelogram. The product of the slopes of consecutive sides is –1, so the consecutive sides are perpendicular and form right angles. Thus, *WXYZ* is a rectangle.



23. $J(3, 3)$, $K(–5, 2)$, $L(–4, –4)$, $M(4, –3)$; Distance Formula

SOLUTION:



First, Use the Distance formula to find the lengths of the sides of the quadrilateral.

$$JK = \sqrt{(-5-3)^2 + (2-3)^2} = \sqrt{\left(-8\right)^2 + (-1)^2} = \sqrt{64+1} = \sqrt{65}$$

$$KL = \sqrt{(-4-(-5))^2 + (-4-2)^2} = \sqrt{1^2 + (-6)^2} = \sqrt{37}$$

$$LM = \sqrt{(4-(-4))^2 + (-3-(-4))^2} = \sqrt{8^2 + 1^2} = \sqrt{65}$$

$$JM = \sqrt{(4-3)^2 + (3-(-3))^2} = \sqrt{1^2 + 6^2} = \sqrt{37}$$

$JK = \sqrt{65} = LM, KL = 8 = JM,$ so *JKLM* is a parallelogram.

A parallelogram is a rectangle if the diagonals are congruent. Use the Distance formula to find the lengths of the diagonals.

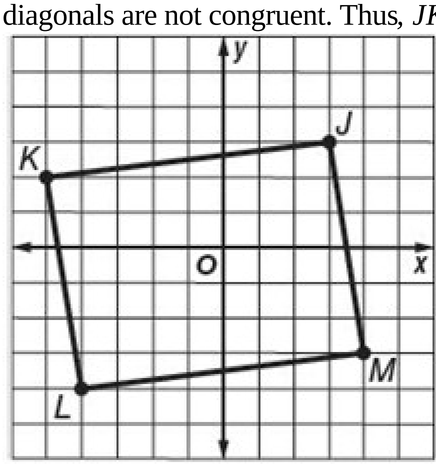
$$KM = \sqrt{(4-(-5))^2 + (-3-2)^2} = \sqrt{9^2 + (-5)^2} = \sqrt{81+25} = \sqrt{106}$$

$$JL = \sqrt{(-4-3)^2 + (-4-3)^2} = \sqrt{(-7)^2 + (-7)^2} = \sqrt{49+49} = \sqrt{98}$$

So the diagonals are not congruent. Thus, *JKLM* is not a rectangle.

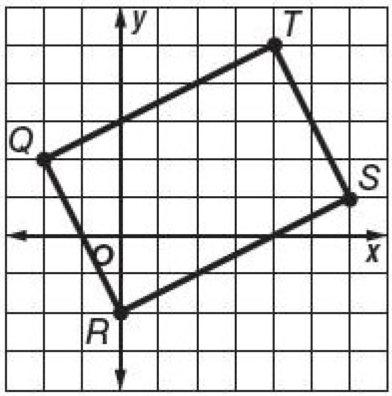
ANSWER:

No; $JK = \sqrt{65} = LM, KL = \sqrt{37} = MJ,$ so *JKLM* is a parallelogram; $KM = \sqrt{106}; JL = \sqrt{98}. KM \neq JL,$ so the diagonals are not congruent. Thus, *JKLM* is not a rectangle.



24. $Q(-2, 2)$, $R(0, -2)$, $S(6, 1)$, $T(4, 5)$; Distance Formula

SOLUTION:



First, use the Distance Formula to find the lengths of the sides of the quadrilateral.

$$QR = \sqrt{(0-(-2))^2 + (-2-2)^2} = \sqrt{2^2 + (-4)^2} = \sqrt{4+16} = \sqrt{20}$$

$$RS = \sqrt{(6-0)^2 + (1-(-2))^2} = \sqrt{6^2 + 3^2} = \sqrt{36+9} = \sqrt{45}$$

$$ST = \sqrt{(4-6)^2 + (5-1)^2} = \sqrt{(-2)^2 + 4^2} = \sqrt{4+16} = \sqrt{20}$$

$$QT = \sqrt{(4-(-2))^2 + (5-2)^2} = \sqrt{6^2 + 3^2} = \sqrt{36+9} = \sqrt{45}$$

$QR = \sqrt{20} = ST, RS = \sqrt{45} = QT,$ so *QRST* is a parallelogram.

A parallelogram is a rectangle if the diagonals are congruent. Use the Distance Formula to find the lengths of the diagonals.

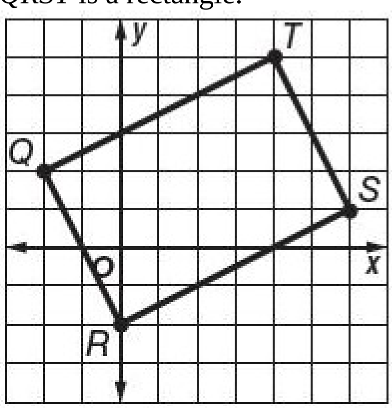
$$QS = \sqrt{(6-(-2))^2 + (2-1)^2} = \sqrt{8^2 + 1^2} = \sqrt{64+1} = \sqrt{65}$$

$$RT = \sqrt{(4-0)^2 + (5-(-2))^2} = \sqrt{4^2 + 7^2} = \sqrt{16+49} = \sqrt{65}$$

So the diagonals are congruent. Thus, *QRST* is a rectangle.

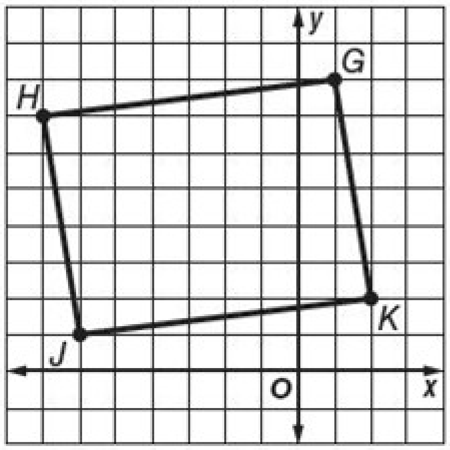
ANSWER:

Yes; $QR = \sqrt{20} = ST, RS = \sqrt{45} = TQ,$ so *QRST* is a parallelogram. $QS = \sqrt{65} = RT,$ so the diagonals are congruent. *QRST* is a rectangle.



25. $G(1, 8)$, $H(–7, 7)$, $J(–6, 1)$, $K(2, 2)$; Slope Formula

SOLUTION:



First, use the slope formula to find the lengths of the sides of the quadrilateral.

$$m_{GH}=\frac{7-8}{-7-1}=\frac{1}{8}$$

$$m_{HJ}=\frac{1-7}{-6-(-7)}=-6$$

$$m_{JK}=\frac{2-1}{2-(-6)}=\frac{1}{8}$$

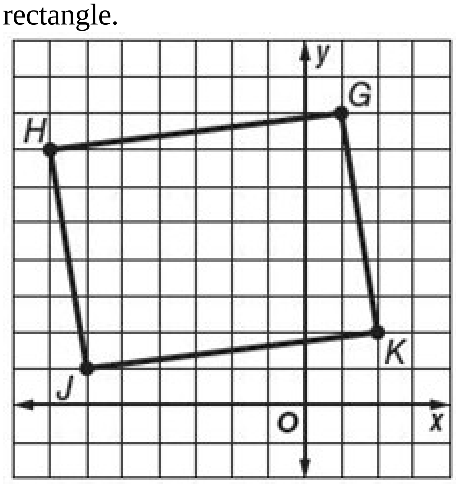
$$m_{GK}=\frac{2-8}{2-1}=-6$$

The slopes of each pair of opposite sides are equal. So, the two pairs of opposite sides are parallel. Therefore, the quadrilateral *GHJK* is a parallelogram.

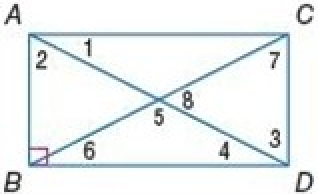
None of the adjacent sides have slopes whose product is –1. So, the angles are not right angles. Therefore, *GHJK* is not a rectangle.

ANSWER:

No; slope of $\overline{GH}=\frac{1}{8}=$ slope of $\overline{JK}$ and slope of $\overline{HJ}=-6=$ slope of $\overline{KG}=-6$. So, *GHJK* is a parallelogram. The product of the slopes of consecutive sides $\neq -1$, so the consecutive sides are not perpendicular. Thus, *GHJK* is not a rectangle.



Quadrilateral *ABCD* is a rectangle. Find each measure if $m\angle 2 = 40$.



26. $m\angle 1$

SOLUTION:

All four angles of a rectangle are right angles. So, $m\angle 1 + m\angle 2 = 90.$

$$m\angle 1 = 90 - 40$$
$$= 50$$

ANSWER:

50

27. $m\angle 7$

SOLUTION:

The measures of angles 2 and 3 are equal as they are alternate interior angles.
Since the diagonals of a rectangle are congruent and bisect each other, the triangle with the angles 3, 7 and 8 is an isosceles triangle. So, $m\angle 7 = m\angle 3$.
Therefore, $m\angle 7 = m\angle 2 = 40$.

ANSWER:

40

28. $m\angle 3$

SOLUTION:

The measures of angles 2 and 3 are equal as they are alternate interior angles.
Therefore, $m\angle 3 = m\angle 2 = 40$.

ANSWER:

40

29. $m\angle 5$

SOLUTION:

All four angles of a rectangle are right angles. So, $m\angle 1 + m\angle 2 = 90.$

$$\begin{aligned} m\angle 1 &= 90 - 40 \\ &= 50 \end{aligned}$$

The measures of angles 1 and 4 are equal as they are alternate interior angles.
Therefore, $m\angle 4 = m\angle 1 = 50$.
Since the diagonals of a rectangle are congruent and bisect each other, the triangle with the angles 4, 5 and 6 is an isosceles triangle. So, $m\angle 6 = m\angle 4 = 50$.
The sum of the three angles of a triangle is 180.
Therefore, $m\angle 5 = 180 – (50 + 50) = 80$.

ANSWER:

80

30. $m\angle 6$

SOLUTION:

All four angles of a rectangle are right angles. So, $m\angle 1 + m\angle 2 = 90.$

$$\begin{aligned} m\angle 1 &= 90 - 40 \\ &= 50 \end{aligned}$$

The measures of angles 1 and 4 are equal as they are alternate interior angles.
Therefore, $m\angle 4 = m\angle 1 = 50$.
Since the diagonals of a rectangle are congruent and bisect each other, the triangle with the angles 4, 5 and 6 is an isosceles triangle. Therefore, $m\angle 6 = m\angle 4 = 50$.

ANSWER:

50

31. $m\angle 8$

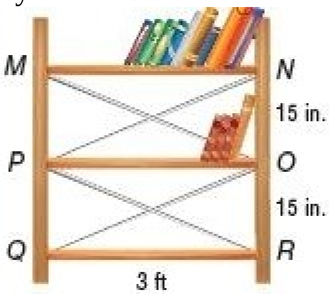
SOLUTION:

The measures of angles 2 and 3 are equal as they are alternate interior angles.
Since the diagonals of a rectangle are congruent and bisect each other, the triangle with the angles 3, 7 and 8 is an isosceles triangle. So, $m\angle 3 = m\angle 7 = 40$.
The sum of the three angles of a triangle is 180. Therefore, $m\angle 8 = 180 – (40 + 40) = 100$.

ANSWER:

100

32. **CCSS MODELING** Jody is building a new bookshelf using wood and metal supports like the one shown. To what length should she cut the metal supports in order for the bookshelf to be *square*, which means that the angles formed by the shelves and the vertical supports are all right angles? Explain your reasoning.



SOLUTION:

In order for the bookshelf to be square, the lengths of all of the metal supports should be equal. Since we know the length of the bookshelves and the distance between them, use the Pythagorean Theorem to find the lengths of the metal supports.

Let x be the length of each support. Express 3 feet as 36 inches.

$$15^2 + 36^2 = x^2$$
$$1521 = x^2$$
$$\sqrt{1521} = x$$
$$39 = x$$

Therefore, the metal supports should each be 39 inches long or 3 feet 3 inches long.

ANSWER:

3 ft 3 in.; Sample answer: In order for the bookshelf to be square, the lengths of all of the metal supports should be equal. Since I knew the length of the bookshelves and the distance between them, I used the Pythagorean Theorem to find the lengths of the metal supports.

PROOF Write a two-column proof.

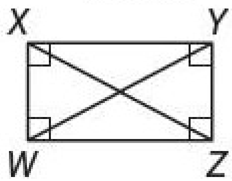
33. Theorem 6.13

SOLUTION:

You need to walk through the proof step by step. Look over what you are given and what you need to prove. Here, you are given *WXYZ* is a rectangle with diagonals $\overline{WY}$ and $\overline{XZ}$. You need to prove $\overline{WY} \cong \overline{XZ}$. Use the properties that you have learned about rectangles to walk through the proof.

Given: *WXYZ* is a rectangle with diagonals $\overline{WY}$ and $\overline{XZ}$.

Prove: $\overline{WY} \cong \overline{XZ}$



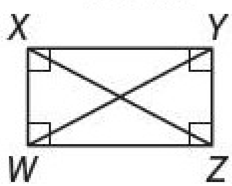
Proof:

1. *WXYZ* is a rectangle with diagonals $\overline{WY}$ and $\overline{XZ}$. (Given)
2. $\overline{WX} \cong \overline{ZY}$ (Opp. sides of a ▭ are ≅.)
3. $\overline{WZ} \cong \overline{WZ}$ (Refl. Prop.)
4. $\angle XWZ$ and $\angle YZW$ are right angles. (Def. of rectangle)
5. $\angle XWZ \cong \angle YZW$ (All right ∠s are ≅.)
6. $\Delta XWZ \cong \Delta YZW$ (SAS)
7. $\overline{WY} \cong \overline{XZ}$ (CPCTC)

ANSWER:

Given: *WXYZ* is a rectangle with diagonals $\overline{WY}$ and $\overline{XZ}$.

Prove: $\overline{WY} \cong \overline{XZ}$



Proof:

1. *WXYZ* is a rectangle with diagonals $\overline{WY}$ and $\overline{XZ}$. (Given)
2. $\overline{WX} \cong \overline{ZY}$ (Opp. sides of a ▭ are ≅.)
3. $\overline{WZ} \cong \overline{WZ}$ (Refl. Prop.)
4. $\angle XWZ$ and $\angle YZW$ are right angles. (Def. of rectangle)
5. $\angle XWZ \cong \angle YZW$ (All right ∠s are ≅.)
6. $\Delta XWZ \cong \Delta YZW$ (SAS)
7. $\overline{WY} \cong \overline{XZ}$ (CPCTC)

6-4 Rectangles

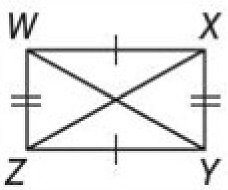
34. Theorem 6.14

SOLUTION:

You need to walk through the proof step by step. Look over what you are given and what you need to prove. Here, you are given $\overline{WX} \cong \overline{YZ}, \overline{XY} \cong \overline{WZ}$, and $\overline{WY} \cong \overline{XZ}$. You need to prove that *WXYZ* is a rectangle.. Use the properties that you have learned about rectangles to walk through the proof.

Given: $\overline{WX} \cong \overline{YZ}, \overline{XY} \cong \overline{WZ}$, and $\overline{WY} \cong \overline{XZ}$

Prove: *WXYZ* is a rectangle.



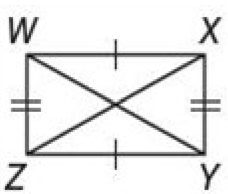
Proof:

1. $\overline{WX} \cong \overline{YZ}, \overline{XY} \cong \overline{WZ}$, and $\overline{WY} \cong \overline{XZ}$ (Given)
2. $\Delta WZX \cong \Delta XYW$ (SSS)
3. $\angle ZWX \cong \angle YXW$ (CPCTC)
4. $m\angle ZWX = m\angle YXW$ (Def. of $\cong \angle$s)
5. *WXYZ* is a parallelogram. (If both pairs of opp. sides are $\cong$, then quad. is ▱.)
6. $\angle ZWX$ and $\angle YXW$ are supplementary. (Cons. $\angle$s of ▱ are suppl.)
7. $m\angle ZWX + m\angle YXW = 180$ (Def of suppl.)
8. $\angle ZWX$ and $\angle YXW$ are right angles. (If 2 $\angle$ are $\cong$ and suppl., each $\angle$ is a rt. $\angle$.)
9. $\angle WZY$ and $\angle XYZ$ are right angles. (If a ▱ has 1 rt. $\angle$, it has 4 rt. $\angle$s.)
10. *WXYZ* is a rectangle. (Def. of rectangle)

ANSWER:

Given: $\overline{WX} \cong \overline{YZ}, \overline{XY} \cong \overline{WZ}$, and $\overline{WY} \cong \overline{XZ}$

Prove: *WXYZ* is a rectangle.



Proof:

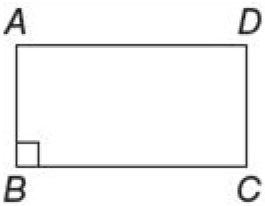
1. $\overline{WX} \cong \overline{YZ}, \overline{XY} \cong \overline{WZ}$, and $\overline{WY} \cong \overline{XZ}$ (Given)
2. $\Delta WZX \cong \Delta XYW$ (SSS)
3. $\angle ZWX \cong \angle YXW$ (CPCTC)
4. $m\angle ZWX = m\angle YXW$ (Def. of $\cong \angle$s)
5. *WXYZ* is a parallelogram. (If both pairs of opp. sides are $\cong$, then quad. is ▱.)
6. $\angle ZWX$ and $\angle YXW$ are supplementary. (Cons. $\angle$s of ▱ are suppl.)
7. $m\angle ZWX + m\angle YXW = 180$ (Def of suppl.)
8. $\angle ZWX$ and $\angle YXW$ are right angles. (If 2 $\angle$ are $\cong$ and suppl., each $\angle$ is a rt. $\angle$.)
9. $\angle WZY$ and $\angle XYZ$ are right angles. (If a ▱ has 1 rt. $\angle$, it has 4 rt. $\angle$s.)
10. *WXYZ* is a rectangle. (Def. of rectangle)

PROOF Write a paragraph proof of each statement.

35. If a parallelogram has one right angle, then it is a rectangle.

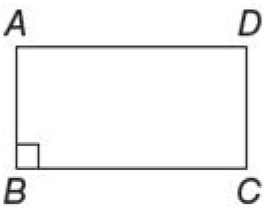
SOLUTION:

You need to walk through the proof step by step. Look over what you are given and what you need to prove. Here, you are given a parallelogram with one right angle. You need to prove that it is a rectangle. Let *ABCD* be a parallelogram with one right angle. Then use the properties of parallelograms to walk through the proof.



ABCD is a parallelogram, and $\angle B$ is a right angle. Since *ABCD* is a parallelogram and has one right angle, then it has four right angles. So by the definition of a rectangle, *ABCD* is a rectangle.

ANSWER:

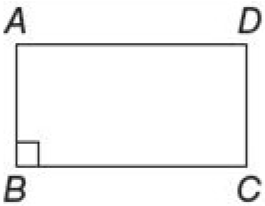


ABCD is a parallelogram, and $\angle B$ is a right angle. Since *ABCD* is a parallelogram and has one right angle, then it has four right angles. So by the definition of a rectangle, *ABCD* is a rectangle.

36. If a quadrilateral has four right angles, then it is a rectangle.

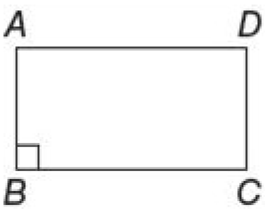
SOLUTION:

You need to walk through the proof step by step. Look over what you are given and what you need to prove. Here, you are given a quadrilateral with four right angles. You need to prove that it is a rectangle. Let *ABCD* be a quadrilateral with four right angles. Then use the properties of parallelograms and rectangles to walk through the proof.



ABCD is a quadrilateral with four right angles. *ABCD* is a parallelogram because both pairs of opposite angles are congruent. By definition of a rectangle, *ABCD* is a rectangle.

ANSWER:



ABCD is a quadrilateral with four right angles. *ABCD* is a parallelogram because both pairs of opposite angles are congruent. By definition of a rectangle, *ABCD* is a rectangle.

37. **CONSTRUCTION** Construct a rectangle using the construction for congruent segments and the construction for a line perpendicular to another line through a point on the line. Justify each step of the construction.

SOLUTION:

Sample answer:

Step 1: Graph a line and place points *P* and *Q* on the line.

Step 2: Place the compass on point *P*. Using the same compass setting, draw an arc to the left and right of *P* that intersects .

Step 3: Open the compass to a setting greater than the distance from *P* to either point of intersection. Place the compass on each point of intersection and draw arcs that intersect above $\overleftrightarrow{PQ}$.

Step 4: Draw line *a* through point *P* and the intersection of the arcs drawn in step 3.

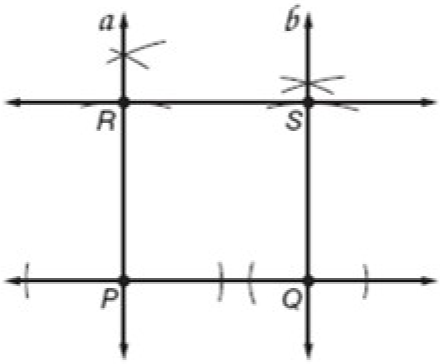
Step 5: Repeat steps 2 and 3 using point *Q*.

Step 6: Draw line *b* through point *Q* and the intersection of the arcs drawn in step 5.

Step 7: Using any compass setting, put the compass at *P* and draw an arc above it that intersects line *a*. Label the point of intersection *R*.

Step 8: Using the same compass setting, put the compass at *Q* and draw an arc above it that intersects line *b*. Label the point of intersection *S*.

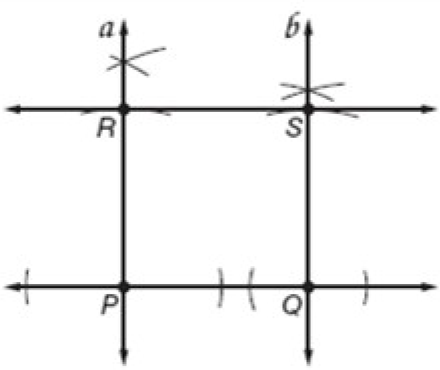
Step 9: Draw $\overline{RS}$.



Since $\overline{RP} \perp \overline{PQ}$ and $\overline{SQ} \perp \overline{PQ}$, the measure of angles *P* and *Q* is 90 degrees. Lines that are perpendicular to the same line are parallel, so $\overline{RP} \parallel \overline{SQ}$. The same compass setting was used to locate points *R* and *S*, so $\overline{RP} \cong \overline{SQ}$. If one pair of opposite sides of a quadrilateral is both parallel and congruent, then the quadrilateral is a parallelogram. A parallelogram with right angles is a rectangle. Thus, *PRSQ* is a rectangle.

ANSWER:

Sample answer: Since $\overline{RP} \perp \overline{PQ}$ and $\overline{SQ} \perp \overline{PQ}$, the measure of angles *P* and *Q* is 90 degrees. Lines that are perpendicular to the same line are parallel, so $\overline{RP} \parallel \overline{SQ}$. The same compass setting was used to locate points *R* and *S*, so $\overline{RP} \cong \overline{SQ}$. If one pair of opposite sides of a quadrilateral is both parallel and congruent, then the quadrilateral is a parallelogram. A parallelogram with right angles is a rectangle. Thus, *PRSQ* is a rectangle.



38. **SPORTS** The end zone of a football field is 160 feet wide and 30 feet long. Kyle is responsible for painting the field. He has finished the end zone. Explain how Kyle can confirm that the end zone is the regulation size and be sure that it is also a rectangle using only a tape measure.

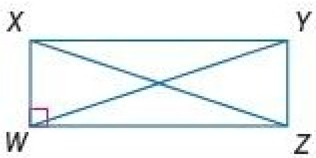
SOLUTION:

Sample answer: If the diagonals of a parallelogram are congruent, then it is a rectangle. He should measure the diagonals of the end zone and both pairs of opposite sides. If both pairs of opposite sides are congruent, then the quadrilateral is a parallelogram. If the diagonals are congruent, then the end zone is a rectangle.

ANSWER:

Sample answer: He should measure the diagonals of the end zone and both pairs of opposite sides. If the diagonals and both pairs of opposite sides are congruent, then the end zone is a rectangle.

ALGEBRA Quadrilateral *WXYZ* is a rectangle.



39. If $XW = 3$, $WZ = 4$, and $XZ = b$, find YW.

SOLUTION:

The diagonals of a rectangle are congruent to each other. So, $YW = XZ = b$. All four angles of a rectangle are right angles. So, ΔXWZ is a right triangle. By the Pythagorean Theorem, $XZ^2 = XW^2 + WZ^2$.

$b^2 = (3)^2 + (4)^2 = 25$

$b = \sqrt{25} = 5$

Therefore, $YW = 5$.

ANSWER:

5

40. If $XZ = 2c$ and $ZY = 6$, and $XY = 8$, find WY.

SOLUTION:

The diagonals of a rectangle are congruent to each other. So, $WY = XZ = 2c$. All four angles of a rectangle are right angles. So, ΔXYZ is a right triangle. By the Pythagorean Theorem, $XZ^2 = XY^2 + ZY^2$.

$(2c)^2 = (6)^2 + (8)^2$

$4c^2 = 100$

$c^2 = 25$

$c = \sqrt{25} = 5$

Therefore, $WY = 2(5) = 10$.

ANSWER:

10

41. **SIGNS** The sign below is in the foyer of Nyoko's school. Based on the dimensions given, can Nyoko be sure that the sign is a rectangle? Explain your reasoning.

SOLUTION:

Both pairs of opposite sides are congruent, so the sign is a parallelogram, but no measure is given that can be used to prove that it is a rectangle. If the diagonals could be proven to be congruent, or if the angles could be proven to be right angles, then we could prove that the sign was a rectangle.

ANSWER:

No; sample answer: Both pairs of opposite sides are congruent, so the sign is a parallelogram, but no measure is given that can be used to prove that it is a rectangle.

PROOF Write a coordinate proof of each statement.

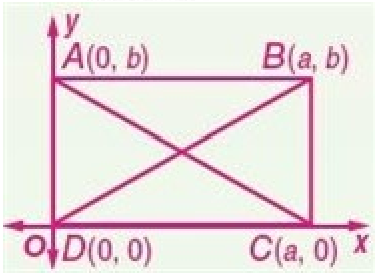
42. The diagonals of a rectangle are congruent.

SOLUTION:

Begin by positioning quadrilateral *ABCD* on a coordinate plane. Place vertex *D* at the origin. Let the length of the bases be *a* units and the height be *b* units. Then the rest of the vertices are *A*(0, *b*), *B*(*a*, *b*), and *C*(*a*, 0). You need to walk through the proof step by step. Look over what you are given and what you need to prove. Here, you are given *ABCD* is a rectangle and you need to prove $\overline{AC} \cong \overline{DB}$. Use the properties that you have learned about rectangles to walk through the proof.

Given: *ABCD* is a rectangle.

Prove: $\overline{AC} \cong \overline{DB}$



Proof:

Use the Distance Formula to find the lengths of the diagonals.

$AC = \sqrt{(0-a)^2 + (b-0)^2} = \sqrt{a^2 + b^2}$

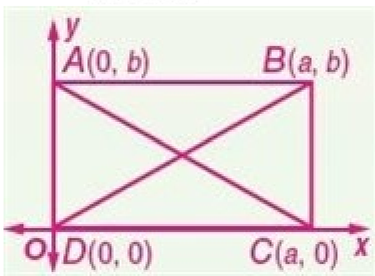
$BD = \sqrt{(a-0)^2 + (b-0)^2} = \sqrt{a^2 + b^2}$

The diagonals, $\overline{AC}$ and $\overline{BD}$ have the same length, so they are congruent.

ANSWER:

Given: *ABCD* is a rectangle.

Prove: $\overline{AC} \cong \overline{DB}$



Proof:

Use the Distance Formula to find

$AC = \sqrt{a^2 + b^2}$ and $BD = \sqrt{a^2 + b^2}$. $\overline{AC}$ and $\overline{DB}$ have the same length, so they are congruent.

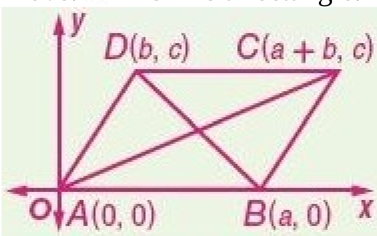
43. If the diagonals of a parallelogram are congruent, then it is a rectangle.

SOLUTION:

Begin by positioning parallelogram *ABCD* on a coordinate plane. Place vertex *A* at the origin. Let the length of the bases be *a* units and the height be *c* units. Then the rest of the vertices are *B*(*a*, 0), *C*(*b* + *a*, *c*), and *D*(*b*, *c*). You need to walk through the proof step by step. Look over what you are given and what you need to prove. Here, you are given ▱*ABCD* and $\overline{AC} \cong \overline{BD}$ and you need to prove that *ABCD* is a rectangle. Use the properties that you have learned about rectangles to walk through the proof.

Given: ▱$ABCD$ and $\overline{AC} \cong \overline{BD}$

Prove: ▱$ABCD$ is a rectangle.



Proof:

$$AC = \sqrt{(a+b-0)^2 + (c-0)^2} = \sqrt{(a+b)^2 + c^2}$$

$$BD = \sqrt{(b-a)^2 + (c-0)^2} = \sqrt{(b-a)^2 + c^2}$$

$AC = BD$. So,

$$\begin{aligned} (a+b)^2 + c^2 &= (b-a)^2 + c^2 \\ a^2 + 2ab + b^2 + c^2 &= b^2 - 2ab + a^2 + c^2 \\ 2ab &= -2ab \\ 4ab &= 0 \\ a = 0 &\text{ or } b = 0 \end{aligned}$$

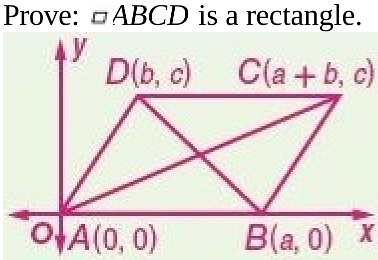
Because A and B are different points,

$a \neq 0$. Then $b = 0$. The slope of $\overline{AD}$ is undefined and the slope of $\overline{AB} = 0$. Thus, $\overline{AD} \perp \overline{AB}$. $\angle DAB$ is a right angle and $ABCD$ is a rectangle.

ANSWER:

Given: ▱$ABCD$ and $\overline{AC} \cong \overline{BD}$

Prove: ▱$ABCD$ is a rectangle.



Proof:

$$AC = \sqrt{(a+b-0)^2 + (c-0)^2} = \sqrt{(a+b)^2 + c^2}$$

$$BD = \sqrt{(b-a)^2 + (c-0)^2} = \sqrt{(b-a)^2 + c^2}$$

But $AC = BD$. So, $\sqrt{(a+b)^2 + c^2} = \sqrt{(b-a)^2 + c^2}$.

$$(a+b)^2+c^2=(b-a)^2+c^2$$
$$a^2+2ab+b^2+c^2=b^2-2ab+a^2+c^2$$
$$2ab=-2ab$$
$$4ab=0$$
$$a=0 \text{ or } b=0$$

Because A and B are different points,
$a \neq 0$. Then $b = 0$. The slope of $\overline{AD}$ is undefined and the slope of $\overline{AB}=0$. Thus, $\overline{AD} \perp \overline{AB}$. $\angle DAB$ is a right angle and $ABCD$ is a rectangle..

6-4 Rectangles

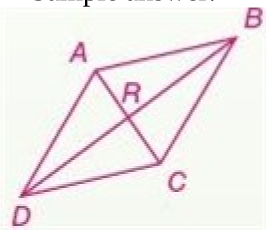
44. **MULTIPLE REPRESENTATIONS** In the problem, you will explore properties of other special parallelograms.
a. GEOMETRIC Draw three parallelograms, each with all four sides congruent. Label one parallelogram *ABCD*, one *MNOP*, and one *WXYZ*. Draw the two diagonals of each parallelogram and label the intersections *R*.
b. TABULAR Use a protractor to measure the appropriate angles and complete the table below.

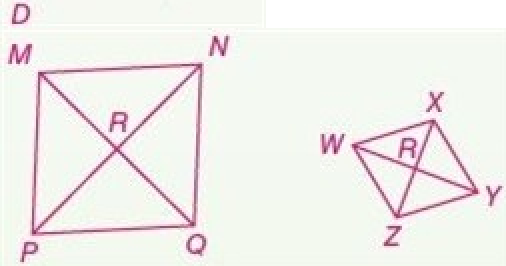
Parallelogram	ABCD		MNOP		WXYZ	
Angle	∠ARB	∠BRC	∠MRN	∠NRO	∠WRX	∠XRY
Angle Measure						

c. VERBAL Make a conjecture about the diagonals of a parallelogram with four congruent sides.

SOLUTION:

a. Sample answer:





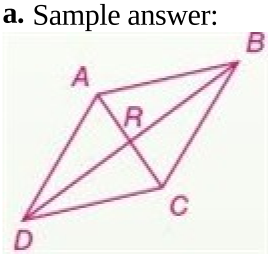
b. Use a protractor to measure each angle listed in the table.

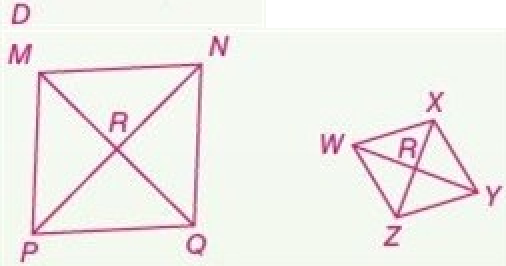
Parallelogram	ABCD		MNOP		WXYZ	
Angle	∠ARB	∠BRC	∠MRN	∠NRO	∠WRX	∠XRY
Angle Measure	90°	90°	90°	90°	90°	90°

c. Sample answer: Each of the angles listed in the table are right angles. These angles were each formed by diagonals. The diagonals of a parallelogram with four congruent sides are perpendicular.

ANSWER:

a. Sample answer:



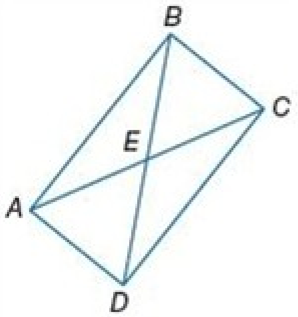


b.

Parallelogram	ABCD		MNOP		WXYZ	
Angle	∠ARB	∠BRC	∠MRN	∠NRO	∠WRX	∠XRY
Angle Measure	90°	90°	90°	90°	90°	90°

c. Sample answer: The diagonals of a parallelogram with four congruent sides are perpendicular.

45. **CHALLENGE** In rectangle *ABCD*, $m\angle EAB = 4x + 6, m\angle DEC = 10 - 11y,$ and $m\angle EBC = 60$ Find the values of *x* and *y*.



SOLUTION:

All four angles of a rectangle are right angles. So, $m\angle EBC + m\angle EBA = 90.$

$60 + m\angle EBC = 90$

$m\angle EBC = 30$

The diagonals of a rectangle are congruent and bisect each other. So, ΔEAB is an isosceles triangle with $\angle A \cong \angle B.$ So,

$m\angle EAB = 30.$

$4x + 6 = 30$

$4x = 24$

$x = 6$

The sum of the angles of a triangle is 180.

So, $m\angle AEB = 180 - (30 + 30) = 120.$

By the Vertical Angle Theorem,

$\angle AEB \cong \angle DEC.$

$m\angle DEC = 10 - 11y = 120$

$-11y = 110$

$y = -10$

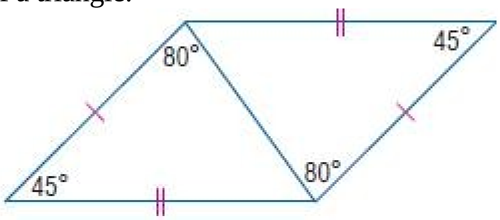
ANSWER:

$x = 6, y = -10$

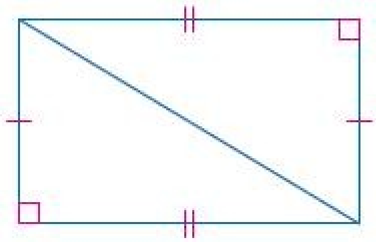
46. **CCSS CRITIQUE** Parker says that any two congruent acute triangles can be arranged to make a rectangle. Tamika says that only two congruent right triangles can be arranged to make a rectangle. Is either of them correct? Explain your reasoning.

SOLUTION:

When two congruent triangles are arranged to form a quadrilateral, two of the angles are formed by a single vertex of a triangle.



In order for the quadrilateral to be a rectangle, one of the angles in the congruent triangles has to be a right angle.

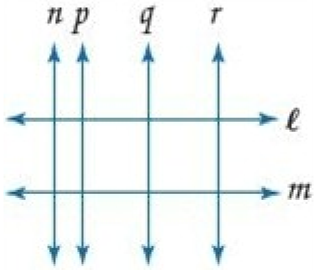


So, Tamika is correct.

ANSWER:

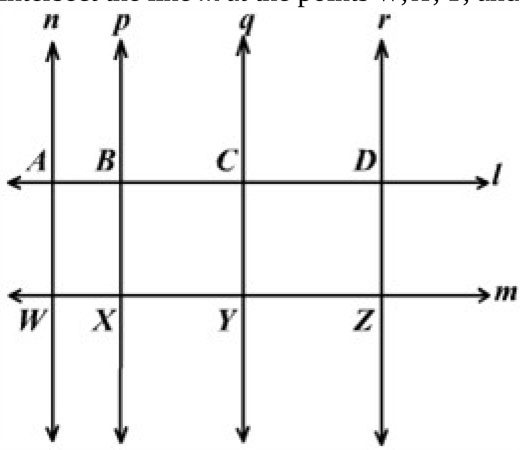
Tamika; Sample answer: When two congruent triangles are arranged to form a quadrilateral, two of the angles are formed by a single vertex of a triangle. In order for the quadrilateral to be a rectangle, one of the angles in the congruent triangles has to be a right angle.

47. **REASONING** In the diagram at the right, lines n, p, q, and r are parallel and lines l and m are parallel. How many rectangles are formed by the intersecting lines?



SOLUTION:

Let the lines n, p, q, and r intersect the line l at the points A, B, C, and D respectively. Similarly, let the lines intersect the line m at the points W, X, Y, and Z.



Then the rectangles formed are $ABXW$, $ACYW$, $BCYX$, $BDZX$, $CDZY$, and $ADZW$. Therefore, 6 rectangles are formed by the intersecting lines.

ANSWER:

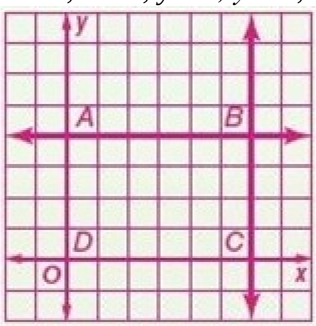
6

48. **OPEN ENDED** Write the equations of four lines having intersections that form the vertices of a rectangle. Verify your answer using coordinate geometry.

SOLUTION:

Sample answer:
A rectangle has 4 right angles and each pair of opposite sides is parallel and congruent. Lines that run parallel to the x-axis will be parallel to each other and perpendicular to lines parallel to the y-axis. Selecting $y = 0$ and $x = 0$ will form 2 sides of the rectangle. The lines $y = 4$ and $x = 6$ form the other 2 sides.

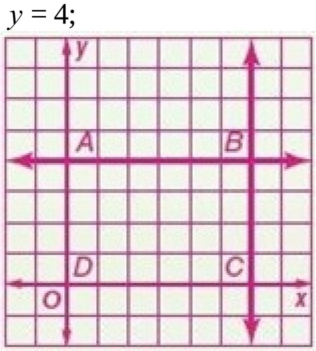
$x = 0$, $x = 6$, $y = 0$, $y = 4$;



The length of $\overline{AB}$ is 6 − 0 or 6 units and the length of $\overline{CD}$ is 6 – 0 or 6 units. The slope of AB is 0 and the slope of DC is 0. Since a pair of sides of the quadrilateral is both parallel and congruent, by Theorem 6.12, the quadrilateral is a parallelogram. Since $\overline{AB}$ is horizontal and $\overline{BC}$ is vertical, the lines are perpendicular and the measure of the angle they form is 90. By Theorem 6.6, if a parallelogram has one right angle, it has four right angles. Therefore, by definition, the parallelogram is a rectangle.

ANSWER:

Sample answer: $x = 0$, $x = 6$, $y = 0$,
$y = 4$;



The length of $\overline{AB}$ is 6 − 0 or 6 units and the length of $\overline{CD}$ is 6 – 0 or 6 units. The slope of AB is 0 and the slope of DC is 0. Since a pair of sides of the quadrilateral is both parallel and congruent, by Theorem 6.12, the quadrilateral is a parallelogram. Since $\overline{AB}$ is horizontal and $\overline{BC}$ is vertical, the lines are perpendicular and the measure of the angle they form is 90. By Theorem 6.6, if a parallelogram has one right angle, it has four right angles. Therefore, by definition, the parallelogram is a rectangle.

49. **WRITING IN MATH** Explain why all rectangles are parallelograms, but all parallelograms are not rectangles.

SOLUTION:

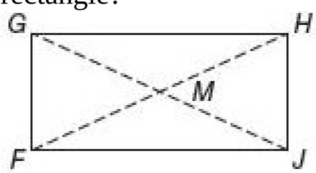
Sample answer: A parallelogram is a quadrilateral in which each pair of opposite sides are parallel. By definition of rectangle, all rectangles have both pairs of opposite sides parallel. So rectangles are always parallelograms.

Parallelograms with right angles are rectangles, so some parallelograms are rectangles, but others with non-right angles are not.

ANSWER:

Sample answer: All rectangles are parallelograms because, by definition, both pairs of opposite sides are parallel. Parallelograms with right angles are rectangles, so some parallelograms are rectangles, but others with non-right angles are not.

50. If $FJ = -3x + 5y$, $FM = 3x + y$, $GH = 11$, and $GM = 13$, what values of x and y make parallelogram *FGHJ* a rectangle?



A $x = 3, y = 4$
B $x = 4, y = 3$
C $x = 7, y = 8$
D $x = 8, y = 7$

SOLUTION:

The opposite sides of a parallelogram are congruent. So, $FJ = GH$.
$-3x + 5y = 11$ $FJ = GH$

If the diagonals of a parallelogram are congruent, then it is a rectangle. Since *FGHJ* is a parallelogram, the diagonals bisect each other. So, if it is a rectangle, then ΔGMF will be an isosceles triangle with $GM \cong FM$. That is, $3x + y = 13$.

Add the two equations to eliminate the x-term and solve for y.

$$\begin{array}{rl} -3x + 5y = 11 & \\ \underline{3x + y = 13} & \\ 6y = 24 & \text{Add to eliminate } x\text{-term.} \\ y = 4 & \text{Divide each side by 6.} \end{array}$$

Use the value of y to find the value of x.

$$\begin{array}{rl} 3x + 4 = 13 & \text{Substitute.} \\ 3x = 9 & \text{Subtract 4 from each side.} \\ x = 3 & \text{Divide each side by 3.} \end{array}$$

So, $x = 3, y = 4$, the correct choice is A.

ANSWER:

A

51. **ALGEBRA** A rectangular playground is surrounded by an 80-foot fence. One side of the playground is 10 feet longer than the other. Which of the following equations could be used to find r, the shorter side of the playground?

F $10r + r = 80$
G $4r + 10 = 80$
H $r(r + 10) = 80$
J $2(r + 10) + 2r = 80$

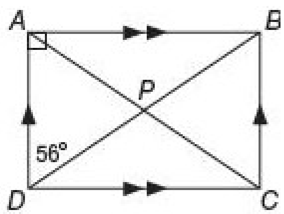
SOLUTION:

The formula to find the perimeter of a rectangle is given by $P = 2(l + w)$. The perimeter is given to be 80 ft. The width is r and the length is $r + 10$. So, $2r + 2(r + 10) = 80$.
Therefore, the correct choice is J.

ANSWER:

J

52. **SHORT RESPONSE** What is the measure of $\angle APB$?



SOLUTION:

The diagonals of a rectangle are congruent and bisect each other. So, ΔPAD is an isosceles triangle with $\angle A \cong \angle D$. So, $m\angle PAD = 56$.
By the Exterior Angle Theorem,
$m\angle APB = m\angle PAD + m\angle PDA = 56 + 56 = 112$.

ANSWER:

112

53. **SAT/ACT** If p is odd, which of the following must also be odd?

A $2p$
B $2p + 2$
C $\frac{p}{2}$
D
E $p + 2$

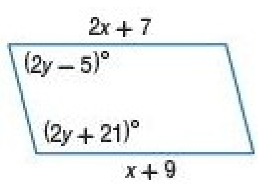
SOLUTION:

Analyze each answer choice to determine which expression is odd when p is odd.
Choice A $2p$
This is a multiple of 2 so it is even.
Choice B $2p + 2$
This can be rewritten as $2(p + 1)$ which is also a multiple of 2 so it is even.
Choice C $\frac{p}{2}$
If p is odd , is not a whole number.
Choice D $2p - 2$
This can be rewritten as $2(p - 1)$ which is also a multiple of 2 so it is even.
Choice E $p + 2$
Adding 2 to an odd number results in an odd number. This is odd.
The correct choice is E.

ANSWER:

E

ALGEBRA Find x and y so that the quadrilateral is a parallelogram.

54. 

SOLUTION:

The opposite sides of a parallelogram are congruent.

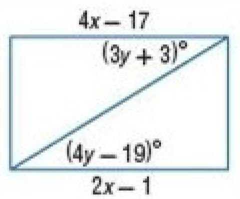
$$2x + 7 = x + 9$$
$$x = 2$$

In a parallelogram, consecutive interior angles are supplementary.

$$2y - 5 + 2y + 21 = 180$$
$$4y + 16 = 180$$
$$4y = 164$$
$$y = 41$$

ANSWER:

$x = 2$, $y = 41$

55. 

SOLUTION:

The opposite sides of a parallelogram are congruent.

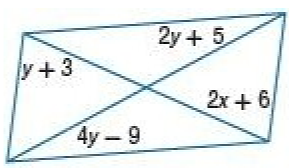
$$4x - 17 = 2x - 1$$
$$2x = 16$$
$$x = 8$$

The opposite sides of a parallelogram are parallel to each other. So, the angles with the measures $(3y + 3)°$ and $(4y - 19)°$ are alternate interior angles and hence they are equal.

$$3y + 3 = 4y - 19$$
$$-y = -22$$
$$y = 22$$

ANSWER:

$x = 8$, $y = 22$

56. 

SOLUTION:

The diagonals of a parallelogram bisect each other. So, $4y - 9 = 2y + 5$ and $y + 3 = 2x + 6$.
Solve the first equation to find the value of y.

$2y = 14$

$y = 7$

Use the value of y in the second equation and solve for x.

$7 + 3 = 2x + 6$

$4 = 2x$

$2 = x$

ANSWER:

$x = 2$, $y = 7$

57. **COORDINATE GEOMETRY** Find the coordinates of the intersection of the diagonals of parallelogram *ABCD* with vertices *A*(1, 3), *B*(6, 2), *C*(4, –2), and *D*(–1, –1).

SOLUTION:

The diagonals of a parallelogram bisect each other, so the intersection is the midpoint of either diagonal. Find the midpoint of $\overline{AC}$ or $\overline{BD}$.

$$M = \left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right) \quad \text{Midpoint Formula}$$

$$M_{\overline{AC}} = \left(\frac{1+4}{2}, \frac{3+(-2)}{2}\right) \quad (x_1, y_1) = (1, 3), (x_2, y_2) = (4, -2)$$

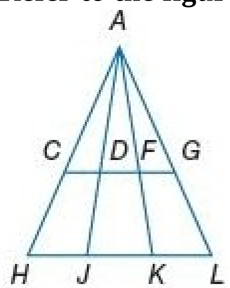
$$= (2.5, 0.5) \quad \text{Simplify.}$$

Therefore, the coordinates of the intersection of the diagonals are (2.5, 0.5).

ANSWER:

(2.5, 0.5)

Refer to the figure.



58. If $\overline{AC} \cong \overline{AF}$, name two congruent angles.

SOLUTION:

Since $\overline{AC} \cong \overline{AF}$, ΔACF is an isosceles triangle. So, $\angle ACF \cong \angle AFC$.

ANSWER:

$\angle ACF$ and $\angle AFC$

59. If $\angle AHJ \cong \angle AJH$, name two congruent segments.

SOLUTION:

In a triangle with two congruent angles, the sides opposite to the congruent angles are also congruent. So, $\overline{AH} \cong \overline{AJ}$.

ANSWER:

$\overline{AH}$ and $\overline{AJ}$

60. If $\angle AJL \cong \angle ALJ$, name two congruent segments.

SOLUTION:

In a triangle with two congruent angles, the sides opposite to the congruent angles are also congruent. So, $\overline{AJ} \cong \overline{AL}$.

ANSWER:

$\overline{AJ}$ and $\overline{AL}$

61. If $\overline{JA} \cong \overline{KA}$, name two congruent angles.

SOLUTION:

Since $\overline{JA} \cong \overline{KA}$, ΔAJK is an isosceles triangle. So, $\angle AJK \cong \angle AKJ$.

ANSWER:

$\angle AJK$ and $\angle AKJ$

Find the distance between each pair of points.

62. (4, 2), (2, –5)

SOLUTION:

Use the Distance Formula to find the distance between the two points.

$$\begin{aligned} D &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\ &= \sqrt{(2-4)^2 + (-5-2)^2} \\ &= \sqrt{53} \end{aligned}$$

ANSWER:

$\sqrt{53}$

63. (0, 6), (–1, –4)

SOLUTION:

Use the Distance Formula to find the distance between the two points.

$$\begin{aligned} D &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\ &= \sqrt{(-1-0)^2 + (-4-6)^2} \\ &= \sqrt{101} \end{aligned}$$

ANSWER:

$\sqrt{101}$

64. (–4, 3), (3, –4)

SOLUTION:

Use the Distance Formula to find the distance between the two points.

$$\begin{aligned} D &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\ &= \sqrt{(3-(-4))^2 + (-4-3)^2} \\ &= \sqrt{98} \\ &= 7\sqrt{2} \end{aligned}$$

ANSWER:

$7\sqrt{2}$