

Mid-Chapter Quiz: Lessons 2-1 through 2-5

Write a conjecture that describes the pattern in each sequence. Then use your conjecture to find the next item in the sequence.

1. 5, 5, 10, 15, 25,...

SOLUTION:

Each element in the pattern is the sum of the previous two elements. The last two elements are 15 and 25, so, the next element will be $15 + 25 = 40$.

ANSWER:

Each element in the pattern is the sum of the previous two elements; 40.



SOLUTION:

Each succeeding figure is contained inside another square. There are 3 squares in the last figure. So, the next figure will have 4 squares.



ANSWER:



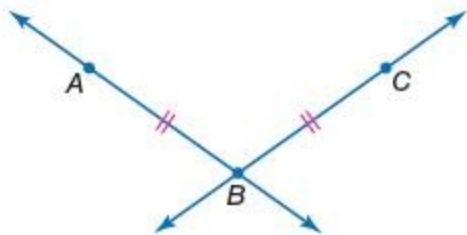
Each succeeding figure is contained inside another square.

Find a counterexample to show that each conjecture is false.

3. If $AB = BC$, then B is the midpoint of \overline{AC} .

SOLUTION:

Points A , B , and C do not have to lie on the same line. The points A , B , and C are not collinear.



ANSWER:

False; A , B , and C do not have to be collinear.

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4. If n is a real number, then $n^3 > n$.

SOLUTION:

Suppose n is the real number 1. When n is 1, $1^3 = 1$. The cubed value is not greater, both values are equal, which makes the conjecture false.

ANSWER:

When n is 1, the conjecture is false, since $1^3 = 1$.

Use the following statements to write a compound statement for each conjunction or disjunction. Then find its truth value. Explain your reasoning.

p : A dollar is equal to 100 cents.

q : There are 4 quarters in a dollar.

r : February is the month before January.

5. $p \wedge r$

SOLUTION:

This statement is read p and r , so both p and r have to be true for the conjunction to be true. A dollar is equal to 100 cents, this is true. February is the month before January, is not true. That is, statement p is true and statement r is false. Therefore, $p \wedge r$ is false.

ANSWER:

False; \$1 = 100¢ and January is the month before February.

6. p and q

SOLUTION:

This statement is an and statement, so both p and q have to be true for the conjunction to be true. A dollar is equal to 100 cents, this is true. There are 4 quarters in a dollar is also true. That is, statement p and statement q are true. Therefore, p and q is true.

ANSWER:

True; \$1 = 100¢ and 4 quarters = \$1.

7. $p \wedge \sim r$

SOLUTION:

This statement is an and statement read p and not r have to be true for the conjunction to be true. The negation of r is that February is the month not before January, which is true. A dollar is equal to 100 cents is also true. That is, statement p and statement $\sim r$ are true. Therefore, $p \wedge \sim r$ is true.

ANSWER:

True; \$1 = 100¢ and February is *not* the month before January.

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8. Copy and complete the truth table.

p	q	$\sim q$	$p \vee \sim q$
T	F		
F	T		
F	F		
T	T		

SOLUTION:

Negate q , finding the opposite truth value. Find the disjunction $p \vee \sim q$. A disjunction is true if at least one of the statements is true.

p	q	$\sim q$	$p \vee \sim q$
T	F	If q is false than not q is true	If p is true and not q is true than the disjunction is true
F	T	If q is true than not q is false	If p is false and not q is false than the disjunction is false
F	F	If q is false than not q is true	If p is false and not q is true than the disjunction is true
T	T	If q is true than not q is false	If p is true and not q is false than the disjunction is true

ANSWER:

p	q	$\sim q$	$p \vee \sim q$
T	F	T	T
F	T	F	F
F	F	T	T
T	T	F	T

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Identify the hypothesis and conclusion of each conditional statement.

9. If a polygon has five sides, then it is a pentagon.

SOLUTION:

The hypothesis of a conditional statement is the phrase immediately following the word *if*. The conclusion of a conditional statement is the phrase immediately following the word *then*.

Hypothesis: a polygon has five sides

Conclusion: it is a pentagon.

ANSWER:

H: a polygon has five sides; C: it is a pentagon

10. If $4x - 6 = 10$, then $x = 4$.

SOLUTION:

The hypothesis of a conditional statement is the phrase immediately following the word *if*. The conclusion of a conditional statement is the phrase immediately following the word *then*.

Hypothesis: $4x - 6 = 10$

Conclusion: $x = 4$

ANSWER:

H: $4x - 6 = 10$; C: $x = 4$

11. An angle with a measure less than 90 is an acute angle.

SOLUTION:

The hypothesis of a conditional statement is the phrase immediately following the word *if*. The conclusion of a conditional statement is the phrase immediately following the word *then*.

Hypothesis: an angle has a measure less than 90

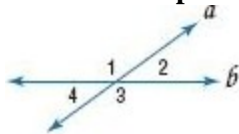
Conclusion: it is an acute angle

ANSWER:

H: an angle has a measure less than 90

C: it is an acute angle

Determine the truth value of each conditional statement. If true, explain your reasoning. If false, give a counterexample.



12. If $\angle 1$ and $\angle 2$ form a linear pair, they are supplementary angles.

SOLUTION:

In the conditional statement "If $\angle 1$ and $\angle 2$ form a linear pair, they are supplementary angles." p is " $\angle 1$ and $\angle 2$ form a linear pair" and q is "they are supplementary angles." From the diagram, $\angle 1$ and $\angle 2$ form a linear pair. So, p is true. Since they form a linear pair, they are supplementary. So, q is true. Therefore, the statement is true.

ANSWER:

True; $m\angle 1 + m\angle 2 = 180$.

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13. If $\angle 1$ and $\angle 4$ form a linear pair, they are congruent angles.

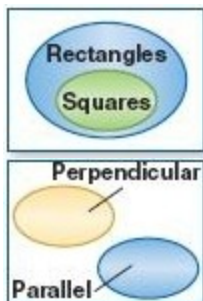
SOLUTION:

In the conditional statement "If $\angle 1$ and $\angle 4$ form a linear pair, then they are congruent angles." p is " $\angle 1$ and $\angle 4$ form a linear pair" and q is "they are congruent angles". p is true, since they form a linear pair. However, $\angle 1$ is an obtuse angle and $\angle 4$ is an acute angle. They are not congruent. So, the statement is false. As a counterexample $\angle 1$ and $\angle 3$ are congruent angles.

ANSWER:

False; $\angle 1$ and $\angle 3$ are congruent angles.

Use the Venn diagrams below to determine the truth value of each conditional. Explain your reasoning.



14. If a polygon is a square, then it is a rectangle.

SOLUTION:

In the Venn diagram, the oval representing squares comes inside that of rectangles. So, all squares are rectangles. Therefore, the statement is true.

ANSWER:

True; All squares are rectangles.

15. If two lines are perpendicular, then they cannot be parallel.

SOLUTION:

In the Venn diagram, There is no common region between the ovals representing parallel and perpendicular. So, two perpendicular lines cannot be parallel. Therefore, the statement is true.

ANSWER:

True; Perpendicular lines intersect and parallel lines never intersect.

16. **FOOTBALL** The Indianapolis Colts played the Chicago Bears in the 2007 Super Bowl. Determine whether the stated conclusion is valid based on the given information. If not, write *invalid*. Explain your reasoning.

Given: The Super Bowl winner has the highest score at the end of the game. The Colts had a score of 29 and the Bears had a score of 17.

Conclusion: The Colts won the Super Bowl.

SOLUTION:

The Colts had the highest score in the Super Bowl. The team with the highest score is the winner, therefore the Colts won the Super Bowl. The statement is valid.

ANSWER:

Valid; the Colts had the highest score in the Super Bowl. The team with the highest score is the winner, therefore the Colts won the Super Bowl.

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17. **MULTIPLE CHOICE** Determine which statement follows logically from the given statements.

(1) If you are a junior in high school, then you are at least 16 years old.

(2) If you are at least 16 years old, then you are old enough to drive.

A If you are old enough to drive, then you are a junior in high school.

B If you are not old enough to drive, then you are a sophomore in high school.

C If you are a junior in high school, then you are old enough to drive.

D No valid conclusion possible.

SOLUTION:

By the Law of Syllogism, if $p \rightarrow q$ and $q \rightarrow r$ are true statements, then $p \rightarrow r$ is a true statement. So, the statements (1) and (2) implies, if you are a junior in high school, , then you are old enough to drive. Therefore, the correct choice is C.

ANSWER:

C

Determine whether each statement is *always*, *sometimes*, or *never* true. Explain your reasoning.

18. Points J , K , L , and N are noncollinear and lie in the same plane M .

SOLUTION:

Postulate 2.4 states that a plane contains at least 3 non-collinear points. Plane M contains 4 noncollinear points. So, the statement is *sometimes* true.

ANSWER:

Sometimes; Postulate 2.4 states that a plane contains at least 3 non-collinear points.

19. There is exactly one line through points R and S .

SOLUTION:

Postulate 2.1 states through any two points, there is exactly one line. So there is exactly one line through points R and S . So, the statement is *always* true.

ANSWER:

Always; Postulate 2.1 states through any two points, there is exactly one line.

20. Line a contains only point Q .

SOLUTION:

Postulate 2.3 states that a line contains at least two points. Therefore, line a must contain another point. So, the statement is *never* true.

ANSWER:

Never; Postulate 2.3 states a line contains at least two points