Mid-Chapter Quiz: Lessons 4-1 through 4-4

1. **COORDINATE GEOMETRY** Classify ΔABC with vertices $A(-2, -1)$, $B(-1, 3)$, and $C(2, 0)$ as scalene, equilateral, or isosceles.

SOLUTION:

Use the Distance Formula to find the lengths of $\overline{AB}, \overline{BC}$ and $\overline{CA}$.

$\overline{AB}$ has endpoints $A(-2, -1)$ and $B(-1, 3)$.

$$AB = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Substitute.

$$\begin{aligned} AB &= \sqrt{(-1-(-2))^2 + (3-(-1))^2} \\ &= \sqrt{(1)^2 + (4)^2} \\ &= \sqrt{1+16} \\ &= \sqrt{17} \end{aligned}$$

$\overline{BC}$ has endpoints $B(-1, 3)$ and $C(2, 0)$.

$$BC = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Substitute.

$$\begin{aligned} BC &= \sqrt{(2-(-1))^2 + (0-3)^2} \\ &= \sqrt{(3)^2 + (-3)^2} \\ &= \sqrt{9+9} \\ &= \sqrt{18} \end{aligned}$$

$\overline{CA}$ has endpoints $C(2, 0)$ and $A(-2,-1)$.

$$CA = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Substitute.

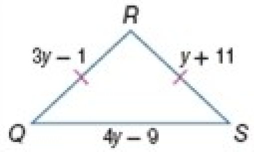
$$\begin{aligned} CA &= \sqrt{(-2-2)^2 + (-1-0)^2} \\ &= \sqrt{(-4)^2 + (-1)^2} \\ &= \sqrt{16+1} \\ &= \sqrt{17} \end{aligned}$$

Here, $AB = CA$. This triangle has two congruent sides. So, it is isosceles.

ANSWER:

isosceles

2. **MULTIPLE CHOICE** Which of the following are the measures of the sides of isosceles triangle *QRS*?



A 17, 17, 15
B 15, 15, 16
C 14, 15, 14
D 14, 14, 16

SOLUTION:
Here, *QR* = *SR*.

$$3y-1=y+11$$
$$3y-1-y=y+11-y$$
$$2y-1=11$$
$$2y-1+1=11+1$$
$$2y=12$$
$$y=6$$

Substitute $y=6$ in *QR*.

$$QR=3y-1$$
$$=3(6)-1$$
$$=18-1$$
$$=17$$

Substitute $y=6$ in *RS*.

$$RS=y+11$$
$$=6+11$$
$$=17$$

Substitute $y=6$ in *QS*.

$$QS=4y-9$$
$$=4(6)-9$$
$$=24-9$$
$$=15$$

ANSWER:
A

3. **ALGEBRA** Find x and the length of each side if ΔWXY is an equilateral triangle with sides $\overline{WX}$ = 6x – 12, $\overline{XY}$ = 2x + 10, and $\overline{WY}$ = 4x – 1.

SOLUTION:

Since ΔWXY is equilateral, $WX = XY = YW$.

$$\begin{aligned} WX &= XY \\ 6x-12 &= 2x+10 \\ 6x-2x-12 &= 2x-2x+10 \\ 4x-12 &= 10 \\ 4x-12+12 &= 10+12 \\ 4x &= 22 \\ x &= 5.5 \end{aligned}$$

Substitute x =5.5.

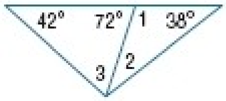
$$\begin{aligned} WX &= 6x-12 \\ &= 6(5.5)-12 \\ &= 33-12 \\ &= 21 \end{aligned}$$

Since all the sides are congruent, $WX = XY = YW = 21$.

ANSWER:

$x = 5.5$; $WX = XY = WY = 21$

Find the measure of each angle indicated.



4. $m\angle 1$

SOLUTION:

In the figure, $m\angle 1 + 72 = 180$.
Solve.

$$\begin{aligned} m\angle 1 + 72 &= 180 \\ m\angle 1 + 72 - 72 &= 180 - 72 \\ m\angle 1 &= 108 \end{aligned}$$

ANSWER:

108

5. $m\angle 2$

SOLUTION:

In the figure, $m\angle 1 + 72 = 180$.
Solve.

$$\begin{aligned} m\angle 1 + 72 &= 180 \\ m\angle 1 + 72 - 72 &= 180 - 72 \\ m\angle 1 &= 108 \end{aligned}$$

The sum of the measures of the angles of a triangle is 180. In the figure, $m\angle 1 + m\angle 2 + 38 = 180$.

Substitute.

$$\begin{aligned} 108 + m\angle 2 + 38 &= 180 \\ 146 + m\angle 2 &= 180 \\ 146 + m\angle 2 - 146 &= 180 - 146 \\ m\angle 2 &= 34 \end{aligned}$$

ANSWER:

34

6. $m\angle 3$

SOLUTION:

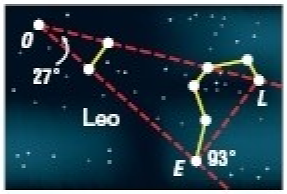
The sum of the measures of the angles of a triangle is 180. In the figure, $42 + 72 + m\angle 3 = 180$.
Substitute.

$$\begin{aligned} 42 + 72 + m\angle 3 &= 180 \\ 114 + m\angle 3 &= 180 \\ 114 + m\angle 3 - 114 &= 180 - 114 \\ m\angle 3 &= 66 \end{aligned}$$

ANSWER:

66

7. **ASTRONOMY** Leo is a constellation that represents a lion. Three of the brighter stars in the constellation form ΔLEO. If the angles have measures as shown in the figure, find $m\angle OLE$.



SOLUTION:

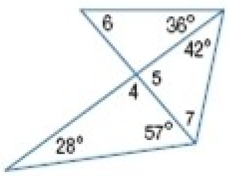
By the Exterior Angle Theorem, $93 = m\angle LOE + m\angle OLE$.
Find $m\angle OLE$.

$$93 = m\angle LOE + m\angle OLE$$
$$93 = 27 + m\angle OLE$$
$$93 - 27 = 27 + m\angle OLE - 27$$
$$66 = m\angle OLE$$

So, $m\angle OLE = 66$.

ANSWER:
66

Find the measure of each numbered angle.



8. $m\angle 4$

SOLUTION:
In the figure, $28 + 57 + m\angle 4 = 180$.

$$85 + m\angle 4 = 180$$
$$85 + m\angle 4 - 85 = 180 - 85$$
$$m\angle 4 = 95$$

ANSWER:
95

9. $m\angle 5$

SOLUTION:
By the Exterior Angle Theorem, $m\angle 5 = 28 + 57$.
Simplify.
$m\angle 5 = 85$

ANSWER:
85

10. $m\angle 6$

SOLUTION:

By the Exterior Angle Theorem, $m\angle 5 = 28 + 57$.
Simplify.
$m\angle 5 = 85$

By the Exterior Angle Theorem, $m\angle 5 = m\angle 6 + 36$.

Substitute.

$$85 = m\angle 6 + 36$$
$$85 - 36 = m\angle 6 + 36 - 36$$
$$49 = m\angle 6$$

Therefore, $m\angle 6 = 49$.

ANSWER:

49

11. $m\angle 7$

SOLUTION:

By the Exterior Angle Theorem, $m\angle 5 = 28 + 57$.
Simplify.
$m\angle 5 = 85$

In the figure, $m\angle 5 + m\angle 7 + 42 = 180$.

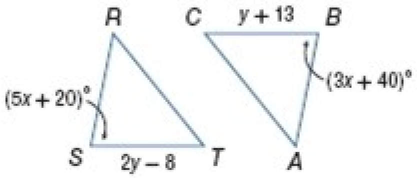
Substitute.

$$85 + m\angle 7 + 42 = 180$$
$$127 + m\angle 7 = 180$$
$$127 + m\angle 7 - 127 = 180 - 127$$
$$m\angle 7 = 53$$

ANSWER:

53

In the diagram, $\Delta RST \cong \Delta ABC$.



12. Find x.

SOLUTION:

If two triangles are congruent then their corresponding parts are congruent.

$$5x + 20 = 3x + 40$$
$$5x + 20 - 3x = 3x + 40 - 3x$$
$$2x + 20 = 40$$
$$2x + 20 - 20 = 40 - 20$$
$$2x = 20$$
$$x = 10$$

ANSWER:

10

13. Find y.

SOLUTION:

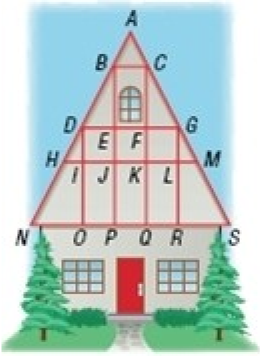
If two triangles are congruent then their corresponding parts are congruent.

$$2y - 8 = y + 13$$
$$2y - 8 - y = y + 13 - y$$
$$y - 8 = 13$$
$$y - 8 + 8 = 13 + 8$$
$$y = 21$$

ANSWER:

21

14. **ARCHITECTURE** The diagram shows an A-frame house with various points labeled. Assume that segments and angles that appear to be congruent in the diagram are congruent. Indicate which triangles are congruent.



SOLUTION:

$\Delta BED \cong \Delta CFG$; $\Delta BJH \cong \Delta CKM$; $\Delta BPN \cong \Delta CQS$;
$\Delta DIH \cong \Delta GLM$; $\Delta DON \cong \Delta GRS$;

ANSWER:

$\Delta BED \cong \Delta CFG$; $\Delta BJH \cong \Delta CKM$; $\Delta BPN \cong \Delta CQS$;
$\Delta DIH \cong \Delta GLM$; $\Delta DON \cong \Delta GRS$;

15. **MULTIPLE CHOICE** Determine which statement is true given that $\Delta CBX \cong \Delta SML$.

F $\overline{MO} \cong \overline{SL}$
G $\overline{XC} \cong \overline{ML}$
H $\angle X \cong \angle S$
J $\angle XCB \cong \angle LSM$

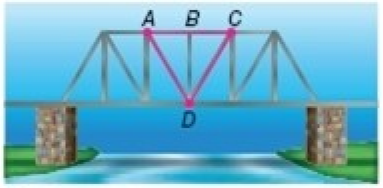
SOLUTION:

Test choice F. None of the vertices are labeled O. This is incorrect
Test choice G: Segment *XC* connects the third vertex of the triangle to the first vertex. However; segment *ML* connects the second vertex of the triangle to the third vertex. This is incorrect.
Test choice H: Angle *X* is the third vertex of the triangle and angle *S* is the first vertex. This is incorrect.
Test choice J: Angle *XCB* represents vertex *C* and angle *LSM* represents vertex *S*. Both of these letters are listed first in the naming the triangles. The correct answer is J.

ANSWER:

J

16. **BRIDGES** A bridge truss is shown in the diagram below, where $\overline{AC} \perp \overline{BD}$ and *B* is the midpoint of $\overline{AC}$. What method can be used to prove that $\Delta ABD \cong \Delta CBD$?



SOLUTION:

If *B* is the midpoint of *AC*, then segment *AB* is equal to segment *BC*. So, those sides are congruent.
If *BD* is perpendicular to *AC*, then angle *ABD* is a right angle and angle *CBD* is a right angle. So, those angles are congruent.
BD is congruent to itself, so that side is also congruent.
Triangle *ABD* and triangle *CBD* are congruent by SAS.

ANSWER:

SAS

Determine whether $\Delta PQR \cong \Delta XYZ$.

17. *P*(3, –5), *Q*(11, 0), *R*(1, 6), *X*(5, 1), *Y*(13, 6), *Z*(3, 12)

SOLUTION:

Use the Distance Formula to find the lengths of $\overline{PQ}$, $\overline{QR}$ and $\overline{RP}$.

$\overline{PQ}$ has endpoints *P*(3, –5) and *Q*(11, 0).

$$PQ = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Substitute.

$$\begin{aligned} PQ &= \sqrt{(11-3)^2 + (0-(-5))^2} \\ &= \sqrt{(8)^2 + (5)^2} \\ &= \sqrt{64+25} \\ &= \sqrt{89} \end{aligned}$$

$\overline{QR}$ has endpoints *Q*(11, 0) and *R*(1, 6).

$$QR = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Substitute.

$$\begin{aligned} QR &= \sqrt{(1-11)^2 + (6-0)^2} \\ &= \sqrt{(-10)^2 + (6)^2} \\ &= \sqrt{100+36} \\ &= \sqrt{136} \end{aligned}$$

$\overline{RP}$ has endpoints *R*(1, 6) and *P*(3, –5).

$$RP = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Substitute.

$$\begin{aligned} RP &= \sqrt{(3-1)^2 + (-5-6)^2} \\ &= \sqrt{(2)^2 + (-11)^2} \\ &= \sqrt{4+121} \\ &= \sqrt{125} \end{aligned}$$

Similarly, find the lengths of $\overline{XY}, \overline{YZ}$ and $\overline{ZX}$.

$\overline{XY}$ has endpoints $X(5, 1)$ and $Y(13, 6)$.

$$XY = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Substitute.

$$\begin{aligned} XY &= \sqrt{(13-5)^2 + (6-1)^2} \\ &= \sqrt{(8)^2 + (5)^2} \\ &= \sqrt{64+25} \\ &= \sqrt{89} \end{aligned}$$

$\overline{YZ}$ has endpoints $Y(13, 6)$ and $Z(3, 12)$.

$$YZ = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Substitute.

$$\begin{aligned} YZ &= \sqrt{(3-13)^2 + (12-6)^2} \\ &= \sqrt{(-10)^2 + (6)^2} \\ &= \sqrt{100+36} \\ &= \sqrt{136} \end{aligned}$$

$\overline{ZX}$ has endpoints $Z(3, 12)$ and $X(5, 1)$.

$$ZX = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Substitute.

$$\begin{aligned} ZX &= \sqrt{(5-3)^2 + (1-12)^2} \\ &= \sqrt{(2)^2 + (-11)^2} \\ &= \sqrt{4+121} \\ &= \sqrt{125} \end{aligned}$$

So, $\overline{PQ} \cong \overline{XY}, \overline{QR} \cong \overline{YZ}$ and $\overline{RS} \cong \overline{ZX}$.

Each pair of corresponding sides has the same measure so they are congruent. $\Delta PQR \cong \Delta XYZ$ by SSS.

ANSWER:

Yes

18. $P(-3, -3)$, $Q(-5, 1)$, $R(-2, 6)$, $X(2, -6)$, $Y(3, 3)$, $Z(5, -1)$

SOLUTION:

Use the Distance Formula to find the lengths of $\overline{PQ}$, $\overline{QR}$ and $\overline{RP}$.

$\overline{PQ}$ has endpoints $P(-3, -3)$ and $Q(-5, 1)$.

$$PQ = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Substitute.

$$\begin{aligned} PQ &= \sqrt{(-5-(-3))^2 + (1-(-3))^2} \\ &= \sqrt{(-2)^2 + (4)^2} \\ &= \sqrt{4+16} \\ &= \sqrt{20} \end{aligned}$$

$\overline{QR}$ has endpoints $Q(-5, 1)$ and $R(-2, 6)$.

$$QR = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Substitute.

$$\begin{aligned} QR &= \sqrt{(-2-(-5))^2 + (6-1)^2} \\ &= \sqrt{(3)^2 + (5)^2} \\ &= \sqrt{9+25} \\ &= \sqrt{34} \end{aligned}$$

$\overline{RP}$ has endpoints $R(-2, 6)$ and $P(-3, -3)$.

$$RP = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Substitute.

$$\begin{aligned} RP &= \sqrt{(-3-(-2))^2 + (-3-6)^2} \\ &= \sqrt{(-1)^2 + (-9)^2} \\ &= \sqrt{1+81} \\ &= \sqrt{82} \end{aligned}$$

Similarly, find the lengths of $\overline{XY}$, $\overline{YZ}$ and $\overline{ZX}$.

$\overline{XY}$ has endpoints $X(2, -6)$ and $Y(3, 3)$.

$$XY = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Substitute.

$$\begin{aligned} XY &= \sqrt{(3-2)^2 + (3-(-6))^2} \\ &= \sqrt{(1)^2 + (9)^2} \\ &= \sqrt{1+49} \\ &= \sqrt{50} \end{aligned}$$

$\overline{YZ}$ has endpoints $Y(3, 3)$ and $Z(5, -1)$.

$YZ = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

Substitute.

$$\begin{aligned} YZ &= \sqrt{(5-3)^2 + (-1-3)^2} \\ &= \sqrt{(2)^2 + (-4)^2} \\ &= \sqrt{4+16} \\ &= \sqrt{20} \end{aligned}$$

$\overline{ZX}$ has endpoints $Z(5, -1)$ and $X(2, -6)$.

$ZX = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

Substitute.

$$\begin{aligned} ZX &= \sqrt{(2-5)^2 + (-6-(-1))^2} \\ &= \sqrt{(-3)^2 + (-5)^2} \\ &= \sqrt{9+25} \\ &= \sqrt{34} \end{aligned}$$

The corresponding sides are not congruent, so the triangles are not congruent.

ANSWER:

No

19. P(8, 1), Q(–7, –15), R(9, –6), X(5, 11), Y(–10, –5), Z(6, 4)

SOLUTION:

Use the Distance Formula to find the lengths of $\overline{PQ}$, $\overline{QR}$ and $\overline{RP}$.

$\overline{PQ}$ has endpoints $P(8, 1)$ and $Q(-7, -15)$.

$PQ = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

Substitute.

$$\begin{aligned} PQ &= \sqrt{(-7-8)^2 + (-15-1)^2} \\ &= \sqrt{(-15)^2 + (-16)^2} \\ &= \sqrt{225+256} \\ &= \sqrt{481} \end{aligned}$$

$\overline{QR}$ has endpoints $Q(-7, -15)$ and $R(9, -6)$.

$$QR = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Substitute.

$$\begin{aligned} QR &= \sqrt{(9-(-7))^2 + (-6-(-15))^2} \\ &= \sqrt{(16)^2 + (9)^2} \\ &= \sqrt{256 + 81} \\ &= \sqrt{337} \end{aligned}$$

$\overline{RP}$ has endpoints $R(9, -6)$ and $P(8, 1)$.

$$RP = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Substitute.

$$\begin{aligned} RP &= \sqrt{(8-9)^2 + (1-(-6))^2} \\ &= \sqrt{(-1)^2 + (7)^2} \\ &= \sqrt{1 + 49} \\ &= \sqrt{50} \end{aligned}$$

Similarly, find the lengths of $\overline{XY}$, $\overline{YZ}$ and $\overline{ZX}$.

$\overline{XY}$ has endpoints $X(5, 11)$ and $Y(-10, -5)$.

$$XY = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Substitute.

$$\begin{aligned} XY &= \sqrt{(-10-5)^2 + (-5-11)^2} \\ &= \sqrt{(-15)^2 + (-16)^2} \\ &= \sqrt{225 + 256} \\ &= \sqrt{481} \end{aligned}$$

$\overline{YZ}$ has endpoints $Y(-10, -5)$ and $Z(6, 4)$.

$$YZ = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Substitute.

$$\begin{aligned} YZ &= \sqrt{(6-(-10))^2 + (4-(-5))^2} \\ &= \sqrt{(16)^2 + (9)^2} \\ &= \sqrt{256 + 81} \\ &= \sqrt{337} \end{aligned}$$

$\overline{ZX}$ has endpoints $Z(6, 4)$ and $X(5, 11)$.

$$ZX = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Substitute.

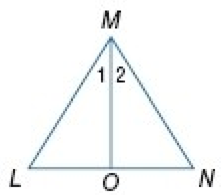
$$\begin{aligned} ZX &= \sqrt{(5-6)^2+(11-4)^2} \\ &= \sqrt{(-1)^2+(7)^2} \\ &= \sqrt{1+49} \\ &= \sqrt{50} \end{aligned}$$

So, $\overline{PQ} \cong \overline{XY}$, $\overline{QR} \cong \overline{YZ}$ and $\overline{RS} \cong \overline{ZX}$.
Each pair of corresponding sides has the same measure so they are congruent. $\Delta PQR \cong \Delta XYZ$ by SSS.

ANSWER:
Yes

Write a two-column proof.

20. Given: ΔLMN is isosceles with $\overline{LM} \cong \overline{NM}$, and $\overline{MO}$ bisects $\angle LMN$.
Prove: $\Delta MLO \cong \Delta MNO$



SOLUTION:
You need to walk through the proof step by step. Look over what you are given and what you need to prove. Here you are given the triangle is isosceles and one segment is an angle bisector. So, you have two congruent sides and two congruent angles. Use what you have learned about triangle congruency to prove the triangles are congruent.
Statements (Reasons)
1. ΔLMN is isos. with $\overline{LM} \cong \overline{NM}$. (Given)
2. $\overline{MO}$ bisects $\angle LMN$ (Given)
3. $\angle 1 \cong \angle 2$ (Defn. of $\angle$ bisector)
4. $\overline{MO} \cong \overline{MO}$ (Refl. Prop.)
5. $\Delta MLO \cong \Delta MNO$ (SAS)

ANSWER:
Statements (Reasons)
1. ΔLMN is isos. with $\overline{LM} \cong \overline{NM}$. (Given)
2. $\overline{MO}$ bisects $\angle LMN$ (Given)
3. $\angle 1 \cong \angle 2$ (Defn. of $\angle$ bisector)
4. $\overline{MO} \cong \overline{MO}$ (Refl. Prop.)
5. $\Delta MLO \cong \Delta MNO$ (SAS)