Mid-Chapter Quiz: Lessons 6-1 through 6-3

Find the sum of the measures of the interior angles of each convex polygon.

1. pentagon

SOLUTION:

A pentagon has five sides. Use the Polygon Interior Angles Sum Theorem to find the sum of its interior angle measures.

Substitute n = 5 in $(n-2)180$.

$$\begin{aligned}(n-2)180 &= (5-2)180 \\ &= 3\cdot 180 \\ &= 540\end{aligned}$$

ANSWER:

540

2. heptagon

SOLUTION:

A heptagon has seven sides. Use the Polygon Interior Angles Sum Theorem to find the sum of its interior angle measures.

Substitute n = 7 in $(n-2)180$.

$$\begin{aligned}(n-2)180 &= (7-2)180 \\ &= 5\cdot 180 \\ &= 900\end{aligned}$$

ANSWER:

900

3. 18-gon

SOLUTION:

An 18-gon has eighteen sides. Use the Polygon Interior Angles Sum Theorem to find the sum of its interior angle measures.

Substitute n = 18 in $(n-2)180$.

$$\begin{aligned}(n-2)180 &= (18-2)180 \\ &= 16\cdot 180 \\ &= 2880\end{aligned}$$

ANSWER:

2880

4. 23-gon

SOLUTION:

A 23-gon has twenty-three sides. Use the Polygon Interior Angles Sum Theorem to find the sum of its interior angle measures.

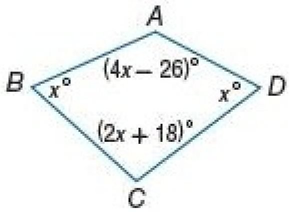
Substitute n = 23 in $(n-2)180$.

$$\begin{aligned}(n-2)180 &= (23-2)180 \\ &= 21 \cdot 180 \\ &= 3780\end{aligned}$$

ANSWER:

3780

Find the measure of each interior angle.

5. 

SOLUTION:

The sum of the interior angle measures is $(4-2)180$ or 360.

$$\begin{aligned}m\angle A + m\angle B + m\angle C + m\angle D &= 360 \\ (4x-26) + x + (2x+18) + x &= 360 \\ 8x - 8 &= 360 \\ 8x &= 368 \\ x &= 46\end{aligned}$$

Use the value of x to find the measure of each angle.

$$\begin{aligned}m\angle A &= 4x - 26 \\ &= 4(46) - 26 \\ &= 184 - 26 \\ &= 158\end{aligned}$$

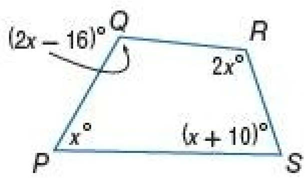
$$\begin{aligned}m\angle B &= x \\ &= 46\end{aligned}$$

$$\begin{aligned}m\angle C &= 2x + 18 \\ &= 2(46) + 18 \\ &= 92 + 18 \\ &= 110\end{aligned}$$

$$\begin{aligned}m\angle D &= x \\ &= 46\end{aligned}$$

ANSWER:

$m\angle A = 158, m\angle B = 46, m\angle C = 110, m\angle D = 46$

6. 

SOLUTION:

The sum of the interior angle measures is $(4-2)180$ or 360.

$$\begin{aligned} m\angle P + m\angle Q + m\angle R + m\angle S &= 360 \\ x + (2x-16) + 2x + (x+10) &= 360 \\ 6x - 6 &= 360 \\ 6x &= 366 \\ x &= 61 \end{aligned}$$

Use the value of *x* to find the measure of each angle.

$$m\angle P = x$$
$$= 61$$
$$\begin{aligned} m\angle Q &= 2x - 16 \\ &= 2(61) - 16 \\ &= 122 - 16 \\ &= 106 \end{aligned}$$
$$\begin{aligned} m\angle R &= 2x \\ &= 2(61) \\ &= 122 \end{aligned}$$
$$\begin{aligned} m\angle S &= x + 10 \\ &= 71 \end{aligned}$$

ANSWER:

$m\angle P = 61, m\angle Q = 106, m\angle R = 122, m\angle S = 71$

The sum of the measures of the interior angles of a regular polygon is given. Find the number of sides in the polygon.

7. 720

SOLUTION:

Let *n* = the number of sides in the polygon. By the Polygon Interior Angles Sum Theorem, the sum of the interior angle measures can also be expressed as $(n-2)180$.

$$\begin{aligned} 720 &= (n-2)180 \\ 720 &= 180n - 360 \\ 180n &= 1080 \\ n &= 6 \end{aligned}$$

ANSWER:

6

8. 1260

SOLUTION:
Let n = the number of sides in the polygon. By the Polygon Interior Angles Sum Theorem, the sum of the interior angle measures can also be expressed as $(n-2)180$.

$$1260 = (n-2)180$$
$$1260 = 180n - 360$$
$$180n = 1620$$
$$n = 9$$

ANSWER:
9

9. 1800

SOLUTION:
Let n = the number of sides in the polygon. By the Polygon Interior Angles Sum Theorem, the sum of the interior angle measures can also be expressed as $(n-2)180$.

$$1800 = (n-2)180$$
$$1800 = 180n - 360$$
$$180n = 2160$$
$$n = 12$$

ANSWER:
12

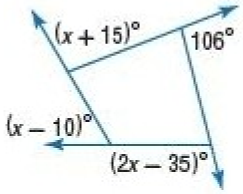
10. 4500

SOLUTION:
Let n = the number of sides in the polygon. By the Polygon Interior Angles Sum Theorem, the sum of the interior angle measures can also be expressed as $(n-2)180$.

$$4500 = (n-2)180$$
$$4500 = 180n - 360$$
$$180n = 4860$$
$$n = 27$$

ANSWER:
27

Find the value of *x* in each diagram.

11. 

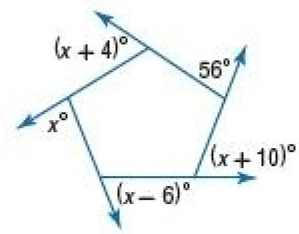
SOLUTION:

Use the Polygon Exterior Angles Sum Theorem to write an equation. Then solve for *x*.

$$\begin{aligned} 106+(2x-35)+(x-10)+(x+15) &= 360 \\ 106+2x-35+x-10+x+15 &= 360 \\ 4x+76 &= 360 \\ 4x &= 284 \\ x &= 71 \end{aligned}$$

ANSWER:

71

12. 

SOLUTION:

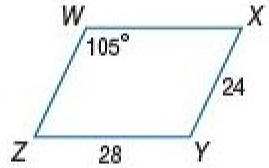
Use the Polygon Exterior Angles Sum Theorem to write an equation. Then solve for *x*.

$$\begin{aligned} x+(x+4)+56+(x+10)+(x-6) &= 360 \\ x+x+4+56+x+10+x-6 &= 360 \\ 4x+64 &= 360 \\ 4x &= 296 \\ x &= 74 \end{aligned}$$

ANSWER:

74

Use ▱ *WXYZ* to find each measure.



13. $m\angle WZY$

SOLUTION:

We know that consecutive angles in a parallelogram are supplementary.
So, $105 + m\angle WZY = 180.$
Solve for $m\angle WZY$.

$$\begin{aligned} 105 + m\angle WZY &= 180 \\ 105 + m\angle WZY - 105 &= 180 - 105 \\ m\angle WZY &= 75 \end{aligned}$$

ANSWER:

75

14. WZ

SOLUTION:

We know that opposite sides of a parallelogram are congruent.
So, $WZ = XY = 24$.

ANSWER:

24

15. $m\angle XYZ$

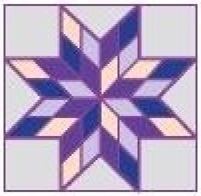
SOLUTION:

We know that opposite angles of a parallelogram are congruent.
So, $m\angle XYZ = m\angle XWZ = 105.$

ANSWER:

105

16. **DESIGN** Describe two ways to ensure that the pieces of the design shown would fit properly together.



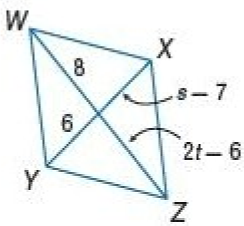
SOLUTION:

The pieces of the design would fit properly together if the opposite sides are congruent or if the opposite angles are congruent. Sample answer: Make sure that opposite sides are congruent or make sure that opposite angles are congruent.

ANSWER:

Sample answer: Make sure that opposite sides are congruent or make sure that opposite angles are congruent.

ALGEBRA Find the value of each variable.

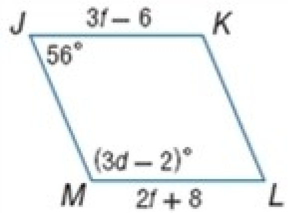
17. 

SOLUTION:

We know that diagonals of a parallelogram bisect each other.
So, $s - 7 = 6$ and $2t - 6 = 8$.
Solve for s and b.
So, $s = 13$ and $t = 7$.

ANSWER:

$s = 13$, $t = 7$

18. 

SOLUTION:

We know that opposite sides of a parallelogram are congruent.

$3f - 6 = 2f + 8$

$f = 14$

We know that consecutive angles in a parallelogram are supplementary.

So, $56 + 3d - 2 = 180.$

Solve for d.

$$\begin{aligned} 56 + 3d - 2 &= 180 \\ 3d &= 126 \\ d &= 42 \end{aligned}$$

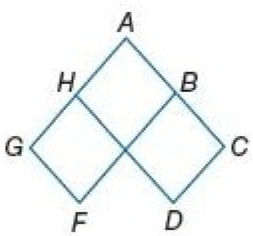
ANSWER:

$d = 42$ and $f = 14$

19. **PROOF** Write a two-column proof.

Given: $\square GFBA$ and $\square HACD$

Prove: $\angle F \cong \angle D$



SOLUTION:

You need to walk through the proof step by step. Look over what you are given and what you need to prove. Here, you are given $\square GFBA$ and $\square HACD$. You need to prove $\angle F \cong \angle D$. Use the properties that you have learned about parallelograms to walk through the proof.

Proof:

<u>Statements (Reasons)</u>

1. $\square GFBA$ and $\square HACD$ (Given)
2. $\angle F \cong \angle A, \angle A \cong \angle D$ (Opp. $\angle s$ of a $\square$ are $\cong$)
3. $\angle F \cong \angle D$ (Transitive Prop.)

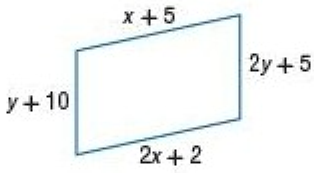
ANSWER:

Proof:

<u>Statements (Reasons)</u>

1. $\square GFBA$ and $\square HACD$ (Given)
2. $\angle F \cong \angle A, \angle A \cong \angle D$ (Opp. $\angle s$ of a $\square$ are $\cong$)
3. $\angle F \cong \angle D$ (Transitive Prop.)

Find x and y so that each quadrilateral is a parallelogram.

20. 

SOLUTION:

We know that opposite sides of a parallelogram are congruent.

$2x + 2 = x + 5$

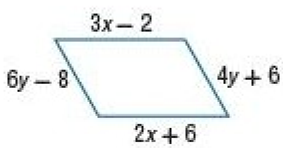
$x = 3$

Similarly, $2y + 5 = y + 10$.

So, $y = 5$.

ANSWER:

$x = 3,\ y = 5$

21. 

SOLUTION:

We know that opposite sides of a parallelogram are congruent.

$3x – 2 = 2x + 6$

$x = 8$

Similarly, $6y – 8 = 4y + 6$.

So, $y = 7$.

ANSWER:

$x = 8, y = 7$

22. **MUSIC** Why will the keyboard stand shown below always remain parallel to the floor?

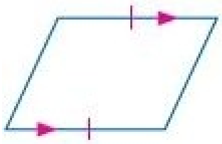
SOLUTION:

Let the top of the stand represent one side of the quadrilateral and let the bottom of the stand represent the opposite side of the same quadrilateral. The legs can now represent the diagonals of the quadrilateral. Since the legs are joined at their midpoints, the legs bisect each other. If the diagonals of a quadrilateral bisect each other, than the quadrilateral is a parallelogram. The opposite sides of a parallelogram are always parallel. Therefore, the keyboard will always be parallel to the floor.

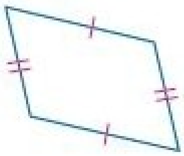
ANSWER:

Sample answer: The legs are made so that they will bisect each other, so the quadrilateral formed by the ends of the legs is always a parallelogram. Therefore, the top of the stand is parallel to the floor.

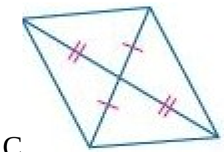
23. **MULTIPLE CHOICE** Which of the following quadrilaterals is not a parallelogram?



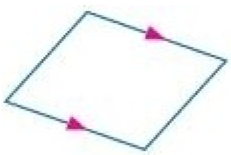
A



B



C



D

SOLUTION:

Test choice A. This quadrilateral has one pair of opposite sides both parallel and congruent. This is sufficient to prove that this quadrilateral is a parallelogram.
Test choice B. This quadrilateral has both pairs of opposite sides congruent. This is sufficient to prove that this quadrilateral is a parallelogram.
Test choice C. This quadrilateral has diagonals that bisect each other. This is sufficient to prove that this quadrilateral is a parallelogram.
Test choice D. This quadrilateral has one pair of opposite sides parallel. That is not sufficient proof that this quadrilateral is a parallelogram.
The correct answer is D.

ANSWER:

D

COORDINATE GEOMETRY Determine whether the figure is a parallelogram. Justify your answer with the method indicated.

24. $A(-6, -5)$, $B(-1, -4)$, $C(0, -1)$, $D(-5, -2)$; Distance Formula

SOLUTION:

Use the Distance Formula to find the distance.

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$\begin{aligned} AB &= \sqrt{(-1-(-6))^2 + (-4-(-5))^2} \\ &= \sqrt{(5)^2 + (1)^2} \\ &= \sqrt{26} \end{aligned}$$

So, the distance between A and B is $\sqrt{26}$.

$$\begin{aligned} BC &= \sqrt{(0-(-1))^2 + (-1-(-4))^2} \\ &= \sqrt{(1)^2 + (3)^2} \\ &= \sqrt{10} \end{aligned}$$

So, the distance between B and C is $\sqrt{10}$.

$$\begin{aligned} CD &= \sqrt{(-5-0)^2 + (-2-(-1))^2} \\ &= \sqrt{(-5)^2 + (-1)^2} \\ &= \sqrt{26} \end{aligned}$$

Therefore, the distance between C and D is $\sqrt{26}$.

$$\begin{aligned} DA &= \sqrt{(-5-(-6))^2 + (-2-(-5))^2} \\ &= \sqrt{(1)^2 + (3)^2} \\ &= \sqrt{10} \end{aligned}$$

The distance between D and A is $\sqrt{10}$. Since both pairs of opposite sides are congruent, $ABCD$ is a parallelogram.

ANSWER:

Yes; both pairs of opposite sides must be congruent. The distance between A and B is $\sqrt{26}$. The distance between B and C is $\sqrt{10}$. The distance between C and D is $\sqrt{26}$. The distance between D and A is $\sqrt{10}$. Since both pairs of opposite sides are congruent, $ABCD$ is a parallelogram.

25. $Q(-5, 2)$, $R(-3, -6)$, $S(2, 2)$, $T(-1, 6)$; Slope Formula

SOLUTION:

Use the slope formula.

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$\begin{aligned}\text{Slope of } \overline{QR} &= \frac{-6-2}{-3-(-5)} \\ &= \frac{-8}{2} \\ &= -4\end{aligned}$$

$$\begin{aligned}\text{Slope of } \overline{TS} &= \frac{6-2}{-1-2} \\ &= \frac{4}{-3} \\ &= -\frac{4}{3}\end{aligned}$$

Since the slope of $\overline{QR} \neq$ slope of $\overline{TS}$, *QRST* is not a parallelogram.

ANSWER:

No; both pairs of opposite sides must be parallel; since the slope of $\overline{QR} \neq$ slope of $\overline{TS}$, *QRST* is not a parallelogram.